Globally Induced Forest: A Prepruning Compression Scheme

Jean-Michel Begon, Arnaud Joly, Pierre Geurts

Systems and Modeling, Dept. of EE and CS, University of Liege, Belgium

ICML 2017

Goal and motivation

What ? Is it possible to build accurate yet lightweight random forests without building the whole model first ?

Why ? Random forests are heavy models memory-wise :

$\propto$ Number of nodes in a tree is (at worst) linear with the size of the data ;

$\propto$ number of required trees grows with the problem complexity.

What for ?

- Big data ;
- small memory devices ;
- better interpretability, less overfitting, faster prediction, ...

How ?

GIF algorithm — High level view

Build an additive model corresponding to a forest by introducing decision nodes sequentially until a node budget constraint is met.

- Splits in decision nodes are optimized locally.
- Nodes are taken from a candidate list.
- The best node is added . . .
- . . . together with its weight.

GIF algorithm — High level view

Build an additive model corresponding to a forest by introducing decision nodes sequentially until a node budget constraint is met.

- Splits in decision nodes are optimized locally.
- Nodes are taken from a candidate list.
- The best node is added ...
- ... together with its weight.

GIF algorithm — Additive model

The model prediction $\hat{y}^{(t)}(x)$ at step t for instance x is given by :

$$\hat{y}^{(t)}(x) = \hat{y}^{(t-1)}(x) + \lambda w_t z_t(x) = w_0 + \lambda \sum_{\tau=1}^{t} w_\tau z_\tau(x) \qquad (1)$$

where

w_0 is some initial bias.

w_τ is the weight of node j $(1 \leq \tau \leq t)$.

λ is the learning rate.

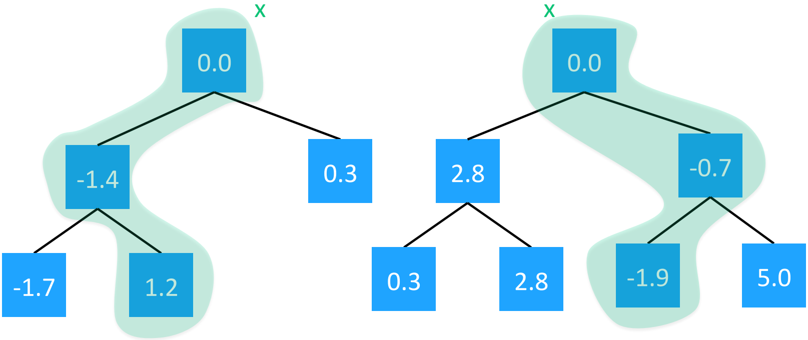
$$z_\tau(x) = \begin{cases} 1, & \text{if } x \text{ reaches node } \tau \\ 0, & \text{otherwise} \end{cases}$$

$(1 \leq \tau \leq t)$

i.e. node τ indicator function

For classification, the sum of weights represents the class probability vector (*i.e.* the weights are multidimensional).

GIF algorithm — Additive model



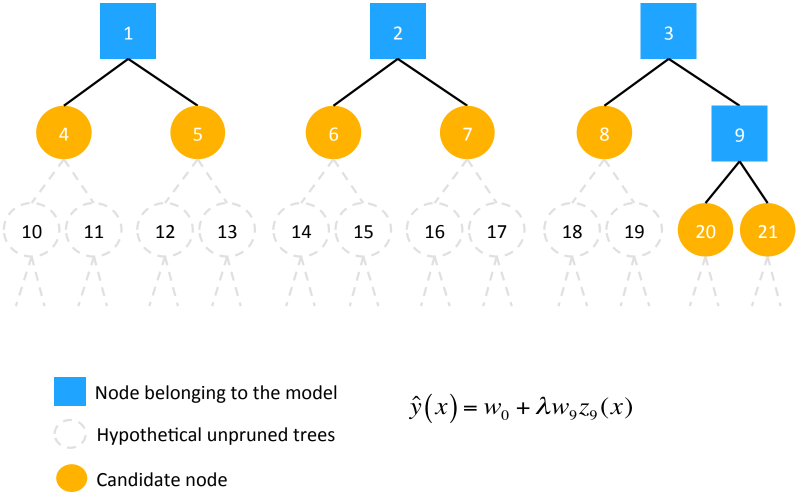
$$\hat{y}(x) = w_0 + \lambda(1.4 + 1.2) + \lambda(-0.7 + -1.9) \qquad (2)$$

GIF algorithm — High level view

Build an additive model corresponding to a forest by introducing decision nodes sequentially until a node budget constraint is met.

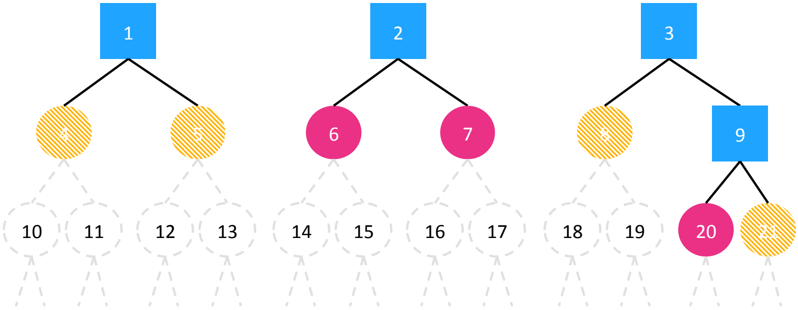
- Splits in decision nodes are optimized locally.
- Nodes are taken from a candidate list.
- The best node is added . . .
- . . . together with its weight.

GIF algorithm — Illustration (regression)



$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x)$$

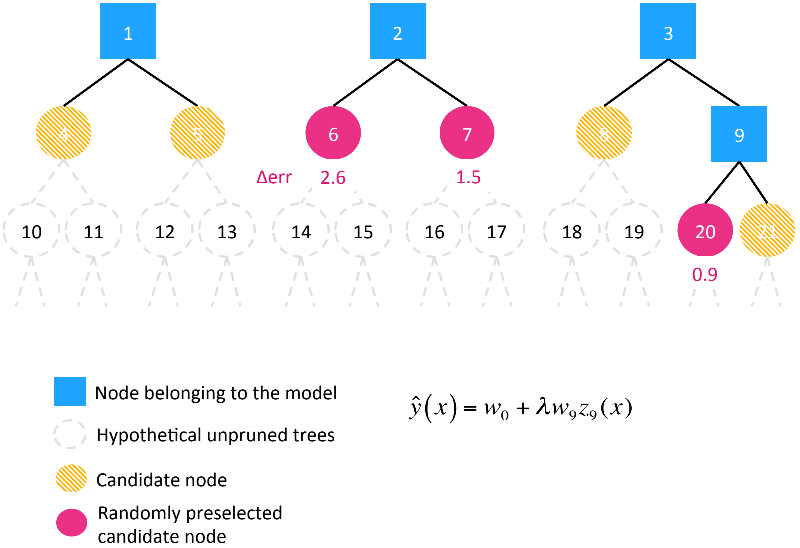
GIF algorithm — Illustration (regression)



- Node belonging to the model
- Hypothetical unpruned trees
- Candidate node
- Randomly preselected candidate node

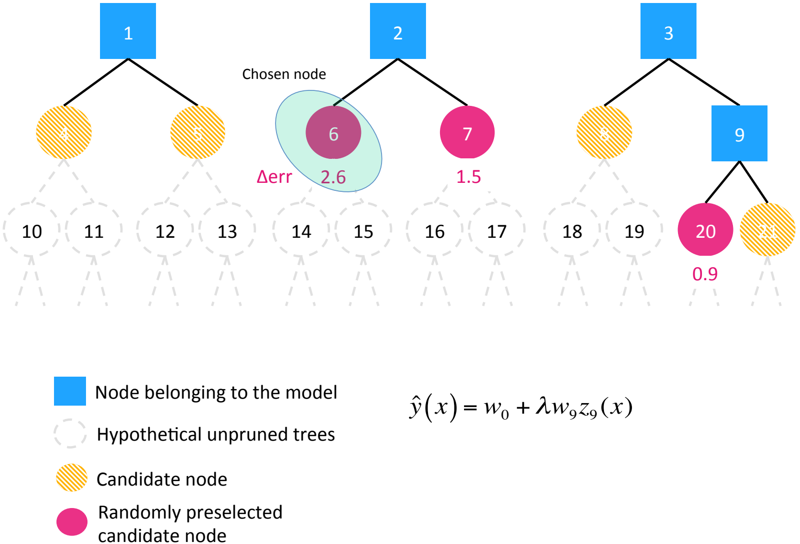
$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x)$$

GIF algorithm — Illustration (regression)



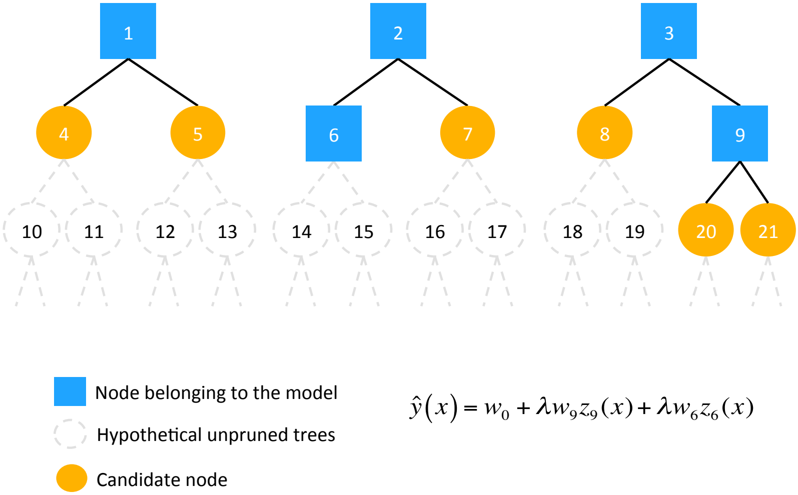
$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x)$$

GIF algorithm — Illustration (regression)



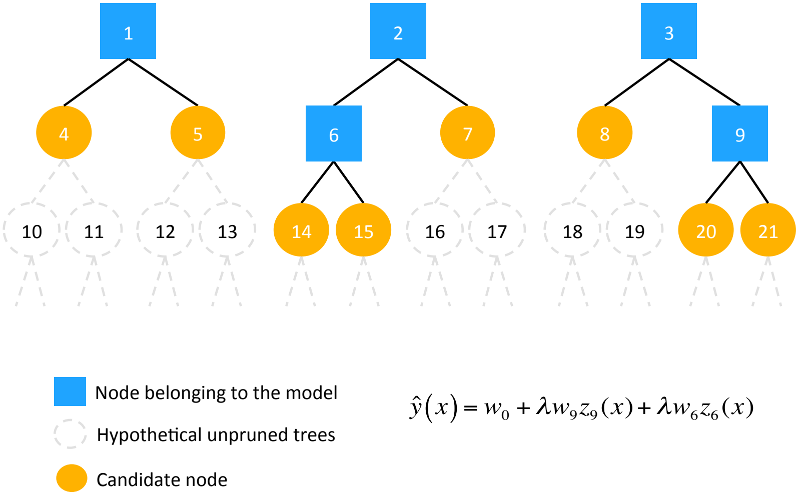
$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x)$$

GIF algorithm — Illustration (regression)



$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x) + \lambda w_6 z_6(x)$$

GIF algorithm — Illustration (regression)



$$\hat{y}(x) = w_0 + \lambda w_9 z_9(x) + \lambda w_6 z_6(x)$$

GIF algorithm — High level view

Build an additive model corresponding to a forest by introducing decision nodes sequentially until a node budget constraint is met.

- Splits in decision nodes are optimized locally.
- Nodes are taken from a candidate list.
- The best node is added . . .
- . . . together with its weight.

Candidate list

Each time a node is added to the model, the learning instances reaching that node are split according to a local criterion. The resulting children are placed in the candidate list.

GIF algorithm — High level view

Build an additive model corresponding to a forest by introducing decision nodes sequentially until a node budget constraint is met.

- Splits in decision nodes are optimized locally.
- Nodes are taken from a candidate list.
- The best node is added . . .
- . . . together with its weight.

GIF algorithm — Node selection

The best node j^*, together with its optimal weight w_j^*, are the ones that minimize some loss L over the training set $(x_i, y_i)_{i=1}^N$:

$$(j^*, w_j^*) = \underset{j \in C_t, w \in \mathbb{R}^K}{\arg\min} \sum_{i=1}^{N} L\left(y_i, \hat{y}^{(t-1)}(x_i) + w z_j(x_i)\right) \qquad (3)$$

where C_t is the subsample of candidates.

This problem is solved in two steps

1. for a candidate j, compute the best weight w_j^* :

$$w_j^* = \underset{w \in \mathbb{R}^K}{\arg\min} \sum_{i=1}^{N} L\left(y_i, \hat{y}^{(t-1)}(x_i) + w z_j(x_i)\right) \qquad (4)$$

2. select the best candidate with exhaustive search :

$$j^* = \underset{j \in C_t}{\arg\min} \sum_{i=1}^{N} L\left(y_i, \hat{y}^{(t-1)}(x_i) + w_j^* z_j(x_i)\right) \qquad (5)$$

GIF algorithm — Boostrapping

Candidate list

Since all root nodes see all the learning examples, they would produce the same loss reduction. To increase diversity, we grow T stumps and use the leaves to form the initial candidate list.

Initial bias

The initial bias w_0 is the best constant that fits the training set

$$w_0 = \arg\min_{y \in \mathbb{R}^K} \sum_{i=1}^{N} L(y_i, y) \tag{6}$$

Results — Experimental setting

1. Grow a forest of a thousand fully-developed Extremely randomized trees ($ET_{100\%}$) and count the number of nodes M.
2. Compare how different methods fare (in average over ten runs) under a constraint of 1% and 10% of that budget.

   $GIF_{x\%}$ grow the forest of a thousand trees until the node budget is met with the GIF algorithm.

   $RAND_{x\%}$ grow a forest of a thousand trees randomly.

   $ET_{x\%}$ grow only $10x$ fully-developed trees.

We used the following values for the hyper-parameters :

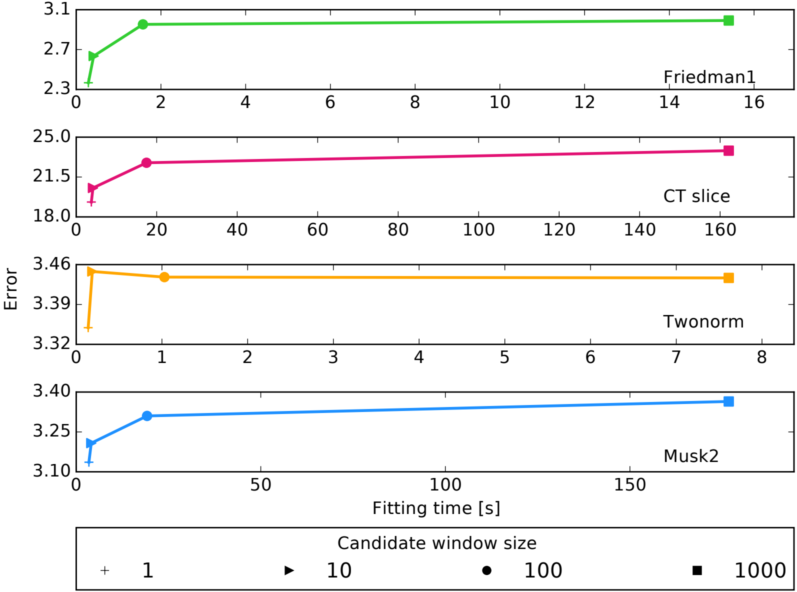
Number of trees : 1000

Candidate window size : 1

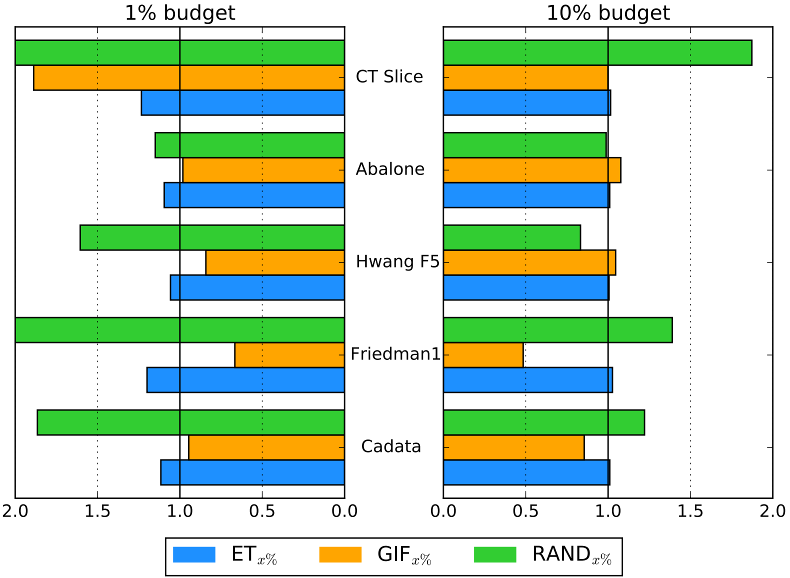
Learning rate : $10^{-1.5}$

Splitting algorithm : Extremely randomized trees (ET)

Results — Candidate window size

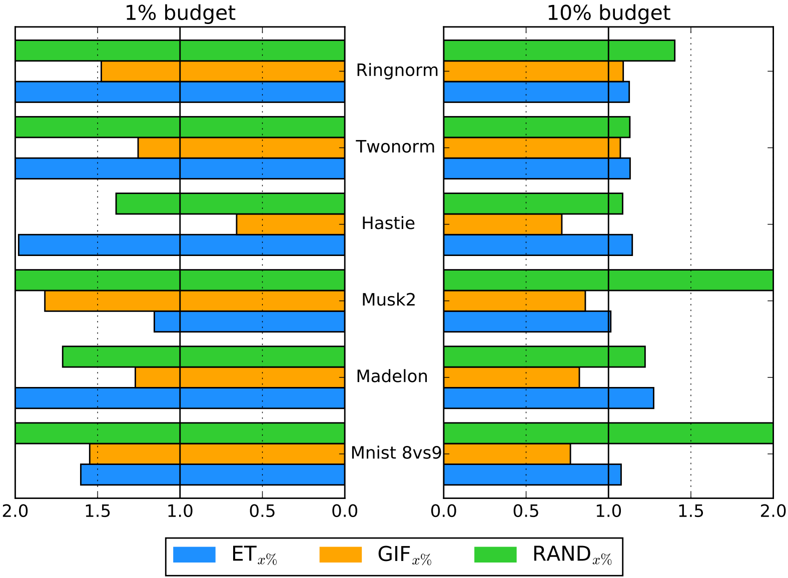


Results — Regression



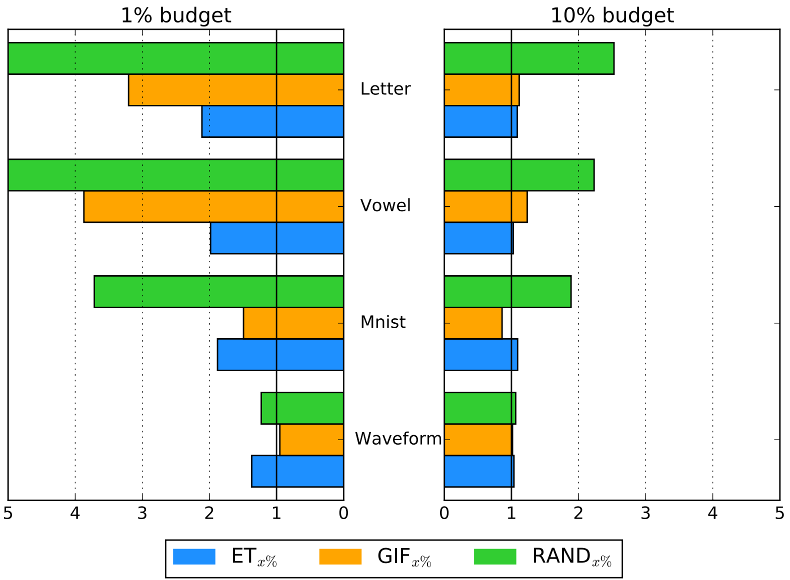
Relative average mean square error to $ET_{100\%}$.

Results — Binary classification



Relative average misclassification rate to $ET_{100\%}$.

Results — Multi-class problems



Relative average misclassification rate to $ET_{100\%}$.

Conclusion and future works

Performances

- GIF allows for lightweight yet accurate forests.
- Global optimization usually helps.
- Surprisingly, optimizing the shapes hurts.

TODOs

- Handle multiclass problems better.
- In depth comparison with Boosting methods.

GIF algorithm

1: **Input :** $D = (x_i, y_i)_{i=1}^N$, the learning set with $x_i \in \mathbb{R}^P$ and $y_i \in \mathbb{R}^K$; $\mathcal{A}$, the tree learning algorithm ; L, the loss function ; B, the node budget ; T, the number of trees ; CW, the candidate window size ; λ, the learning rate.

2: **Output :** An ensemble S of B tree nodes with their corresponding weights.

3: **Algorithm :**

4: $S = \emptyset$; $C = \emptyset$; $t = 1$

5: $\hat{y}^{(0)}(.) = \arg\min_{y \in \mathbb{R}^K} \sum_{i=1}^N L(y_i, y)$

6: Grow T stumps with $\mathcal{A}$ on D and add the left and right successors of all stumps to C.

7: **repeat**

8: C_t is a subset of size $\min\{CW, |C|\}$ of C chosen uniformly at random.

9: Compute :

$$(j^*, w_j^*) = \arg\min_{j \in C_t, w \in \mathbb{R}^K} \sum_{i=1}^N L\left(y_i, \hat{y}^{(t-1)}(x_i) + w z_j(x_i)\right)$$

10: $S = S \cup \{(j^*, w_j^*)\}$; $C = C \setminus \{j^*\}$;
$y^{(t)}(.) = y^{(t-1)}(.) + \lambda w_j^* z_{j^*}(.)$

11: Split j^* using $\mathcal{A}$ to obtain children j_l and j_r

12: $C = C \cup \{j_l, j_r\}$; $t = t + 1$

13: **until** budget B is met

GIF algorithm — Regularization

Node split

Although the weight are optimized globally, the splitting elements of a node are still determined locally.

Learning rate

A learning rate λ was introduced to prevent overfitting :

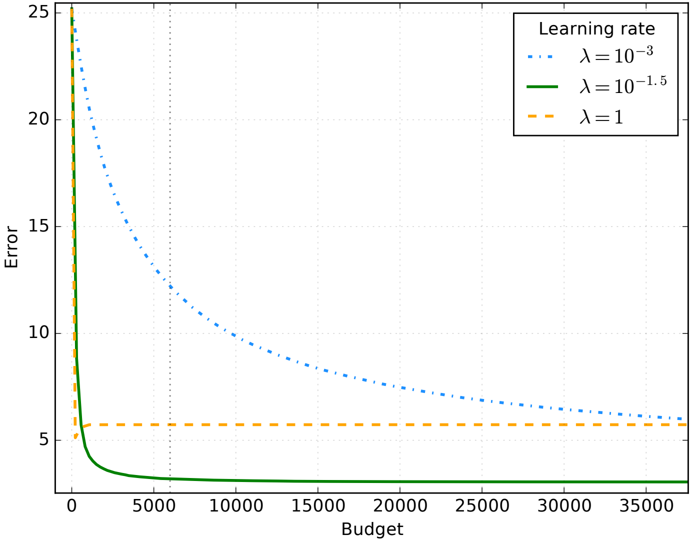
$$y^{(t)}(.) = y^{(t-1)}(.) + \lambda w_j^* z_{j^*}(.) \tag{7}$$

Candidate window

Only a uniformly drawn subset of candidates are examined at each iterations.

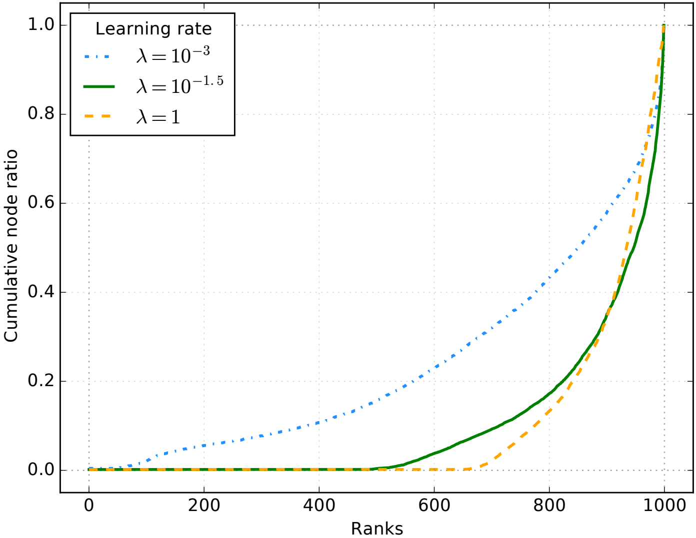
- This also serves to speed up computations

Result — Error rates droupout with iteration



Friedman1 : average test set error with respect to the budget B ($CW = +\infty$, m=$\sqrt{10}$, $T = 1000$).

Result — Cumulative node distribution



Friedman1 : cumulative node distribution with respect to the size-ranks ($CW = \infty$, m=$\sqrt{10}$, $T = 1000$, $B = 10\%$)

Datasets

Table: Characteristics of the datasets. N is the learning sample size, TS stands for testing set, and p is the number of features.

Dataset	N	$\|TS\|$	p	# classes
Friedman1	300	2000	10	-
Abalone	2506	1671	10	-
CT slice	2000	51500	385	-
Hwang F5	2000	11600	2	-
Cadata	12384	8256	8	-
Ringnorm	300	7100	20	2
Twonorm	300	7100	10	2
Hastie	2000	10000	10	2
Musk2	2000	4598	166	2
Madelon	2200	2200	500	2
Mnist8vs9	11800	1983	784	2
Waveform	3500	1500	40	3
Vowel	495	495	10	11
Mnist	50000	10000	784	10
Letter	16000	4000	8	26