Diametral dimension and property $(\overline{\Omega})$ for spaces S^{ν}

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Introduction: spaces S^ν and diametral dimension

The concave case

Local p-convexity

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- $\Omega := \mathbb{C}^\Lambda$.

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The space S^ν is the set of all $\vec{c} \in \Omega$ s.t.

$$\forall \alpha \in \mathbb{R}, \forall \varepsilon > 0, \forall C > 0, \exists J \in \mathbb{N}_0 :$$
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Properties

- Metric spaces: topological vector spaces (tvs), separable, complete, Schwartz, non-nuclear.
- If
$$p_0 := \min\left\{1, \inf_{\alpha_{\min} \leq \alpha < \alpha_{\max}} \underline{\partial}^+ \nu(\alpha)\right\}$$
with $\underline{\partial}^+ \nu(\alpha) = \liminf_{h \to 0^+} \frac{\nu(\alpha+h)-\nu(\alpha)}{h}$, then
  - if $p_0 > 0$, S^ν is "exactly" **locally p_0-convex**;
  - if $p_0 = 0$, S^ν is only **locally pseudoconvex**.

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$$\Delta(E) := \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \exists V \in \mathcal{U}, V \subseteq U, \text{ s.t. } \xi_n \delta_n(V, U) \to 0\right\},$$

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Theorem (J.M. Aubry, F. Bastin, 2010)

If S^ν is locally p-convex, then

$$\Delta(S^\nu) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0, \xi_n (n+1)^{-s} \to 0\right\}.$$

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When ν is concave... it's nice! ☺

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Definition (Besov spaces)

For $p > 0$ and $s \in \mathbb{R}$,

$$b^s_{p,\infty} := \left\{ \vec{c} \in \Omega : \|\vec{c}\|_{b^s_{p,\infty}} := \sup_{j \in \mathbb{N}_0} \left[2^{(s-\frac{1}{p})j} \left(\sum_{k=0}^{2^j-1} |c_{j,k}|^p \right)^{1/p} \right] < \infty \right\}.$$

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If $(p_n)_{n\in\mathbb{N}}$ is a dense sequence of $(0,\infty)$ and $\varepsilon_m \to 0^+$, then

$$S^\nu = \bigcap_{n\in\mathbb{N}} \bigcap_{m\in\mathbb{N}} b^{\frac{\eta(p_n)}{p_n}-\varepsilon_m}_{p_n,\infty},$$

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For us: $(p_n)_{n\in\mathbb{N}} \equiv \mathbb{Q}^+$ and $\varepsilon_m = 1/m$.

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Topology defined by

$$P_m^{(I)}(\vec{c}) = \sup_{i\in I}\sup_{j\in\mathbb{N}_0}\left[2^{\left(\frac{\eta(p_i)}{p_i}-\frac{1}{p_i}-\varepsilon_m\right)j}\left(\sum_{k=0}^{2^j-1}|c_{j,k}|^{p_i}\right)^{1/p_i}\right]$$

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Numbers for weights

$j=0$	$(0,0)$			
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$j = 1$ 1 $(1, 1)$

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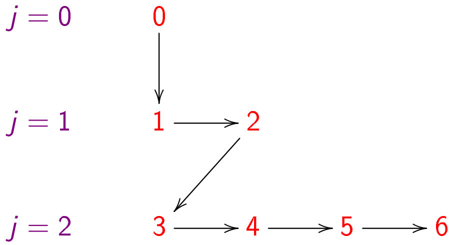

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$j=0$ 0
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$j=1$ 1 → 2
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$j=2$ 3 → 4 → 5 → 6

Scale j associated to the number n: $j(n)$ s.t.

$$2^{j(n)}-1\leq n\leq 2^{j(n)+1}-2.$$

A first property

Proposition

If $I \subseteq \mathbb{N}$ is finite and if $k_0 \geq m$,

$$\delta_n \left(B_{P_{k_0}^{(I)}}, B_{P_m^{(I)}} \right) \leq 2^{(\varepsilon_{k_0} - \varepsilon_m) j(n)}.$$

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$$\leq 2^{(\varepsilon_{k_0}-\varepsilon_m)j(n)}P^{(I)}_{k_0}(\vec{c}) \leq 2^{(\varepsilon_{k_0}-\varepsilon_m)j(n)}.$$

$$\Rightarrow \vec{c} = \vec{c} - \pi_n(\vec{c}) + \pi_n(\vec{c}) \in 2^{(\varepsilon_{k_0}-\varepsilon_m)j(n)}B_{P^{(I)}_m} + \pi_n(S^\nu).\ \blacksquare$$

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Corollary

$$\left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0,\ \xi_n(n+1)^{-s} \to 0 \text{ if } n \to \infty\right\} \subseteq \Delta(S^\nu).$$

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Proof: If $k_0 > m$,

$$\delta_n\left(B_{P^{(I)}_{k_0}}, B_{P^{(I)}_m}\right) \leq \underbrace{2^{(\varepsilon_{k_0}-\varepsilon_m)j(n)} \leq 2^{\varepsilon_m-\varepsilon_{k_0}}(n+2)^{\varepsilon_{k_0}-\varepsilon_m}}_{\text{because } n \leq 2^{j(n)+1}-2}$$

$$\leq 2^{\varepsilon_m-\varepsilon_{k_0}}(n+1)^{\varepsilon_{k_0}-\varepsilon_m}. \quad \blacksquare$$

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Lemma (L.D., 2017)

$$\lim_{p\to 0^+} \frac{\eta(p)}{p} = \alpha_{\max}.$$

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Examples

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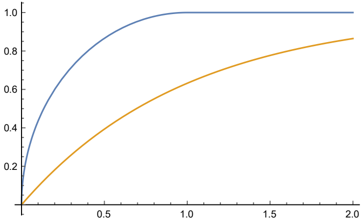
- 

Pseudoconvexity, with $\alpha_{\max} < \infty$ (blue) and $\alpha_{\max} = \infty$ (orange).

Construction of the I_ε's

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1. Because $\alpha_{\max} < \infty$, $\exists i_0 \in \mathbb{N}$ s.t.

$$0 < p < p_{i_0} \Rightarrow \frac{\eta(p)}{p} - \frac{\eta(p_{i_0})}{p_{i_0}} \leq \varepsilon.$$

NB: $p > 0 \mapsto \frac{\eta(p)}{p} = \inf_{\alpha \geq \alpha_{\min}} \left\{ \alpha + \frac{1-\nu(\alpha)}{p} \right\}$ is decreasing and $(p_n)_{n \in \mathbb{N}} \equiv \mathbb{Q}^+$.

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2. $\exists \ell \in \mathbb{N}_0$ s.t. $\ell\varepsilon < \dfrac{1}{p_{i_0}} \leq (\ell+1)\varepsilon$. For $k = 0, ..., \ell$, we define $i_k \in \mathbb{N}$ by

$$\frac{1}{p_{i_k}} = \frac{1}{p_{i_0}} - k\varepsilon.$$

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   NB: $p > 0 \mapsto \frac{\eta(p)}{p} = \inf_{\alpha \geq \alpha_{\min}} \left\{\alpha + \frac{1-\nu(\alpha)}{p}\right\}$ is decreasing and $(p_n)_{n \in \mathbb{N}} \equiv \mathbb{Q}^+$.

2. $\exists \ell \in \mathbb{N}_0$ s.t. $\ell\varepsilon < \frac{1}{p_{i_0}} \leq (\ell + 1)\varepsilon$. For $k = 0, ..., \ell$, we define $i_k \in \mathbb{N}$ by

$$\frac{1}{p_{i_k}} = \frac{1}{p_{i_0}} - k\varepsilon.$$

3. $I_\varepsilon := \{i_0, ..., i_\ell\}$.

The main property of the I_ε's

Proposition (L.D., 2017)

If $m, n \in \mathbb{N}$ and $\vec{c} \in S^\nu$,

$$\|\vec{c}\|_{b^{\eta(p_n)/p_n-\varepsilon_m}_{p_n,\infty}} \leq \sup_{i\in I_\varepsilon} \sup_{j\in\mathbb{N}_0} \left[2^{\left(\frac{\eta(p_i)}{p_i}-\frac{1}{p_i}+\varepsilon-\varepsilon_m\right)j} \left(\sum_{k=0}^{2^j-1} |c_{j,k}|^{p_i} \right)^{1/p_i} \right].$$

Proof: Admitted. ■

Consequences

Proposition

Let $m, k_0 \in \mathbb{N}$ be given, $k_0 \geq m$. If $I_\varepsilon \subseteq J \subseteq \mathbb{N}$, J finite, then,

$$\delta_n \left(B_{P_{k_0}^{(J)}}, B_{P_m^{(I_\varepsilon)}} \right) \geq 2^{(\varepsilon_{k_0} - \varepsilon_m - \varepsilon) j(n)}.$$

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Proof: Follows from:

Proposition (A. Pietsch, 1972)

Let $(E, \|.\|)$ be a normed space, with closed unit ball U, and B be bounded. If $\exists\ P : E \to E$ proj. with $\|P\| \leq 1$, $\dim P(E) = n+1$, then

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Here: $U = B_{P_m^{(I_\varepsilon)}}$, $B = B_{P_{k_0}^{(J)}}$, $P = \pi_{n+1}$ projection on the first $n+1$ $\overrightarrow{e_{j,k}}$ and $\delta = 2^{(\varepsilon_{k_0} - \varepsilon_m - \varepsilon) j(n)}$.

Thesis

$$2^{(\varepsilon_{k_0}-\varepsilon_m-\varepsilon)j(n)}B_{P_m^{(I_\varepsilon)}}\cap\pi_{n+1}(S^\nu)\subseteq B_{P_{k_0}^{(J)}}$$
$$\Leftrightarrow P_{k_0}^{(J)}(\vec{c})\leq 2^{(\varepsilon_m+\varepsilon-\varepsilon_{k_0})j(n)}P_m^{(I_\varepsilon)}(\vec{c})\qquad\forall\vec{c}\in\pi_{n+1}(S^\nu).$$

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Diametral dimension and spaces S^ν

Theorem (L.D., 2017)

If ν is concave and $\alpha_{\max} < \infty$,

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Property $(\overline{\Omega})$

Definition

A Fréchet space E, with a fundamental system of semi-norms $(\|.\|_n)_{n\in\mathbb{N}}$, has the *property* $(\overline{\Omega})$ if

$$\forall m\ \exists k\ \forall j\ \exists C > 0 : \left(\|x'\|_k^*\right)^2 \leq C\|x'\|_m^*\|x'\|_j^* \quad \forall x' \in E'$$

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Characterization

A Fréchet space E, with a basis of 0-neighbourhoods $(U_n)_{n\in\mathbb{N}}$, has the property $(\overline{\Omega})$ iff

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$\rightsquigarrow$ "Property (Ω_{id})"

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For $\vec{c} \in B_{P_{k_0}^{(I_{k_0})}}$,

$$\vec{c_1} := \sum_{j=0}^{J} \sum_{k=0}^{2^j-1} c_{j,k}\, \overrightarrow{e_{j,k}} \qquad \text{and} \qquad \vec{c_2} := \sum_{j=J+1}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k}\, \overrightarrow{e_{j,k}}.$$

So, since $I_\varepsilon \subseteq I_{k_0}$ and $\varepsilon = \varepsilon_m/2 - \varepsilon_{k_0}$,

$$P_{j_0}^{(I_{j_0})}(\vec{c_1}) \leq \sup_{i \in I_{k_0}} \sup_{j \leq J} \left[2^{(\varepsilon+\varepsilon_{k_0}-\varepsilon_{j_0})j} 2^{\left(\frac{\eta(p_i)}{p_i} - \frac{1}{p_i} - \varepsilon_{k_0}\right)j} \left(\sum_{k=0}^{2^j-1} |c_{j,k}|^{p_i} \right)^{1/p_i} \right]$$

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so $\vec{c_1} \in r B_{P_{j_0}^{(I_{j_0})}}$.

Because $I_m \subseteq I_{k_0}$ and $\varepsilon_{k_0} < \varepsilon_m/2$,

$$\begin{aligned} P_m^{(I_m)}(\vec{c_2}) &\leq \sup_{i\in I_{k_0}} \sup_{j\geq J+1} \left[2^{(\varepsilon_{k_0}-\varepsilon_m)j} 2^{\left(\frac{\eta(p_i)}{p_i}-\frac{1}{p_i}-\varepsilon_{k_0}\right)j} \left(\sum_{k=0}^{2^j-1} |c_{j,k}|^{p_i} \right)^{1/p_i} \right] \\ &\leq 2^{\frac{-\varepsilon_m}{2}(J+1)} P_{k_0}^{(I_{k_0})}(\vec{c}) \\ &\leq \frac{1}{r} \qquad (2^{\frac{\varepsilon_m}{2}J} \leq r \leq 2^{\frac{\varepsilon_m}{2}(J+1)}), \end{aligned}$$

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$$\begin{aligned}
P_m^{(I_m)}(\vec{c_2}) &\leq \sup_{i \in I_{k_0}} \sup_{j \geq J+1} \left[2^{(\varepsilon_{k_0} - \varepsilon_m) j} 2^{\left(\frac{\eta(p_i)}{p_i} - \frac{1}{p_i} - \varepsilon_{k_0}\right) j} \left(\sum_{k=0}^{2^j - 1} |c_{j,k}|^{p_i} \right)^{1/p_i} \right] \\
&\leq 2^{\frac{-\varepsilon_m}{2}(J+1)} P_{k_0}^{(I_{k_0})}(\vec{c}) \\
&\leq \frac{1}{r} \qquad (2^{\frac{\varepsilon_m}{2} J} \leq r \leq 2^{\frac{\varepsilon_m}{2}(J+1)}),
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so $\vec{c_2} \in \frac{1}{r} B_{P_m^{(I_m)}}$.

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$$\leq \frac{1}{r} \qquad (2^{\frac{\varepsilon_m}{2}J} \leq r \leq 2^{\frac{\varepsilon_m}{2}(J+1)}),$$

so $\vec{c_2} \in \frac{1}{r} B_{P_m^{(I_m)}}$.

Finally, $\vec{c} = \vec{c_1} + \vec{c_2} \in rB_{P_{j_0}^{(I_{j_0})}} + \frac{1}{r} B_{P_m^{(I_m)}}$. ■

Introduction: spaces S^ν and diametral dimension

The concave case

Local p-convexity

When $p_0 > 0$...

Reminder: S^ν locally p_0-convex iff $p_0 = \min\left\{1, \inf_{\alpha_{\min} \leq \alpha < \alpha_{\max}} \underline{\partial}^+ \nu(\alpha)\right\} > 0$.

Definition

For $\vec{c} \in S^\nu$, $\alpha, s \in \mathbb{R}$,

$$\|\vec{c}\|_{\alpha,s} := \inf\left\{\|\vec{c'}\|_{b^s_{p_0,\infty}} + \|\vec{c''}\|_{b^\alpha_{\infty,\infty}} : \vec{c} = \vec{c'} + \vec{c''}\right\}$$

where

$$\|\vec{c'}\|_{b^s_{p_0,\infty}} = \sup_{j \in \mathbb{N}_0} \left[2^{(s-\frac{1}{p_0})j} \left(\sum_{k=0}^{2^j-1} |c'_{j,k}|^{p_0}\right)^{1/p_0}\right],$$

$$\|\vec{c''}\|_{b^\alpha_{\infty,\infty}} = \sup_{j \in \mathbb{N}_0} \sup_{0 \leq k \leq 2^j-1} \left(2^{\alpha j} |c''_{j,k}|\right).$$

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Proposition (J.M. Aubry, F. Bastin, 2010)

The topology of S^ν is defined by the pseudonorms

$$|||\vec{c}|||_{A,\varepsilon} := \sup_{\alpha \in A}\|\vec{c}\|_{\alpha-\varepsilon,\alpha-\varepsilon+(1-\nu(\alpha))/p_0}, \quad A \subseteq (-\infty, \alpha_{\max}) \text{ finite and } \varepsilon > 0.$$

Diametral dimension and property $(\overline{\Omega})$

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Key ideas:

- Definition of a set A_{ε_0} for any $\varepsilon_0 > 0$ ($\equiv I_\varepsilon$)

Diametral dimension and property $(\overline{\Omega})$

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- Definition of a set A_{ε_0} for any $\varepsilon_0 > 0$ ($\equiv I_{\varepsilon}$)
- Gives a proof for $\Delta\left(S^{\nu}\right) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0, \xi_n(n+1)^{-s} \to 0\right\}$.
- If $p_0 > 0$, S^{ν} has (Ω_{id}) (L.D., 2017).

Thank you for your attention!

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