Incremental-secant mean-field-homogenization method for elasto-visco-plastic materials systems

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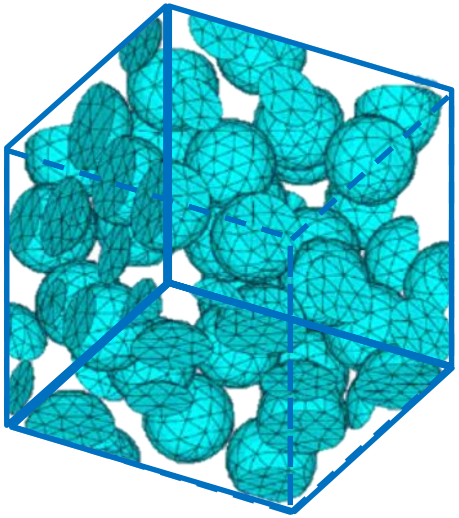

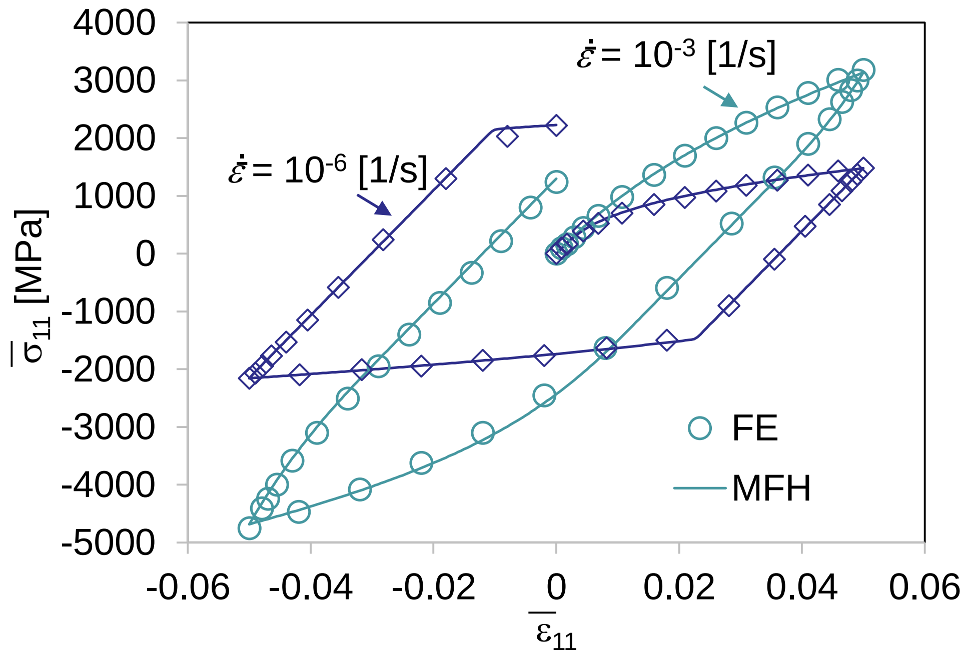

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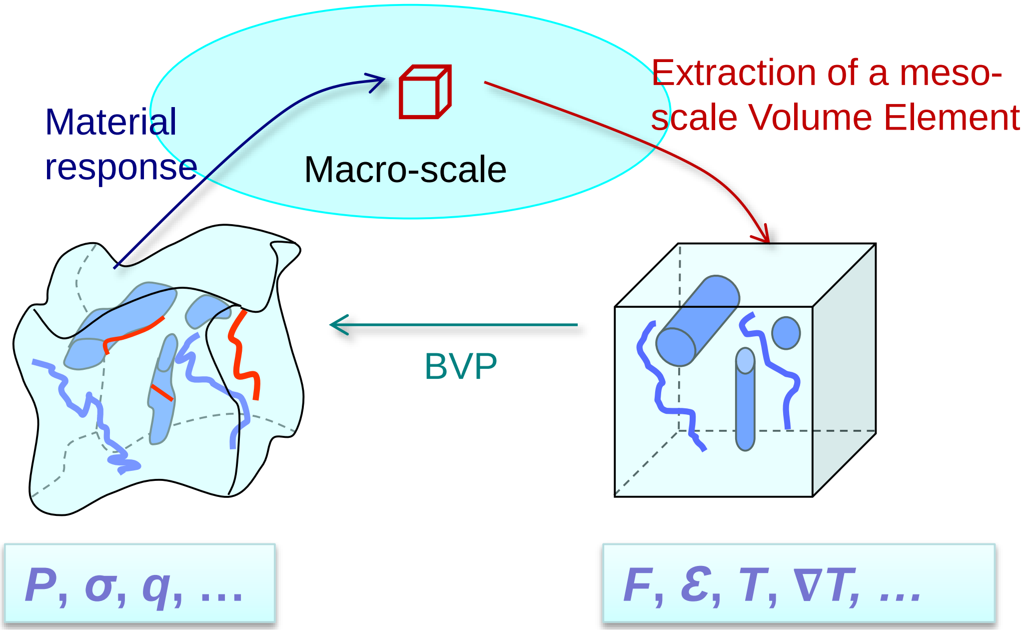
  - Experimental validation on laminate failure

Multi-scale modelling

- Two-scale modelling
  - One way: homogenization
  - 2 problems are solved (concurrently)
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)

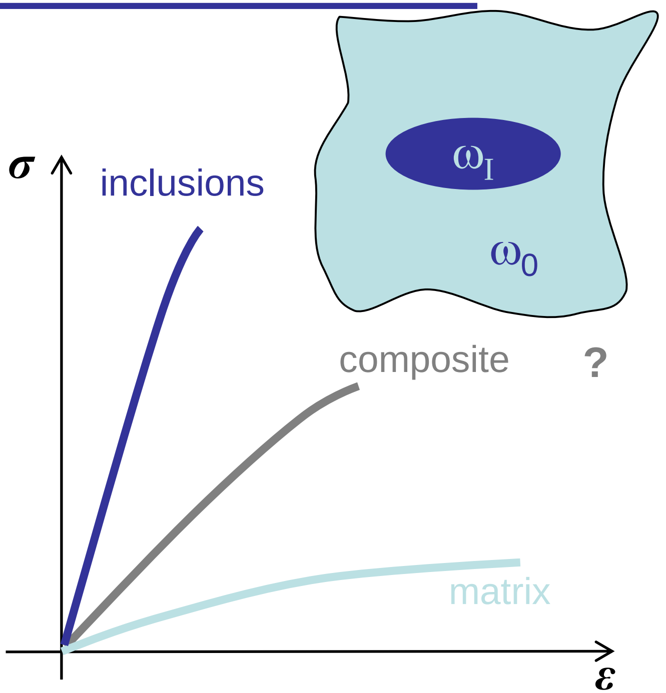


- Mean-Field-Homogenization: Key principles
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$



    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon} \ ?$$

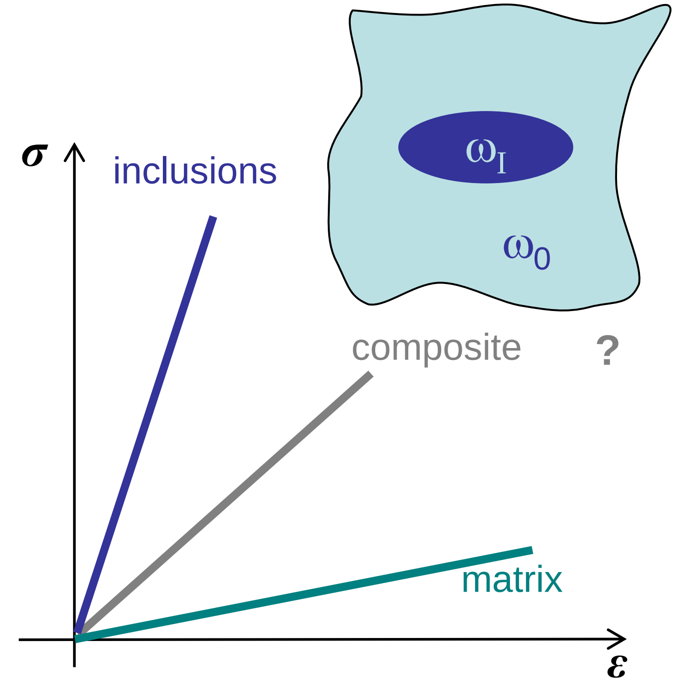
- Mean-Field-Homogenization: Key principles (2)
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{0} = \overline{\boldsymbol{C}}_{0} : \boldsymbol{\varepsilon}_{0} \end{cases}$$

    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby tensor

$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$



- Mean-Field-Homogenization: Key principles (2)
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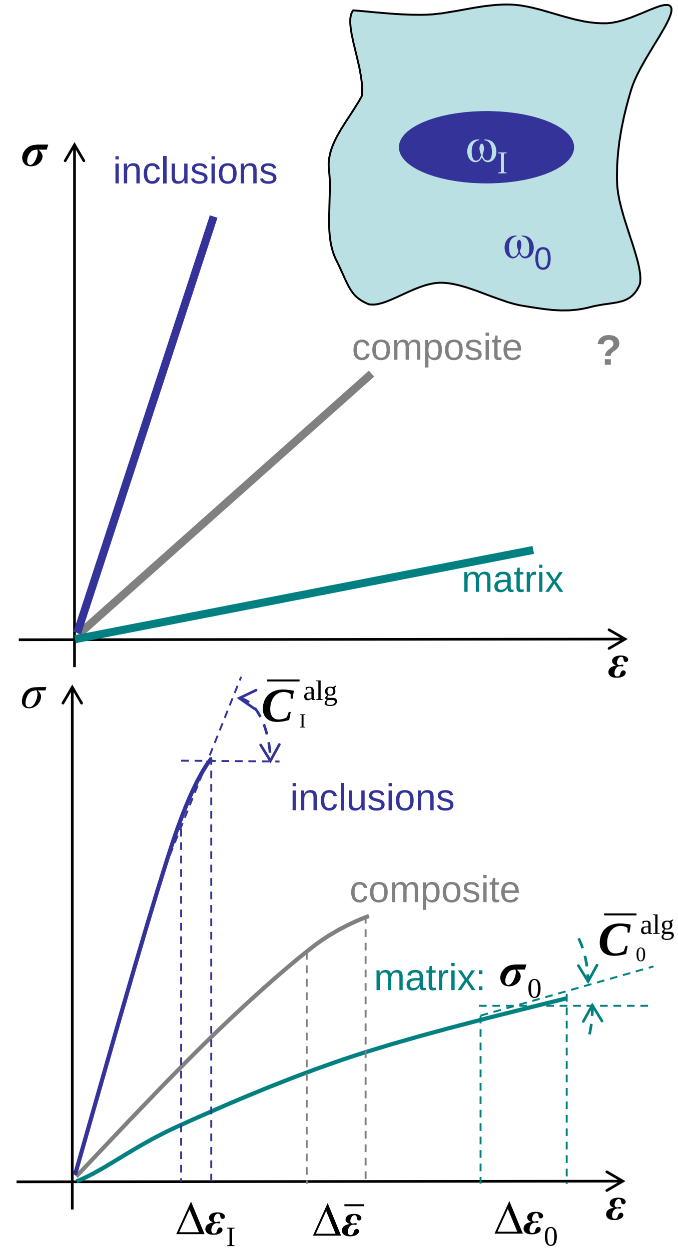
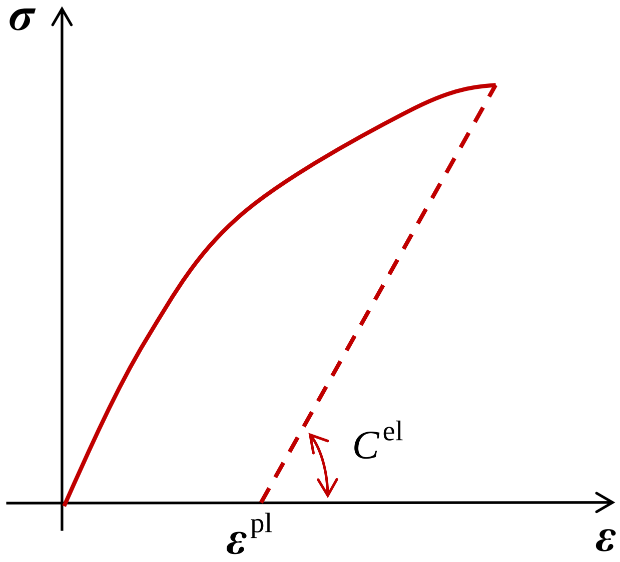
with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

  - Non-linear materials
    - Define a Linear Comparison Composite (LCC)
    - Common approach: incremental tangent

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$



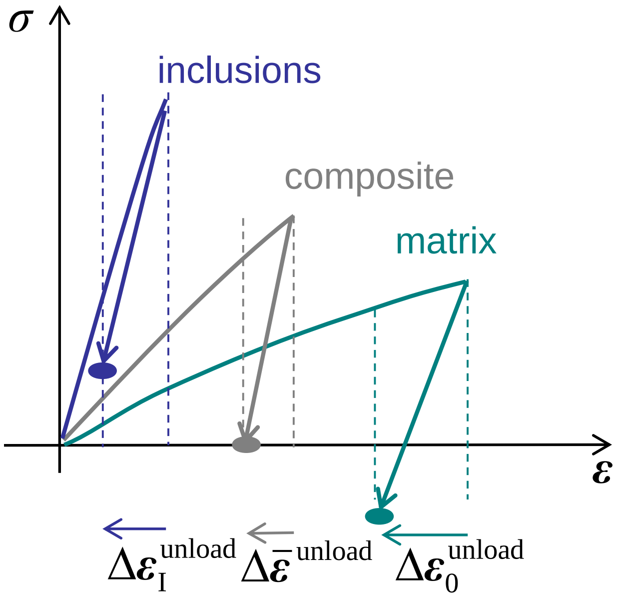
- Material model
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$



Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

New Linear Comparison Composite (LCC)



- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

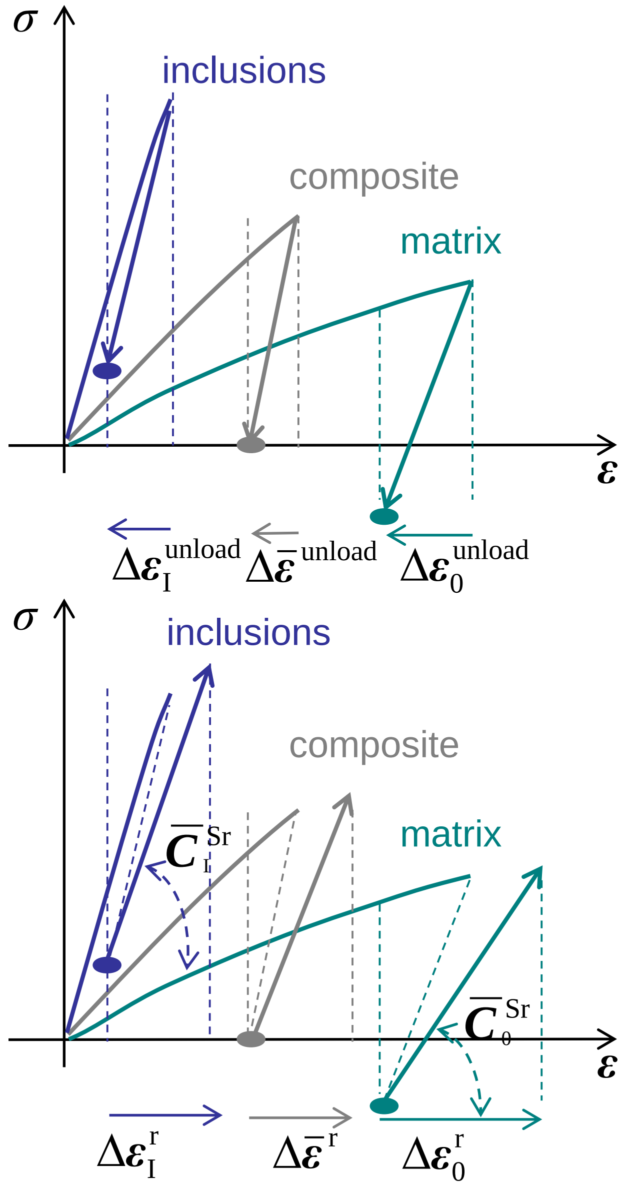
New Linear Comparison Composite (LCC)

  - Apply MFH from unloaded state
    - New strain increments (>0)

$$\Delta \boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}/0} = \Delta \boldsymbol{\varepsilon}_{\mathrm{I}/0} + \Delta \boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

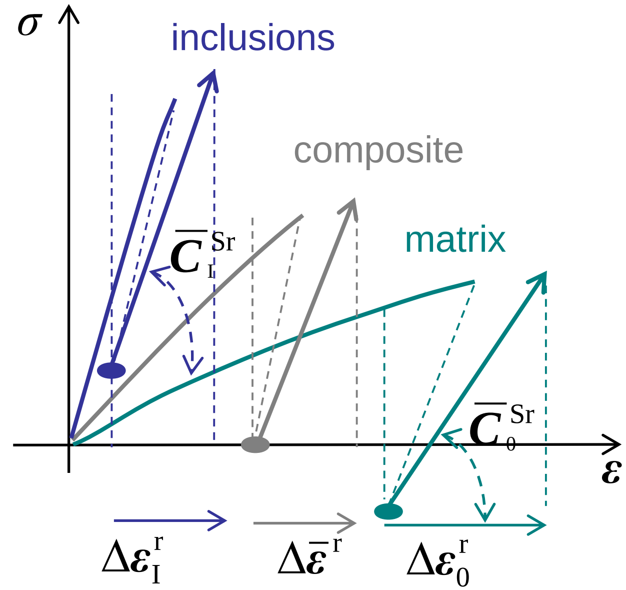
    - Use of secant operators

$$\Delta \boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta \boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$

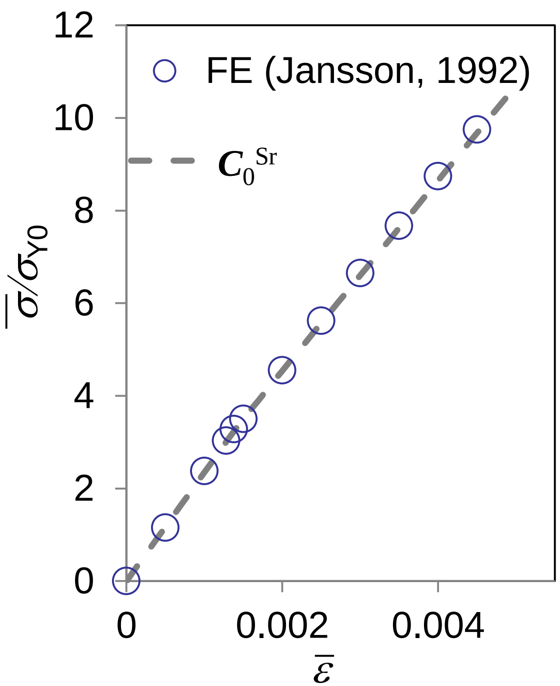


Incremental-secant mean-field-homogenization

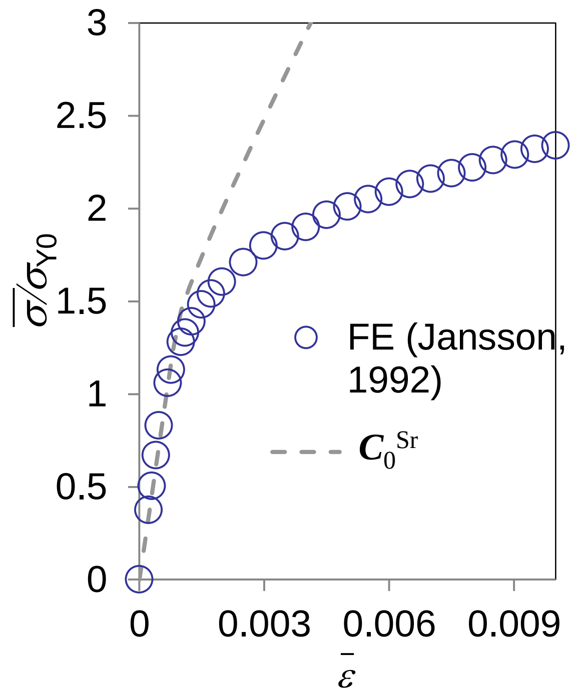
- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - For inclusions with high hardening (elastic)
    - Model is too stiff



Longitudinal tension



Transverse loading



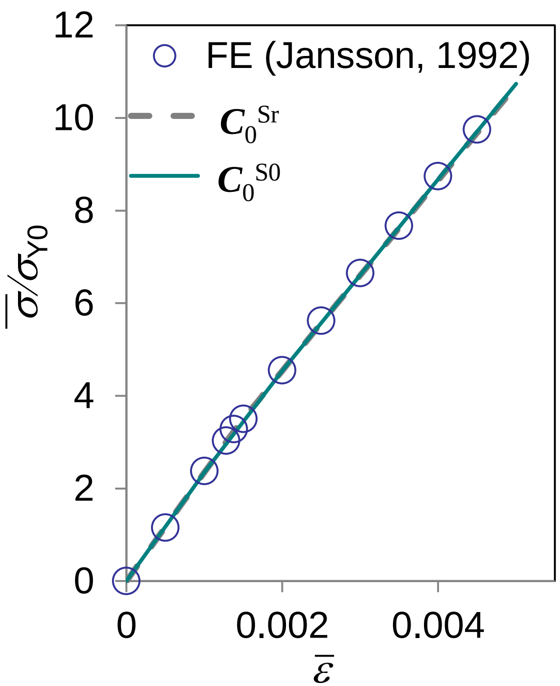
$$f(\boldsymbol{\sigma}, p) = \overline{\boldsymbol{\sigma}}^{eq} - \sigma^{Y} - R(\overline{p}) \leq 0$$

$\overline{\boldsymbol{\sigma}}^{eq}$ is underestimated

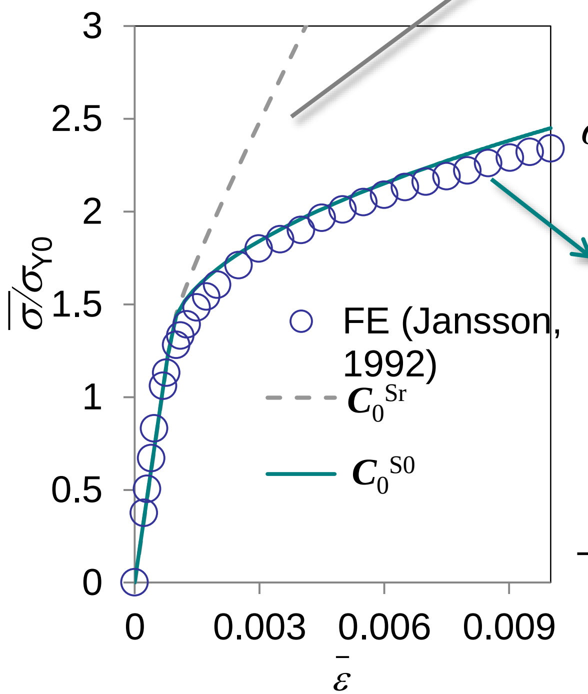
Incremental-secant mean-field-homogenization

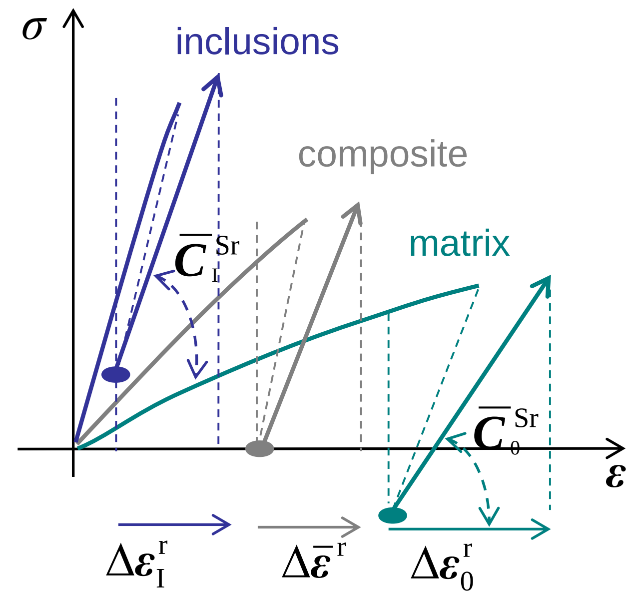
- Zero-incremental-secant method (2)
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if negative residual stress

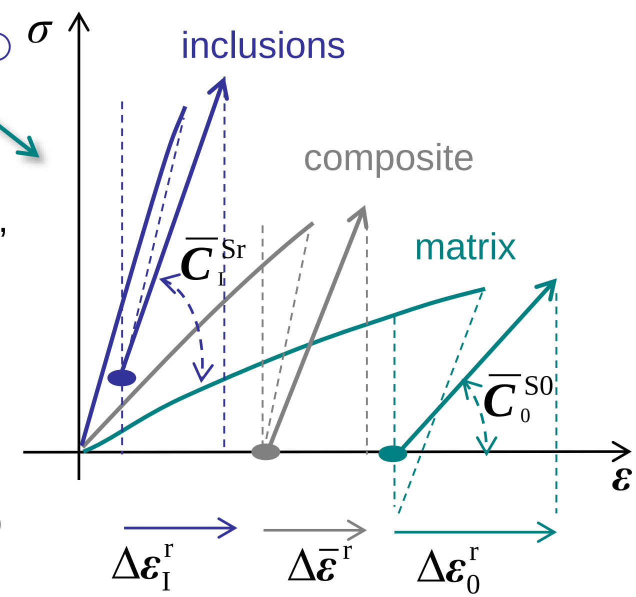
Longitudinal tension



Transverse loading







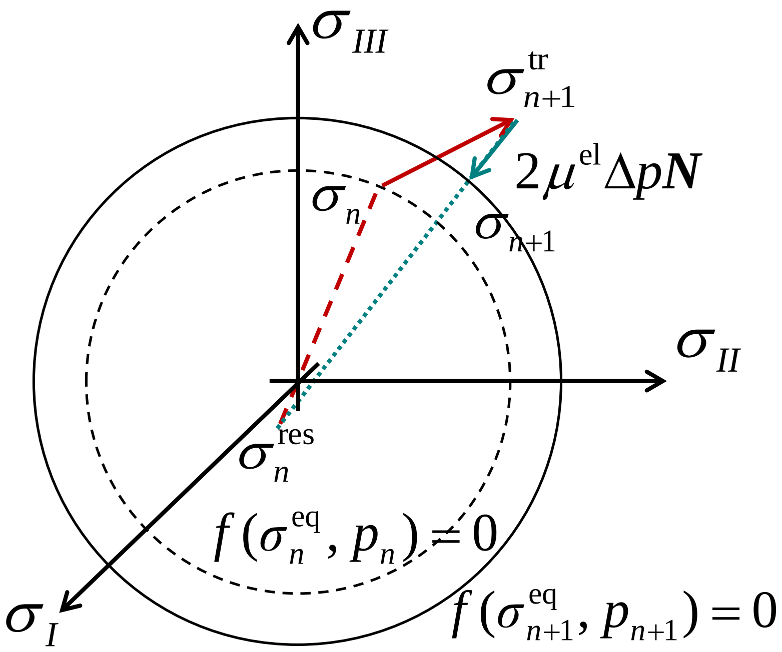
Incremental-secant mean-field-homogenization

- The secant operators
  - Stress tensor (2 forms)

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I/0}} = \boldsymbol{\sigma}_{\mathrm{I/0}}^{\mathrm{res}} + \overline{\boldsymbol{C}}_{\mathrm{I/0}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I/0}}^{\mathrm{r}} \\ \boldsymbol{\sigma}_{\mathrm{I/0}} = \overline{\boldsymbol{C}}_{\mathrm{I/0}}^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I/0}}^{\mathrm{r}} \end{cases}$$

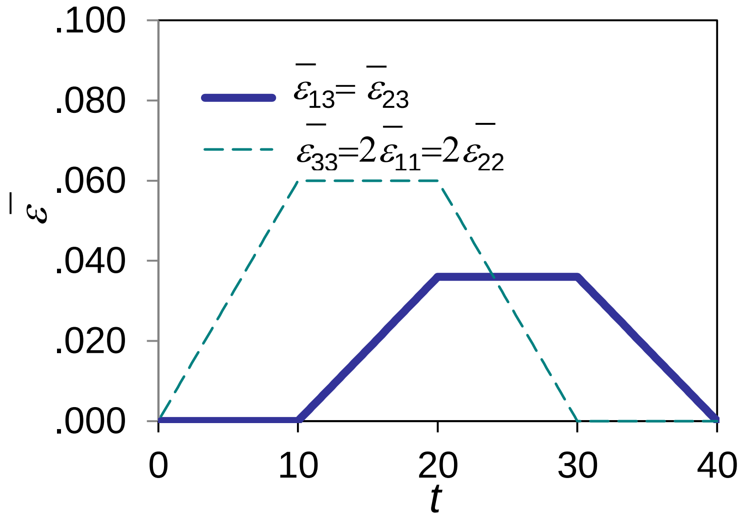
  - Radial return direction toward residual stress
    - First order approximation in the strain increment (and not in the total strain)
    - Exact for the zero-incremental-secant method
  - The secant operators are naturally isotropic

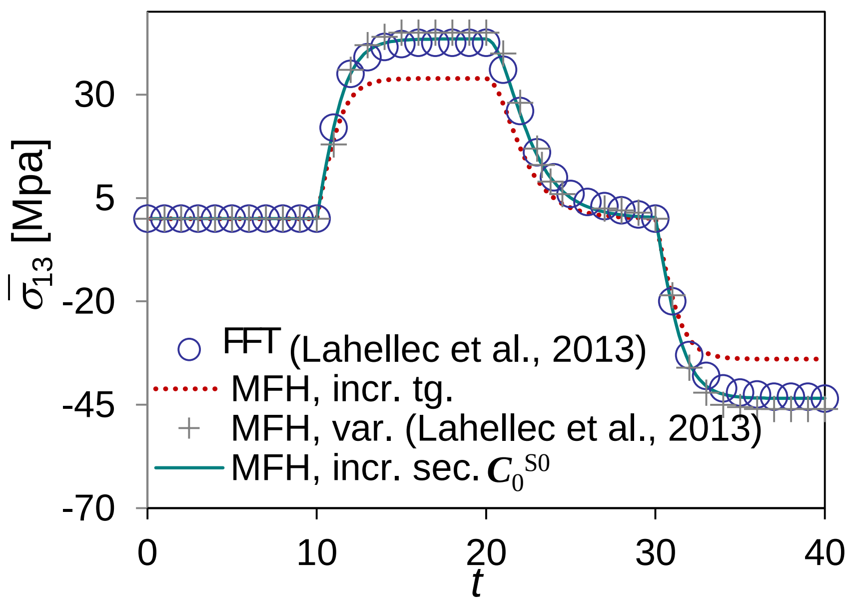
$$\begin{cases} \overline{\boldsymbol{C}}^{\mathrm{Sr}} = 3\kappa^{\mathrm{el}}\boldsymbol{I}^{\mathrm{vol}} + 2\left(\mu^{\mathrm{el}} - 3\dfrac{\mu^{\mathrm{el}^2}\Delta p}{\left(\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_n^{\mathrm{res}}\right)^{\mathrm{eq}}}\right)\boldsymbol{I}^{\mathrm{dev}} \\ \overline{\boldsymbol{C}}^{\mathrm{S0}} = 3\kappa^{\mathrm{el}}\boldsymbol{I}^{\mathrm{vol}} + 2\left(\mu^{\mathrm{el}} - 3\dfrac{\mu^{\mathrm{el}^2}\Delta p}{\boldsymbol{\sigma}_{n+1}^{\mathrm{eq}}}\right)\boldsymbol{I}^{\mathrm{dev}} \end{cases}$$

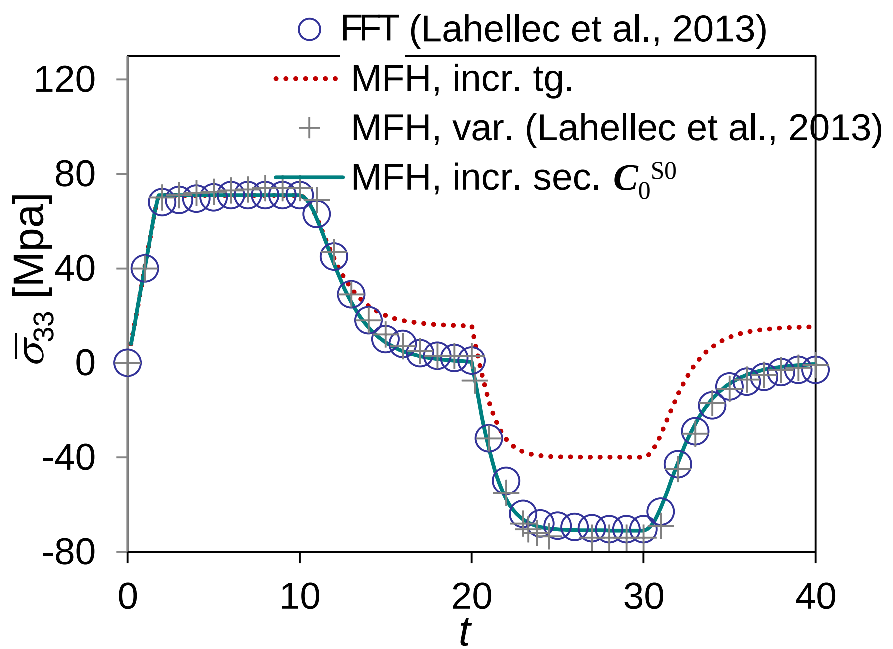


Incremental-secant mean-field-homogenization

- Verification of the method
  - Spherical inclusions
    - 17 % volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-proportional loading







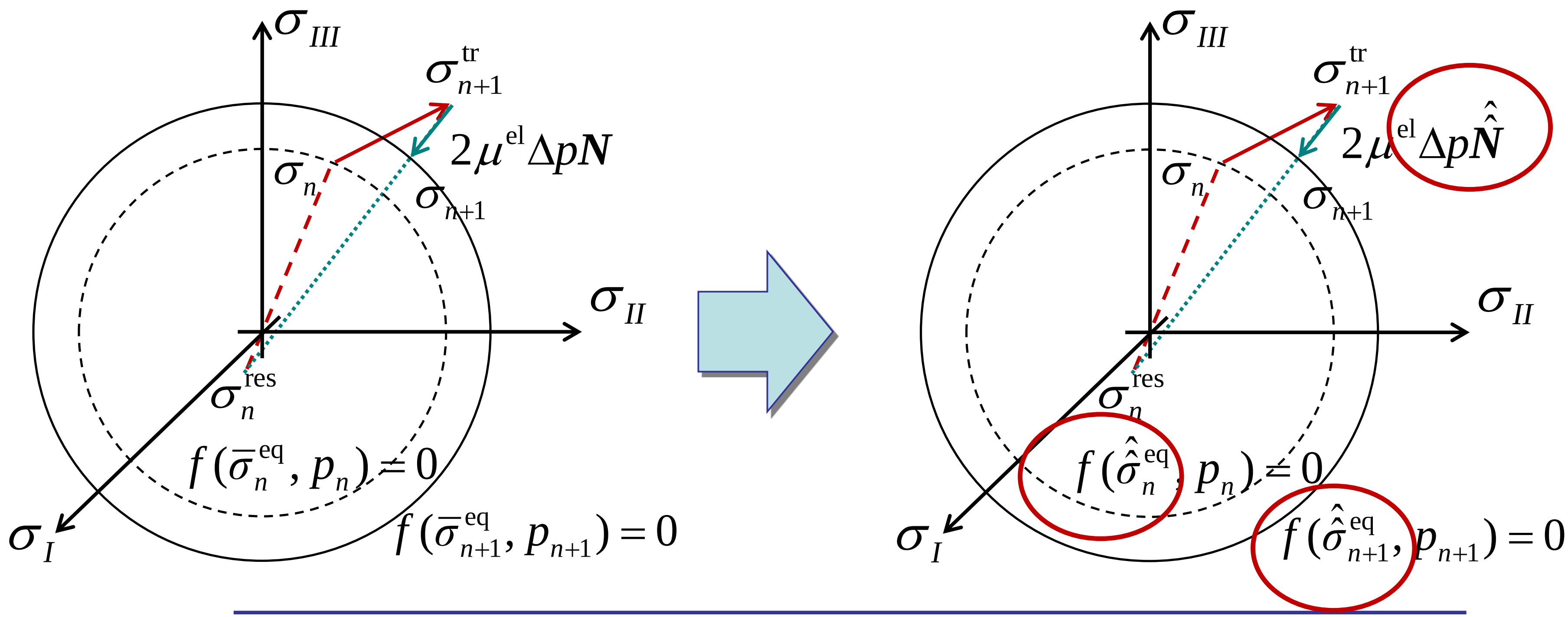
Incremental-secant mean-field-homogenization

- Second-statistical moment estimation of the von Mises stress
  - J2-plasticity involves quadratic terms
    - First statistical moment (mean value) not fully representative

$$\bar{\sigma}^{\text{eq}}_{\text{I/0}} = \sqrt{\frac{3}{2}\bar{\boldsymbol{\sigma}}^{\text{dev}}_{\text{I/0}} : \bar{\boldsymbol{\sigma}}^{\text{dev}}_{\text{I/0}}}$$

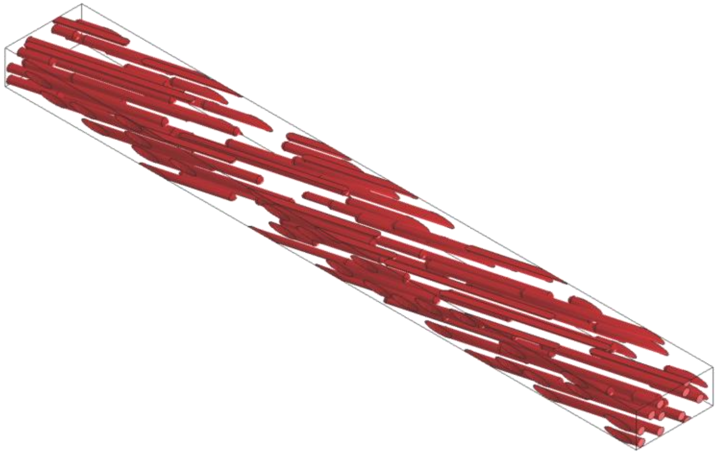
    - Use second statistical moment estimations to define the yield surface

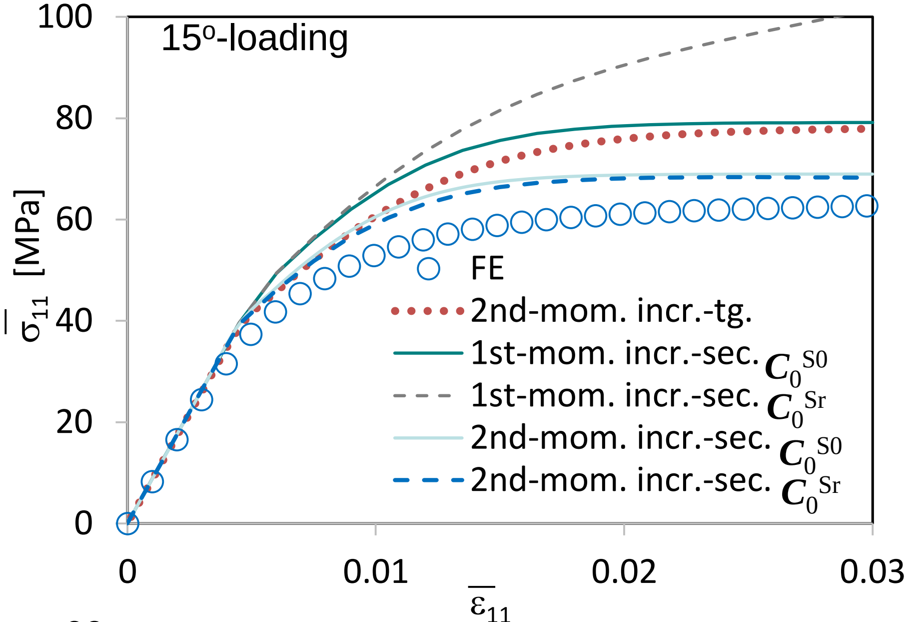
$$\hat{\hat{\sigma}}^{\text{eq}}_{\text{I/0}} = \sqrt{\frac{3}{2}\mathbf{I}^{\text{dev}} :: \left\langle \boldsymbol{\sigma}_{\text{I/0}} \otimes \boldsymbol{\sigma}_{\text{I/0}} \right\rangle_{\omega_{\text{I/0}}}} \geq \bar{\sigma}^{\text{eq}}_{\text{I/0}}$$

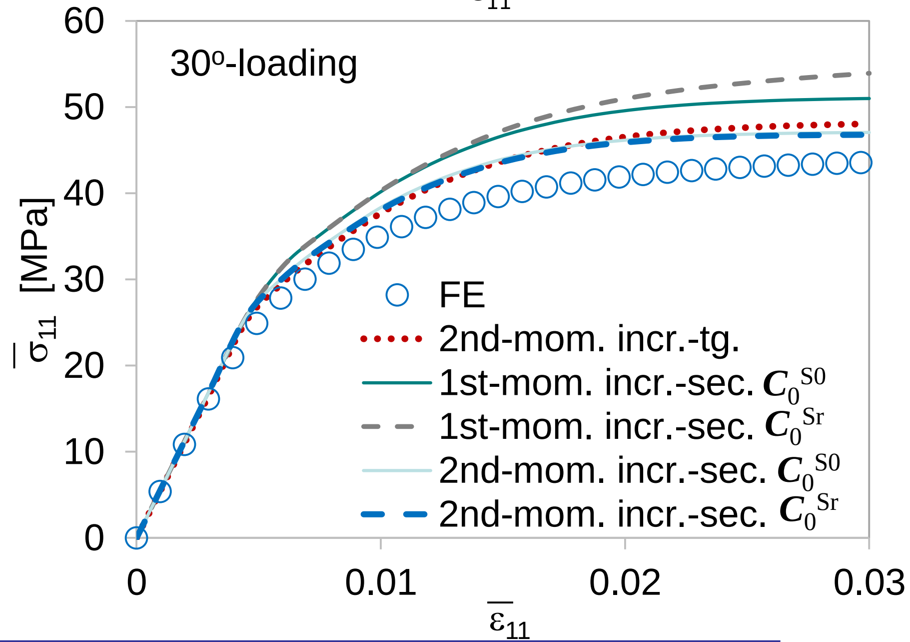


Incremental-secant mean-field-homogenization

- Short fibre reinforced matrix
  - Elastic short fibres
    - Aspect ratio of 20
    - 15.87 % volume fraction
    - 15º- & 30º- orientation
  - Elastic-plastic matrix

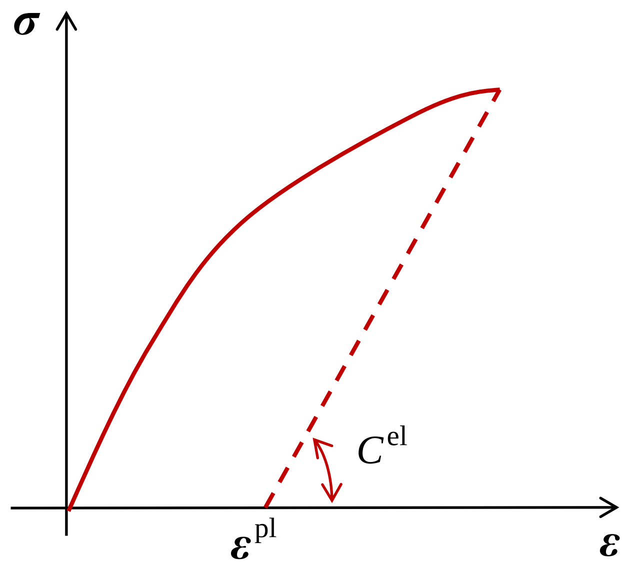


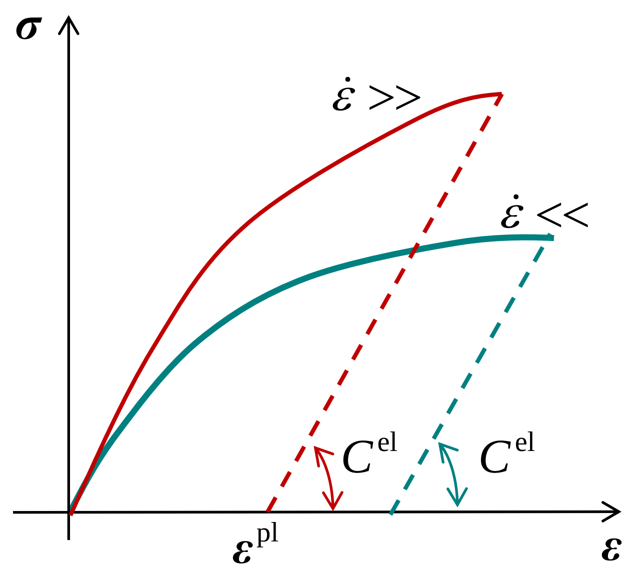




- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Elasto-visco-plastic material
    - Plastic flow $\dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}} = \dot{p}\mathbf{N}$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p)$
    - Flow function $\dot{p} = g_v(\boldsymbol{\sigma}^{eq}, p)$
    - Perzyna visco-plasticity model

$$g_v(\sigma^{eq}, p) = \kappa\left(\frac{f(\boldsymbol{\sigma}^{eq}, p)}{\sigma^{Y} + R(p)}\right)^{m}$$





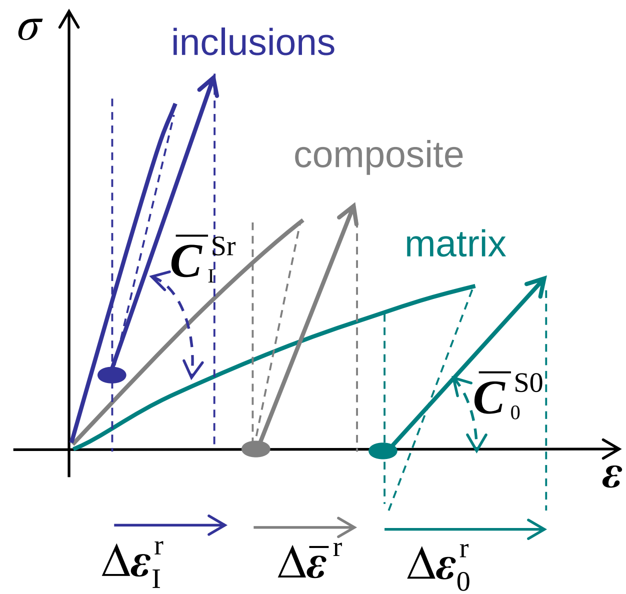
- Incremental-secant mean-field-homogenization
  - For soft matrix response
    - Remove residual stress in matrix
    - Or use second moment estimates
  - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{\mathrm{r}} = v_0 \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} + v_{\mathrm{I}} \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \mathbf{B}^{\varepsilon}\left(\mathrm{I}, \bar{\mathbf{C}}_0^{\mathrm{Sr}}, \bar{\mathbf{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \text{ or } \\ \qquad \mathbf{B}^{\varepsilon}\left(\mathrm{I}, \bar{\mathbf{C}}_0^{\mathrm{S0}}, \bar{\mathbf{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

  - With the stress tensors

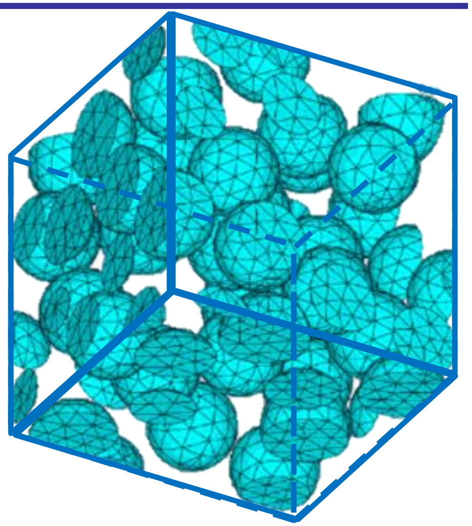
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{\mathrm{I}} = \boldsymbol{\sigma}_{\mathrm{I}}^{\mathrm{res}} + \bar{\mathbf{C}}_{\mathrm{I}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = \boldsymbol{\sigma}_0^{\mathrm{res}} + \bar{\mathbf{C}}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \text{ or } \bar{\mathbf{C}}_0^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

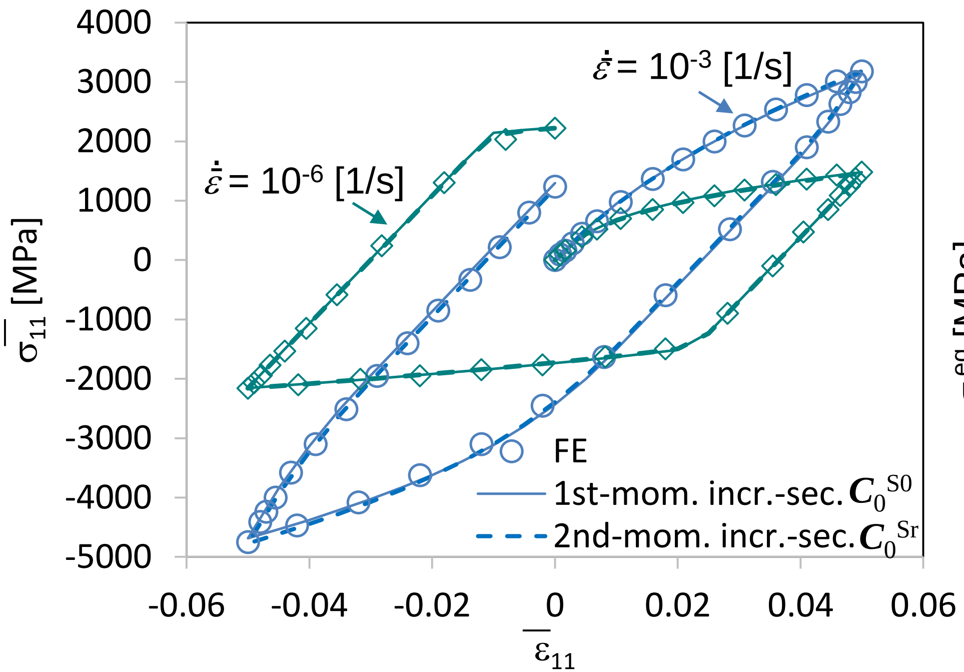
Same as for elasto-plasticity

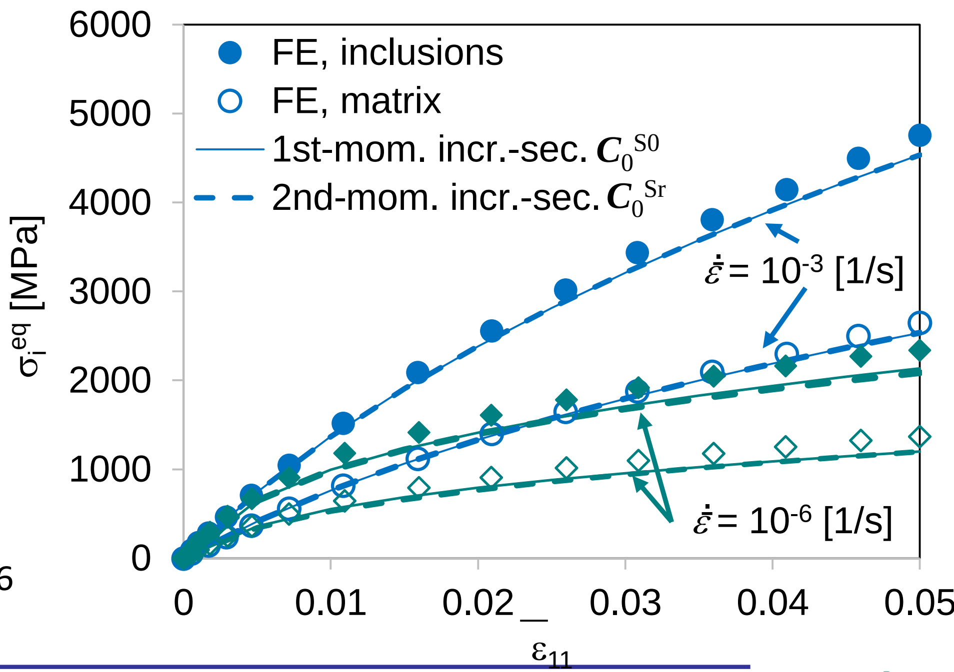


Visco-plasticity

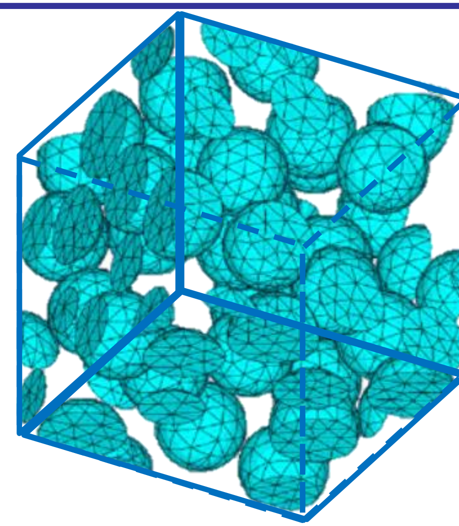
- Short fibre reinforced matrix (1)
  - Elasto-visco-plastic short fibres
    - Spherical
    - 30 % volume fraction
  - Elasto-visco-plastic matrix

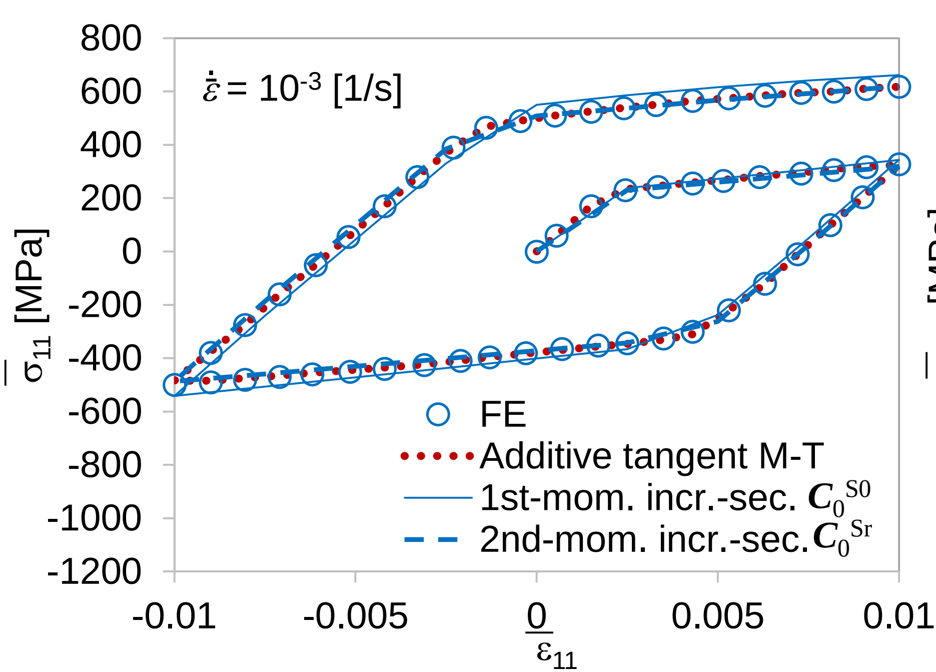


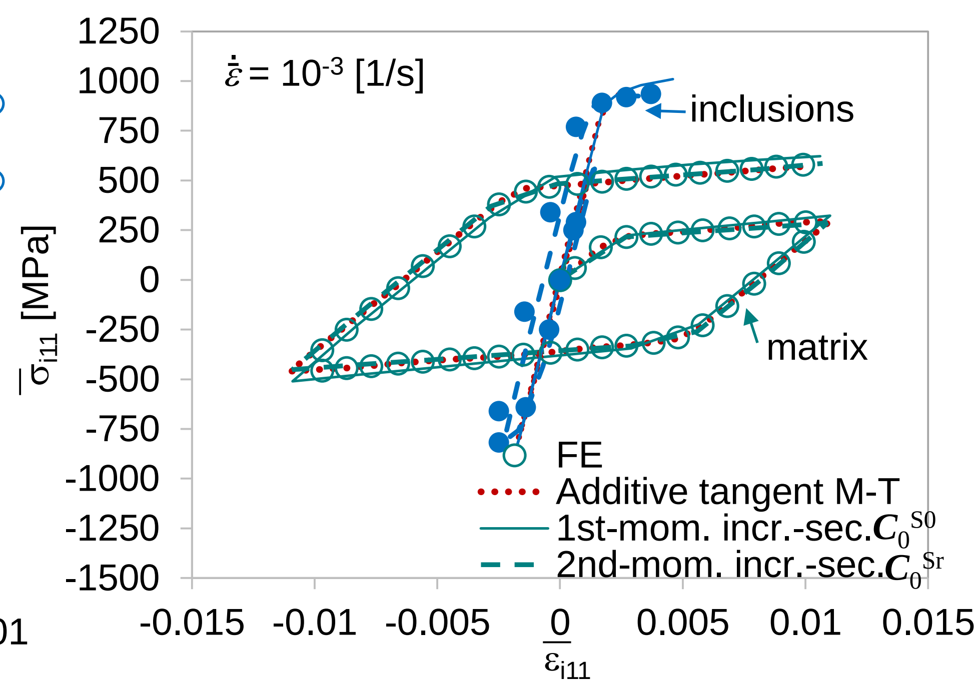




- Short fibre reinforced matrix (2)
  - Elasto-visco-plastic short fibres
    - Spherical
    - 10 % volume fraction
  - Elasto-visco-plastic matrix







Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
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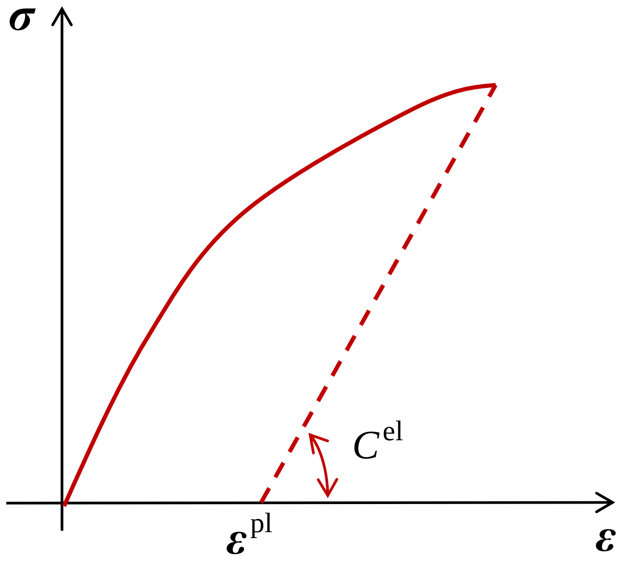
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$



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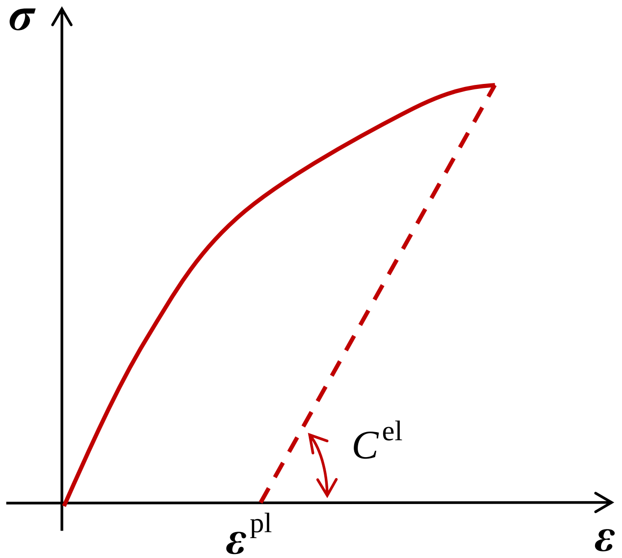
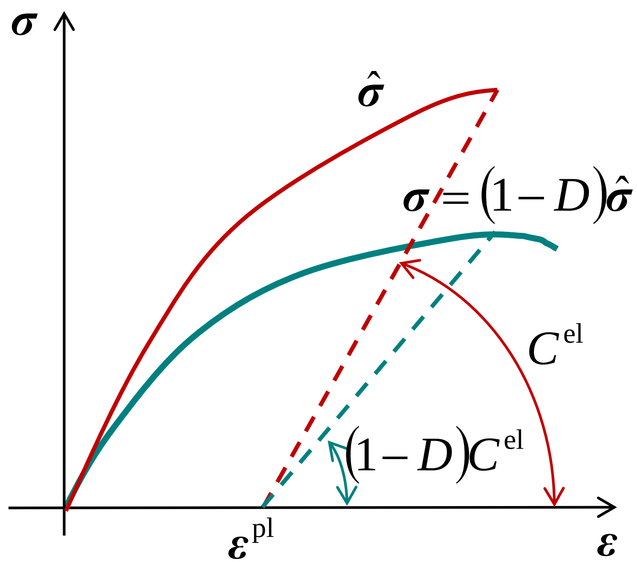
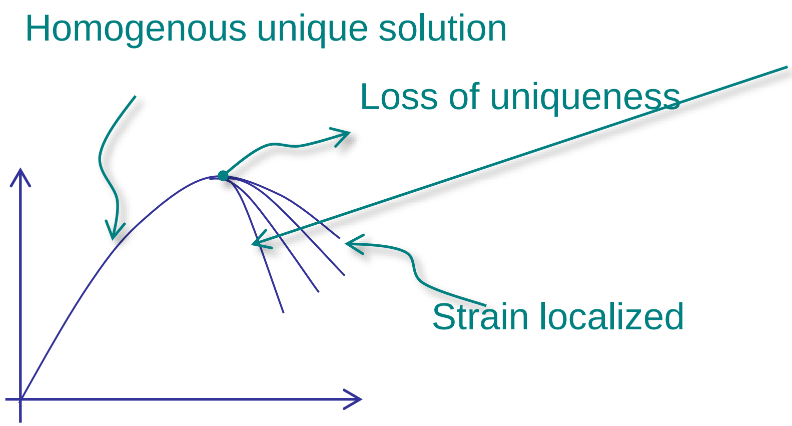
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$



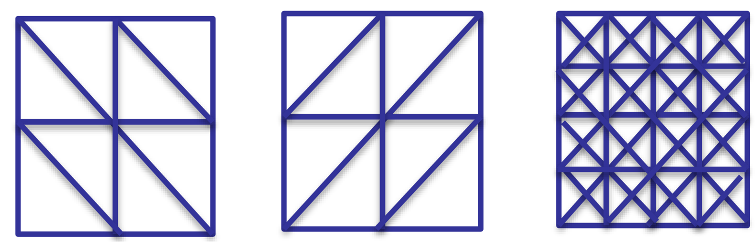


Non-local damage-enhanced MFH

- Finite element solutions with strain softening suffer from:
  - The loss of the solution uniqueness and strain localization
  - Mesh dependency



The numerical results change with the size of mesh and direction of mesh



The numerical results change without convergence

- Solution: Implicit non-local approach [Peerlings et al 96, Geers et al 97, …]
  - A state variable is replaced by a non-local value reflecting the interaction between neighboring material points

$$\tilde{a}(\boldsymbol{x}) = \frac{1}{V_C}\int_{V_C} a(\boldsymbol{y})w(\boldsymbol{y};\boldsymbol{x})\mathrm{d}V$$

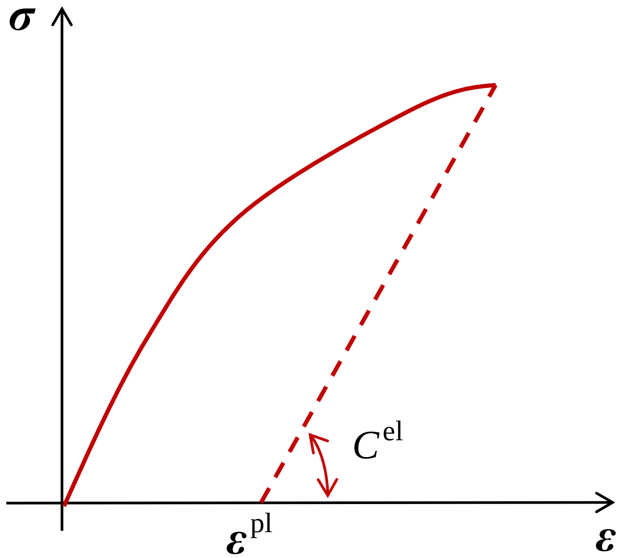
  - Use Green functions as weight $w(y; x)$

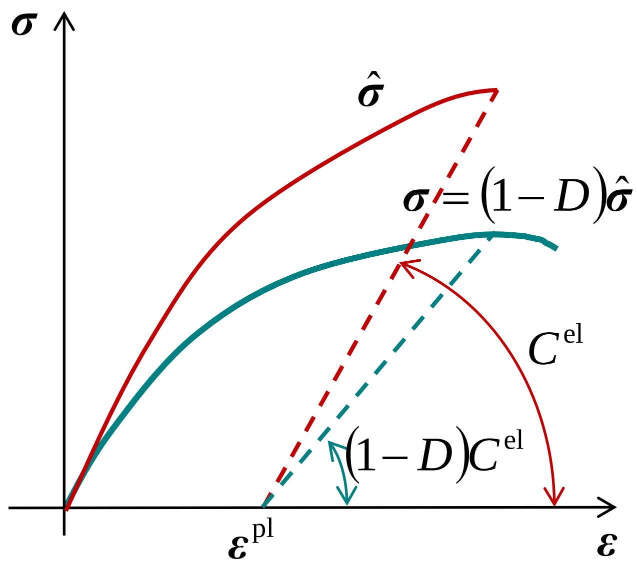
⇒ $\tilde{a} - c\nabla^2\tilde{a} = a$ ⇒ New degrees of freedom

Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$
  - Non-Local damage model
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta\tilde{p})$
    - Anisotropic governing equation $\tilde{p} - \nabla \cdot (\boldsymbol{c}_{\mathrm{g}} \cdot \nabla\tilde{p}) = p$
    - Linearization

$$\delta\boldsymbol{\sigma} = \left[(1 - D)\boldsymbol{C}^{\mathrm{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}}\right] : \delta\boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta\tilde{p}$$

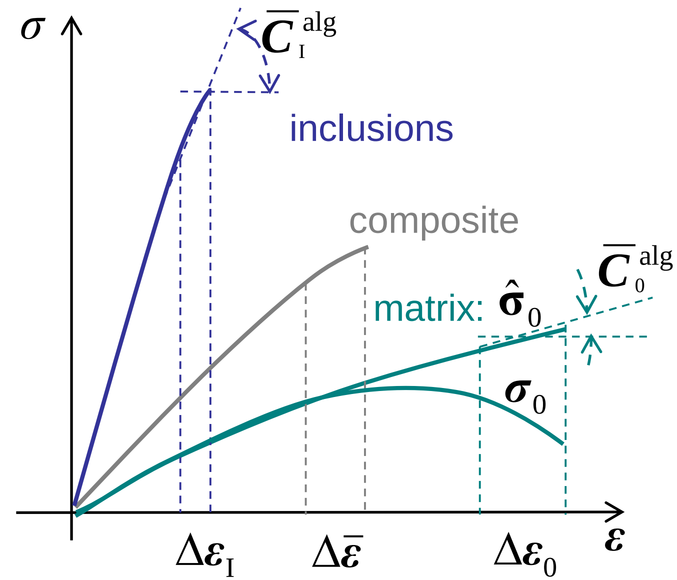




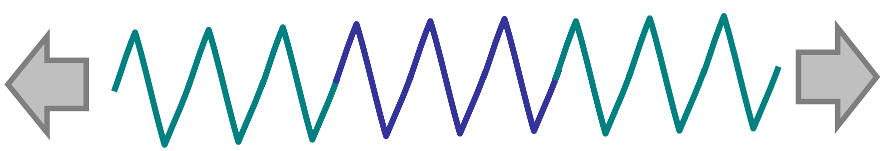
Mean-Field-Homogenization

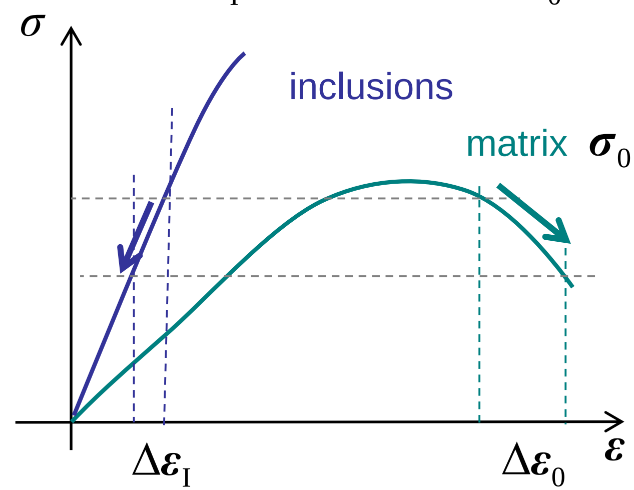
- Problem for materials with strain softening
  - Strain increments in the same direction

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_{0}^{\,\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\,\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{0}$$



  - Because of the damaging process, the fiber phase is elastically unloaded during matrix softening

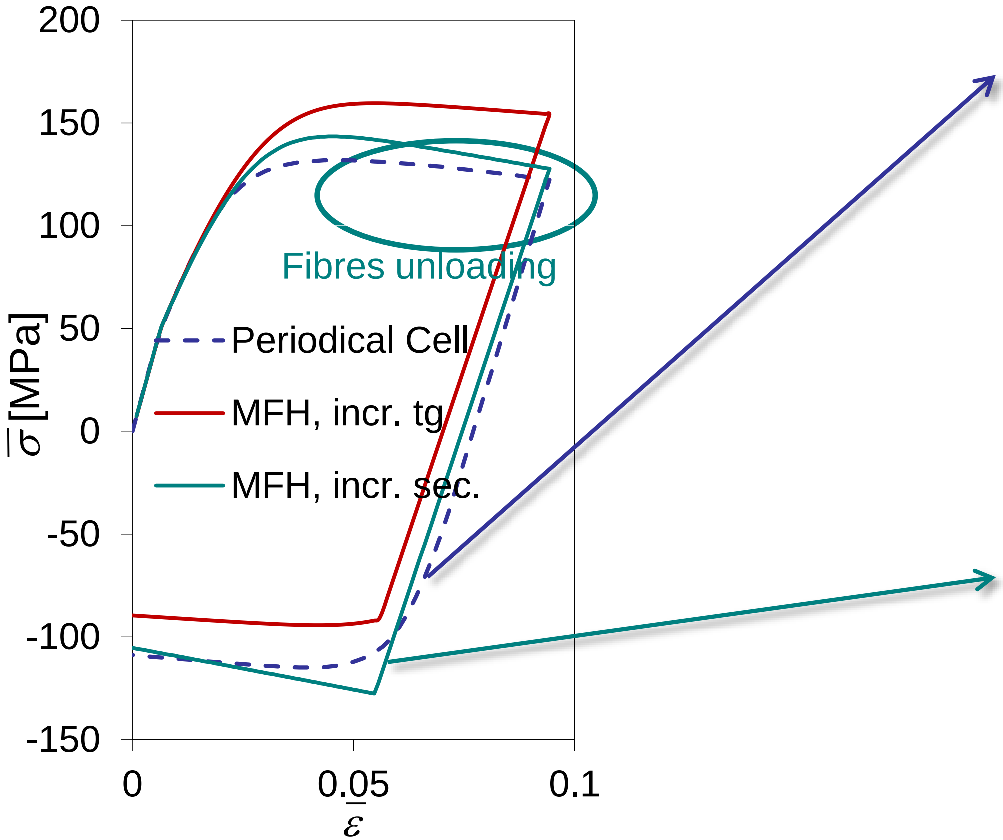


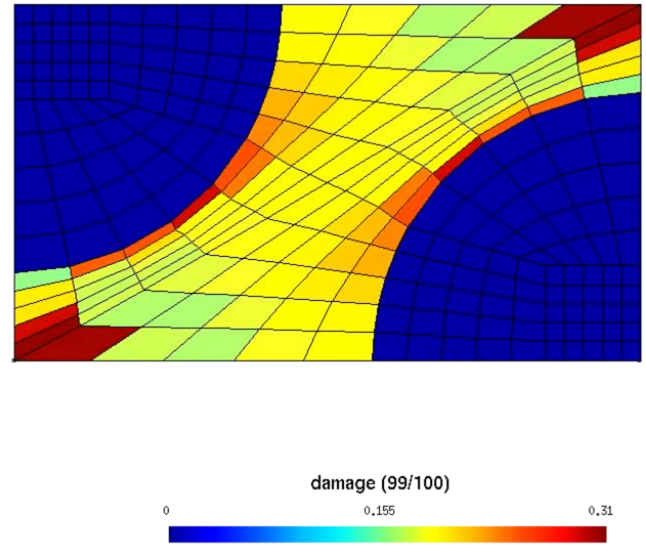


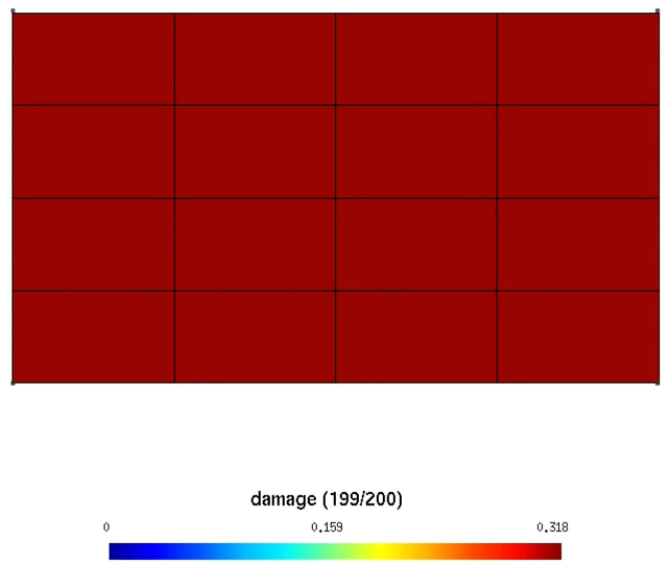
- Solution: new incremental-secant method
  - We need to define the LCC from another stress state

Non-local damage-enhanced MFH

- New results for damage
  - Fictitious composite
    - 50%-UD fibres
  - Elasto-plastic matrix with damage

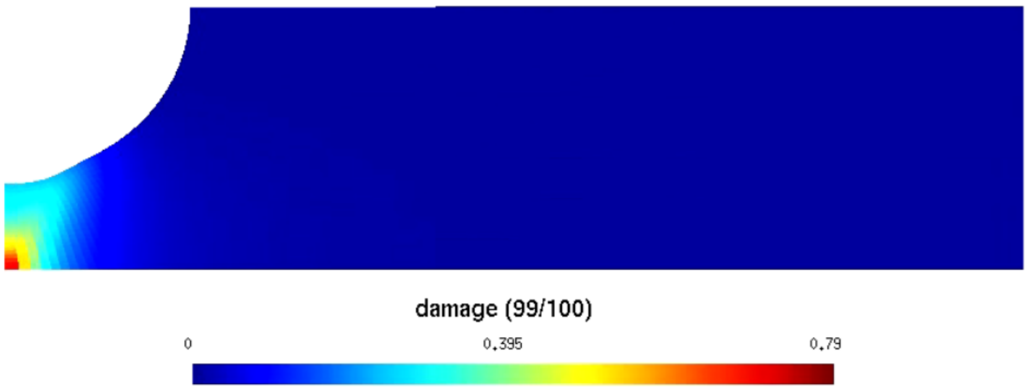


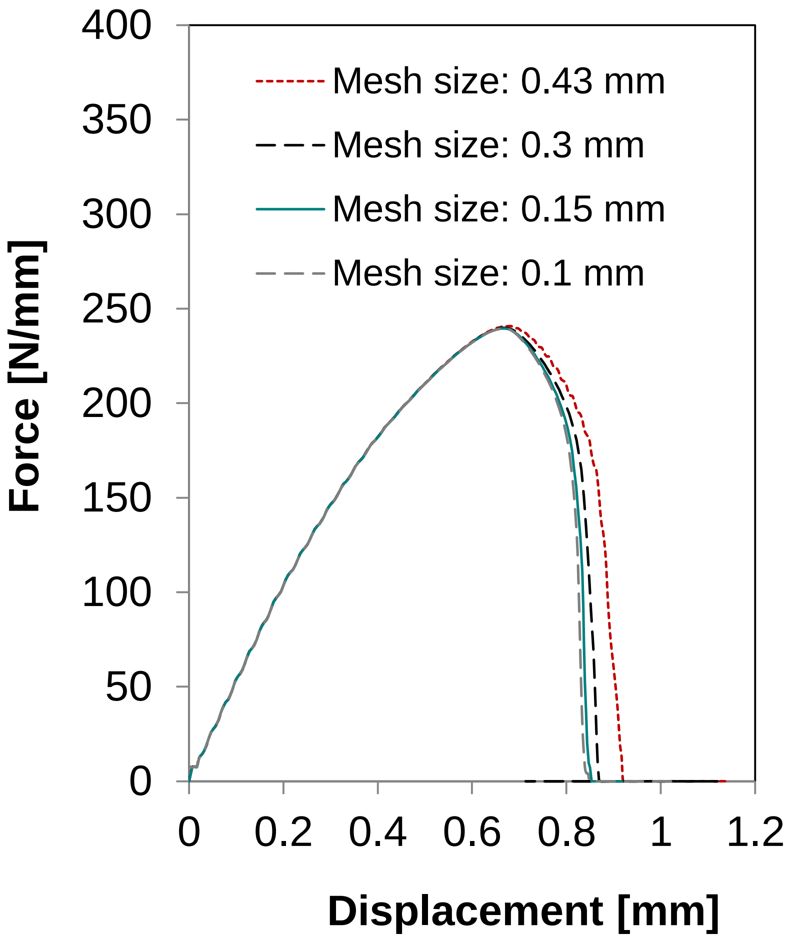




Non-local damage-enhanced MFH

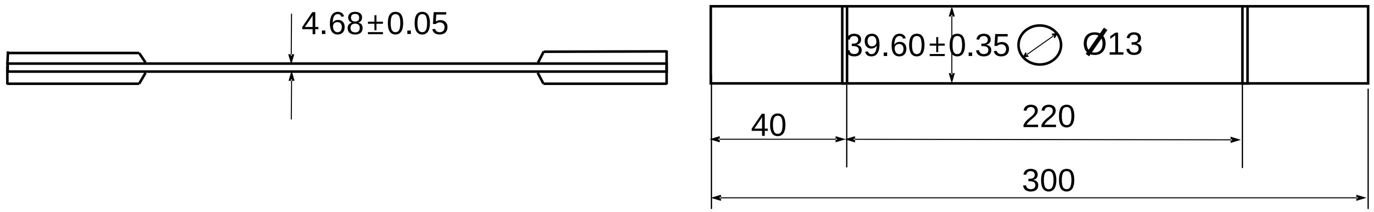
- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply



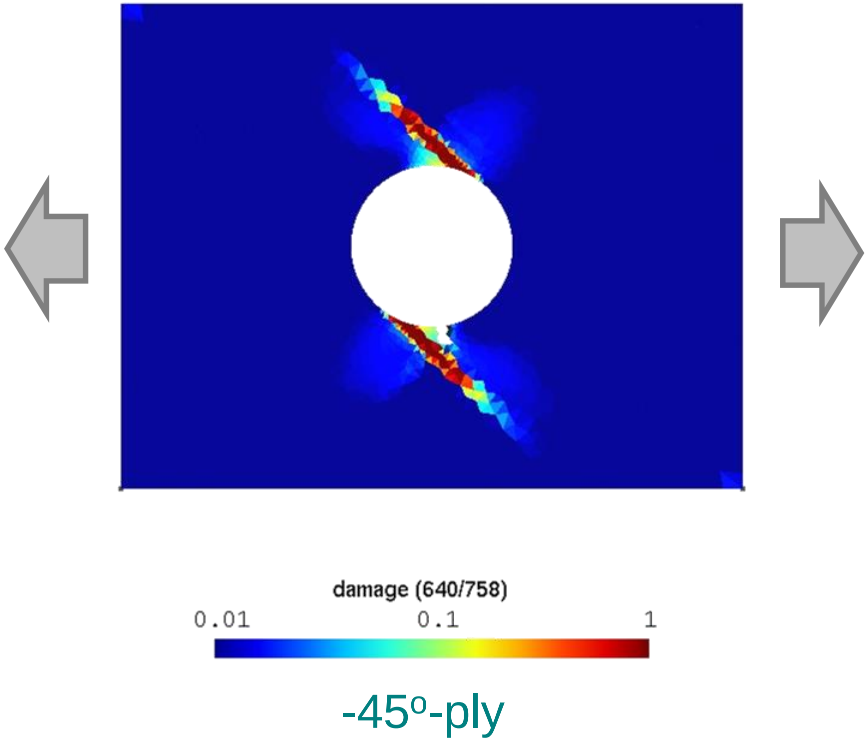


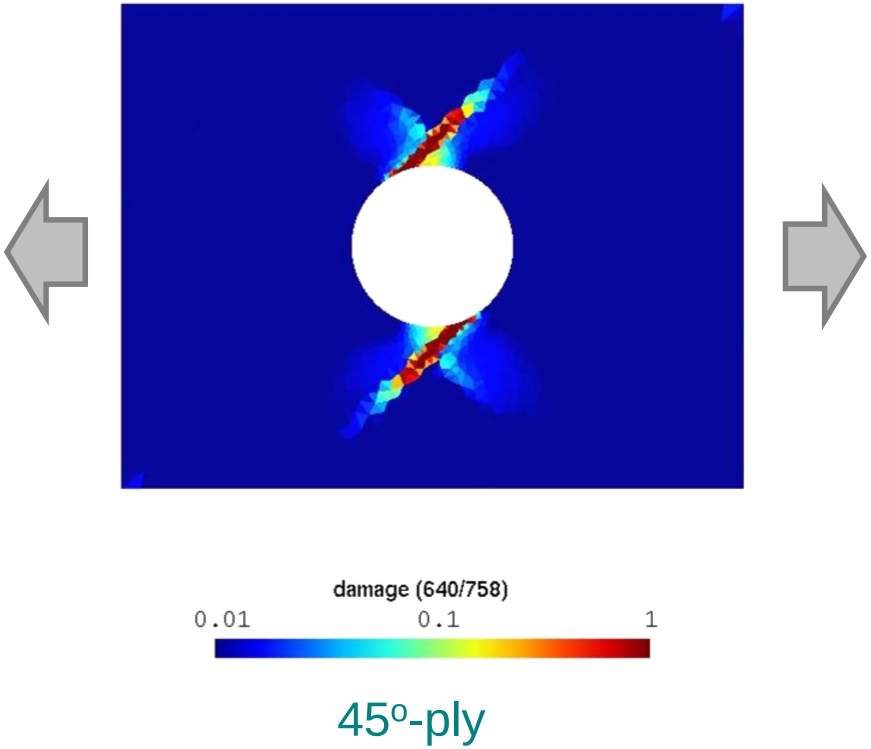
Non-local damage-enhanced MFH

- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate
  - Tensile test on several coupons



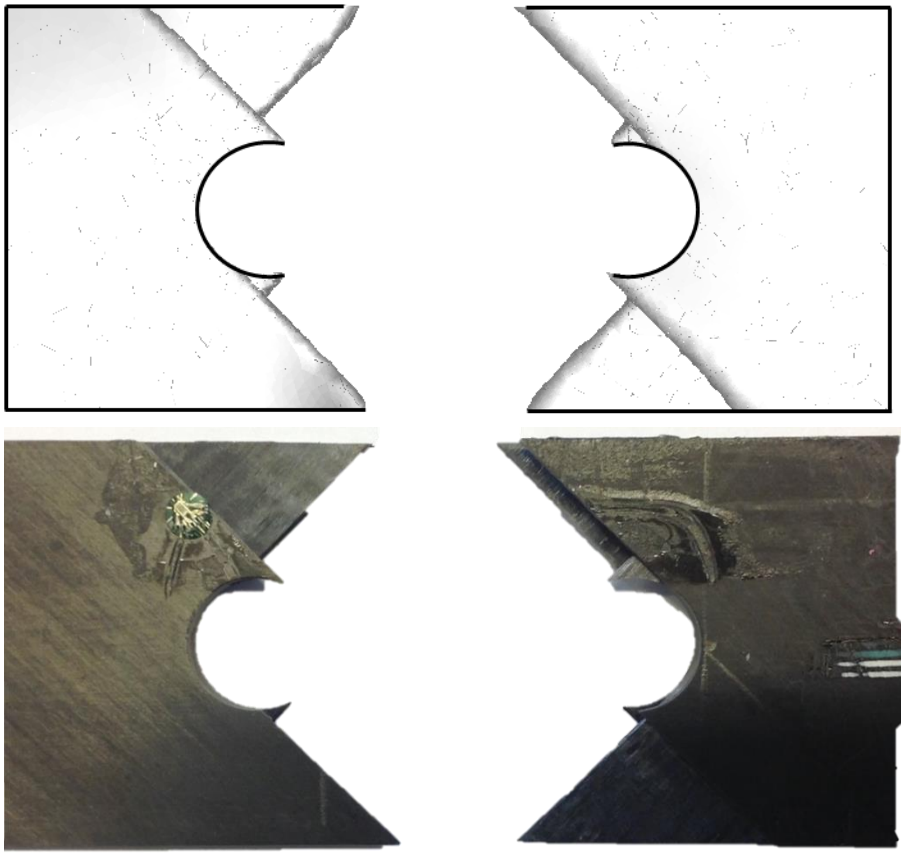
  - Propagation of the damaged zones in agreement with the fibre direction

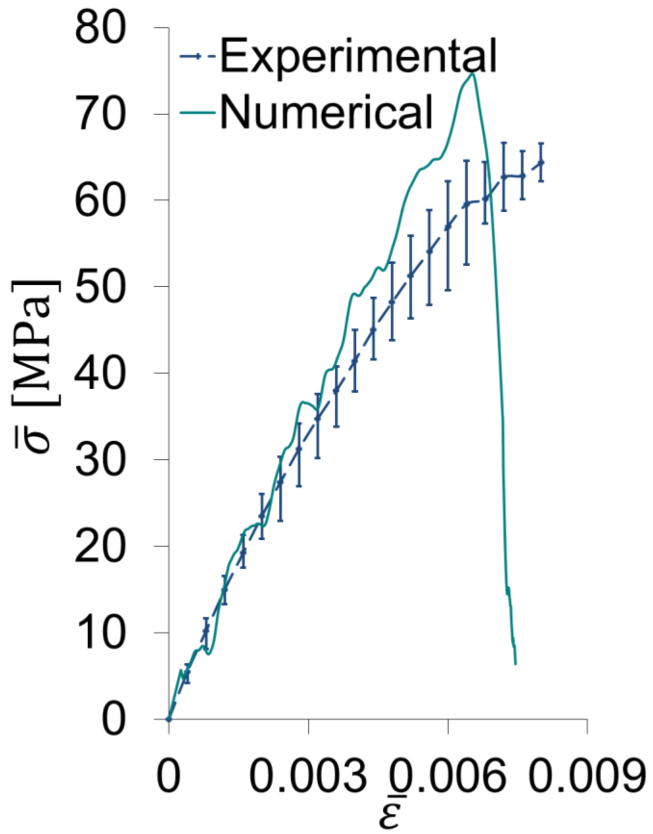




Non-local damage-enhanced MFH

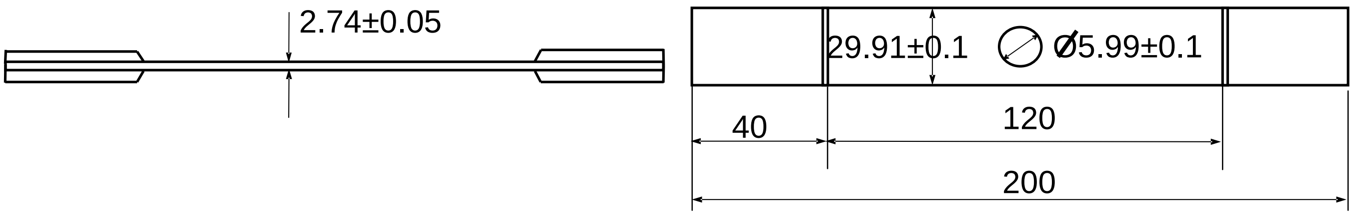
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate (2)
  - Predicted delamination zones in agreement with experiments
  - Tensile stress within 15 %



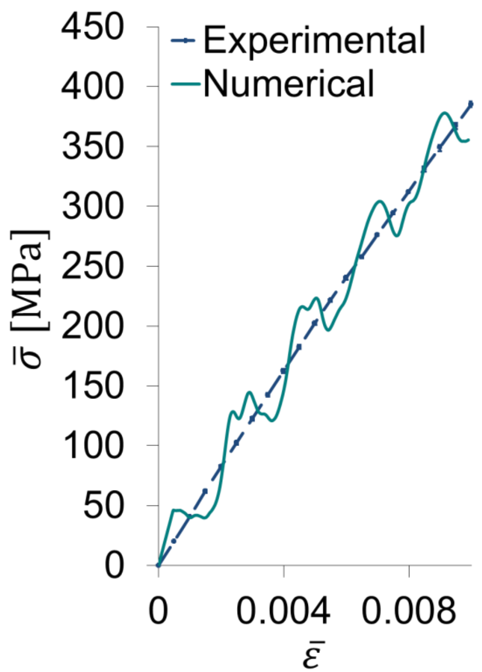


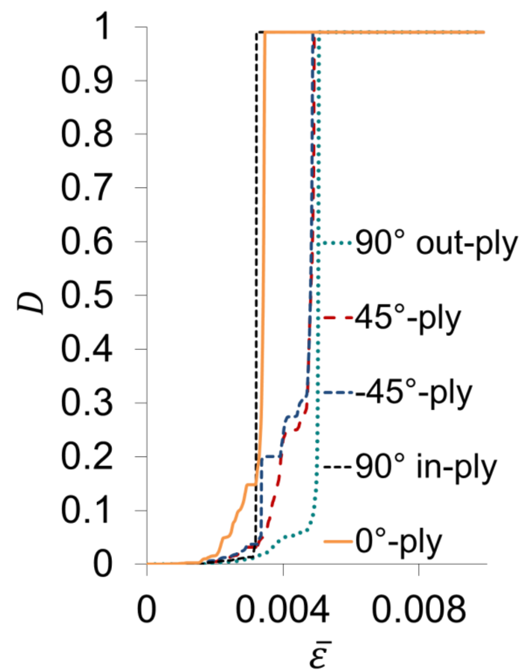
Non-local damage-enhanced MFH

- [90º / 45º / -45º / 90º / 0º]$_S$- open hole laminate
  - Tensile test on several coupons



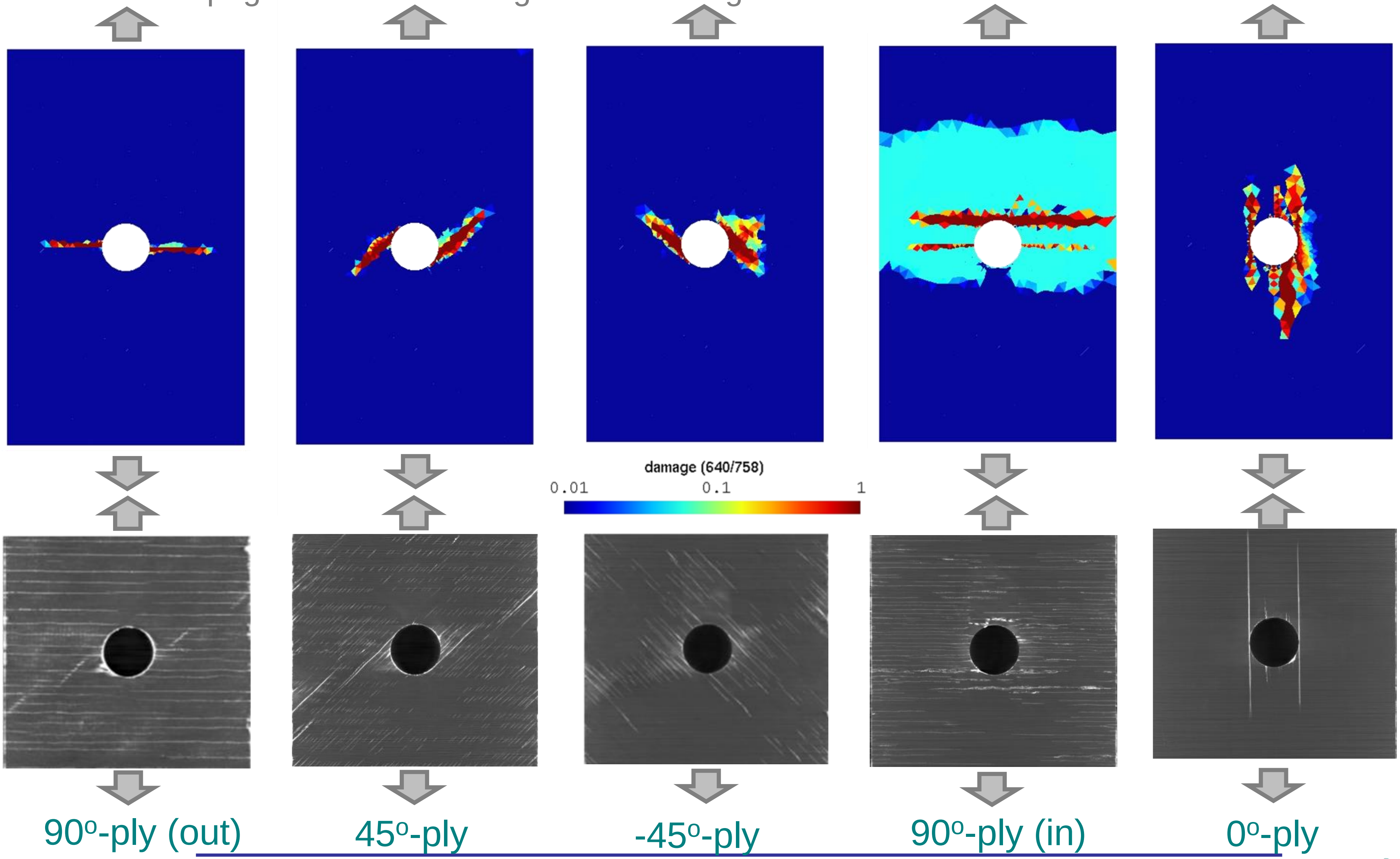
  - Predicted response (stress & maximum damage in each ply)





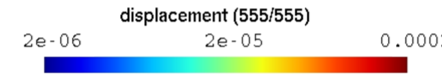
Non-local damage-enhanced MFH

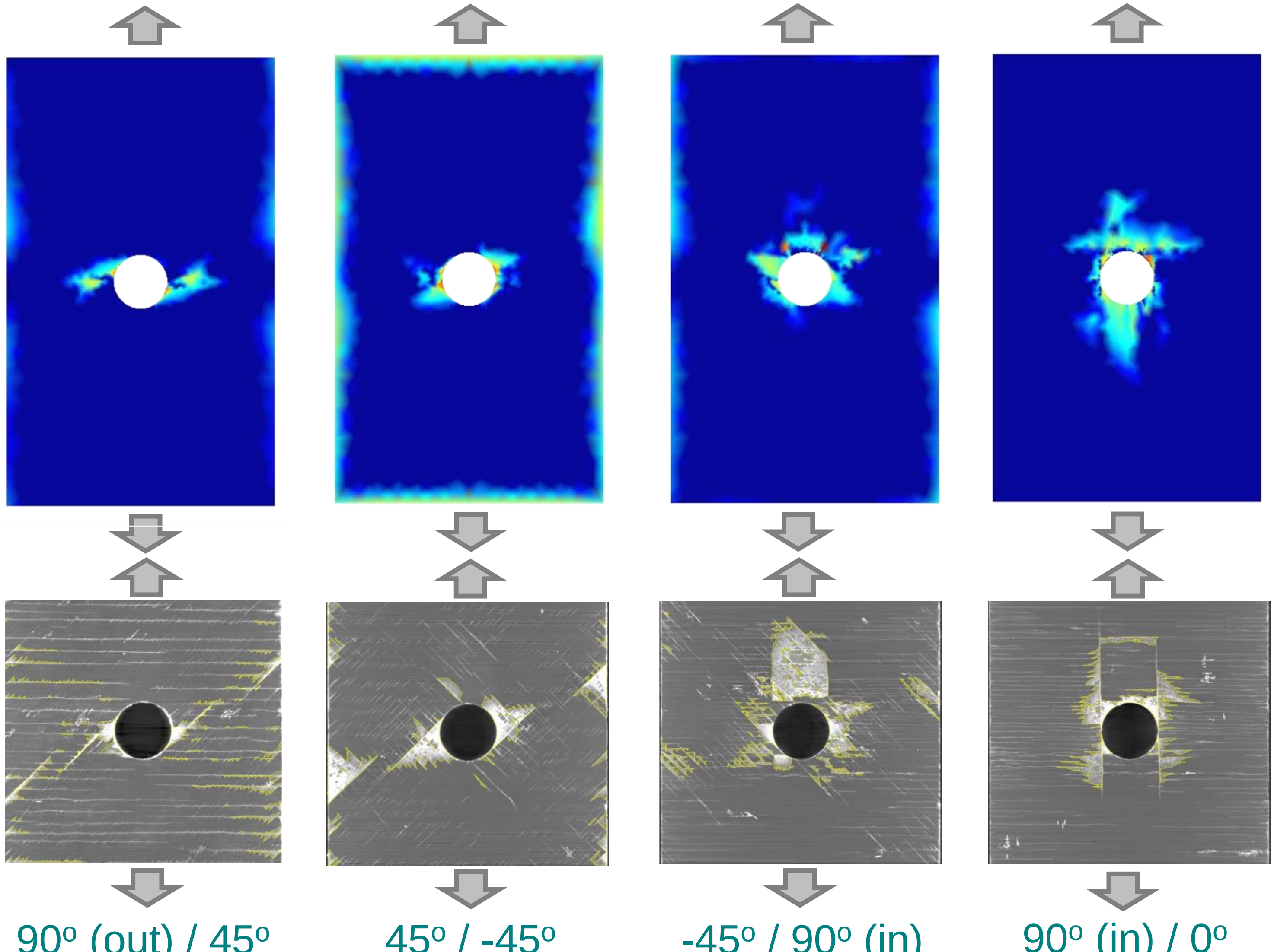
- $[90^o / 45^o / -45^o / 90^o / 0^o]_S$- open hole laminate (2)
  - Propagation of the damaged zones in agreement with the fibre direction



Non-local damage-enhanced MFH

- $[90^o / 45^o / -45^o / 90^o / 0^o]_S$- open hole laminate (3)
  - Predicted delamination zones in agreement with experiments





Conclusions

- New incremental secant Mean-Field-Homogenization
  - EP & EVP phases
  - Non-local damage EP phases
  - First and second statistical moment estimates

- Multi-scale methods
  - Computationally efficient
  - Verified with direct numerical simulations
  - Experimentally validated

- Papers
  - On www.ltas-cm3.ulg.ac.be