Damage Detection in Structures Based on Principal Component Analysis of Forced Harmonic Responses

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Outline

- **Principal Component Analysis**
  - Definition and mathematical formulation
  - Computation of the POMs
  - Interpretation of the POMs
  - Detection of damage
  - Localization of damage
- **Example of a truss structure**
- **Conclusion**

Principal Component Analysis (PCA)

Principal component analysis is a multi-variate statistical method.

Aim: to obtain a compact representation of the data.

Principal Component Analysis = Proper Orthogonal Decomposition

PCA has been successfully applied for operational modal analysis (OMA) of civil engineering structures including temperature effects [1-2].

1. A-M Yan, G. Kerschen, P. De Boe, J.-C. Golinval, Structural damage diagnosis under varying environmental conditions - Part I: A linear analysis, Mechanical Systems & Signal Processing, Academic Press Ltd Elsevier Science Ltd, (2005) 19(4), 847—864.
2. V.H. Nguyen, J. Mahowald, J.-C. Golinval, S. Maas, Damage Detection in Civil Engineering Structure Considering Temperature Effect, International Modal Analysis Conference (IMAC) XXXII, Society of Experimental Mechanics, Orlando (FL), 3-6 February 2014.

Principal Component Analysis (PCA)

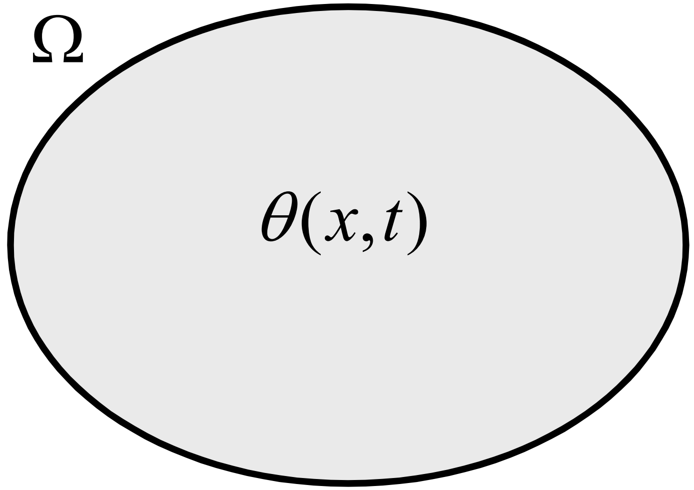
Mathematical formulation

Let $\theta(x,t)$ be a random field on a domain Ω

mean

$$\theta(x,t) = \mu(x) + \vartheta(x,t)$$

time varying part



At time t_k , the system displays a snapshot $\vartheta^k(x) = \vartheta(x,t_k)$

Principal Component Analysis (PCA)

The POD aims at obtaining the most characteristic structure $\phi(x)$ of an ensemble of snapshots i.e.

$$\text{Maximize } \left\langle \left| \left(\vartheta^k , \phi \right) \right|^2 \right\rangle \quad \text{with} \quad \left\| \phi \right\|^2 = 1$$

where $\left(f , g \right) = \int_{\Omega} f(x)\, g(x)\, d\Omega$

$\langle \cdot \rangle$ denotes the averaging operation

$\| \cdot \|$ denotes the norm

It can be shown that the problem reduces to the following integral eigenvalue problem

averaged auto-correlation function

$$\int_{\Omega} \left\langle \vartheta^k(x)\, \vartheta^k(x') \right\rangle \phi(x')\, dx' = \lambda\, \phi(x)$$

Principal Component Analysis (PCA)

Thus the solution of the optimization problem

$$\text{Maximize } \left\langle \left| \left(\vartheta^k, \phi \right) \right|^2 \right\rangle \quad \text{with} \quad \left\| \phi \right\|^2 = 1$$

is given by the orthogonal eigenfunctions $\phi_i(x)$ of the integral equation

$$\int_\Omega \left\langle \vartheta^k(x)\, \vartheta^k(x') \right\rangle \phi(x')\, dx' = \lambda\, \phi(x)$$

$\phi_i(x)$ are called the proper orthogonal modes (POM)

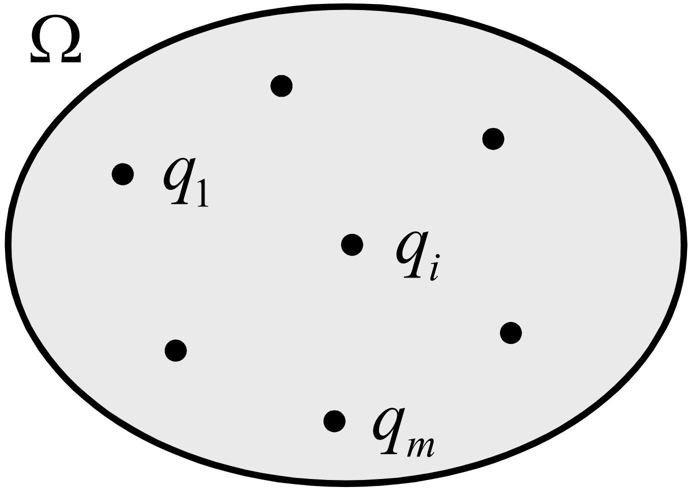
λ_i are called the proper orthogonal values (POV)

and we have $\vartheta(x,t) = \sum_{i=1}^{\infty} a_i(t)\, \phi_i(x)$ where $a_i(t) = \left(\vartheta(x,t), \phi_i(x) \right)$

→ uncorrelated coefficients

In practice, the data are discretized in space and time.

Instrumented structure



n snapshots

Observation matrix:

$$\mathbf{Q}_{m\times n} = \begin{bmatrix} q_1(t_1) & \dots & q_1(t_n) \\ \vdots & \ddots & \vdots \\ q_m(t_1) & \dots & q_m(t_n) \end{bmatrix}$$

m measurement coordinates

Principal Component Analysis (PCA)

The m x m covariance matrix Σ is built

$$\mathbf{\Sigma} = \frac{1}{m}\mathbf{Q}\,\mathbf{Q}^T$$

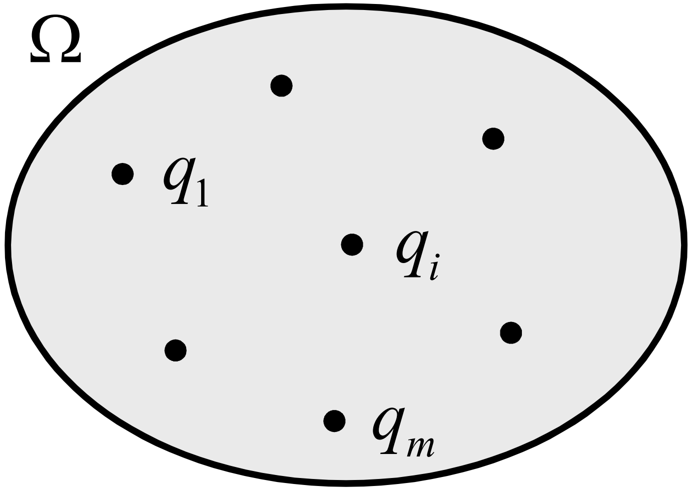
The eigenvalue problem is solved

$$\mathbf{\Sigma}\,\mathbf{u} = \sigma\,\mathbf{u}$$

eigenvalues (POVs)

eigenvectors of $\mathbf{Q}\,\mathbf{Q}^T$ (POMs)

Computation of the POMs using SVD



m measurement co-ordinates

n time samples

$$\mathbf{Q}_{m\times n} = \begin{bmatrix} q_1(t_1) & \dots & q_1(t_n) \\ \vdots & \ddots & \vdots \\ q_m(t_1) & \dots & q_m(t_n) \end{bmatrix}$$

Using SVD

$$\mathbf{Q}_{m\times n} = \mathbf{U}_{m\times m}\,\mathbf{S}_{m\times n}\,\mathbf{V}^T_{n\times n}$$

→ $diag(\sqrt{\sigma_i})$ $(\sigma_i \equiv \text{POV})$

→ eigenvectors of $\mathbf{Q}\,\mathbf{Q}^T$ (POM)

Harmonic excitation

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{F}\sin(\omega\, t)$$

Forced response: $$\mathbf{q}(t) = \Im\left(\mathbf{H}(i\,\omega)\,\mathbf{F}\,e^{i\,\omega\, t}\right)$$

where $$\mathbf{H}(i\,\omega) = (\mathbf{K} - \omega^2\,\mathbf{M} + i\,\omega\,\mathbf{C})^{-1}$$ is the FRF matrix

The $(m$ x $n)$ response matrix writes $\mathbf{Q} = \left[\mathbf{q}(t_1) \quad \ldots \quad \ldots \quad \mathbf{q}(t_n)\right]$

$$\mathbf{Q} = \Re(\mathbf{H})\,\mathbf{F}\begin{bmatrix}\sin(\omega\, t_1)\\ \vdots \\ \sin(\omega\, t_n)\end{bmatrix}^T + \Im(\mathbf{H})\,\mathbf{F}\begin{bmatrix}\cos(\omega\, t_1)\\ \vdots \\ \cos(\omega\, t_n)\end{bmatrix}^T$$

Harmonic excitation

$$\mathbf{Q} = \Re(\mathbf{H})\,\mathbf{F}\,\mathbf{e}_s^T + \Im(\mathbf{H})\,\mathbf{F}\,\mathbf{e}_c^T$$

$$\mathbf{Q} = \underbrace{\begin{bmatrix} \dfrac{\Re(\mathbf{H})\,\mathbf{F}}{\|\Re(\mathbf{H})\,\mathbf{F}\|} & \dfrac{\Im(\mathbf{H})\,\mathbf{F}}{\|\Im(\mathbf{H})\,\mathbf{F}\|} & \mathbf{A} \end{bmatrix}}_{\mathbf{U}} \underbrace{\begin{bmatrix} \|\Re(\mathbf{H})\,\mathbf{F}\|\,\|\mathbf{e}_s\| & 0 & 0 & \ldots & 0 \\ 0 & \|\Im(\mathbf{H})\,\mathbf{F}\|\,\|\mathbf{e}_c\| & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}}_{\mathbf{S}} \underbrace{\begin{bmatrix} \mathbf{e}_s / \|\mathbf{e}_s\| \\ \mathbf{e}_c / \|\mathbf{e}_c\| \\ \mathbf{B} \end{bmatrix}}_{\mathbf{V}^T}$$

It follows that there are only two non-zero singular values
→ the forced harmonic response is captured by two POMs which reflect the real and imaginary parts of the ODS.

Damage detection problem

Concept of subspace angle

Key idea

- Use PCA to extract the structural response subspace
- Use the concept of subspace angles to compare the hyperplanes associated with the reference (undamaged) state and with the current (possibly damaged?) state of the structure.

Damage detection problem

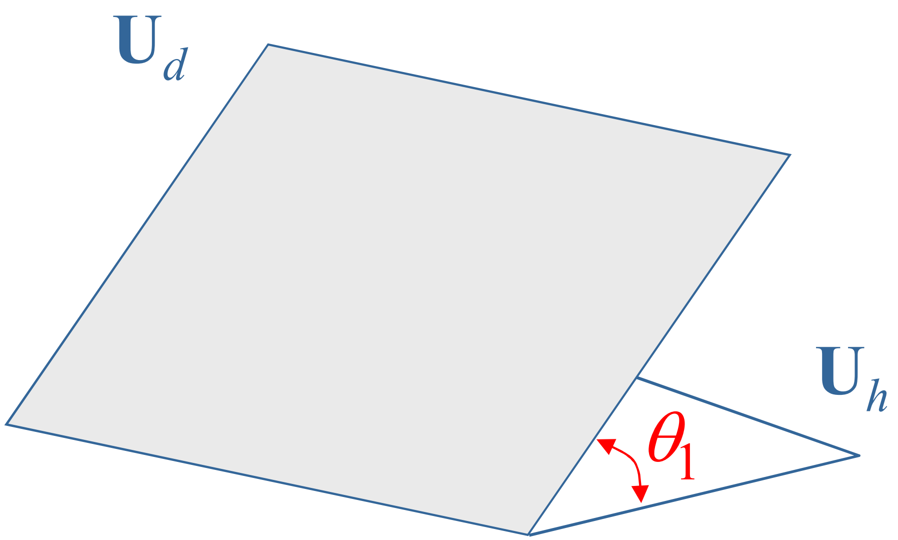
Concept of subspace angle

Given two subspaces $\mathbf{U}_h \in \Re^{m\times n}$ and $\mathbf{U}_d \in \Re^{m\times n}$

Carry out the QR-factorizations

$$\mathbf{U}_h = \mathbf{Q}_h \mathbf{R}_h \text{ and } U_d = \mathbf{Q}_d \mathbf{R}_d$$

define orthonormal bases

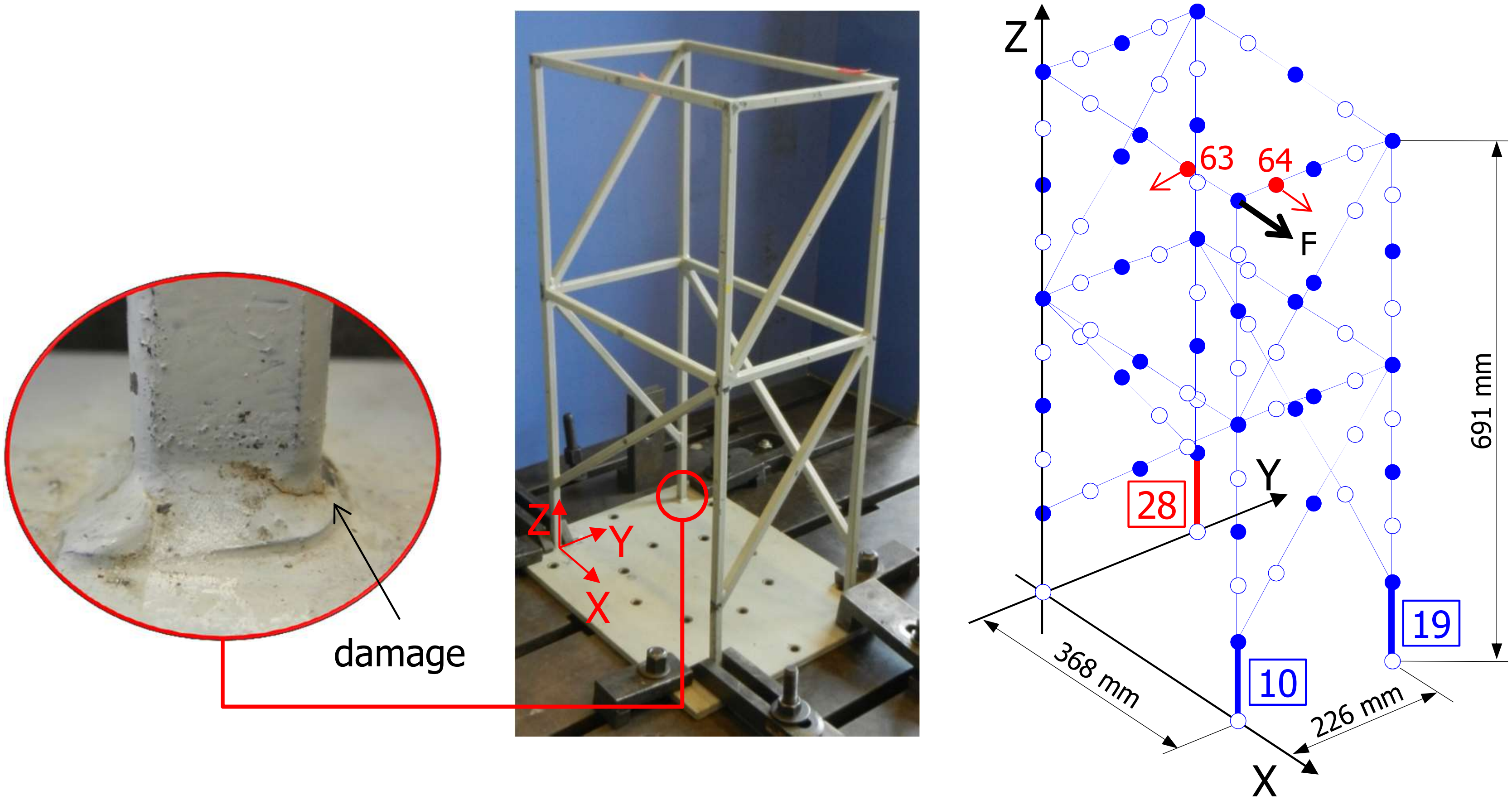


The angles θ_i between subspaces $\mathbf{U}_h$ and $\mathbf{U}_d$ are defined through the singular values associated to

$$\mathbf{Q}_h^T \mathbf{Q}_d = \mathbf{U}_{hd} \mathbf{\Sigma}_{hd} \mathbf{V}_{hd}^T$$

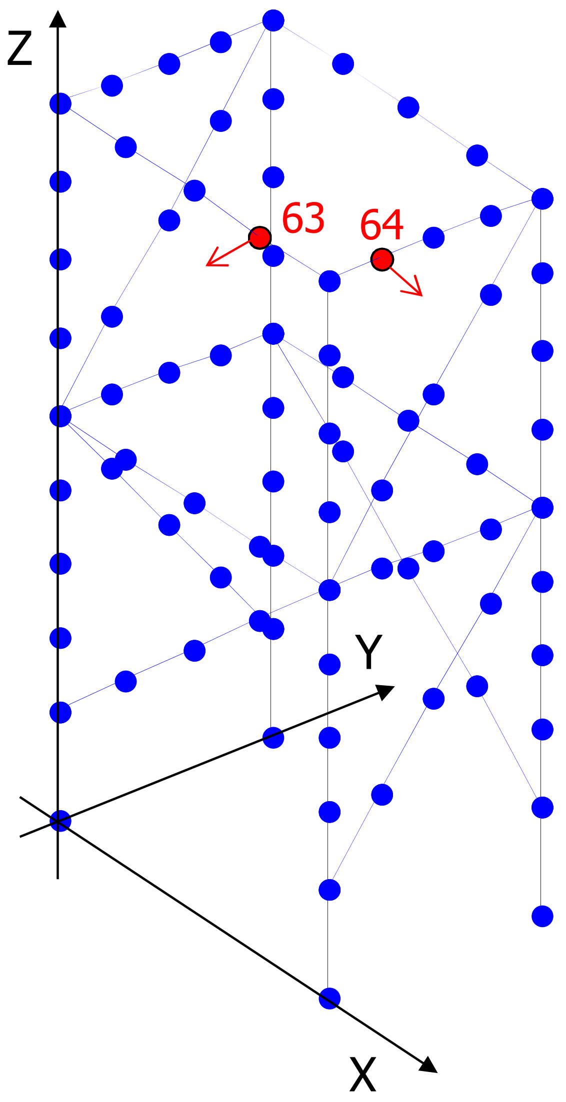
$$\mathbf{\Sigma}_{hd} = diag(\cos\theta_i) \quad (i = 1,\ldots,2)$$

Example of a truss structure



Presence of rust → reduction of stiffness of 40 % of element 28

Measurement set-up

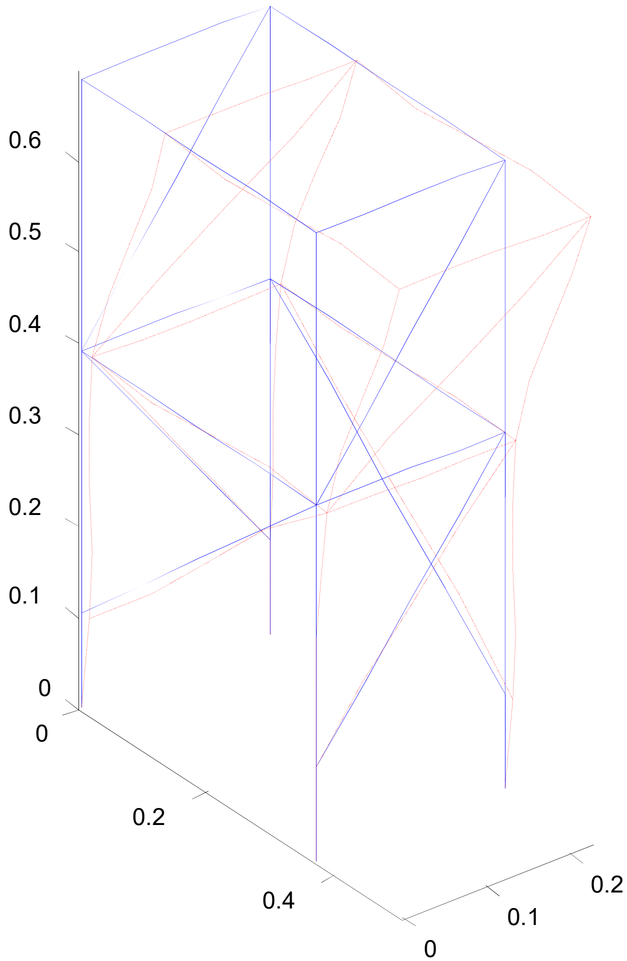


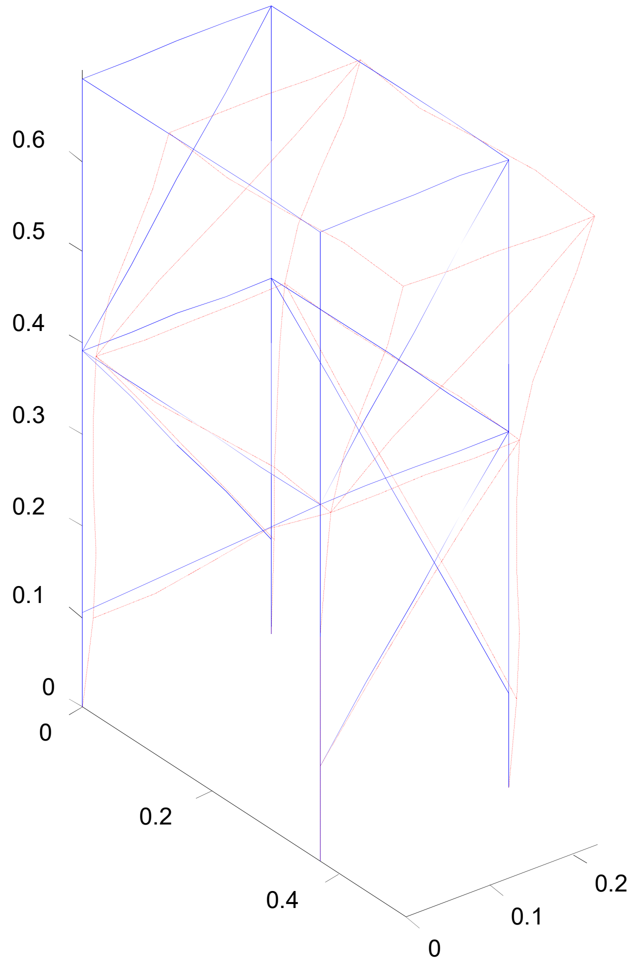
Impact excitation (roving hammer technique)

- Accelerometer at node 63 (Y) and at node 64 (X)
- Impact at 36 nodes in directions X and Y
- Frequency range: [0 - 400] Hz
- Resolution: 4096 lines → 0.1 Hz
- Identification using the PolyMAX method

Experimental Modal Analysis

Healthy structure	**Damaged structure**
FE Mode n° 1 (78.21 Hz)	Experimental mode n° 1 (77.50 Hz)

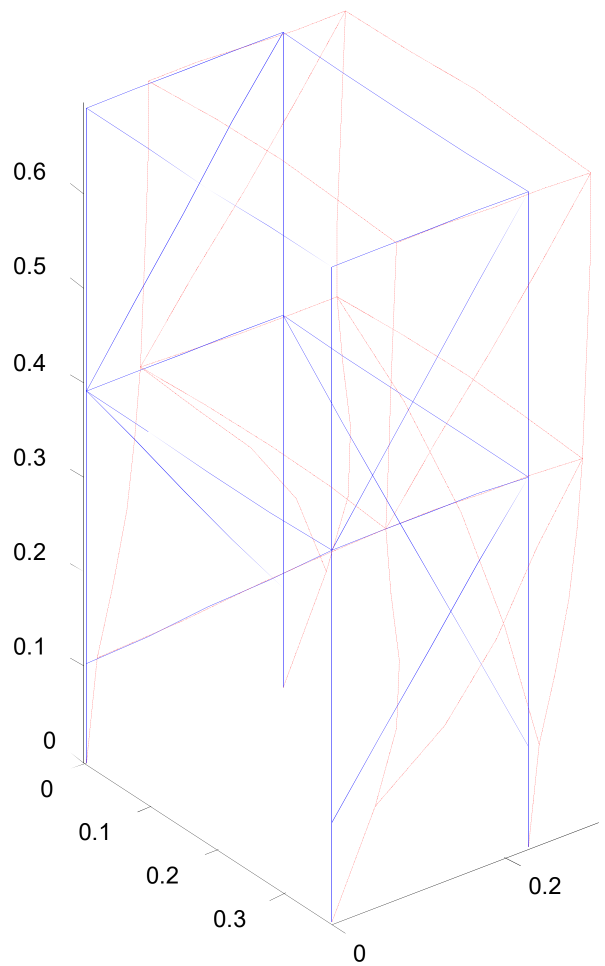




Experimental Modal Analysis

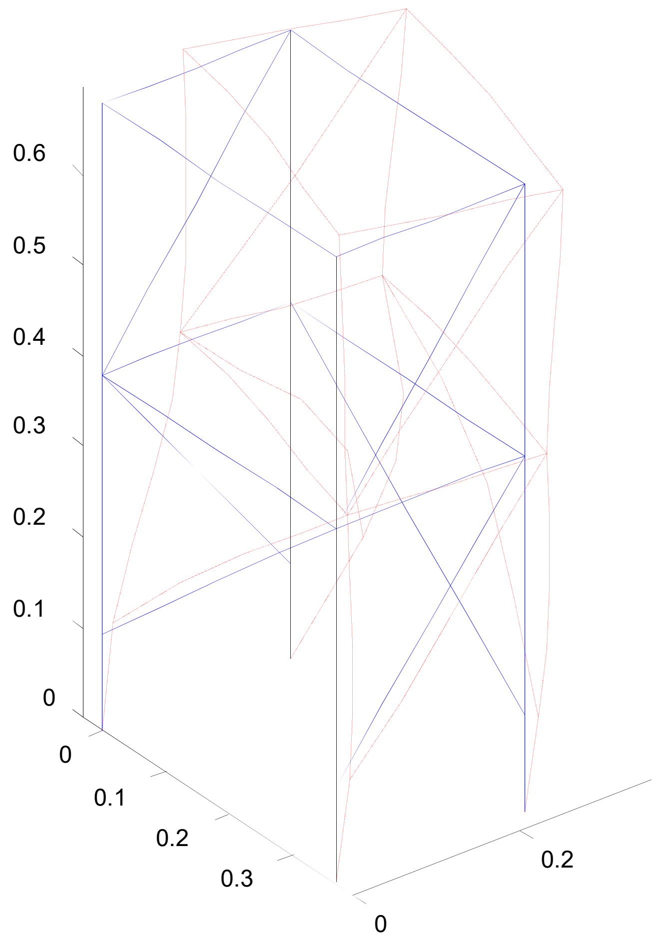
Healthy structure

FE mode n° 2 (124.59 Hz)



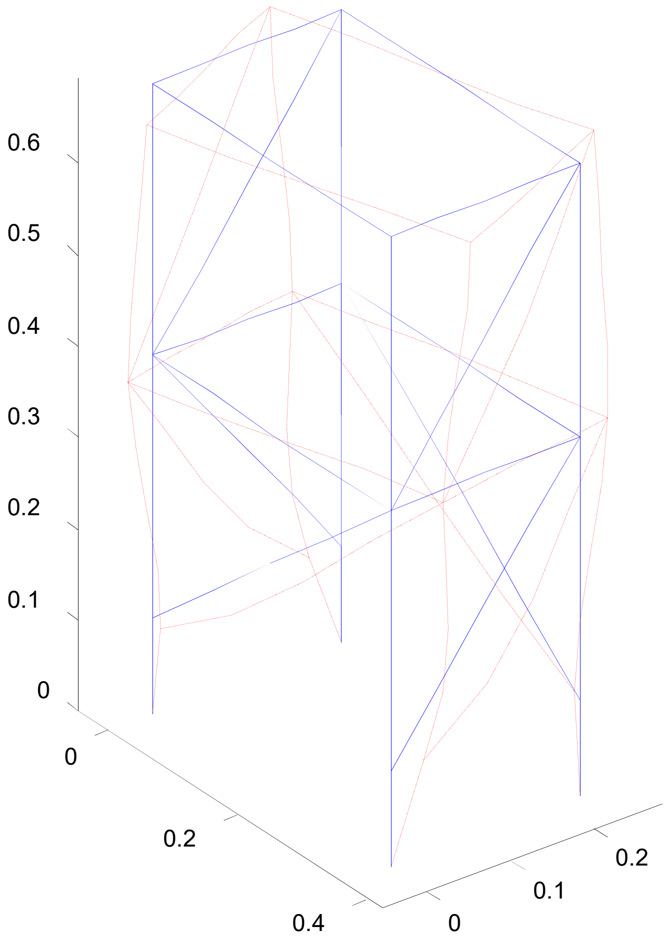
Damaged structure

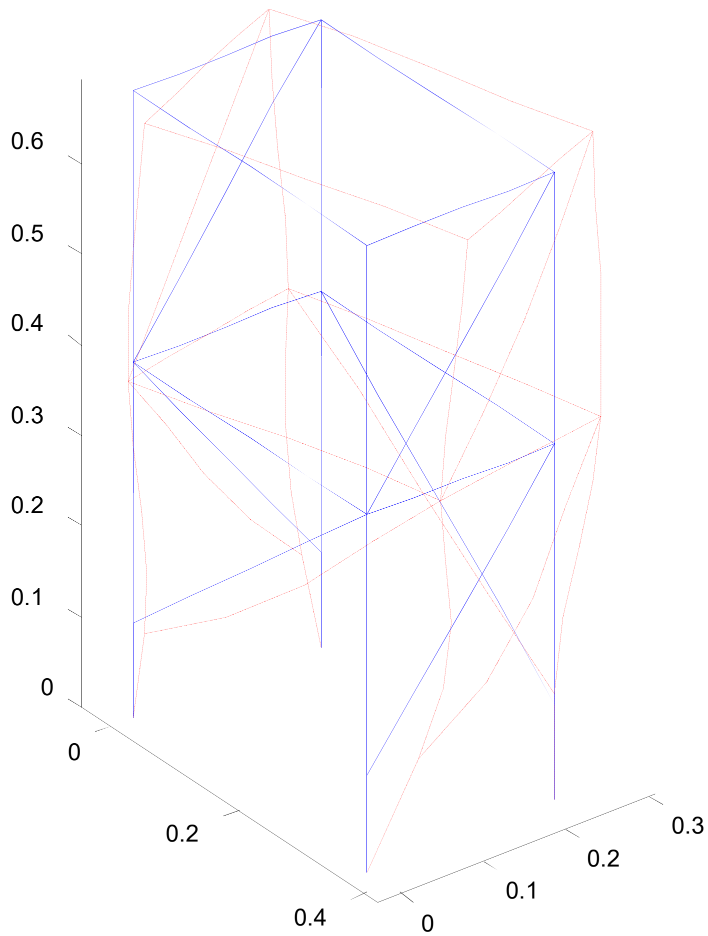
Experimental mode n° 2 (113.95 Hz)



Experimental Modal Analysis

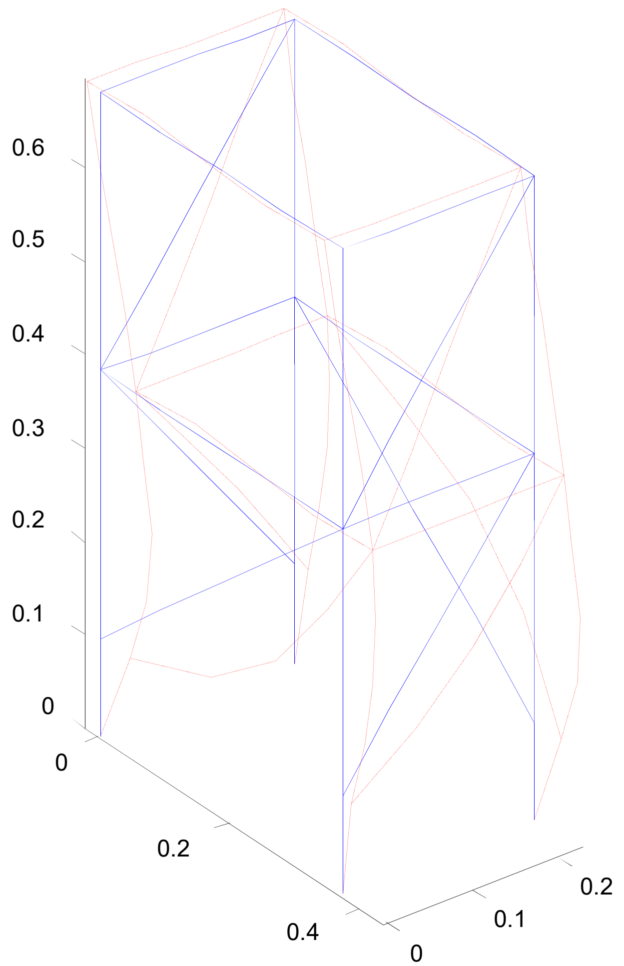
Healthy structure	**Damaged structure**
FE mode n° 3 (137.66 Hz)	Experimental mode n° 3 (136.10 Hz)

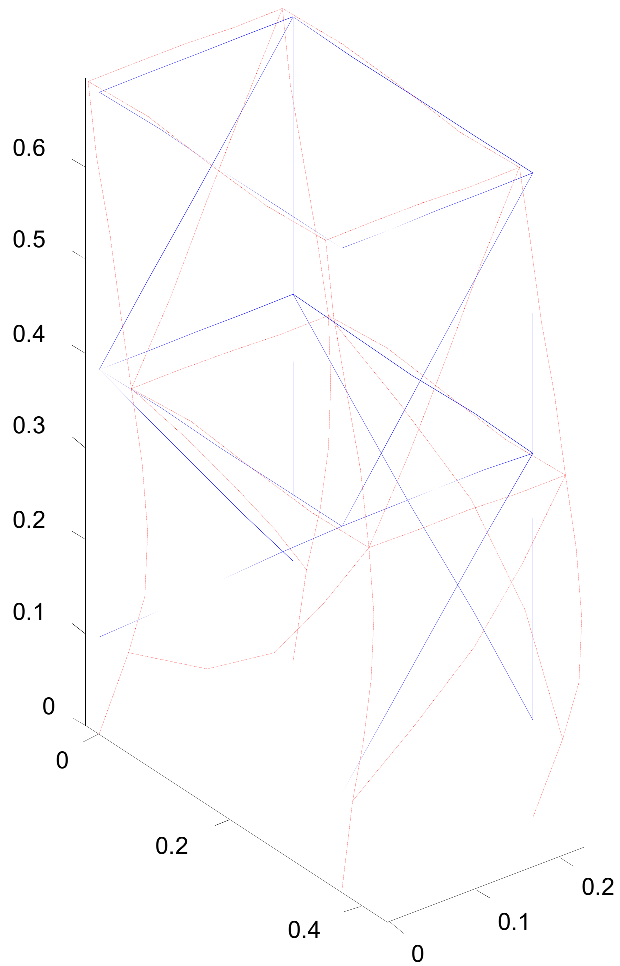




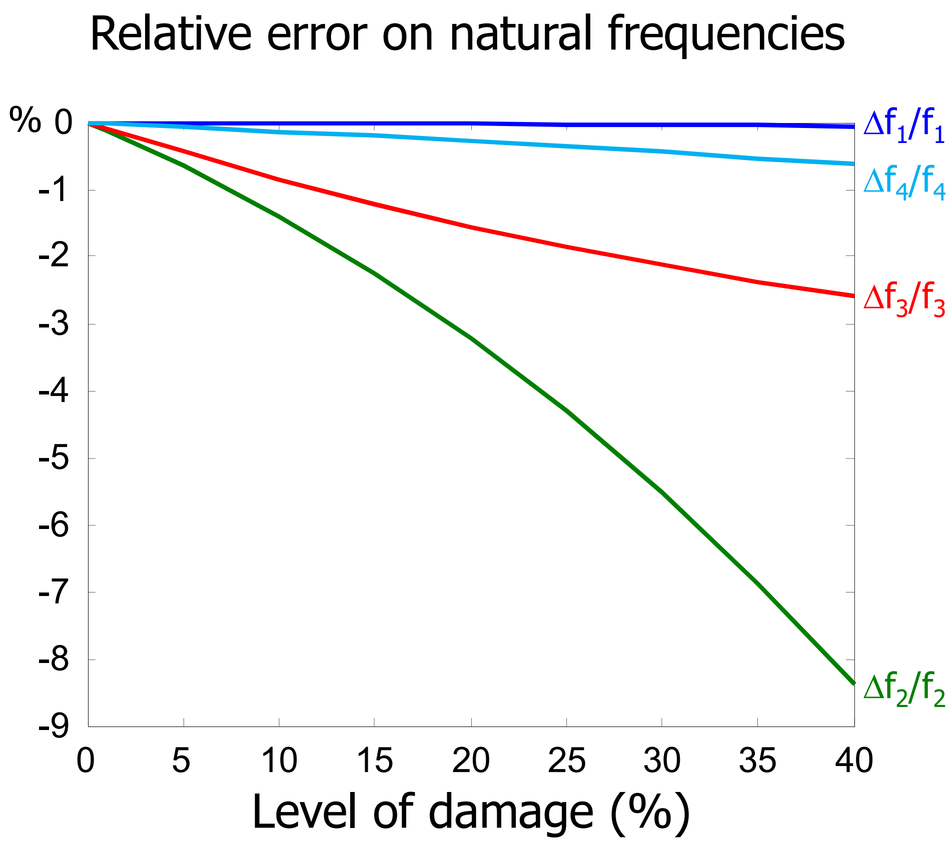
Experimental Modal Analysis

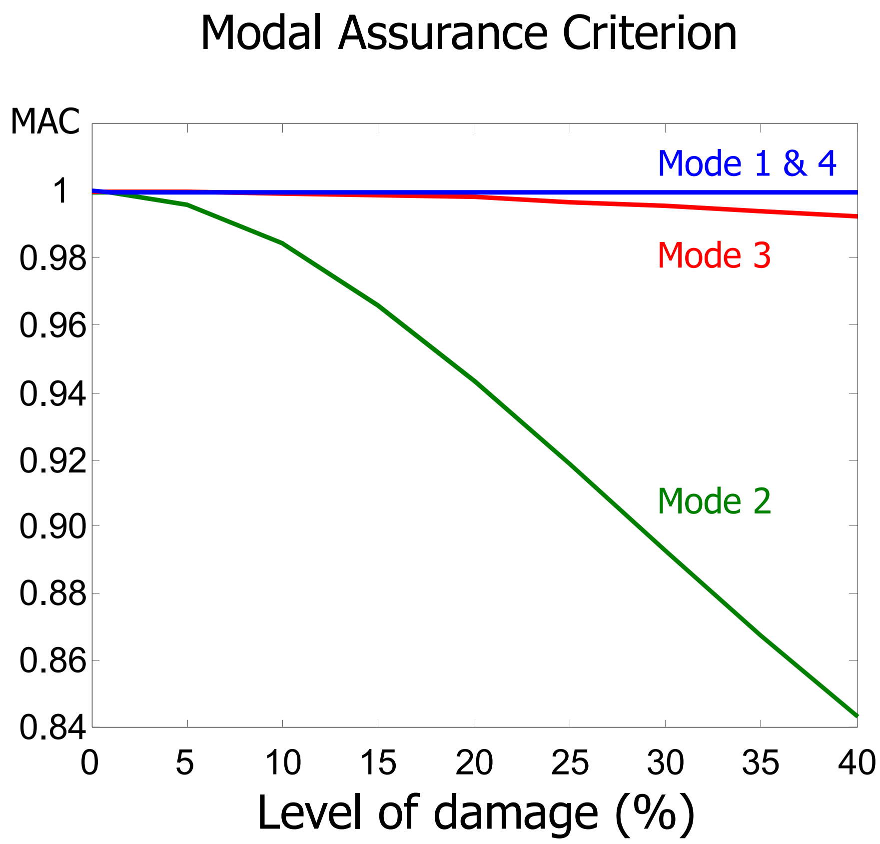
Healthy structure	**Damaged structure**
FE mode n° 4 (160.19 Hz)	Experimental mode n° 4 (158.59 Hz)





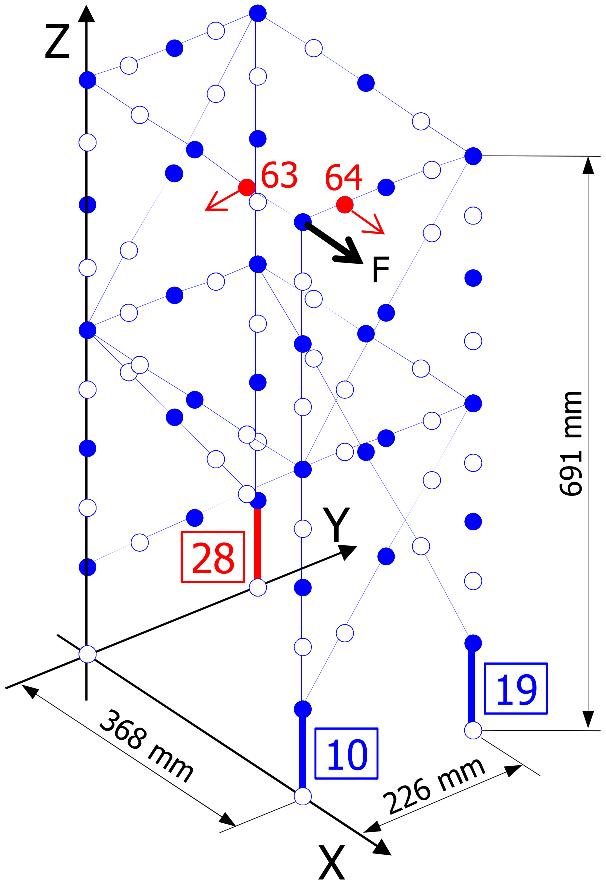
Damage assessment



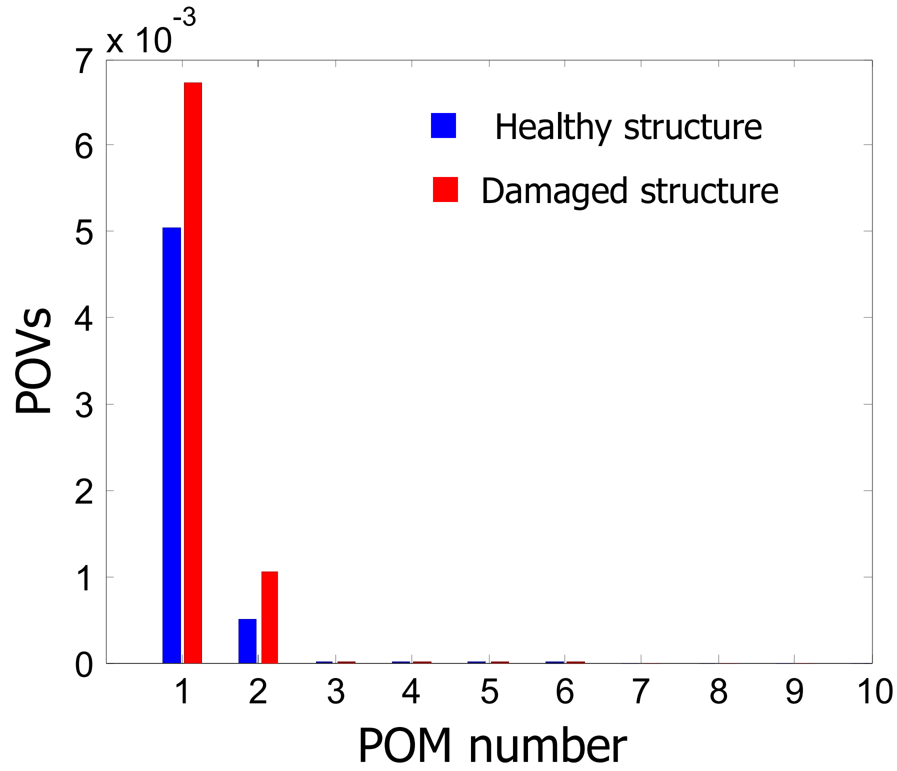


PCA Analysis of forced harmonic responses

Harmonic force **F** of 1 N at 130 Hz



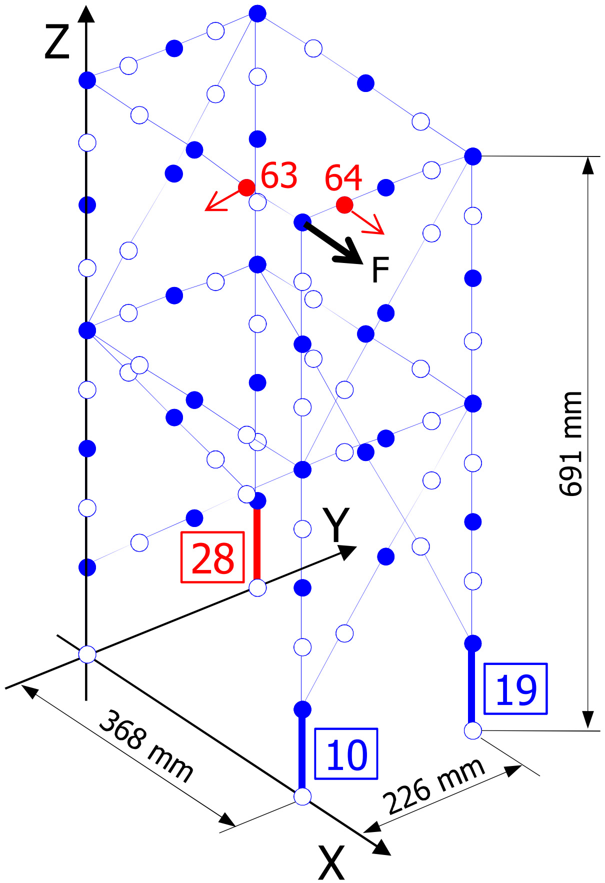
Proper orthogonal values



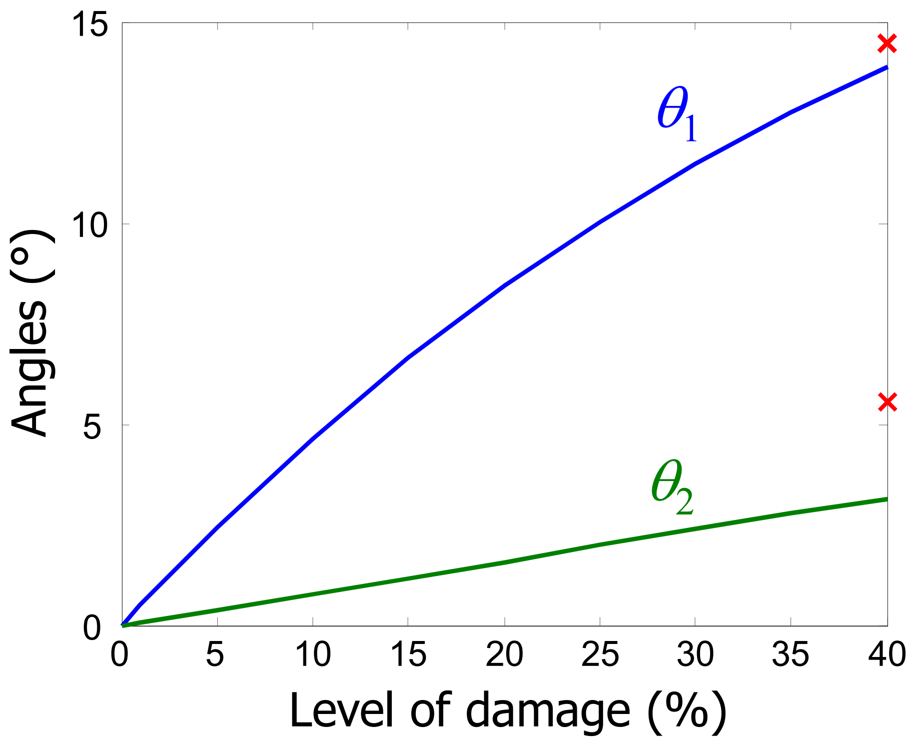
→ Presence of two POMs

PCA Analysis of forced harmonic responses

Harmonic force **F** of 1 N at 130 Hz



Detection of damage



Damage localization

The strain energy function may be used to localize damage.

For the j^{th} element, it writes

stiffness matrix

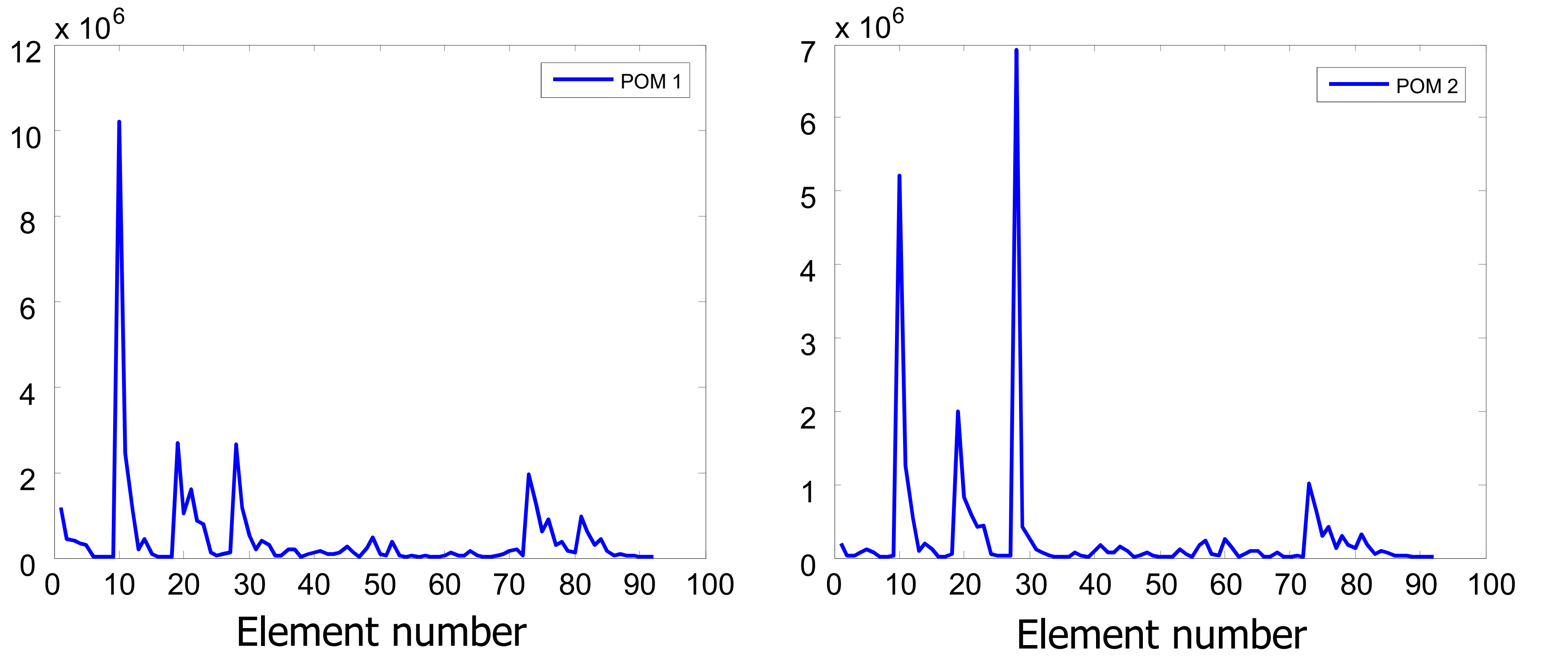
$$\Delta E_{(k)}^{j} = \left(\mathbf{u}_{d,(k)}^{j} - \mathbf{u}_{h,(k)}^{j}\right)^{T} \mathbf{K}^{j} \left(\mathbf{u}_{d,(k)}^{j} - \mathbf{u}_{h,(k)}^{j}\right) \quad (k = 1, 2)$$

k^{th} POM (damaged structure)

k^{th} POM (healthy structure)

Damage localization

Strain energy functions



Conclusion

- Use of PCA for damage detection problem in the case of harmonically excited structures
- Advantages:
- Direct processing of time-responses (EMA is not required).
- The damage indicator based on the concept of subspace angle is a global indicator (selection of modes is not required).
- The POMs are scaled by the excitation force → they can be interpreted as 'deformation modes' enabling damage localization.

Thank you for your attention.