

# A STOCHASTIC 3-SCALE METHOD TO PREDICT THE THERMO-ELASTIC BEHAVIORS OF POLYCRYSTALLINE STRUCTURES

LING WU\*, VINCENT LUCAS\*, JEAN-CLAUDE GOLINVAL\*,  
STEPHANE PAQUAY<sup>†</sup> AND LUDOVIC NOELS\*

\*University of Liege, Department of Aerospace and Mechanical Engineering  
Allée de la découverte 9, Belgium  
e-mail: {l.wu,JC.Golinval,l.noels}@ulg.ac.be

<sup>†</sup>Open Engineering SA  
Rue Bois Saint-Jean 15/1, B4102 Seraing, Belgium  
e-mail: s.paquay@open-engineering.com

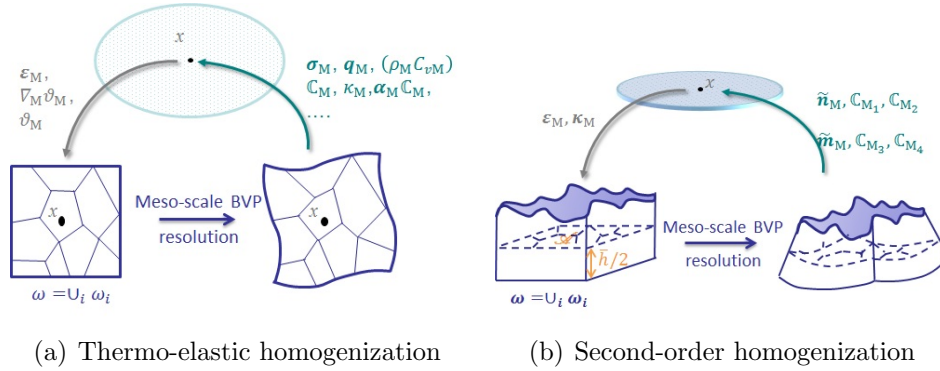
**Key words:** Stochastic multiscale, Thermo-elasticity, Polycrystalline

**Abstract.** The purpose of this work is to upscale material uncertainties in the context of thermo-elastic response of polycrystalline structures.

The probabilistic behavior of micro-resonators made of polycrystalline materials is evaluated using a stochastic multi-scale approach defined using the following methodology.

1. Stochastic volume elements (SVEs) [1] are defined from Voronoï tessellations using experimental measurements of the grain size, orientation, and surface roughness [2];
2. Mesoscopic apparent thermo-elastic properties such as elasticity tensor, thermal conductivity tensor, and thermal dilatation tensor are extracted using a coupled homogenization theory [3, 4] applied on the SVE realizations;
3. A stochastic model of the homogenized properties extracted from Voronoï tessellations using a moving window technique is then constructed in order to be able to generate spatially correlated meso-scale random fields;
4. These meso-scale random fields are then used as input for stochastic finite element simulations.

As a result, the probabilistic distribution of micro-resonator properties can be extracted. The applications are two-fold:



**Figure 1:** Stochastic homogenization on SVEs

1. A stochastic thermo-elastic homogenization, see Fig. 1(a), is coupled to thermo-elastic 3D models of the micro-resonator in order to extract the probabilistic distribution of the quality factor of micro-resonators [5];
2. A stochastic second-order mechanical homogenization, see Fig. 1(b), is coupled to a plate model of the micro-resonator in order to extract the effect of the uncertainties related to the surface roughness of the polycrystalline structures [2].

## REFERENCES

- [1] Ostoja-Starzewski, M., Wang, X. Stochastic finite elements as a bridge between random material microstructure and global response. *Comput. Meth. in Appl. Mech. and Eng.* (1999) **168**: 35–49.
- [2] Lucas, V., Golinval, J.-C., Voicu, R., Danila, M., Gravila, R., Muller, R., Dinescu, A., Noels, L., Wu, L. Propagation of material and surface profile uncertainties on MEMS micro-resonators using a stochastic second-order computational multi-scale approach. *Int. J. for Num. Meth. in Eng.* (2017).
- [3] Temizer, I., Wriggers, P. Homogenization in finite thermoelasticity. *J. of the Mech. and Phys. of Sol.* (2011) **59**, 344–372.
- [4] Nguyen, V. D., Wu, L., Noels, L. Unified treatment of boundary conditions and efficient algorithms for estimating tangent operators of the homogenized behavior in the computational homogenization method. *Computat. Mech.* (2017) **59**, 483–505.
- [5] Wu, L., Lucas, V., Nguyen, V. D., Golinval, J.-C., Paquay, S., Noels, L. A Stochastic Multi-Scale Approach for the Modeling of Thermo-Elastic Damping in Micro-Resonators. *Comput. Meth. in Appl. Mech. and Eng.* (2016) **310**, 802–839.