Damage Detection in Structures Based on Principal Component Analysis of Forced Harmonic Responses

GOLINVAL Jean-Claude

Université de Liège, Belgique

Département d’Aérospatiale et Mécanique
9, Allée de la Découverte, Bât. B 52
B-4000 Liège (Belgium)
E-mail : JC.Golinval@ulg.ac.be
Outline

• **Principal Component Analysis**
  - Definition and mathematical formulation
  - Computation of the POMs
  - Interpretation of the POMs
  - Detection of damage
  - Localization of damage

• **Example of a truss structure**

• **Conclusion**
Principal component analysis is a multi-variate statistical method.

**Aim**: to obtain a compact representation of the data.

**Principal Component Analysis = Proper Orthogonal Decomposition**

PCA has been successfully applied for operational modal analysis (OMA) of civil engineering structures including temperature effects [1-2].

Mathematical formulation

Let $\theta(x,t)$ be a random field on a domain $\Omega$

$$\theta(x,t) = \mu(x) + \mathcal{G}(x,t)$$

At time $t_k$, the system displays a snapshot

$$\mathcal{G}^k(x) = \mathcal{G}(x,t_k)$$
The POD aims at obtaining the most characteristic structure $\phi(x)$ of an ensemble of snapshots i.e.

$$\text{Maximize } \left\langle \left| \left( G_k, \phi \right) \right|^2 \right\rangle \text{ with } \| \phi \|^2 = 1$$

where  $$(f, g) = \int_{\Omega} f(x) g(x) \, d\Omega$$

$$\langle \cdot \rangle \text{ denotes the averaging operation}$$

$$\| \cdot \| \text{ denotes the norm}$$

It can be shown that the problem reduces to the following integral eigenvalue problem

$$\int_{\Omega} \left\langle G_k(x) G_k(x') \right\rangle \phi(x') \, dx' = \lambda \phi(x)$$

\text{averaged auto-correlation function}
Thus the solution of the optimization problem

\[
\text{Maximize } \left\langle \left\| \left( G^k, \phi \right) \right\|^2 \right\rangle \quad \text{with} \quad \left\| \phi \right\|^2 = 1
\]

is given by the orthogonal eigenfunctions \( \phi_i(x) \) of the integral equation

\[
\int_{\Omega} \left\langle \mathcal{G}^k(x), \mathcal{G}^k(x') \right\rangle \phi(x') \, dx' = \lambda \, \phi(x)
\]

\( \phi_i(x) \) are called the proper orthogonal modes (POM)

\( \lambda_i \) are called the proper orthogonal values (POV)

and we have

\[
\mathcal{G}(x,t) = \sum_{i=1}^{\infty} a_i(t) \phi_i(x)
\]

where

\[
a_i(t) = \left( \mathcal{G}(x,t), \phi_i(x) \right)
\]

uncorrelated coefficients
In practice, the data are discretized in space and time.

Instrumented structure

Observation matrix:

\[ Q_{m \times n} = \begin{bmatrix} q_1(t_1) & \cdots & q_1(t_n) \\ \vdots & \ddots & \vdots \\ q_m(t_1) & \cdots & q_m(t_n) \end{bmatrix} \]
The $m \times m$ covariance matrix $\Sigma$ is built

$$\Sigma = \frac{1}{m} Q Q^T$$

The eigenvalue problem is solved

$$\Sigma u = \sigma u$$

eigenvalues (POVs)
eigenvectors of $Q Q^T$ (POMs)
**Principal Component Analysis (PCA)**

**Computation of the POMs using SVD**

\[ Q_{m \times n} = U_{m \times m} S_{m \times n} V_{n \times n}^T \]

- *m* measurement co-ordinates
- *n* time samples

Using SVD

\[ diag(\sqrt{\sigma_i}) \quad (\sigma_i \equiv \text{POV}) \]

- Eigenvectors of \( QQ^T \) (POM)
Harmonic excitation

\[ M \ddot{q} + C \dot{q} + K q = F \sin(\omega t) \]

Forced response:

\[ q(t) = \Im \left( H(i \omega) F e^{i \omega t} \right) \]

where

\[ H(i \omega) = (K - \omega^2 M + i \omega C)^{-1} \]

is the FRF matrix

The \((m \times n)\) response matrix writes

\[ Q = \begin{bmatrix} q(t_1) & \ldots & \ldots & q(t_n) \end{bmatrix} \]

\[ Q = \Re(H) F \begin{bmatrix} \sin(\omega t_1) \\ \vdots \\ \sin(\omega t_n) \end{bmatrix}^T + \Im(H) F \begin{bmatrix} \cos(\omega t_1) \\ \vdots \\ \cos(\omega t_n) \end{bmatrix}^T \]
It follows that there are only two non-zero singular values → the forced harmonic response is captured by two POMs which reflect the real and imaginary parts of the ODS.
Concept of subspace angle

Key idea

- Use PCA to extract the structural response subspace
- Use the concept of subspace angles to compare the hyperplanes associated with the reference (undamaged) state and with the current (possibly damaged?) state of the structure.
Damage detection problem

Concept of subspace angle

Given two subspaces \( U_h \in \mathbb{R}^{m \times n} \) and \( U_d \in \mathbb{R}^{m \times n} \)

Carry out the QR-factorizations

\[
U_h = Q_h \ R_h \quad \text{and} \quad U_d = Q_d \ R_d
\]

define orthonormal bases

The angles \( \theta_i \) between subspaces \( U_h \) and \( U_d \) are defined through the singular values associated to

\[
Q_h^T Q_d = U_{hd} \ \Sigma_{hd} \ V_{hd}^T
\]

\[
\Sigma_{hd} = \text{diag}(\cos \theta_i) \quad (i = 1, \ldots, 2)
\]
Presence of rust $\rightarrow$ reduction of stiffness of 40 % of element 28
Experimental Modal Analysis

Measurement set-up

Impact excitation (roving hammer technique)

- Accelerometer at node 63 (Y) and at node 64 (X)
- Impact at 36 nodes in directions X and Y
- Frequency range: [0 - 400] Hz
- Resolution: 4096 lines → 0.1 Hz
- Identification using the PolyMAX method
Experimental Modal Analysis

Healthy structure

FE Mode n° 1 (78.21 Hz)

Damaged structure

Experimental mode n° 1 (77.50 Hz)
Experimental Modal Analysis

Healthy structure
FE mode n° 2 (124.59 Hz)

Damaged structure
Experimental mode n° 2 (113.95 Hz)
Experimental Modal Analysis

Healthy structure
FE mode n° 3 (137.66 Hz)

Damaged structure
Experimental mode n° 3 (136.10 Hz)
Experimental Modal Analysis

Healthy structure
FE mode n° 4 (160.19 Hz)

Damaged structure
Experimental mode n° 4 (158.59 Hz)
Damage assessment

Relative error on natural frequencies

- $\Delta f_1/f_1$
- $\Delta f_4/f_4$
- $\Delta f_3/f_3$
- $\Delta f_2/f_2$

Modal Assurance Criterion

- Mode 1 & 4
- Mode 3
- Mode 2
Harmonic force $\mathbf{F}$ of 1 N at 130 Hz

→ Presence of two POMs
PCA Analysis of forced harmonic responses

Harmonic force $\mathbf{F}$ of 1 N at 130 Hz

Detection of damage

Harmonic force $\mathbf{F}$ of 1 N at 130 Hz
Damage localization

The strain energy function may be used to localize damage.

For the \( j^{th} \) element, it writes

\[
\Delta E_{(k)}^j = \left( u_{d,(k)}^j - u_{h,(k)}^j \right)^T K^j \left( u_{d,(k)}^j - u_{h,(k)}^j \right) \quad (k = 1, 2)
\]
Damage localization

Strain energy functions

Element number

$10^6$

$10^6$

POM 1

POM 2
• Use of PCA for damage detection problem in the case of harmonically excited structures

• Advantages:
  • Direct processing of time-responses (EMA is not required).
  • The damage indicator based on the concept of subspace angle is a global indicator (selection of modes is not required).
  • The POMs are scaled by the excitation force → they can be interpreted as ‘deformation modes’ enabling damage localization.
Thank you for your attention.