NUMERICAL SIMULATION OF AN ARC COLUMN

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Summary : This paper presents a numerical model of a SF6 arc column. This model, which is a 1-dimensional one, allows to compute the thermodynamical evolution of an arc crossed by a given current. The simulation is able to treat unstationnary situations and to take into account compressibility and real gas properties, different energy transport mechanisms such as convection, conduction and radiation,.... The developed code is based on Godounov scheme. It can give information about temperature profiles, pressure rise around the arc, arc voltage ,....

1. Introduction.

This paper presents a numerical simulation of a free burning SF6 arc column. In this model, the arc and the surrounding cold gas are considered as a single continuous medium. So the same mathematical treatment is used in the core and in the cold gas. The arc is modelled as a gas region at very high temperature heated by the energy dissipated in the arc, i.e. by Joule effect.

The numerical simulation is based on the fundamental equations of physics : the mass, momentum and energy conservation laws, the local Ohm's law and the SF6 state equation.

In this model, the current is forced by an external circuit. Joule losses in the arc column are then computed according to the local electrical conductivity (depending on the instantaneous temperature). The calculated electric field is assumed to be axial and independant of the radius.

SF6 properties are taken into account : density, internal energy, electrical conductivity, thermal conductibility,... are retained in a database [1]. This allows to consider the real gas characteristics and not to adopt the classical "perfect gas" assumption.

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2. Basic equations.

Considering an infinite length cylindrical arc, one has to solve the following equations :

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho v\\ E\end{pmatrix}+\frac{\partial}{\partial r}\begin{pmatrix}\rho v\\ P+\rho v^2\\ (E+P)\,v\end{pmatrix}-\frac{1}{r}\frac{\partial}{\partial r}\begin{pmatrix}0\\ 0\\ r\,k\,\frac{\partial T}{\partial r}\end{pmatrix}$$
$$=-\frac{1}{r}\begin{pmatrix}\rho v\\ \rho v^2\\ (E+P)\,v\end{pmatrix}+\begin{pmatrix}0\\ 0\\ \sigma E_l^2\end{pmatrix}-\begin{pmatrix}0\\ 0\\ u\end{pmatrix} \qquad (1)$$

where "ρ" is the mass density, "v" is the radial velocity, "E" is the total energy (internal energy plus kinetic energy), "P" is the pressure, "T" is the temperature, "k" is the thermal conductibility, "E_l" is the electric field, "σ" is the electrical conductivity and "u" represents the radiative losses.

These equations are Euler's equations extended in order to represent Joule losses, diffusive and radiative heat processes. These classical conservation laws must be completed by the SF6 gas state equation. This state equation is written in the following form : $P = f(\rho,e)$.

The numerical method used to solve these equations is based on Godounov scheme [2], [3], usually used in fluid dynamics. But this classical method is modified with a finite difference scheme to evaluate diffusive fluxes.

3. Godounov scheme and convective fluxes computation.

Forgetting temporarily Joule effects and radiative and diffusive transfers, equations (1) can be written in the following form :

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho v\\ E\end{pmatrix}+\frac{\partial}{\partial r}\begin{pmatrix}\rho v\\ P+\rho v^2\\ (E+P)\,v\end{pmatrix}=-\frac{1}{r}\begin{pmatrix}\rho v\\ \rho v^2\\ (E+P)\,v\end{pmatrix}$$

The Godounov method used to solve these equations is a "finite volume" one.

At a given moment, all the unknowns are supposed to be constant on each subdivision of the radial meshing. To move forward in the time, at each step, a local mass, momentum and energy balance allows to compute the evolution of the unknowns. For instance, the mass contained in a given finite volume at time "t+dt", is equal to the mass contained in that volume at time "t" increased by the quantity convected through the boundary during the interval "dt". The same reasoning can be applied to the momentum and energy variables.

To compute these convective fluxes (of mass, momentum and energy) a Riemann problem is solved. A difficulty lies in the manner of taking into account the real SF6 gas properties at high temperatures. A special Riemann problem's solver has been adapted (based on reference [4]).

4. Computation of diffusive fluxes.

As the arc is a very hot medium, there is a very strong temperature gradient at the boundary between the core and the surrounding cold gas. Thus the thermal diffusion is an important phenomenon which cannot be neglected.

In the presented model, the diffusive fluxes are estimated by a finite difference method taking into account the dependance of the thermal conductibility on temperature :

$$k\frac{\partial T}{\partial r}=k\left(\frac{T_L+T_R}{2}\right)\left(\frac{T_L-T_R}{dr}\right)$$

where T_L and T_R are respectively the temperature on the left and the right sides of a given boundary between two finite volumes.

5. Computation of Joule losses.

To estimate the energy dissipated in the arc core, we have to calculate the electric field and the local electrical conductivity.

In each finite volume, the local electrical conductivity "σ" is computed as a function of the temperature. Integrating all these conductivities on the arc section, one can evaluate the arc conductance "G". As the current "I" is forced by the external circuit (i.e. a given function of time), the electric field E_ℓ in the arc is calculated in the following way :

$$E_\ell=\frac{I}{G}$$

Then, knowing electric field and conductivity, one can estimate the local energy sources to be included in the energy balance, in order to represent Joule losses:

$$\sigma E_\ell^2$$

6. Computation of radiative fluxes.

It is well known that, in high current arcs, radiative losses constitute important phenomena [5], whereas their quantitative estimation is very difficult.

A first and quite simple approach consists in assuming that the arc core radiates energy which is not at all absorbed by the surrounding cold gas (optically thin approximation).

In that case, radiative effects are simulated through a negative term in the energy conservation equation. This "negative source" is evaluated as a function of local temperature and pressure of the gas [5].

Finally, the discretized conservation equations are written in the following form :

$$\rho^{n+1}=\rho^n+\frac{dt}{dr\,r}\left(\rho_g V_g R_g-\rho_d V_d R_d\right)$$

$$(\rho V)^{n+1}=(\rho V)^n+\frac{dt}{dr\,r}\left(\left(\rho_g V_g^2+P_g\right)R_g-\left(\rho_d V_d^2+P_d\right)R_d+P^n dr\right)$$

$$V^{n+1}=\frac{(\rho V)^{n+1}}{\rho^{n+1}}$$

$$e^{n+1}=$$

$$e^n+\left(\sigma E_\ell^2-u\right)dt+\frac{dt}{dr\,r}\left[\left(\left(\rho_g\left(e_g+\frac{1}{2}\left(V_g^2\right)\right)+P_g\right)V_g+F_g\right)R_g\right]$$

$$-\left[\left(\left(\rho_d\left(e_d+\frac{1}{2}\left(V_d^2\right)\right)+P_d\right)V_d+F_d\right)R_d\right]$$

The sub-indexes "g" and "d" respectively point out quantities calculated on the left and right faces of a given finite volume. "r" is the radial coordinate of the barycenter of that volume. "R" is the radial coordinate of the boundaries and "F" represents the diffusive fluxes.

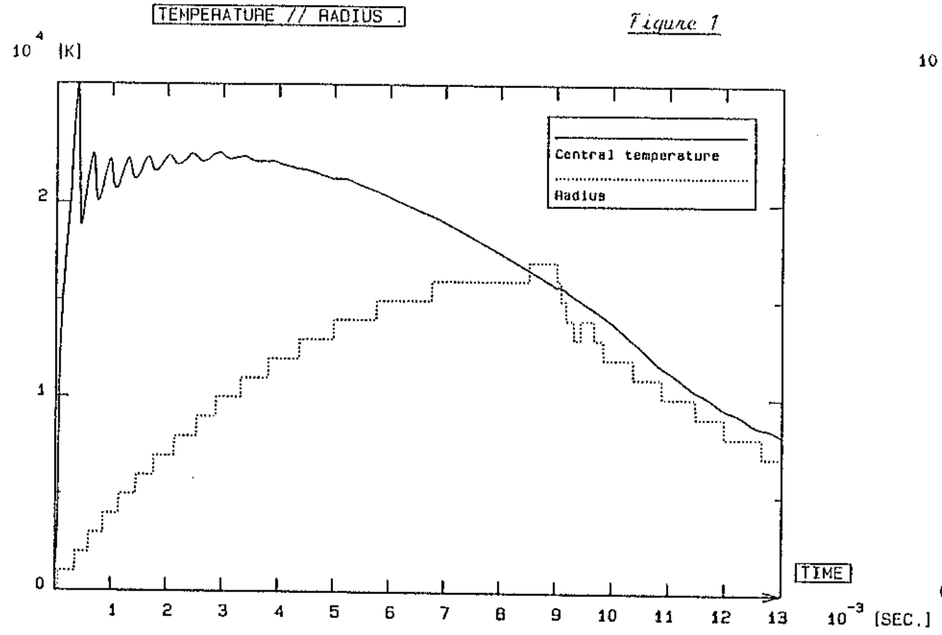


Figure 1

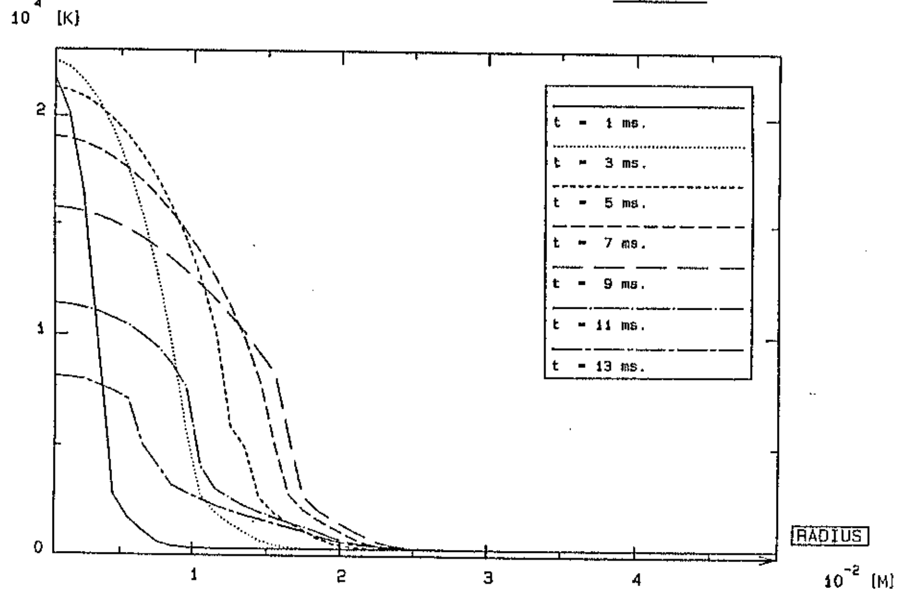


Figure 2

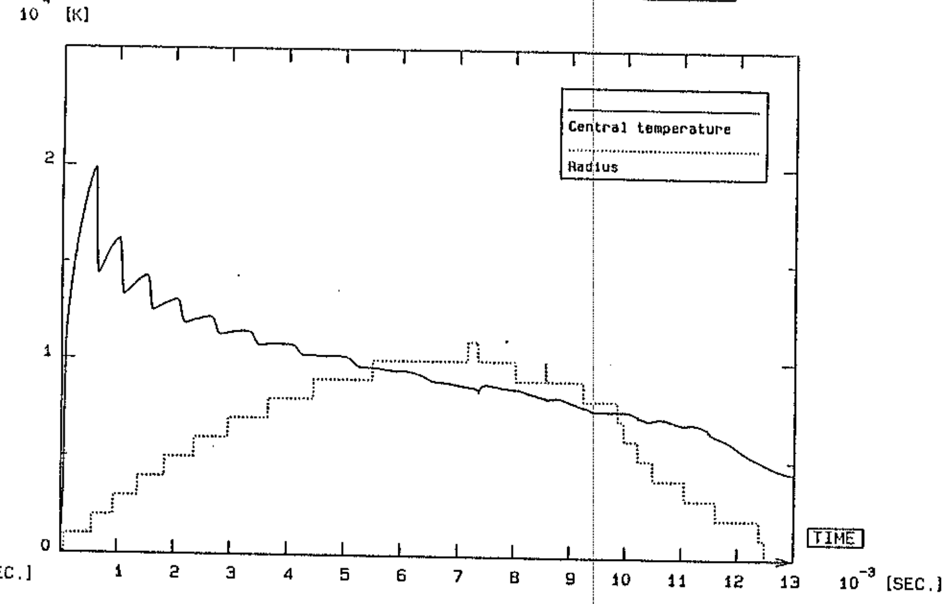
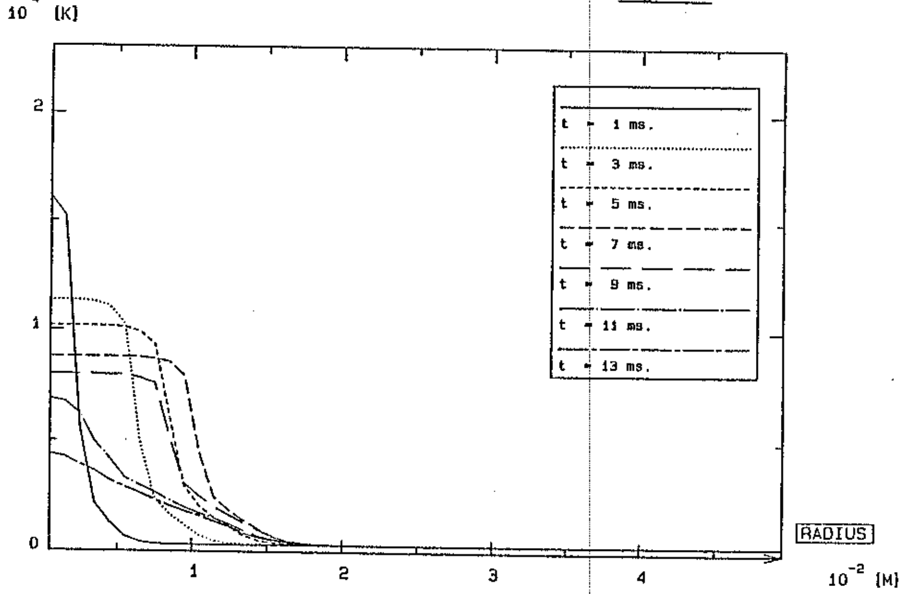


Figure 3



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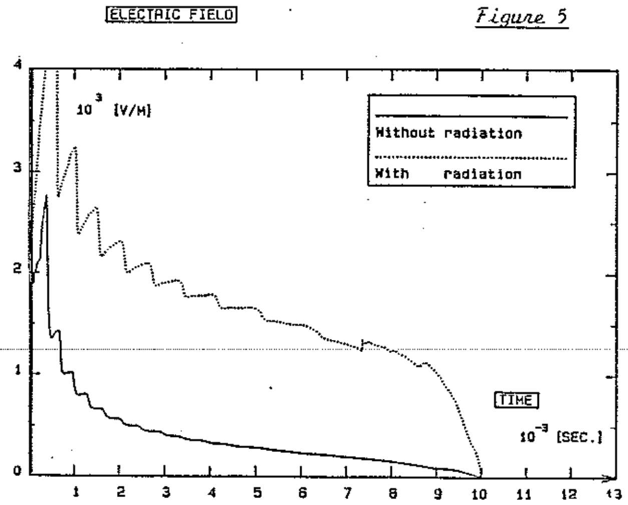

Figure 5

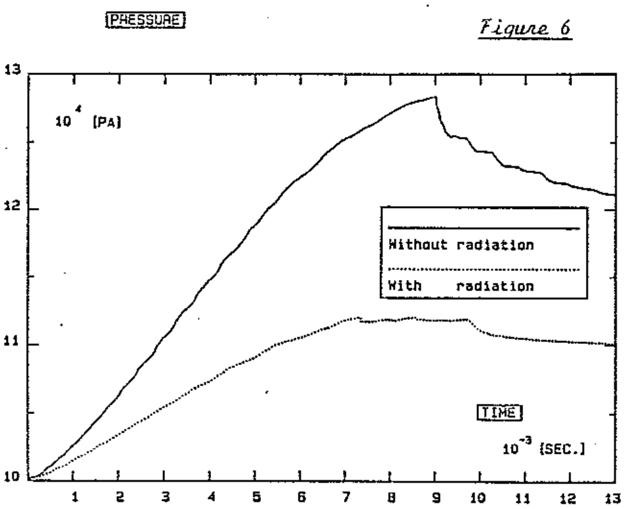

Figure 6

7. Results.

The developed model has been used to compute an arc crossed by a 1 kA current at 50 Hz interrupted after 10 milliseconds.

Figure 1 shows the temporal evolution of the arc central temperature and of the arc radius. These plots are obtained while completely neglecting radiative processes. Figure 2 gives the temperature profiles at different times in the same conditions.

Figures 3 and 4 represent the same quantities (respectively arc central temperature, radius and temperature profiles) while adopting the hypothesis of the optically thin radiation model.

Figures 5 and 6 respectively show electric field and pressure rise as functions of time when neglecting or taking into account radiation.

The comparison of the two numerical simulations results emphasizes the radiative transfers importance. It is thus necessary to take them into account and a more complete model (with absorption) should be developed in order to give a better representation of reality.

8. Conclusions.

The developed model allows to compute the thermodynamical evolution of an arc crossed by a given current. The different phenomena taken into account are :

- unstationnary situations,
- compressibility and real gas properties ,
- energy transport by convection and conduction,
- some kind of radiative transport.

In the developed code, which can be considered as a physical model, only the current is a "forced quantity". So the calculations can give information on the different thermodynamic phenomena such as temperature profiles, relative importance of energetic processes, pressure rise around the arc, temporal evolution of the arc voltage,...

Finally, the model may be helpful for theoretical investigations of quenching chambers. Nevertheless, the complexity of the physical phenomena and the difficulty of their modelling require a real complementarity between calculation and experiment.

9. References.

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