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Childbearing Age, Family Allowances, and Social Security

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Although the optimal public policy under an endogenous number of children has been widely studied, the optimal public intervention under an endogenous timing of births has remained largely unexplored. This paper examines the optimal family policy when the timing of births is chosen by individuals who differ as to how early fertility weakens future earnings. We analyze the design of a policy of family allowances and of public pensions in such a setting, under distinct informational environments. Endogenous childbearing ages is shown to affect the optimal policy through the redistribution across the earnings dimension and the internalization of fertility externalities. Contrary to common practice, children benefits differentiated according to the age of parents can be part of the optimal family policy. Our results are robust to introducing: (i) children as durable ''goods''; (ii) education choices; (iii) varying total fertility.

JEL Classification: J13, D10, H21, H55

1. Introduction

As is well known among demographers, there has been a continuous postponement of fertility in European economies since the late 1970s. That strong demographic trend is well illustrated by the case of France. As shown in Figure 1, the average age at motherhood has increased from about 26.9 years in 1977 to about 29.7 years in 2005. The postponement of fertility appears even more strongly when we look at the mode of the distribution of the age at motherhood: The mode age has grown from 25 years in 1977 to 29 years today. With the development of assisted reproductive technologies, that evolution is likely to be sustained—if not reinforced—in the future.

The tendency toward a postponement of births is a widespread phenomenon, which can be observed in many economies around the world. To illustrate this, Figure 2 shows, for France, Germany, Japan, the United Kingdom, and the United States, the decomposition of the average total fertility rate between the number of children born during the *early* part of the reproduction period (i.e., when women are between 15 and 34 years old) and the number of children born during the *late* part of the reproduction period (i.e., when women are between 35 and 49 years old).[1] During the period 1985–2005, the proportion of children from mothers

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[1] Here ''reproduction period'' means ''period of fertility.''

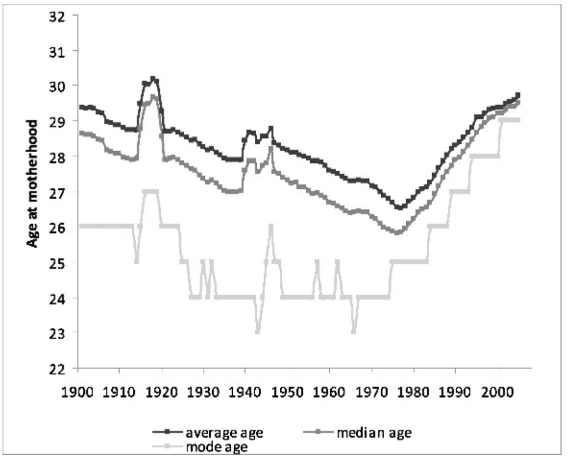

Figure 1. Age at motherhood in France. Source: INED (2011)

older than 34 years within the total number of children has been continuously increasing in all economies under study. Hence, we observe an unambiguous rise in the proportion of what we can call "late births" (i.e., children born from older parents), in comparison to what we can call "early births" (i.e., children born from younger parents).

The causes of the postponement of fertility have been widely explored.[2] Empirical studies highlighted that women's earnings opportunities and educational achievements are a major determinant of fertility behavior. The rise in women's wages, by implying a higher opportunity cost of motherhood, appears to be a major factor of fertility decline, which coincides also with later motherhood.[3]

Besides those empirical studies, theoretical models have also been developed to explain the choice of a particular fertility age pattern over the lifecycle. Those models, such as Happel, Hill, and Low (1984), Cigno and Ermisch (1989), and Cigno (1991), highlighted the central influence of lifecycle effects on the timing of births. Whereas Happel, Hill, and Low (1984) emphasized that the

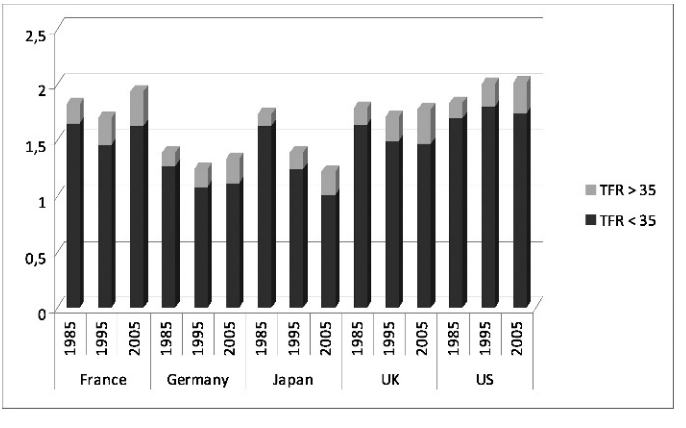

Figure 2. Decomposition of the total fertility rate (before and after 35 years), 1985–2005. Source: UNDP (2012)

[2] See Gustafsson (2001) for a survey on the literature on childbearing age.

[3] Empirical studies include Schultz (1985), Heckman and Walker (1990), and Tasiran (1995) on Sweden, Ermisch and Ogawa (1994) on Japan, Merrigan and St Pierre (1998) on Canada, and Joshi (2002) on Great Britain.

consumption smoothing induced by the maximization of lifetime welfare may lead to delaying births, Cigno and Ermisch (1989) focused on the shape of the earnings profile induced by human capital investment and argued that steeper earnings profiles lead to the postponement of births later in life.[4]

The various influences of governments on the timing of births have also been studied. While the income tax rate and parental leave benefits reduce the opportunity cost of early children, children allowances reduce the net direct expenditures, and the income tax rate also reduces the forgone returns to forgone human capital investment. Those governmental influences have been confirmed by various empirical studies.[5] Macroeconometric studies, such as Ermisch (1988) on the United Kingdom and Walker (1995) on Sweden, identified a positive impact of child allowances on early motherhood. That effect was confirmed by microeconometric studies, such as Laroque and Salanié (2004) on France, who found that cash benefits increase the probability of having a first child.

In the light of those influences of governments on fertility behavior, a natural question to raise concerns the optimal design of family policies. What should governments do in front of the postponement of fertility? Should governments implement a transfer policy in such a way as to reinforce (or weaken) that demographic trend? Should governments subsidize births differently depending on the age of parents? By which channels are the optimal fertility policy and the optimal pension policy related to each other?

The literature on the optimal policy under endogenous fertility has examined various aspects of the problem, but without paying attention to the optimal *timing* of births. Cigno (1986) showed that family allowances aimed at reducing child poverty may, by raising fertility, have quite the opposite effects, so that the optimal policy may consist in taxing—instead of subsidizing—the number of children.[6] More recently, Cremer, Gahvari, and Pestieau (2006) studied the design of the optimal pension system under endogenous and stochastic fertility, and showed that, under positive fertility externalities, one should grant parents who have more children larger pension benefits.[7]

The goal of this article is to complement that literature, by examining the design of the optimal public intervention in a context where the *timing* of births is chosen by parents. For that purpose, we develop a three-period model. In the first two periods, individuals are active, and they can have children. In the third period, individuals retire. In that framework, having a child in the first period relative to the second is cheaper, but it imposes a cost in terms of professional achievement, so that some trade-off exists between lower fertility costs and higher future wages. Moreover, when parents retire, and rely on some help from their children, they will, if earnings increase with age, receive fewer resources per child under late childbearing than under early childbearing. Retired agents will also benefit from the contributions of fewer individuals under late childbearing than under early childbearing (where they benefit from the contributions of their working children *and* grandchildren). Finally, in order to account for the widely observed effect of heterogeneity in education on the timing of births (Cigno and Ermisch 1989), we assume that agents differ in the level and the slope of their earnings profile, as well as in the extent to which future earnings are decreasing in the number of early children.

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[4] Cigno and Ermisch (1989) also found, on the basis of U.K. data, empirical support for that explanation of the observed heterogeneity in terms of fertility patterns.

[5] On this, see the survey by Gauthier (2007).

[6] This was confirmed by Balestrino, Cigno, and Pettini (2002) in a model where households differ in their productivities and in their ability to raise children.

[7] That result is also obtained in Cremer, Gahvari, and Pestieau (2008), who examined the optimal pension scheme when fertility is endogenous and parents differ in their ability to raise children.

In the following, we make, for the sake of presentation, several simplifying assumptions. First, we assume, in the baseline model, that the total number of children over the whole reproduction period is equal to the replacement fertility level, in such a way as to focus only on the timing of births.[8] Second, we abstract here from intrafamily decision-making issues and treat a couple as a single individual capable of working, saving, and having children. Third, we assume away the childhood period. Fourth, we focus on a static overlapping-generations model and do not explore here the effects of childbearing ages on the long-run dynamics.[9] Fifth, we assume here that the number of children is deterministic, and thus rule out uncertainty about the number of children.[10] Sixth, we do not consider investment in the quality of children and the impact of birth spacing on this.[11]

Anticipating our results, we first show that, at the laissez-faire, agents with low earnings tend, under general conditions, to have children earlier than agents with high earnings. Then, we characterize the first-best utilitarian social optimum and show that this can be decentralized by means of lump-sum transfers from agents with high earnings toward agents with low earnings, but leave fertility profiles unchanged. Then, we consider the second-best optimal policy under linear taxation instruments and show that the optimal family allowances are positive only if subsidizing children favors early childbearing. Those family allowances are here justified on redistributive grounds, that is, as a way to achieve indirect redistribution toward low-earnings agents, who tend to favor early births. The study of the second-best problem under nonlinear instruments shows that children allowances should concern only the children of young parents, in such a way as to solve the incentive problem. We also examine the optimal family policy under the presence of a Pay-As-You-Go (PAYG) pension system and show that the social planner faces, under endogenous childbearing age, a trade-off between efficiency (favoring late childbearing) and equity (helping early childbearing). Finally, we examine the robustness of our results to alternative settings, including parental preferences treating children as durable consumption goods, the introduction of parental education decisions, and the addition of a third working period, as well as relaxing the replacement fertility assumption. Our results are shown to be qualitatively robust to those changes.

The rest of the article is organized as follows. Section 2 presents the baseline model. The first-best utilitarian optimum is derived in section 3, where its decentralization is also examined. In section 4, we characterize the optimal child benefits when there is a perfect capital market. In section 5, we introduce the idea that pensions can be of the PAYG type, which may give an incentive to both the government and individuals to have early children. Several extensions are studied in section 6. Section 7 concludes.

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[8] That assumption is relaxed later, in section 6.

[9] Those effects are discussed in Pestieau and Ponthiere (2013), where it is shown that, compared to the two-period OLG model, the conditions for optimal capital accumulation and optimal fertility differ quite significantly under varying childbearing ages. However, Samuelson's (1975) Serendipity Theorem still holds in that broader demographic environment. On the dynamics, see also DÁlbis, Augeraud-Véron, and Schubert (2010) on the continuous-time OLG setup.

[10] See Cigno and Luporini (2011) on the optimal family policy when the number and the future earnings of children are random. The absence of risk rules out a case for early childbearing: the possibility to insure oneself against a total number of children lower than desired.

[11] On that issue, see Schultz (1997).

2. Basic Model

Environment

Whereas reproduction can, in reality, occur at various points in the lifetime, we will focus here, for the sake of simplicity, on a three-period model with two reproduction periods.[12] The three life periods are labeled a, ℓ, and o (i.e., advanced, late, old). In the first two periods, individuals work, consume, save, and have children. It is assumed that a perfect capital market exists. In the last period, individuals are retired and consume the proceeds of their savings.

The specificity of the model lies in the existence of two, instead of one, periods at which parents can give birth to children. The number of early births is denoted by n_a, whereas the number of late births is denoted by n_ℓ. As we are concerned with the timing of fertility rather than with the level of total fertility, we assume, for the sake of simplicity, that total fertility is fixed to its replacement level, which is, in the absence of mortality, equal to 1: $n_a + n_\ell = 1$.[13]

When considering the optimal fertility timing, an important issue consists of the cost imposed by early fertility on future career and earnings opportunities. Young childbearing ages make career progression more difficult. In order to capture this, we assume that all individuals have, at period 2, a productivity that is larger than in period 1, to an extent that is *decreasing* in the number of children they had in period 1. Hence, while first-period earnings is equal to w, second-period earnings is equal to $wh(n_a)$, where the function $h(n_a) > 1$ accounts for the extra productivity and extra earnings due to career advancement. That function exhibits $h'(n_a) < 0$ and $h''(n_a) < 0$. This reflects the idea that early childbearing has a cost on earnings in period ℓ.[14] Indeed, various empirical studies, such as Joshi (1990, 1998) on Great Britain and Dankmeyer (1996) on the Netherlands, show that having children in an early stage of a career slows down human capital accumulation and professional promotion.[15]

To do justice to the observed heterogeneity in terms of career and childbearing ages, we assume that individuals differ in the shape of the earnings profile, that is, in the function $h(n_a)$. We consider two types of agents:[16]

- Type-1 agents: low earnings profile, with $h_1(n_{ai}) < h_2(n_{ai})$ and $|h_1'(n_{ai})| < |h_2'(n_{ai})|$ for an equal n_{ai}.
- Type-2 agents: high earnings profile, with $h_2(n_{ai}) > h_1(n_{ai})$ and $|h_2'(n_{ai})| > |h_1'(n_{ai})|$ for an equal n_{ai}.

The earnings profile of type-2 agents lies above the one of type-1 agents for an equal level of early births. Moreover, the earnings profile of type-2 agents is more sensitive to early fertility than the one of type-1 agents.

The lifetime welfare of an individual of type $i \in \{1,2\}$ is expressed as

$$U_i = u(c_i) + \beta u(d_i) + \beta^2 u(b_i) + v(n_{ai} + n_{\ell i}) \tag{1}$$

[12] Adding more periods to the model would not bring additional insights or results.

[13] That assumption is relaxed in section 6.

[14] Note that late births can also have a negative impact on earnings. The consequences of that additional effect are discussed in section 6.

[15] The causes of the influence of early childbearing are unknown. For instance, Ermisch and Pevalin (2005) showed that very early births worsen later outcomes on the marriage market.

[16] This is a reduced form model of an economy where agents differ in career profiles, due to past education choices. Section 6 studies how adding an education choice affects our results.

where c_i, d_i, and b_i are consumption, $u(\cdot)$ is strictly concave, β is a time preference factor ($0 \leq \beta \leq 1$), and $v(\cdot)$ is the utility for early and late children, which is also strictly concave.[17]

Substituting for $n_{ai} + n_{\ell i} = 1$ and normalizing the utility from children, in such a way that $v(1) = 0$, lifetime welfare can be rewritten as[18]

$$U_i = u(c_i) + \beta u(d_i) + \beta^2 u(b_i) \tag{2}$$

In the first period, consumption is equal to wage w *minus* saving and *minus* the cost of raising children, $n_{ai} e_a$:[19]

$$c_i = w - s_{ai} - n_{ai} e_a. \tag{3}$$

In the second period, the individual earns also savings income Rs_{ai}, where R is the interest factor. He saves also $s_{\ell i}$ for the old age and spends $e_\ell(1 - n_{ai})$ as cost of raising his $n_{\ell i} = 1 - n_{ai}$ late children. The direct cost of children is larger for late children than for early children:

$$e_\ell > e_a$$

That assumption is in conformity with the medical literature showing the larger costs of late motherhood in comparison to the ones under early motherhood. Those additional costs are of various kinds and concern both the mother and the child (see Gilbert, Nesbitt, and Danielsen 1999; Gustafsson 2001).[20] We assume, for simplicity, that the parents face the entire additional cost from late childbearing. Hence, second-period consumption is

$$d_i = wh_i(n_{ai}) + Rs_{ai} - s_{\ell i} - (1 - n_{ai}) e_\ell. \tag{4}$$

In the third period, an agent of type $i \in \{1,2\}$ consumes the proceeds of his savings:

$$b_i = Rs_{\ell i}. \tag{5}$$

Laissez-faire

Each agent of type $i \in \{1,2\}$ chooses first-, second-, and third-period consumptions, as well as first- and second-period children, in such a way as to maximize his lifetime welfare subject to his budget constraint, and subject to $n_{ai} + n_{\ell i} = 1$. For an agent of type $i \in \{1,2\}$, the first-order conditions (FOCs) are

$$u'(c_i) = \beta R u'(d_i) = \beta^2 R^2 u'(b_i)$$

$$u'(c_i) e_a - \beta u'(d_i) w h_i'(n_{ai}) = \beta u'(d_i) e_\ell$$

[17] We assume here a perfect substitutability, in welfare terms, between early and late children. That assumption, which is made for simplicity, amounts to assuming that children are not durable consumption goods. That assumption is relaxed in section 6.

[18] $v(\cdot)$ will be used below in section 6, where we assume variable overall fertility.

[19] For the sake of simplicity, we abstract here from the time cost of children and assume only a physical cost for each child. Note, however, that some additional costs, which could be called "career" costs, are associated with early parenthood (see below).

[20] According to Gustafsson (2001), late births imply, on average, more pregnancy complications, more ceasareans, and more breast cancer for mothers. Moreover, late children are also more subject to somatic and learning problems.

The first equalities, which describe the optimal consumption profile, are standard. However, the second condition, which characterizes the optimal timing for births, is not straightforward. It tells us that the allocation of births along the lifecycle is optimal when there is an equalization of the marginal welfare loss from varying the number of early children (the left-hand side [LHS]) with the marginal welfare gain from varying the number of early children (the right-hand side [RHS]). The marginal welfare loss from early births has two components: It reduces consumption at the young age (first term of the LHS) and reduces future earnings possibilities (second term of the LHS). But having children early in the career has nonetheless the advantage of avoiding late fertility, which is more expensive, since $e_\ell > e_a$ (see the RHS).

To interpret those conditions, let us assume that $\beta = R = 1$. Then we have a perfect consumption smoothing for all agents: $c_i = d_i = b_i$, with higher consumptions for type-2 agents, who have a higher earnings profile.

Note that, under a flat consumption profile, the FOC for n_{ai} becomes

$$e_\ell - e_a = -wh_i'(n_{ai})$$

Hence, for an agent of type i, three possible cases can arise, depending on the size of the cost differential between early and late children and on the impact of early parenthood on future productivity:

Case 1 If $-wh_i'(0) < e_\ell - e_a$ and $-wh_i'(1) > e_\ell - e_a$ for type i, then $0 < n_{ai} < 1$ and $0 < n_{\ell i} = 1 - n_{ai} < 1$.[21]
Case 2 If $-wh_i'(0) > e_\ell - e_a$ for type i, then $n_{ai} = 0$ and $n_{\ell i} = 1$.
Case 3 If $-wh_i'(1) < e_\ell - e_a$ for type i, then $n_{ai} = 1$ and $n_{\ell i} = 0$.

If the cost differential between late and early births $e_\ell - e_a$ is very low, so that the marginal gain from early childbearing is *inferior* to the marginal loss from early children for any n_{ai}, then type-i agents have no early children, and only late children, since this will preserve their future productivity at little additional child costs (Case 2). If, on the contrary, the cost differential $e_\ell - e_a$ is extremely large, so that the marginal gain from early childbearing is *superior* to the marginal loss from early children for any n_{ai} (Case 3), type-i agents opt for early children only. In the intermediate case (Case 1), there is an interior solution with $n_{ai} > 0$ and $n_{\ell i} > 0$, with a shape of fertility profile that depends on how sensitive the earnings profile is to early parenthood.

Let us now compare type-1 and type-2 agents. Six distinct cases can arise:

Case 1-1 If $-wh_i'(0) < e_\ell - e_a$ and $-wh_i'(1) > e_\ell - e_a$ is true for all agents, then all types are in Case 1: $0 < n_{a2} < n_{a1} < 1$ and $0 < n_{\ell 1} < n_{\ell 2} < 1$.[22]
Case 1-2 If $-wh_1'(0) < e_\ell - e_a$ and $-wh_1'(1) > e_\ell - e_a$ and $-wh_2'(0) > e_\ell - e_a$, type-1 is in Case 1, and type-2 in Case 2: $0 < n_{a2} < n_{a1}$ and $0 < n_{\ell 1} < n_{\ell 2} = 1$.
Case 2-2 If $-wh_1'(0) > e_\ell - e_a$, then both types are in the Case 2: $n_{a1} = n_{a2} = 0$ and $n_{\ell i} = n_{\ell 2} = 1$.
Case 3-1 If $-wh_1'(1) < e_\ell - e_a$ and $-wh_2'(0) < e_\ell - e_a$ and $-wh_2'(1) > e_\ell - e_a$, then type-1 is in Case 3, and type-2 is in Case 1: $0 < n_{a2} < n_{a1} = 1$ and $n_{\ell 1} = 0 < n_{\ell 2} < 1$.

[21] To see this, note that, under the above conditions, there exists, by monotonicity and continuity of $h_i(n_{ai})$, a unique level of n_{ai} satisfying $e_\ell - e_a = -wh_i'(n_{ai})$.

[22] Indeed, the interior levels of n_{a1} and n_{a2} satisfy respectively: $e_\ell - e_a = -wh_1'(n_{a1})$ and $e_\ell - e_a = -wh_2'(n_{a2})$. Hence, given that $|h_1'(n_{ai})| < |h_2'(n_{ai})|$ for any $0 \leq n_{ai} \leq 1$, the LHS is, for an equal n_{ai}, larger in the case of type 2, leading to $n_{a2} < n_{a1}$.

Case 3-2 If $-wh_1'(1) < e_\ell - e_a$ and $-wh_2'(0) > e_\ell - e_a$, then type-1 is in Case 3, and type-2 is in Case 2: $n_{a2} = 0 < n_{a1} = 1$ and $n_{\ell 1} = 0 < n_{\ell 2} = 1$.

Case 3-3 If $-wh_2'(1) < e_\ell - e_a$, then both types are in the Case 3: $n_{a1} = n_{a2} = 1$ and $n_{\ell i} = n_{\ell 2} = 0$.

Whatever the case, the number of early children among type-1 parents is never inferior to the number of early children among type-2 parents. Inversely, type-2 parents never have fewer late children than type-1 parents. Moreover, under general conditions (excluding cases 2-2 and 3-3), type-1 agents have more early children and fewer late children than type-2 parents.

Therefore, our baseline model, although simple, allows us to rationalize the observed heterogeneity within the population. Empirical studies (Cigno and Ermisch 1989; Ermisch and Ogawa 1995; Joshi 2002) show that adults with lower career opportunities have, on average, their children earlier than adults with (potentially) steeper earnings profiles. This is clearly the case in our model, where, if one excludes cases 2-2 and 3-3, type-2 parents have their children, on average, at ages that exceed the childbearing ages for type-1 agents.

Our model can also explain the observed postponement of fertility. Indeed, with the development of assisted reproductive technologies, the cost gap $e_\ell - e_a$ has been reduced. When the gap $e_\ell - e_a$ goes down, Case 3 becomes less likely for all agents. Moreover, a reduction of $e_\ell - e_a$ also favors, for all agents, a transition from Case 1 to Case 2. All this leads to a rise in the average age at motherhood, as shown by the data.

3. The First-Best Problem

Let us now characterize the social optimum of our economy. For that purpose, we will focus on a classical utilitarian social objective, whose goal coincides with the maximization of aggregate welfare. We also focus here on the aggregate welfare of a single cohort of individuals, which includes a fixed fraction π_1 of agents of type 1, and a fixed fraction π_2 of agents of type 2.[23] Whereas focusing on a single cohort of given size is an obvious simplification, this allows us to escape from well-known difficulties raised by population ethics.[24]

The first-best optimum is obtained by maximizing the following Lagrangian:

$$\sum \pi_i (U_i - \mu[c_i + d_i + b_i + n_{ai} e_a + (1 - n_{ai}) e_\ell - w(1 + h_i(n_{ai}))])$$

where μ denotes the Lagrange multiplier associated with the resource constraint of the economy. The FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i) = \mu$$

$$e_\ell - e_a = -wh_i'(n_{ai})$$

The utilitarian social optimum involves an equalization of all consumptions across individuals and across periods. Regarding optimal fertility timing, the FOC is exactly the same as under the laissez-faire. Therefore, we have, for each type of agent, three distinct cases, and, in total, six possibilities. As under the laissez-faire, we also have that, under general conditions

[23] For the sake of simplicity, we will also assume, in the rest of this article; that the time preference factor β and the interest factor R are equal to 1.

[24] On this, see Blackorby, Bossert, and Donaldson (2005).

(i.e., excluding Cases 2-2 and 3-3), it is socially optimal that type-2 agents have their children later than type-1 agents. Hence, we have, at the social optimum

$$n_{a1} > n_{a2}$$
$$n_{\ell 1} < n_{\ell 2}$$

Comparing the utilitarian optimum with the laissez-faire, it appears that, on the consumption side, the two allocations are quite different: Whereas type-2 agents enjoy higher consumptions than type-1 agents at the laissez-faire, utilitarianism recommends the equalization of consumptions across individuals and time periods. But on the fertility side, given that the socially optimal timing of births is characterized by the same optimality conditions as under the laissez-faire, and given that those conditions are independent from consumption levels (because of separability), all agents keep, at the social optimum, the *same* fertility profile as at the laissez-faire. However, as a result of consumption equalization, type-1 agents are better off at the utilitarian optimum in comparison to the laissez-faire, whereas the opposite holds for type-2 agents.

Regarding the decentralization of the social optimum, this can be achieved by means of lump-sum transfers from type-2 agents toward type-1 agents, in such a way as to equalize all consumptions. Given that the optimal timing of births is, for both agents, independent from the resource level, those transfers leave the fertility timing unchanged.

4. Family Allowances

The previous section showed how the first-best optimum can be decentralized by means of lump-sum transfers from individuals with a high earnings profile toward individuals with a low earnings profile. However, such an intervention requires both that lump-sum transfers are available policy instruments and that the government can observe the types of individuals. These are strong assumptions. In this section we depart from such a first-best problem and consider instead the design of the second-best policies.

For that purpose, we will proceed in two stages. We will first focus on the optimal policy when the only available fiscal instruments are *uniform* (i.e., nonindividualized) payroll taxes, demogrants, and children allowances. Then, we will consider the optimal policy with asymmetric information, under nonlinear fiscal instruments.

Linear Case

Let us first consider the government's problem when the set of available policy instruments does not include type-specific lump-sum transfers, but includes only a uniform payroll tax, τ; a uniform demogrant, T; and a uniform subsidy on the number of children, σ, which can be understood as a family allowance.[25]

[25] Note that other fiscal instruments could be used instead, but we focus here on those three instruments, in order to keep the analysis simple. See section 5 for the study of the optimal pension benefit in the context of varying childbearing age.

Each agent of type $i \in \{1,2\}$ maximizes

$$U_i = u(c_i) + u(d_i) + u(b_i) - \mu_i[c_i + d_i + b_i - T - (1-\tau)$$

$$w(1 + h_i(n_{ai})) + (1-\sigma)((e_a - e_\ell)n_{ai} + e_\ell)].$$

where μ_i is the Lagrange multiplier associated with the agent's budget constraint. The FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i) = \mu_i$$

$$-(1-\tau)wh_i'(n_{ai}) + (1-\sigma)(e_a - e_\ell) = 0$$

Thus agents tend, here again, to smooth consumption across periods and organize the timing of births in such a way as to equalize the marginal welfare losses and marginal welfare gains from early parenthood.

As to the government, its goal is to maximize the sum of individual utilities subject to its revenue constraint. Namely, it maximizes the Lagrangian:

$$\mathcal{L} = \sum \pi_i \Bigg\{ 3u\left(\frac{(1-\tau)w(1+h_i(n_{ai})) + T - (1-\sigma)((e_a - e_\ell)n_{ai} + e_\ell)}{3}\right)$$

$$+ \mu[\tau w(1 + h_i(n_{ai})) - T - \sigma(e_a - e_\ell)n_{ai} + \sigma e_\ell] \Bigg\}$$

where μ is the Lagrange multiplier associated with the revenue constraint.

Using the envelope theorem, we obtain the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial \tau} = -\sum \pi_i \left[u'(x_i)w(1+h_i(n_{ai})) - \mu w(1+h_i(n_{ai})) - \mu\tau w h_i'(n_{ai})\frac{\partial n_{ai}}{\partial \tau} - \mu\sigma(e_a - e_\ell)\frac{\partial n_{ai}}{\partial \tau}\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} = \sum \pi_i \left[u'(x_i)[(e_a - e_\ell)n_{ai} + e_\ell] + \mu\tau w h_i'(n_{ai})\frac{\partial n_{ai}}{\partial \sigma} - \mu\sigma(e_a - e_\ell)\frac{\partial n_{ai}}{\partial \sigma} - \mu[(e_a - e_\ell)n_{ai} + e_\ell]\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mathrm{T}} = \sum \pi_i \left[u'(x_i) - \mu(1+\sigma)(e_a - e_\ell)\frac{\partial n_{ai}}{\partial T} + \tau w h_i'(n_{ai})\frac{\partial n_{ai}}{\partial T}\right] = 0$$

where $x_i \equiv \dfrac{(1-\tau)w(1+h_i(n_{ai})) + T - (1-\sigma)((e_a - e_\ell)n_{ai} + e_\ell)}{3}$ is the argument of the agent i's temporal utility $u(\cdot)$.

Those conditions characterize the optimal values for our instruments τ, σ, and T. Note, however, that the simultaneous study of the optimal levels of the three taxation tools would be quite laborious, as their values are related to each others through the government's budget constraint. Hence, to keep the analysis simple, we will proceed as follows. To interpret those optimality conditions, we will consider alternative pairs of instruments, holding the other instrument equal to 0. Hence, we will focus on the pairs (τ, T) and (σ, T), while keeping, each time, the other fiscal tool set to zero. This will allow us to derive, *in fine*, closed-form solutions for the optimal levels of the fiscal instruments.

Let us start with the pair (τ, T), composed of a payroll tax on labor earnings and a first-period demogrant. The first FOC from above does not suffice, on its own, to characterize the optimal level of τ, as a rise in τ must, under the government's budget constraint, imply a change in the demogrant T, in such a way as to maintain the budget equilibrium. Therefore, in order to characterize the optimal τ, we will use a compensated Lagrangian expression, whose derivative with respect to the policy instrument τ gives us the effect of a variation of τ on the Lagrangian when that change is compensated by a variation of T that keeps the government's budget balanced. Using the optimality conditions, the derivative of the compensated Lagrangian can be defined as

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial \tau} \equiv \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T}\frac{\partial T}{\partial \tau}$$

$$= \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T} E[w(1+h(n_a))]$$

where $\widetilde{\mathcal{L}}$ denotes the compensated Lagrangian, and where the second term of the RHS accounts for the effect of a change in the tax rate τ on the first-period demogrant T, under the government's budget equilibrium constraint. The operator $E(\cdot)$ denotes the *average* value of its argument among the population.[26]

Substituting for the FOCs for optimal τ and T and equalizing to zero yields

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial \tau} = \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T} E[w(1+h(n_a))]$$

$$= -E\Big[u'(x)w(1+h(n_a))\Big] + E\Big[u'(x)w\Big]E[(1+h(n_a))] - \mu E\left(A\frac{\partial \tilde{n}_a}{\partial \tau}\right) = 0$$

where $A \equiv \tau w h'(n_a) + \sigma(e_\ell - e_a)$ is the effect of n_a on the revenue constraint (the first term is negative and the second positive) and $\frac{\partial \tilde{n}_a}{\partial \tau} \equiv \frac{\partial n_a}{\partial \tau} + \frac{\partial n_a}{\partial T}\frac{\partial T}{\partial \tau}$ denotes the effect of a change in τ on early fertility, when that change is compensated by a change in T so as to maintain the budget equilibrium.

Regarding the pair (σ, T), one can proceed in the same way as with the pair (τ, T) and define the derivative of the compensated Lagrangian as follows:

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial \sigma} \equiv \frac{\partial \mathcal{L}}{\partial \sigma} + \frac{\partial \mathcal{L}}{\partial T}\frac{\partial T}{\partial \sigma}$$

$$= \frac{\partial \mathcal{L}}{\partial \sigma} - \frac{\partial \mathcal{L}}{\partial T}[(e_a - e_\ell)E(n_a) - e_\ell]$$

where the second term is the effect of a change in σ on the demogrant, under the government's budget equilibrium.

Substituting for the FOCs for optimal σ and T and equalizing to zero yields

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial \sigma} = \frac{\partial \mathcal{L}}{\partial \sigma} - \frac{\partial \mathcal{L}}{\partial T}[(e_a - e_\ell)E(n_a) - e_\ell]$$

[26] In the following, we drop the type index i when using the expectation operator.

$$= (e_a - e_\ell)\left[E\left(u'(x)n_a\right) - E\left(u'(x)\right)E(n_a)\right] + \mu E\left(A\frac{\partial \tilde{n}_a}{\partial \sigma}\right) = 0$$

where $\frac{\partial \tilde{n}_a}{\partial \sigma} \equiv \frac{\partial n_a}{\partial \sigma} + \frac{\partial n_a}{\partial T}\frac{\partial T}{\partial \sigma}$ denotes the effect of a change in σ on early fertility when that change is compensated by a change in T in such a way as to maintain the budget equilibrium.

The two compensated Lagrangian conditions can be rewritten as

$$-cov\left(u'(x), w(1+h(n_a))\right) - \mu E\left(A\frac{\partial \tilde{n}_a}{\partial \tau}\right) = 0$$

$$cov\left(u'(x), n_a\right)(e_a - e_\ell) + \mu E\left(A\frac{\partial \tilde{n}_a}{\partial \sigma}\right) = 0$$

To interpret these tax formulae, we look at pairs of instruments: τ and T and σ and T. Then we obtain the following expressions for the optimal payroll tax and the optimal child subsidy:

$$\tau = \frac{-cov\left(u'(x), (1+h(n_a))\right)}{\mu E\left(\frac{\partial \tilde{n}_a}{\partial \tau} h'(n_a)\right)} \tag{6}$$

$$\sigma = \frac{cov\left(u'(x), n_a\right)}{\mu E\left(\frac{\partial \tilde{n}_a}{\partial \sigma}\right)} \tag{7}$$

Regarding the optimal payroll tax, note first that the numerator of (Equation 6) is positive, as the covariance is negative, the correlation between x and $h(n_a)$ being positive, given $h'(n_a) < 0$. This is the standard equity term of the tax formula. However, the sign of τ depends also on that of the denominator, whose sign depends on the one of $-\partial \tilde{n}_a / \partial \tau$. If the (aggregate) compensated effect of the tax on n_a is negative, then the tax has a favorable impact on aggregate earnings and is thus desirable. This is the efficiency term of the tax formula. On the contrary, if the (aggregate) compensated effect of τ on n_a is positive, the denominator is negative, implying a negative optimal τ.

As far as optimal family allowances are concerned, the numerator of (Equation7) is positive (low-income agents have early children). The equity term of the fiscal formula pushes toward the subsidization of children. We thus have positive child benefits if the denominator is also positive, that is, if those children benefits have a positive (aggregate) effect on early childbearing in compensated terms.

In sum, the optimal policy consists, in the absence of lump-sum transfers, in a subsidization of children, to the extent that such family allowances increase early parenthood. However, if subsidizing children reduces early parenthood, then family allowances are no longer justified, and a taxation of children is required instead. The intuition is the following. If subsidizing children fosters early parenthood, this means, given that early parenthood concerns generally the individuals with a low earnings profile, that family allowances are a way to redistribute resources in the right direction (i.e., individuals with a low earnings profile), by subsidizing a good that those agents consume more than others. Inversely, if subsidizing children reduces early parenthood and raises late parenthood, family allowances are regressive, since these amount to subsidizing a good that is mostly consumed by individuals with a high earnings profile.

Note that, as an alternative to this presentation, we could replace the *general* children subsidy σ by a subsidy σ_a on *early* childbearing only.[27] Under such a refined instrument, children would be subsidized differently, according to the age of their parents. In that case, the optimal child allowance would be

$$\sigma_a = \frac{cov\left(u'(x), n_a\right)}{\mu E \dfrac{\partial \tilde{n}_a}{\partial \sigma_a}} \tag{8}$$

where $\partial \tilde{n}_a / \partial \sigma_a$ is the effect, on early fertility, of a change in σ_a compensated by a change in T in such a way as to maintain the budget equilibrium. The compensated effect $\partial \tilde{n}_a / \partial \sigma_a$ is positive, leading to a subsidy on early children.

Hence, the availability of children allowances differentiated according to the age of parents would be an indirect way to redistribute resources toward agents with a low earnings profile, who are usually the ones who have early children. The reason why governments can use children allowances differentiated according to the age of parents in order to achieve redistribution toward low-earning individuals lies in the differentiated fertility profiles adopted by the two types of agents: Early childbearing concerns much more type-1 agents than type-2 agents. Hence subsidizing children from young parents only is an indirect way to redistribute resources toward type-1 agents in a second-best world.

Although the implementation of family allowances differentiated according to the parents' age would not generate large administrative costs (since the age of parents is easy to observe), the implementation of such differentiated family allowances would nonetheless face some other difficulties, which lie on the political economy side. Clearly, family allowances differentiated according to the age of parents would face the same kind of criticisms as other age-differentiated fiscal policies (e.g., age-differentiated income taxation). The reason is that age is often regarded as a characteristic that should not be used by the state to treat its citizens differently. Therefore the implementation, in practice, of such differentiated family allowances would face strong criticisms, and governments may be reluctant to use that policy instrument.[28]

Having said this, it should nonetheless be stressed that, as shown in this subsection, standard family allowances—that is, *not* differentiated according to the age of parents—can also, under mild conditions, be used to achieve indirect redistribution toward low earnings profile agents (even though the redistribution would be better achieved by differentiated family allowances). The reason why those undifferentiated family allowances can be used to achieve redistribution lies in their capacity to raise early fertility, which concerns more type-1 agents than type-2 agents. Thus standard, undifferentiated, family allowances can also, provided these raise early fertility, achieve redistribution toward the low earnings profile individuals in a second-best setting.

Nonlinear Case

Let us now turn to an alternative formulation of the second-best problem, where the available fiscal instruments are not restricted to linear (uniform) instruments, but where the government cannot observe the types of agents. In that alternative framework, the government

[27] In that case, there is no tax nor subsidy on late childbearing.

[28] Note, however, that most tax systems *implicitly* differentiate people according to age through consumption taxation, tax breaks for young workers, early retirement schemes, etc.

cannot distinguish between, on the one hand, low-earnings-profile agents (i.e., type 1), and, on the other hand, high-earnings-profile agents (i.e., type 2).

The problem for an individual of type i is to maximize

$$u(c_i)+u(d_i)+u(b_i)-\mu_i[c_i+d_i+b_i-w(1+h_i(n_{ai}))+n_{ai}(e_a-e_\ell)+e_\ell]$$

This implies $c_i=d_i=b_i$, so that the utility can be reduced to

$$\varphi[w(1+h_i(n_{ai}))-n_{ai}(e_a-e_\ell)-e_\ell+T_i]$$

where $\varphi(\cdot)$ denotes the agent's lifetime welfare ($\varphi'(\cdot)>0$, $\varphi''(\cdot)<0$), whereas T_i denotes a lump-sum transfer such that $\sum\pi_i T_i=0$.

Remember that, for a given level of n_{ai}, we assume $h_2(n_{ai})>h_1(n_{ai})$ with $|h_2'(n_{ai})|>|h_1'(n_{ai})|$. Hence, if the difference between $h_2(\cdot)$ and $h_1(\cdot)$ is large enough, type-1 agents will choose a very high value of n_{a1}, and type-2 agents choose a lower level of n_{a2} and will postpone childbearing.

As we showed above, the first-best optimum involves a perfect equalization of consumptions for all agents and all periods, unlike at the laissez-faire. Hence, the decentralization of the first-best optimum requires a transfers scheme, from type-2 agents toward type-1 agents, namely, $T_1>0>T_2$.

In the second-best, the central planner cannot observe the types of agents. Hence, it is tempting, for an agent of type 2, that is, with a high earnings profile, to pretend to be of type 1 (by adopting his childbearing pattern), in such a way as to benefit from public transfers $T_1>T_2$. Therefore, the social planner must make sure that type-2 agents do not mimic type-1 agents. This leads us to the self-selection constraint:

$$\varphi[w(1+h_2(n_{a2}))+n_{a2}(e_\ell-e_a)-e_\ell+T_2]\geq$$

$$\varphi[w(1+h_2(n_{a1}))+n_{a1}(e_\ell-e_a)-e_\ell+T_1]. \tag{9}$$

The second-best problem can be written by means of the Lagrangian:

$$\mathcal{L} = \sum\pi_i\{\varphi(w(1+h_i(n_{ai}))+n_{ai}(e_\ell-e_a)-e_\ell+T_i)-\gamma T_i\}$$

$$+\lambda\{\varphi(w(1+h_2(n_{a2}))+n_{a2}(e_\ell-e_a)-e_\ell+T_2)-\varphi(w(1+h_2(n_{a1}))+n_{a1}(e_\ell-e_a)-e_\ell+T_1)\}$$

where γ is the Lagrange multiplier associated with the constraint on lump-sum transfers, whereas λ is the Lagrange multiplier associated with the self-selection constraint. The FOCs are

$$n_{a1}:\pi_1\varphi'(x_1)\left[wh_1'(n_{a1})+(e_\ell-e_a)\right]-\lambda\varphi'(x_2)\left[wh_2'(n_{a1})+(e_\ell-e_a)\right]=0$$

$$n_{a2}:\left[\varphi'(x_2)\pi_2+\lambda\right]\left[wh_2'(n_{a2})+(e_\ell-e_a)\right]=0$$

$$T_1:\pi_1\left(\varphi'(x_1)-\gamma\right)-\lambda\varphi'(x_2)=0$$

$$T_2:\pi_2\left(\varphi'(x_2)-\gamma\right)+\lambda\varphi'(x_2)=0$$

We see that, when the self-selection constraint is not binding ($\lambda = 0$), we have the first-best outcome:

$$e_\ell - e_a = -wh_i'(n_{ai})$$

$$\varphi'(x_i) = \gamma$$

On the contrary, when $\lambda > 0$, we have

$$e_\ell - e_a = -wh_2'(n_{a2})$$

$$\varphi'(x_1) > \gamma > \varphi'(x_2)$$

In other words, in the choice of n_{a2}, one has the standard "nondistortion at the top" result. As to redistribution, it is only partial, since we have $x_2 > x_1$. Thus, the presence of asymmetric information prevents the equalization of lifetime welfare across types, unlike in the first-best optimum.

Regarding the optimal early fertility for type-1 agents, we have

$$e_\ell - e_a = -wh_1'(n_{a1}) + \lambda \frac{\varphi'(\tilde{x}_2)}{\varphi'(x_1)\pi_1}\left[wh_2'(n_{a1}) + (e_\ell - e_a)\right]$$

When comparing that condition with the one at the laissez-faire, that is, $e_\ell - e_a = -wh_1'(n_{a1})$, it appears that the above condition includes an additional term on the RHS, which is of positive sign. In the light of that comparison, it follows that, in order to decentralize the second-best optimum, one should have a subsidy on n_{a1}, in such a way as to relax the self-selection constraint. Indeed, given that early children are more important for agents of type 1 in comparison to agents of type 2 (for whom early childbearing has worse effects on the earnings profile), a simple way to prevent type-2 agents from pretending to be of type 1 to get transfers consists of subsidizing early births for those who declare themselves being of type 1, and not for others. As type-2 agents are less interested in early childbearing in comparison to type-1 agents, such a family allowance scheme would allow governments to induce the self-selection of agents: Only true type-1 agents will, under those family allowances, claim to be of type 1.

Hence the optimal policy under asymmetric information involves children allowances on children of young parents and that are *differentiated* according to the parents' earnings profiles. The intuition behind that result lies in the fact that, even if an individual earnings profile is hard to observe, the age at which one has children is actually *observable*, and the government can use the fact that type-2 agents are less interested in early fertility than type-1 agents in order to induce those agents to reveal their true type. Thus, even if the government cannot observe the earnings of agents, it can nonetheless insure the self-selection of types by proposing different children allowances, which make, by construction, any mimicking suboptimal for individuals.[29]

[29] Note that, here again, as in section 4, governments may, in practice, be reluctant to apply such differentiated family allowances, on the ground of a will to treat all individuals equally. But the problem with such reluctance is that it turns out, at the end of the day, to prevent redistribution toward the worst-off agents. This issue is identical to that found in the tagging literature.

5. Childbearing and PAYG Social Security

Up to now, we studied the optimal family policy under a perfect capital market. We will now relax that assumption. For that purpose, we will assume that the only way individuals can provide resources for their old age is through a contract such that when retired they get a fraction of their children's earnings.

Such a contract can take various forms. It can be a standard Pay-As-You-Go pension system (PAYG), which provides a pension to the elderly thanks to the contributions of the young active individuals. It can also take the form of intergenerational trade within the (extended) family, each active child giving some fraction of his income to the inactive elderly in his family. Moreover, the system can be an *individualized* system, where parents internalize the effect of their fertility choices on retirement benefits, or, alternatively, a *collective* system, where parents can free-ride on the system (either PAYG or familial) to get large pensions without supporting the cost of fertility.

As we shall see, relaxing the perfect capital market assumption and replacing it by a pension system is not neutral at all for the design of the optimal family allowances in the context of endogenous childbearing ages. The reason is that, once a (formal or informal) pension system is introduced, there might be, for parents, an incentive in having *early* children. The reasons are twofold. First, children, if born a longer time ago, can, under increasing earnings profiles, provide higher pensions to their parents. Second, and more importantly, when parents with early children are retired, they can rely not on one, but on *two,* generations of workers: their own children *and* their grandchildren.

To illustrate this, we will, first, for the sake of presentation, study fertility choices under an individualized pension system, when there is only one type of agent. We will then reintroduce heterogeneity later in this section.

Optimal Policy with One Type

In that context, the problem faced by an individual amounts to choosing savings s_a, early children n_a, and contribution rate γ to maximize:[30]

$$U = u[(1-\gamma)w - e_a n_a - s_a] + u[(1-\gamma)wh(n_a) - (1-n_a)e_\ell + s_a]$$

$$+ u[\gamma(w(1-n_a) + wh(\bar{n}_a)n_a + wn_a\bar{n}_a)]$$

where γ is the fraction of earnings that is paid to the elderly, while $\bar{n}_a$ means that the individual expects his early children to have the same number of early children as himself (i.e., n_a), without having any control on it. Note that the earnings' basis available for the funding of the elderly's pensions includes three terms: The first two terms (i.e., $w(1-n_a)$ and $wh(\bar{n}_a)n_a$) concern the incomes from the late and the early children, whereas the third term, that is, $wn_a\bar{n}_a$, concerns the income of grandchildren, whose number is $n_a\bar{n}_a$ (as each n_a early child had also $\bar{n}_a$ early children once an adult). The FOCs are

$$s_a : u'(c) = u'(d) \tag{10}$$

[30] To keep things simple, we assume that savings between the first and the second periods is possible.

$$\gamma : u'(c) = u'(b)\frac{1 - n_a + h(\bar{n}_a)n_a + n_a\bar{n}_a}{1 + h(n_a)} \tag{11}$$

$$n_a : u'(c)\left[e_a - (1-\gamma)wh'(n_a) - e_\ell\right] = u'(b)\gamma(-w + wh(\bar{n}_a) + w\bar{n}_a) \tag{12}$$

In a world of identical individuals, these three conditions lead to the optimal decision concerning s_a, n_a, and γ as long as each individual considers that the pension scheme operates at the level of the family. It is interesting to observe that such a PAYG scheme creates a distortion in favor of early childbearing, relative to a setting of a fully funded pension system. The reason is that early children have higher earnings when their parents retire, and that they have themselves some working children, unlike the parents' late children.

If, instead of that individualized pension system, the PAYG scheme were collective, each agent would not, when making his fertility timing choice, perceive his impact on pension benefits. Hence, agents would choose a *lower* number of early births. To see this, note that, under collective pensions, conditions (Equation 10) and (Equation 11) would be unchanged, but conditions (Equation 12) would become

$$u'(c)\left[e_a - (1-\gamma)wh'(n_a) - e_\ell\right] = 0 \tag{12$'$}$$

Given that the RHS of (Equation 12) includes an additional positive term in comparison to (Equation 12), agents have, under a collective pensions system, *fewer* early children—and thus more late children—than under an individualized system.

Therefore, in order to induce the optimal fertility behavior under a collective system, one would need a Pigouvian subsidy σ_a on early children, such that

$$\sigma_a = w\gamma(h(\bar{n}_a) + \bar{n}_a - 1)\frac{u'(b)}{u'(c)} \tag{13}$$

Subsidizing children is standard in economies with PAYG pensions system and endogenous fertility (see Gahvari 2009): Individuals do not internalize the effect that their fertility choice can have on pension benefits, and a subsidy can induce parents to have the right number of children. The specificity of the present approach is that the subsidization does not concern all births (which are here constant, since $n_{ai} + n_{\ell i} = 1$), but only children born from young parents. The reason why such a differentiated subsidization of children is needed is twofold.

First, under $h(\bar{n}_a) > 1$, the *levels* of earnings of contributors born from young parents are larger than the earnings of contributors born from older parents, since only the former can benefit from increasing productivity along the career. Second, the *number* of contributors is strongly affected by early fertility decisions, through the number of grandchildren. On the other hand, in the case of late parenthood, grandchildren cannot fund the pension system, and so the fertility externality is less sizeable.

Those arguments justify a differentiated fiscal treatment of early and late parenthoods, under the form of children allowances that depend on the age of parents: $\sigma_a > 0 = \sigma_\ell$.[31] Thus, even if one assumes a perfect homogeneity of the population in terms of career opportunity and

[31] Here again, governments may be reluctant to treat different births differently, depending on the age of parents. However, if the goal of the policy is to internalize externalities, the two reasons mentioned above legitimate a subsidization that encourages early births and discourages late births, on the ground of their distinct external effects for pensions funding.

earnings profile, there is already a strong argument not only for subsidizing children, but also for subsidizing them differently depending on the age of their parents, in the sense that only early births should be subsidized. The justification is that fertility externalities induced by early and late fertility are not of the same magnitudes, inviting a differentiated fiscal treatment.[32]

Optimal Policy under Two Types

Let us now turn back to the case when individuals differ in their future earnings opportunities (i.e., the function $h_i(n_{ai})$). Given that children allowances differentiated according to the age of parents have been already studied above, we will abstract here from these and assume that the available fiscal instruments are a uniform child benefit of rate σ, a payroll tax τ, and flat rate PAYG pension p.

When agents smooth their consumption over the active period, the lifetime welfare for an agent of type i is

$$U_i = 2u\left(\frac{(1-\tau)w(1+h_i(n_{ai})) - (1-\sigma)(n_{ai}(e_a - e_\ell) - e_\ell)}{2}\right) + u(p)$$

where the pension benefit is, under a balanced government budget, equal to

$$p = \sum \pi_i \left[\tau w\left(1 - n_{ai} + h_i(n_{ai})n_{ai} + n_{ai}^2\right) - \sigma(e_a n_{ai} + e_\ell(1 - n_{ai}))\right]$$

Each individual $i \in \{1,2\}$ chooses n_{ai} without seeing the effect his choice has on his future pension benefits. His choice of n_{ai} is determined by the FOC:

$$u'(c_i)\left[w(1-\tau)h_i'(n_{ai}) - (1-\sigma)(e_a - e_\ell)\right] = 0$$

That condition does not yield the socially optimal level of early children, since this neglects the impact of early fertility on old-age pensions p through its impact on the budget constraint of the economy. That point is similar to the one made in the one-type reduced model studied in section 5.

In order to derive the optimal policy, let us now rewrite the social planner's problem as the Lagrangian:

$$\mathcal{L} = \sum \pi_i \left\{ U_i - \mu\left[p + \sigma(e_a n_{ai} + e_\ell(1 - n_{ai})) - \tau w\left(1 - n_{ai} + h_i(n_{ai})n_{ai} + n_{ai}^2\right)\right] \right\}$$

where μ is the Lagrange multiplier associated with the government's budget constraint. Using the operator $E(\cdot)$, we have the FOCs:

$$\frac{\partial \mathcal{L}}{\partial p} = E\left[u'(p) - \mu\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left(u'(c)w(1+h(n_a))\right) + \mu E\left(w\left(1 - n_a + h(n_a)n_a + n_a^2\right)\right) - \mu E\left(B\frac{\partial n_a}{\partial \tau}\right) = 0$$

[32] Note, however, that we focus here, as above, on a static economy, and that the study of the optimal family allowances could give significantly different results in a dynamic economy. In a dynamic setting, the impact of fertility timing on the existence, at the laissez-faire, of a stable stationary equilibrium is studied in Pestieau and Ponthiere (2013).

$$\frac{\partial \mathcal{L}}{\partial \sigma} = E\left[u'(c)[n_a(e_a - e_\ell) - e_\ell]\right] - \mu E((e_a n_a + e_\ell(1+n_a))) - \mu E\left(B\frac{\partial n_a}{\partial \sigma}\right) = 0$$

where $B \equiv \sigma(e_a - e_\ell) - \tau w\left(-1 + h(n_a) + n_a h'(n_a) + 2n_a\right)$ denotes the effect of n_a on the budget constraint. A marginal increase in early fertility n_a has four revenue effects: (i) a revenue increase thanks to lower child cost (i.e., as $e_a < e_\ell$); (ii) a revenue loss, since $h'(n_a) < 0$ (due to the productivity loss induced by early childbearing); (iii) another revenue gain as $h(n_a) > 1$ (thanks to the larger contributions of the children once older and more productive); (iv) and another revenue gain through grandchildren (which is linear in the number of early children). A sufficient condition for $B < 0$ is $-\frac{n_a h'(n_a)}{h(n_a)} < 1 + \frac{2n_a - 1}{h(n_a)}$. Substituting for μ in the second and third FOCs, we obtain

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau} &= -cov\left(u'(x), 1 + h(n_a)\right) + E\left[u'(p) - v'(x)\right]E(1 + h(n_a)) \\
&\quad + u'(p)E\left(1 - n_a + h(n_a)n_a + n_a^2\right) - \frac{E\left(u'(p)\right)}{w}E\left(B\frac{\partial n_a}{\partial \tau}\right) \\
\frac{\partial \mathcal{L}}{\partial \sigma} &= (e_a - e_\ell)cov\left[u'(x)\right] - E\left(u'(b) - u'(x)\right)E(n_a) \\
&\quad - E\left(u'(b)\right)E\left(B\frac{\partial n_a}{\partial \sigma}\right)
\end{aligned}$$

Together, those optimality conditions characterize the optimal values of p, τ, and σ. Note, however, that the joint study of the optimal levels of those three tax instruments would be quite laborious, since their values are all linked (through the government's budget constraint). Therefore, for the sake of presentation, we will, as above, consider successively the pairs (τ, p) and (σ, p), while keeping, each time, the other fiscal tool set to zero.

Let us start with the pair (τ, p), that is, only a payroll tax and a pension are available instruments, σ being fixed to 0. Equalizing the above FOCs to 0 yields

$$\tau = \frac{-cov\left(u'(x), 1 + h(n_a)\right) + E\left(u'(p) - u'(c)\right)E(1 + h(n_a)) + u'(p)E\left(1 - n_a + h(n_a) + n_a^2\right)}{\frac{E\left(u'(p)\right)}{w}E\left(\frac{\partial n_a}{\partial \tau}\left(-1 + n_a h'(n_a) + h(n_a) + 2n_a\right)\right)} \tag{14}$$

That formula defines the optimal level of the payroll tax τ when the proceeds are used to fund a pension system only. The numerator consists of three terms that are all expected to be positive (assuming that $E\left(u'(p) - u'(c)\right) > 0$). Hence those three terms push toward more redistribution through the income tax. The denominator gives the effect of the tax on public revenue. The sign of this efficiency effect depends on the impact of earnings taxation on aggregate early fertility (i.e., $E(\partial n_a / \partial \tau)$), and on the impact of aggregate early fertility on the government's budget constraint. If the impact of τ on the aggregate early fertility is negative (i.e., $E(\partial n_a / \partial \tau) < 0$), and if the impact of early fertility on fiscal revenues is also negative $\left(\text{i.e.,if } \frac{n_a h'(n_a)}{h(n_a)} < -1 - \frac{2n_a - 1}{h(n_a)}\right)$, the efficiency effect is also positive. As a consequence, the optimal tax is most likely to be positive: $\tau > 0$.

Turning now to the pair (σ, p), under the assumption $\tau = 0$, equalizing the above FOCs to 0 yields[33]

$$\sigma = \frac{(e_\ell - e_a)cov(u'(x), n_a) - E(u'(p) - u'(c))}{(e_\ell - e_a)Eu'(p)E\dfrac{\partial n_a}{\partial \sigma}}. \tag{15}$$

Assume that type-1 have more early children than type-2. Assume further that child allowances have a positive effect on n_a (i.e., $E(\partial n_a/\partial\sigma) > 0$). Then the optimal σ depends on the relative strength of the two terms of the numerator. On the one hand, the positive covariance term pushes for a high σ and a low p. If the poor tend to have more early children, the equity term pushes toward a less negative σ, as a way to achieve indirect redistribution toward type-1 agents. That effect is increasing in the cost differential between late and early childbearing, $e_\ell - e_a$. On the other hand, the positive term $Eu'(p) - Eu'(c)$ pushes toward a more negative σ aimed at financing the pension scheme. Hence, the level of the optimal σ depends on the strengths of those two terms. Finally, the denominator gives the impact of σ on fiscal revenues, through its effect on the timing of births.[34]

In sum, once we reintroduce heterogeneity in terms of earnings profile, the optimal family policy requires an arbitrage between equity and efficiency. Given that agents with lower future earnings tend to have children earlier, the subsidization of fertility is supported by equity concerns. The cost gap between late and early fertility tends to reinforce that equity case for children allowances. However, a higher cost gap between late and early childbearing pushes, from an efficiency point of view, toward lower family allowances. Hence, the observed falling cost gap between late and early childbearing has an ambiguous effect on the optimal family policy.

6. Extensions

This section considers several extensions of the baseline model. First, an alternative modeling of parental preferences, where children are durable goods; second, the introduction of an education decision; third, the addition of a third working period; and, fourth, the introduction of varying total fertility.[35]

Children as Durable Goods

Throughout the article; parental lifetime welfare has taken a simple form, where there is a perfect substitutability between early and late births. That assumption, although analytically convenient, neglects the status of children as durable goods, which can be "consumed" during more than a single period. In the present context, the idea of durable goods amounts to introducing a *qualitative* difference between children born at different points in time: their differentiated capacity to increase their parents' welfare.

Let us now assume that children are durable goods, so that parental lifetime welfare now depends on the duration of coexistence with the children:

[33] Note that fixing $\tau = 0$ does not cause any problem of funding for the government, since σ or p can take a negative value, so as to bring the government's budget equilibrium. Given that p is the only resource in retirement, σ has to be negative. In other words, pension benefits are financed by a tax on children, which is in the spirit of a PAYG scheme.

[34] Indeed the timing of births is not neutral for the government's budget, as the cost of children depends on the timing of births, since $e_a < e_\ell$.

[35] Because of space constraints, we focus, for each extension, on the comparison of the laissez-faire and the first-best with the corresponding ones under the baseline model, but do not formally explore the design of the optimal second-best policy.

$$U_i = u(c_i) + u(d_i) + u(b_i) + 3v(n_{ai}) + 2v(1 - n_{ai}) \tag{16}$$

The intuition behind expression (Equation 16) is that the n_{ai} children born when parents are in period 1 will be "enjoyed" during three periods, while the $n_{\ell i}$ children born later on will only be "enjoyed" during two periods. The FOCs become

$$u'(c_i) = u'(d_i) = u'(b_i)$$

$$u'(c_i)e_a - u'(d_i)wh_i'(n_{ai}) = u'(d_i)e_\ell + 3v'(n_{ai}) - 2v'(1 - n_{ai})$$

In the FOC for optimal fertility, there are now two additional terms on the RHS: On the one hand, a larger number of early children generates a welfare gain during the *entire* lifecycle (second term of the RHS), but, at the same time, it reduces the welfare gain associated with late parenthood (third term of the RHS). One can rewrite that FOC as

$$e_\ell - e_a = -wh_i'(n_{ai}) - \frac{3v'(n_{ai}) - 2v'(1 - n_{ai})}{u'(c_i)}$$

Comparing this FOC with the one in the baseline model $e_\ell - e_a = -wh_i'(n_{ai})$ shows that a central difference is that the optimal fertility timing depends here on the level of consumption, through $u'(c_i)$. Given $u''(c_i) < 0$, viewing children as durable goods especially affects our results when consumption is high.

If one focuses only on the interior solution, the comparison of the fertility profile with the one under the baseline model can be carried out as follows. Given that $3v''(n_{ai}) + 2v''(1 - n_{ai}) < 0$ and that $3v'(0) - 2v'(1) > 0$, the additional term is negative at $n_{ai} = 0$ and increasing in n_{ai}. Therefore the addition of that term has an ambiguous effect on n_{ai}.[36] Additional restrictions on the function $v(\cdot)$ are needed to know the effect with certainty. If, for instance, $3v'(n_{ai}) - 2v'(1 - n_{ai}) > 0$ for any $0 < n_{ai} < 1$, then the added term is negative, so that treating children as durable goods raises the number of early births n_{ai}.

Regarding the comparison of the two types, the additional term in the RHS of the FOC is, *ceteris paribus*, larger, in absolute value, for type-2 agents than for type-1 agents when type-2 agents have a higher consumption.[37] Hence taking children as durable goods affects the fertility profile of type-2 more significantly than the one of type 1. If $3v'(n_{ai}) - 2v'(1 - n_{ai}) > 0$ for any n_{ai}, n_{ai} is increased for all agents, and to a larger extent for type-2 agents. But this modeling does not, in general, lead to an inversion of the fertility profiles across types. If the differential in terms of earnings sensitivity is sufficiently large, we still have $n_{a1} > n_{a2}$ and $n_{\ell 1} < n_{\ell 2}$, as in the baseline model.

The first-best optimum is obtained by maximizing the following Lagrangian:

$$\sum \pi_i (U_i - \mu[c_i + d_i + b_i + n_{ai}e_a + (1 - n_{ai})e_\ell - w(1 + h_i(n_{ai}))])$$

where μ denotes the Lagrange multiplier associated with the resource constraint of the economy. The FOCs are

[36] The reason is that it reduces the RHS for low levels of n_{ai}, and it raises the RHS for high ones, so that the chosen n_{ai} may be higher or lower in comparison to the baseline model.

[37] This case is plausible, given their higher lifetime earnings (whatever their fertility is).

$$u'(c_i) = u'(d_i) = u'(b_i) = \mu$$

$$e_\ell - e_a = -wh_i'(n_{ai}) - \frac{3v'(n_{ai}) - 2v'(1-n_{ai})}{u'(c_i)}$$

The first-best involves, as in the baseline model, an equalization of consumptions for all agents and all periods. But an important difference is that the socially optimal fertility timing is now dependent on consumption profiles. Therefore, the equalization of consumption profiles across all individuals has here some impact on optimal fertility profiles, unlike in the baseline model. Type-1 agents have a higher consumption profile than at the laissez-faire, whereas the opposite holds for type-2. Assuming $3v'(n_{ai}) - 2v'(1-n_{ai}) > 0$ for any $0 < n_{ai} < 1$, the optimal n_{a1} is higher than at the laissez-faire, whereas the optimal n_{a2} is lower.

The first-best can be decentralized by means of lump-sum transfers, from type-2 toward type-1 agents. Those transfers, by equalizing $u'(c_i)$ across agents, also affect the fertility timing within each group, in the direction mentioned above. When such transfers are not available, the decentralization of the second-best can be studied, by means of linear fiscal tools, in line with section 4. Under that alternative modeling, family allowances can still be justified when early children remain mostly consumed by type-1 agents, provided such allowances increase early fertility. Depending on whether the rise in n_{ai} is larger for type-2 or for type-1 agents, the optimal level of family allowances may be reduced or increased in comparison to the baseline model. Thus viewing children as durable goods can—at least quantitatively—affect the optimal family policy.

Education

So far, our study has relied on a model where agents differ in terms of their earnings profiles, because of unexplained reasons. Such a modeling is a reduced form of the real world, where wages differentials are not exogenous, but depend on past education choices. This subsection explores, in a modified framework, the interactions between education choices and fertility choices.

For that purpose, let us assume that individuals of type i can study a fraction k_i of the first period, and thus work a fraction $1-k_i$ of the first period. That education brings a return in period 2, where the total labor earnings is now

$$wh_i(n_{ai})B_i(k_i)$$

where the return from education is $B_i(k_i) = 1 + k_i^{\alpha_i}$, with $0 < \alpha_1 < \alpha_2 < 1$, that is, type-2 agents benefit from a higher elasticity of future earnings with respect to education.[38] The agent's problem is now

$$\max_{c_i, d_i, b_i, n_{ai}, k_i} u(c_i) + u(d_i) + u(b_i) \text{s.t.} \begin{cases} w(1-k_i) + wh_i(n_{ai})B_i(k_i) \\ = c_i + n_{ai}e_a + d_i + (1-n_{ai})e_\ell + b_i \end{cases}$$

The FOCs describing an interior optimum are

[38] Note that $B_i(0) = 1$, in conformity with the baseline model.

$$u'(c_i) = u'(d_i) = u'(b_i)$$

$$e_\ell - e_a = -wh_i'(n_{ai})B_i(k_i)$$

$$1 = h_i(n_{ai})B_i'(k_i)$$

The education choice is described by the third FOC: The optimal education equalizes, at the margin, the welfare loss due to education (LHS), with the welfare gain from education (RHS), which is decreasing in n_{ai}. Note also that, since $B_i(k_i) > 1$, introducing education tends to increase, *ceteris paribus*, the marginal cost from having early children (RHS of the second FOC), implying a lower n_{ai} and a higher $n_{\ell i}$, in conformity with the literature emphasizing the role of education as a cause of births postponement.

Regarding the comparison of the two types, education levels are given by[39]

$$h_1(n_{a1})\alpha_1 k_1^{\alpha_1 - 1} = 1 = h_2(n_{a2})\alpha_2 k_2^{\alpha_2 - 1}$$

which yields $k_1 = (h_1(n_{a1})\alpha_1)^{\frac{1}{1-\alpha_1}}$ and $k_2 = (h_2(n_{a2})\alpha_2)^{\frac{1}{1-\alpha_2}}$. Substituting for education in the FOC for n_{ai} yields

$$e_\ell - e_a = -wh_i'(n_{ai})\left(1 + (h_i(n_{ai})\alpha_i)^{\frac{\alpha_i}{1-\alpha_i}}\right)$$

As $h_1(n_{ai}) < h_2(n_{ai})$, $|h_1'(n_{ai})| < |h_2'(n_{ai})|$ and $\alpha_1 < \alpha_2$, type-2 agents choose, in comparison to type-1 agents, a lower number of early children and a higher education level, because of larger gains from education for them.

The social planner's problem can be written as the Lagrangian:

$$\sum \pi_i(U_i - \mu[c_i + d_i + b_i + n_{ai}e_a + (1 - n_{ai})e_\ell - w(1 - k_i + h_i(n_{ai})B_i(k_i))])$$

where μ denotes the Lagrange multiplier associated with the resource constraint. The FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i) = \mu$$

$$e_\ell - e_a = -wh_i'(n_{ai})B_i(k_i)$$

$$1 = h_i(n_{ai})B_i'(k_i)$$

The first-best involves, here again, an equal consumption across agents and periods. As in the baseline model, the first-best can be decentralized by means of lump-sum transfers, from type-2 toward type-1 agents. Those transfers do not affect fertility timing and education decisions. If the only available fiscal tools are a linear tax on earnings and a linear subsidy on births, the decentralization of the second-best can be achieved by means of a positive subsidy on births (provided it raises early fertility), on the same redistributive grounds as under the baseline model. Hence our results are robust to introducing education.[40]

[39] For the sake of presentation, we focus here only on interior solutions.

[40] Note, however, that if one adds a tax on education as an alternative fiscal tool, then the fact that type-2 agents tend to choose a higher education can justify, in a second-best world, a positive tax on education, on the same redistributive grounds.

A Third Working Period

The baseline model exhibits an asymmetry between early births and late births: Whereas early births affect future earnings opportunities, late births did not have such an effect. To evaluate the robustness of our results to relaxing that asymmetry, let us now consider a four-period model. As above, periods 1 and 2 are periods of labor, consumption, and childbearing, and the last period (now period 4) is a period of retirement. But we add an intermediate period of labor (period 3), during which labor earnings depend negatively on fertility at the previous period. Thus the wage profile is now w in period 1, $wh_i(n_{ai})$ in period 2, and $[wh_i(n_{ai})]H_i(n_{i\ell})$ in period 3, where the function $H_i(n_{i\ell})$ exhibits $H_i'(\cdot)<0$, $H_i''(\cdot)<0$, $H_1(n_{i\ell})<H_2(n_{i\ell})$ and $|H_1'(n_{i\ell})|<|H_2'(n_{i\ell})|$ for an equal $n_{i\ell}$.

For an agent of type $i \in \{1,2\}$, the problem becomes

$$\max_{c_i,d_i,b_i,g_i,n_{ai}} u(c_i)+u(d_i)+u(b_i)+u(g_i) \text{s.t.} \begin{cases} w+wh_i(n_{ai})+wh_i(n_{ai})H_i(1-n_{ai}) \\ =c_i+n_{ai}e_a+d_i+(1-n_{ai})e_\ell+b_i+g_i \end{cases}$$

where g_i denotes consumption in period 4. The FOCs describing an interior optimum are

$$u'(c_i)=u'(d_i)=u'(b_i)=u'(g_i)$$

$$e_\ell-e_a=-wh_i'(n_{ai})-wh_i'(n_{ai})H_i(1-n_{ai})+wh_i(n_{ai})H_i'(1-n_{ai})$$

In the FOC for n_{ai}, we now have two additional terms on the RHS. The term $-wh_i'(n_{ai})H_i(1-n_{ai})$ gives us the marginal cost of early children in terms of reduced earnings in period 3. That term is of positive sign. The second additional term, $wh_i(n_{ai})H_i'(1-n_{ai})$, is of negative sign. It consists of the marginal cost due to postponing births, that is, reduced earnings in period 3. When the first additional term dominates (resp. is dominated by) the second one, the marginal cost of having early children is increased (resp. reduced) in comparison to the baseline model, which pushes, *ceteris paribus*, toward fewer (resp. more) early births and more (resp. fewer) late births.[41]

When comparing type-1 and type-2 agents, we have

$$e_\ell-e_a=-wh_1'(n_{a1})-wh_1'(n_{a1})H_1(1-n_{a1})+wh_1(n_{a1})H_1'(1-n_{a1})$$

$$e_\ell-e_a=-wh_2'(n_{a2})-wh_2'(n_{a2})H_2(1-n_{a2})+wh_2(n_{a2})H_2'(1-n_{a2})$$

For an equal n_{ai}, the first and the second terms of the RHS are larger for type-2 than for type-1, while the third term is smaller (i.e., more negative). Therefore, unlike in the baseline model, the comparison of fertility profiles gives here ambiguous results, and additional assumptions are needed to draw some conclusions. If, for instance, we have, for any equal n_{ai}: $-wh_1'(n_{ai})H_1(1-n_{ai})+wh_1(n_{ai})H_1'(1-n_{ai}) < -wh_2'(n_{ai})H_2(1-n_{ai})+wh_2(n_{ai})H_2'(1-n_{ai})$, then $n_{a1}>n_{a2}$ and $n_{\ell 1}<n_{\ell 2}$, so that the qualitative results of the baseline model would still prevail, but the differential between the two fertility profiles may vary. If $H_i'(1-n_{ai})\approx 0$, the early fertility gap between the two types of agents would be even *larger* than in the baseline model, because of the cumulative effect of early births on earnings during two working periods (instead

[41] For the sake of presentation, we assume here an interior solution.

of one). It would only be under a highly negative $H_i'(1-n_{ai})$ that the results of the baseline model would be inverted. In general, we still have $n_{a1} > n_{a2}$ and $n_{\ell 1} < n_{\ell 2}$. The differential between the two fertility profiles may even be larger here, under large cumulative earnings loss due to early fertility.

The social planner's problem can be written by means of the Lagrangian:

$$\sum \pi_i(U_i - \mu[c_i + d_i + b_i + g_i + n_{ai}e_a + (1-n_{ai})e_\ell - w(1 + h_i(n_{ai}) + h_i(n_{ai})H(1-n_{ai}))])$$

where μ denotes the Lagrange multiplier associated with the resource constraint. The FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i) = u'(g_i) = \mu$$

$$e_\ell - e_a = -wh_i'(n_{ai}) - wh_i'(n_{ai})H_i(1-n_{ai}) + wh_i(n_{ai})H_i'(1-n_{ai})$$

Thus the first-best optimum involves, here again, an equalization of consumptions across agents and periods. As in the baseline model, the first-best social optimum can be decentralized by means of lump-sum transfers, from type-2 agents toward type-1 agents. At the second-best, and focusing on linear fiscal instruments (as in section 4), positive family allowances are still justified here, to the extent that these raise the consumption of a good that is mostly consumed by low earnings profile agents: early children. Thus our results are, at least qualitatively, robust to the introduction of a third working period.

Varying Total Fertility

To conclude this section, let us now consider an alternative fertility decision, where parents choose not only the timing of births, but, also, the total number of children $n_{ai} + n_{\ell i}$, which is no longer fixed to unity. Under that alternative environment, the problem of an individual of type $i \in \{1,2\}$ is

$$\max_{c_i, d_i, b_i, n_{ai}, n_{\ell i}} u(c_i) + u(d_i) + u(b_i) + v(n_{ai} + n_{\ell i}) \text{s.t.} \begin{cases} w + wh_i(n_{ai}) \\ = c_i + n_{ai}e_a + d_i + n_{\ell i}e_\ell + b_i \end{cases}$$

The FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i)$$

$$-u'(c_i)e_a + u'(d_i)wh_i'(n_{ai}) + v'(n_{ai} + n_{\ell i}) = 0$$

$$-u'(d_i)e_\ell + v'(n_{ai} + n_{\ell i}) = 0$$

Substituting the third FOC in the second one and simplifying yields[42]

$$e_\ell - e_a = -wh_i'(n_{ai})$$

[42] For simplicity, we focus here only on the interior optimum, with $n_{ai} > 0$ and $n_{i\ell} > 0$.

Regarding the interior optimal $n_{\ell i}$, this is determined by the condition

$$u'(d_i)e_\ell = v'(n_{ai} + n_{\ell i})$$

Let us compare the two types of agents, while focusing on the interior optimum. Given that $|h_2'(n_{ai})| > |h_1'(n_{ai})|$, it follows from $e_\ell - e_a = -wh_i'(n_{ai})$ that $n_{a1} > n_{a2}$. Regarding late births, we know from the above condition that type-2 agents, who benefit from larger consumption possibilities, exhibit a lower $u'(d_i)$ than type-1 agents and, thus, a lower LHS in the FOC for $n_{\ell i}$. Regarding the RHS, we have $n_{a1} > n_{a2}$, which leads to $v'(n_{a1} + n_{\ell i}) < v'(n_{a2} + n_{\ell i})$ for an equal $n_{\ell i}$. Given that the LHS is smaller for type-2, and that the RHS is larger for type-2 *ceteris paribus*, it must be the case that $n_{\ell 1} < n_{\ell 2}$. In total, given that $n_{a1} > n_{a2}$ and $n_{\ell 1} < n_{\ell 2}$, it is not obvious to see which type of agent has the largest *total* fertility. But we know for sure that the fertility profile differs across types. The previous results are thus robust to relaxing $n_{ai} + n_{\ell i} = 1$.

The first-best optimum is obtained by maximizing the Lagrangian:

$$\sum \pi_i (U_i - \mu[c_i + d_i + b_i + n_{ai}e_a + n_{\ell i}e_\ell - w(1 + h_i(n_{ai}))])$$

where μ denotes the Lagrange multiplier associated with the resource constraint. Assuming interior solutions, the FOCs are

$$u'(c_i) = u'(d_i) = u'(b_i) = \mu$$

$$v'(n_{ai} + n_{\ell i}) = \mu\left[e_a - wh_i'(n_{ai})\right]$$

$$v'(n_{ai} + n_{\ell i}) = \mu e_\ell$$

The utilitarian social optimum involves an equalization of all consumptions across individuals of all types, and across periods. Regarding optimal fertility, substituting the third FOC in the second one yields

$$e_\ell - e_a = -wh_i'(n_{ai})$$

That equality characterizes the optimal n_{ai}. It is the same as in the laissez-faire. This is not true for the FOC for $n_{\ell i}$:

$$v'(n_{ai} + n_{\ell i}) = u'(d_i)e_\ell$$

since the equalization of consumptions affects $u'(d_i)$. As a consequence, and still focusing on an interior social optimum, we obtain that type-1 agents enjoy, at the social optimum, a higher consumption profile than at the laissez-faire, and the opposite is true for type-2 agents. Regarding fertility, whereas both types of agents should have, at the social optimum, the same level of n_{ai} as in the laissez-faire, type-2 agents should have *fewer* late children $n_{\ell 2}$ than at the laissez-faire, whereas type-1 agents should have a higher $n_{\ell 1}$ than at the laissez-faire. Therefore the *total* fertility $n_{ai} + n_{\ell i}$ of type-1 is *higher* than at the laissez-faire, whereas this is reduced for type-2.

The decentralization of the first-best can be achieved by means of lump-sum transfers from type-2 toward type-1 agents, so as to equalize all consumptions. Such transfers, by reducing the marginal utility of consumption for type-1 agents, raise $n_{\ell 1}$ above its laissez-faire level. Inversely, those transfers tend to reduce $n_{\ell 2}$, but without affecting early fertility. At the second-best, and focusing on linear fiscal instruments, the decentralization of the optimum involves positive family allowances provided these contribute to raise n_{a1}. The underlying intuition is that such family allowances can be used a way to achieve redistribution toward the low earnings profile individuals. Therefore our results are globally robust to relaxing the fixed total fertility assumption.

7. Conclusion

As is widely acknowledged among demographers, European economies have been characterized, during the last three decades, by a tendency toward the postponement of fertility. Through its consequences on the economy as a whole, that demographic trend raises various challenges to policymakers.

The present article aimed at examining the design of the optimal family policy in an economy where the timing of births is chosen by individuals. For that purpose, we developed a three-period model with replacement fertility, where fertility can take place during two—instead of one—periods. The population was partitioned in two groups, who differ in the level and slope of their earnings profile, which is decreasing in early fertility. That model can replicate the observed differential of fertility timing across earnings groups.

Regarding the optimal public intervention, we firstly considered the first-best utilitarian optimum, and showed that this can be decentralized by means of lump-sum transfers from high-earnings toward low-earnings agents. Then, considering an economy where only linear uniform policy instruments are available, we showed that the optimal second-best policy involves a subsidy on early children, to the extent that it fosters early parenthood. Indeed, given that early parenthood concerns generally parents with low earnings opportunities, family allowances are justified on redistributive grounds. Children allowances differentiated according to the parent's age would, if available, achieve that redistribution even better. Turning then to the optimal nonlinear policy under asymmetric information, we showed that only early children from low earnings profile parents should be subsidized, to solve the self-selection problem.

We also investigated the impact of childbearing ages on the funding of social security, by introducing, in our setup, a pension system. We showed, in a framework without heterogeneity, that, under a collective pension system, the fertility externalities related to early and late parenthoods differ significantly, inviting a positive Pigouvian subsidy for early children only. The underlying reason is both qualitative (older children, under increasing productivity, contribute more than younger, late children) and quantitative (early parents can, once retired, benefit from the contributions of their grandchildren). Turning back to the 2-type case, we showed that the social planner faces a trade-off between efficiency (favoring late childbearing) and equity (helping early childbearing).

Finally, we also explored the robustness of our results to four distinct variations of the model: first, parental preferences regarding children as durable consumption goods; second, the introduction of education; third, the addition of a third working period; fourth, relaxing the replacement fertility assumption. Those extensions do not, in general, affect our results from a qualitative perspective, since the fertility timing differential between agents is preserved. The same robustness holds when characterizing the social optimum, and when studying its decentralization under various sets of fiscal instruments.

In sum, the present study highlights major challenges faced by policymakers in the context of endogenous childbearing ages. Endogenous childbearing age affects fundamental aspects of the optimal policy design. As far as redistribution is concerned, the observed correlation between the heterogeneity in terms of fertility age pattern and in terms of career opportunities makes children allowances differentiated in terms of the age of parent socially desirable. Moreover, the internalization of fertility externalities in the context of pensions funding would also require children allowances differentiated according to the age of parents. Hence, even though policy debates are usually concerned with changes in *total* fertility, the *timing* of births is far from a detail for the design of the optimal family policy and, as such, will require more attention in the future.

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