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Radiative Decay Parameters for Highly Excited Levels in Ti II

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ABSTRACT
The present work reports new experimental radiative lifetimes of six $3d^2(^3F)5s$ levels in singly ionized titanium, with an energy around 63000 cm^{-1} and four $3d^2(^3F)4p$ odd parity levels where we confirm previous investigations. **Combining the new 5s lifetimes with branching fractions measured previously by Pickering et al. [Astrophys Journal Suppl Ser 132, 403 (2001)], we report 57 experimental log gf values for transitions from the 5s levels.** The lifetime measurements are performed using time-resolved laser-induced fluorescence on ions produced by laser ablation. One- and two-step photon excitation is employed to reach the 4p and 5s levels, respectively. Theoretical calculations of the radiative lifetimes of the measured levels as well as of oscillator strengths for 3336 transitions from these levels are reported. The calculations are carried out by a pseudo-relativistic Hartree-Fock method taking into account core polarization effects. The theoretical results are in a good agreement with the experiments **and are needed for accurate abundance determinations in astronomical objects.**

Key words: atomic data – methods: laboratory: atomic

1 INTRODUCTION

Lines from the iron-group elements are among the most abundant in spectra of astronomical objects. In the B, A and F class stars, the first ions of Fe, Cr and Ti dominate the ultraviolet and visible spectrum. Even so-called forbidden lines in Ti II have been found to be very intense in the strontium filament ejecta of the massive star Eta Carinae (Hartman et al. 2004). Titanium is located among the lighter of the iron group elements and is considered primarily to be an α-element, i.e. produced by successive captures of He nuclei through among others, Mg, Si and Ca. Recent studies of metal poor stars, however, show trends where the titanium abundance is correlated with scandium and vanadium indicating a similar production mechanism as the iron-group elements rather than the α-elements (Sneden et al. 2015). Accurate atomic data are needed to reliably determine the Ti abundance in these objects.

The ground term in Ti II is $3d^24s\ ^4F$, followed by even terms belonging to the $3d^24s$, $3d^3$ and $3d4s^2$ configurations up to an energy of 25000 cm^{-1}. The lowest odd configurations are $3d^24p$ and 3d4s4p spanning the energy interval from 29500 to 59500 cm^{-1} (Huldt et al. 1982). Transitions between these configurations give rise to the most intense spectral lines in Ti II, and have been the subject of most previous experimental and theoretical investigations. The higher lying even configurations $3d^25s$, $3d^26s$, $3d^24d$ and $3d^25d$ cover the energy interval from 62180 cm^{-1} to 84652 cm^{-1}. In total 253 energy levels and 1872 spectral lines ranging from 122 to 2198 nm are reported in the NIST compilations (Kramida et al. 2014; Saloman 2012).

Experimental lifetimes in Ti II obtained with different techniques have been reported in several papers. The first investigation, by Roberts et al. (1973), used the beam-foil method to obtain lifetimes in both the 4p and 4d configurations. In Gosselin et al. (1987) a Ti^+ beam from a heavy ion accelerator was crossed by a laser to selectively excite the 4p $z^4D_{5/2}$ state. The reported lifetime has an uncer-

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tainty of only 1.5 % and remains the most accurately known lifetime in Ti II. Langhans et al. (1995), Kwiatkowski et al. (1985) and Bizzarri et al. (1993) employed time-resolved laser-induced fluorescence (TR-LIF) on ions from a hollow cathode discharge. The metastable states of the $3d^3$, $3d^2(^3P)4s$ and $(3d + 4s)^3$ electron configurations of Ti II have been investigated by a laser probing technique on Ti ions in a storage ring in series of papers (Hartman et al. 2003; Palmeri et al. 2008; Hartman et al. 2005). Along with the lifetimes, transition probabilities for several decay channels from these metastable levels are also reported.

A number of experimental investigations of both absolute and relative transition probabilities and oscillator strengths from 4p levels have been stimulated by astrophysical observations. Various methods and spectral sources have been employed. These include absorption measurements (Wiese et al. 2001), emission studies from a shock tube (Boni 1968; Wolnik & Berthel 1973), from arc- (Tatum 1961; Roberts et al. 1975) and spark-sources (Wobig 1962), hook method (Danzmann & Kock 1980) and Fourier transform spectroscopy (Pickering et al. 2001; Wood et al. 2013).

Several papers report theoretical investigations of radiative parameters in Ti II. The most recent of these are Kurucz (2011) and Ruczkowski et al. (2014). Transition probabilities and oscillator strengths of Ti II spectral lines are also included in several compilations, e.g. Thévenin (1989), Savanov et al. (1990) and Meylan et al. (1993).

This short literature survey shows that a large number of studies have been devoted to transition probabilities and radiative lifetimes for low lying excited states, mainly belonging to the $3d^24p$ electron configuration. There are no experimental data for the high lying 5s levels that are the main subject of the present work.

2 EXPERIMENT

The experimental set-up for single-step experiments at the Lund High Power Laser Facility has recently been described in detail (Engström et al. 2014), and here we focus mainly on the new features involved in the two-step process. The experimental set-up is presented in Fig. 1. The Ti ions are produced in an ablation process, where the second harmonic of a Nd:YAG laser (Continuum Surelite) with 10 ns pulses are focused on a rotating Ti target. The target is placed in a vacuum chamber with a pressure of around 10^{-4} mbar. The generated plasma is crossed by the two excitation laser beams about 1 cm above the target. With the first laser the intermediate odd states in $3d^2(^3F)4p$, around 32000 cm^{-1}, are excited and used as platforms for the excitation of the high-lying even parity 5s states.

The first laser channel consist of a Nd:YAG laser (Continuum NY-82) pumping a Continuum Nd - 60 dye laser operating with DCM dye $(\mathbf{C_{19}H_{17}N_3O})$. The 10 ns long pulses are frequency doubled using a KDP crystal $(\mathbf{KH_2PO_4})$. For the second step, the same type of lasers and dye are used but here the Nd:YAG laser is injection seeded and the pulses are temporally compressed using stimulated Brillouin scattering (SBS) in water. After frequency doubling in a KDP crystal we obtain pulses with a typical FWHM of 1.2 ns.

All three laser systems operate at 10 Hz and are synchronized by a delay generator. The delay generator allows

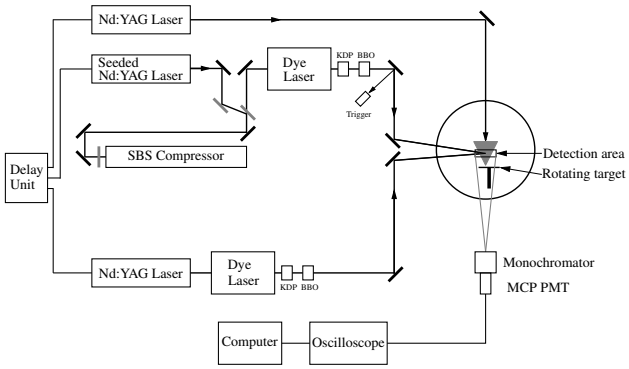

Figure 1. Experimental set-up for TR-LIF at the Lund High Power Laser Facility using two-step excitation. See discussion in text for details.

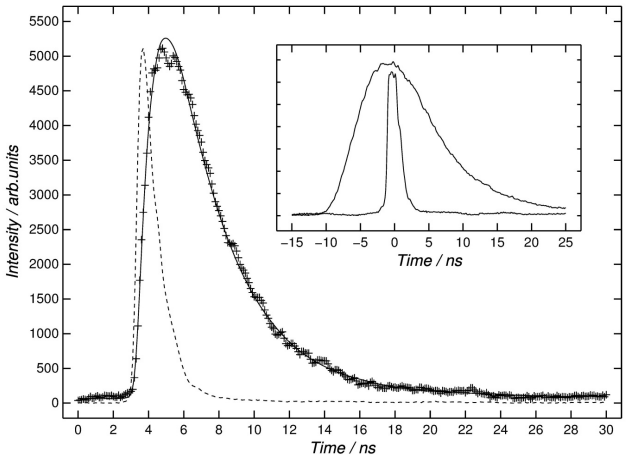

Figure 2. The first 30 ns of the decay of the $e^2F_{5/2}$ level in Ti II following two-step excitation. The measured decay (+) is plotted together with a fitted single exponential function (solid line) convoluted by the recorded second step laser pulse (dashed curve). The insert shows the timing between the fluorescence from the intermediate $z^2D_{3/2}$ level (broad structure) and the second step laser (narrow peak).

us to set the time between the plasma generating laser and the excitation pulses (typically around 1 μs) and also the delay between the first and second excitation steps. The latter timing was checked before every measurement to ensure that the second step occurred at the maximum population of the intermediate level, as determined by the decay of this level in some channel. Since the pulses in the first step are much longer than in the second step this also ensures that the intermediate population is almost constant during the final excitation. The timing between the two steps is illustrated in the insert in Fig. 2.

The fluorescence from the excited states was observed with a 1/8 m grating monochromator, with its 0.28 mm wide entrance slit oriented parallel to the excitation lasers, giving a line width of 0.5 nm in the second spectral order. The

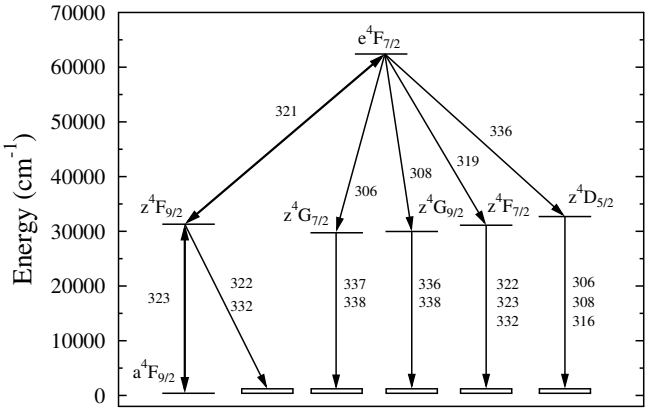

Figure 3. Decay chains from the 5s $e^4F_{7/2}$ level in Ti II. All wavelengths are given in nm. In the experiment the 5s level is excited in two steps using the transitions at 323.4 nm (4s - 4p) and 321.3 nm (4p - 5s).

dispersed light was registered by a fast micro-channel-plate photomultiplier tube (Hamamatsu R3809U) and digitized by a Tektronix DPO 7254 oscilloscope triggered by the second step laser pulses detected with a fast photodiode. The final decay curves and pulse shape were obtained by averaging over 1000 laser pulses. The code DECFIT (Palmeri et al. 2008) was then used to extract the lifetimes by fitting a single exponential function convoluted by the measured shape of the second step laser pulse and a background function to the observed decay. A typical example is shown in Fig. 2.

Table 1 gives the wavelengths and excitation schemes used in the single-step measurements of four 4p levels, performed to allow a comparison with earlier experimental results. Two of these levels are also used as platforms in the two-step experiments. A complete description of the latter is given in Table 2. In the complex level system of the iron group elements, with a parent term structure and multiple ionization limits, the strong transitions group in wavelengths due to the similar energy difference between 4s and 4p for the different parent terms (similar promotion energy). In addition, the energy difference between 4p and 5s is similar to that of 4s and 4p, making the transitions between these configurations also fall in the same region. For Ti II this region is between 300 and 350 nm. Thus, the observed decay from the 5s levels under investigation might be blended with fluorescence from 4p levels. Fig. 3 presents a schematic picture using 5s $e^4F_{7/2}$ as an example. Two types of blending may be distinguished: from a direct fluorescence (4s - 4p) channel from the intermediate level or from the secondary decay of a 4p level populated by the decay from the 5s level studied.

While the fluorescence from the intermediate level is very intense and observable even at a rather large wavelength off-set its perturbing influence is easily handled by recording an additional decay curve, with the second step laser turned off to reveal the blending contribution, which can be subtracted from the primary decay measurement before the lifetime analysis. Two examples of this problem were encountered and are marked in Table 2. Secondary decays, on the other hand, arise from decays along a chain from the level of interest and cannot be corrected for, but must be understood in order to choose an appropriate channel for the primary decay measurement. For example, the lifetime of 5s $e^4F_{7/2}$ is measured in the channel 319 nm, with the final result of (3.02 ± 0.20) ns, after correction for the first step fluorescence at 323 nm. However, the decay could be observed in three other sufficiently intense channels as well: 306, 308 and 336 nm. These are, however, blended by one or more secondary decays, as shown in Fig. 3. The measured lifetimes in these channels are 6.5, 3.8 and 3.3 ns, respectively, clearly demonstrating the importance of choosing an appropriate decay channel.

The final lifetimes obtained are presented in Table 3. The values represent the average of between 10 and 20 measurements performed over a number of days, and the quoted uncertainties take into account both the statistical uncertainty in the fitting process and, primarily, the variation of the results between the different measurements.

In Table 4 we have derived absolute transition probabilities and log gf values for 57 transitions depopulation 5 of the 6 5s levels investigated in this work. The results are obtained by combining our experimental lifetimes in Table 3 with experimental branching fractions reported by Pickering et al (2001).

3 THEORY

The relativistic Hartree-Fock (HFR) approach (Cowan 1981) including core-polarization (CPOL) effects by means of a model potential and a correction to the transition dipole operator (HFR + CPOL) (see e.g. Quinet et al. 1999 and Quinet et al. 2002) has been used to compute the transition probabilities in Ti II. The following 27 configurations have been considered explicitly in our physical model: $3d^3$, $3d^24s$, $3d^25s$, $3d^26s$, $3d^24d$, $3d^25d$, $3d4s^2$, $3d4p^2$, $3d4d^2$, 3d4s4d, 3d4s5d, 3d4s5s, $4s^24d$, $4s^25d$, $4s^25s$ (even parity) and $3d^24p$, $3d^25p$, $3d^24f$, $3d^25f$, 3d4s4p, 3d4s5p, 3d4s4f, 3d4s5f, $4s^24p$, $4s^25p$, $4s^24f$, $4s^25f$ (odd parity). The ionic core considered for the core-polarization effects was an argon-like core, i.e. a $3p^6$ Ti V core. The dipole polarizability, α_d, for such a core is 1.48 a_0^3, according to Johnson et al. (1983). For the cut-off radius, we used the HFR mean value of the outermost 3p core orbital, i.e. 1.08 a_0.

Radial integrals of the $3d^3$, $3d^24s$, $3d^25s$, $3d^24d$, $3d4s^2$, $3d^24p$ and 3d4s4p, considered as free parameters, were then adjusted with a well-established least-squares optimization program minimizing the discrepancies between the calculated Hamiltonian eigenvalues and the experimental energy levels taken from Huldt et al. (1982). More precisely, the average energies (E_{av}), the electrostatic direct (F^k) and exchange (G^k) integrals, the spin-orbit (ζ_{nl}) and effective interaction parameters (α and β) were allowed to vary during the fitting process. We also adjusted the CI parameters (R^k) between the $3d^3$ and $3d^24s$ even configurations and between the $3d^24p$ and 3d4s4p odd configurations. For parameters belonging to other configurations, a scaling factor of 0.80 was applied.

The numerical values of the parameters adopted in the present calculations are reported in Tables 5 and 6

Table 1. Single-step excitation schemes for $3d^2(^3F)4p$ levels in Ti II.

Level	E^a/cm^{-1}	Starting level E^a/cm^{-1}	Excitation^b λ_{air}/nm	λ^c_{obs}/nm
$z^4F_{5/2}$	30958	0	322.92	333
$z^2D_{3/2}$	31756	0	314.80	368
$z^4D_{5/2}$	32698	1087	316.26	307, 308, 316
$z^4D_{7/2}$	32767	393	308.80	217

[a]Huldt et al. (1982).
[b]All levels were excited using the second harmonic of the dye laser.
[c]All fluorescence measurements were performed in the second spectral order.

for even and odd-parity configurations, respectively. This semi-empirical process led to average deviations with experimental energy levels equal to 125 cm^{-1} (even parity) and 78 cm^{-1} (odd parity).

4 RESULTS AND DISCUSSION

Table 3 shows that our lifetimes for the 4p levels agree with the previous investigations using laser excitation within the mutual error bars. Although there is a tendency for the new results to be somewhat shorter. Compared with the old values obtained by the beam-foil technique by Roberts et al. (1973) we note a significant discrepancy. This is most likely caused by the combined problem of line blending and cascades from higher lying states caused by the non-selective excitation in the beam-foil process. In the case of the $3d^25s$ $e^2F_{7/2}$ level, Roberts et al. (1973) measured the lifetime using a transition at 348.4 nm, but assigned the value to the configuration 3d4s4p. This is most likely a typographical error since e^2F denotes an even configuration. The line at 348.4 nm is an intense transition used in the present work to measure the 5s $e^2F_{7/2}$ level (Table 2) and we believe that the **assignment by Roberts et al. (1973)** should be changed. Doing so also results in agreement between the measured lifetimes.

The computed radiative lifetimes obtained in the present work are compared with our experimental values in Table 3. As shown in this table, the overall agreement between theory and experiment is very good (within 10%). For comparison, Table 3 includes the theoretical lifetimes obtained by Kurucz (2011). This work also used a semi-empirical approach based on a superposition of configurations calculation with a modified version of the Cowan (1981) codes and experimental level energies. We note a very good qualitative agreement between the two calculations.

Table 7 gives the HFR+CPOL oscillator strengths (log gf) and weighted transition probabilities (gA), obtained in the present work, for 3336 Ti II spectral lines from 138 to 9966 nm, **combining lower levels in the range 0 – 74000 cm^{-1} and upper levels 30000 – 80000 cm^{-1}.** Only transitions with log gf > -4 are reported in the table in which we also give, in the last column, the value of the cancellation factor (CF), as defined by Cowan (1981). Very small values of this factor (typically < 0.05) indicate strong cancellation effects in the calculation of the line strengths and the corresponding transition rates can be affected by larger uncertainties and should be considered with some care.

Table 2. Two-step excitation schemes for $3d^2(^3F)5s$ levels in Ti II.

Final level	E^a/cm^{-1}	Intermediate level E^a/cm^{-1}	Excitation^b λ_{air}/nm	λ^c_{obs}/nm
$e^4F_{3/2}$	62180	30958	320.20	306, 319
$e^4F_{5/2}$	62272	30958	319.26	305, 318[d]
$e^4F_{7/2}$	62411	31301	321.35	319[e]
$e^4F_{9/2}$	62595	31301	319.46	306
$e^2F_{5/2}$	63169	31756	318.25	312, 349
$e^2F_{7/2}$	63446	32025, 32767	318.17, 325.86	313[f], 348

[a]Huldt et al. (1982).
[b]All levels were excited using the second harmonic of the dye laser.
[c]All fluorescence measurements were performed in the second spectral order.
[d]Corrected for scattered light from the second step laser at 319 nm.
[e]Corrected for fluorescence background from first step laser at 323 nm.
[f]Corrected for fluorescence background from first step laser at 316 nm.

Table 3. Lifetimes of the $3d^2(^3F)4p$ and 5s levels in Ti II.

Level	E^a	τ_{exp}/ns		τ_{calc}/ns	
	cm^{-1}	Our work	Other	Our work	Kurucz[b]
4p $z^4F_{5/2}$	30958	3.87 ± 0.20	4.1 ± 0.2[c] 4.1 ± 0.3[d]	3.76	4.15
4p $z^2D_{3/2}$	31756	6.10 ± 0.20	6.6 ± 0.3[c] 6.3 ± 1[d] 7.8 ± 1[e]	5.87	6.85
4p $z^4D_{5/2}$	32698	3.86 ± 0.20	4.0 ± 0.2[c] 3.9 ± 0.4[d] 5.2 ± 0.8[e] 4.01 ± 0.06[f]	3.47	3.92
4p $z^4D_{7/2}$	32767	3.75 ± 0.20	4.0 ± 0.2[c] 4.1 ± 0.5[d]	3.40	3.85
5s $e^4F_{3/2}$	62180	2.96 ± 0.20		3.19	2.82
5s $e^4F_{5/2}$	62272	3.05 ± 0.20		3.19	2.82
5s $e^4F_{7/2}$	62411	3.02 ± 0.20		3.19	2.82
5s $e^4F_{9/2}$	62595	3.14 ± 0.20		3.19	2.82
5s $e^2F_{5/2}$	63169	3.04 ± 0.15		3.41	3.04
5s $e^2F_{7/2}$	63446	3.02 ± 0.15	3.0 ± 0.6[e1]	3.41	3.05

[a]Huldt et al. (1982).
[b]Kurucz (2011).
[c]Bizzarri et al. (1993), TR-LIF.
[d]Kwiatkowski et al. (1985), TR-LIF.
[e]Roberts et al. (1973), Beam-Foil.
[e1]The original assignment of this level in Roberts et al. (1973) to $3d4s(^3F)4p$ is changed to $3d^2(^3F)5s$.
[f]Gosselin et al. (1987), Beam-Laser.

In the same table, we list the most recent oscillator strengths published by Pickering et al. (2001), Kurucz (2011), Wood et al. (2013) and Ruczkowski et al. (2014). In the work of Pickering et al. (2001), the relative intensities of Ti II emission lines between 187 and 602 nm from 89 levels were measured by high-resolution Fourier transform spectrometry, using a hollow cathode lamp as light source. The branching fractions were then combined with 39 measured and 44 computed lifetimes to give absolute transition probabilities for 624 lines. Fig. 4 shows the good agreement between our calculated loggf values and the values from Pickering et al. (2001), particularly when they used experimental lifetimes (filled circles). **Table 4 gives a more**

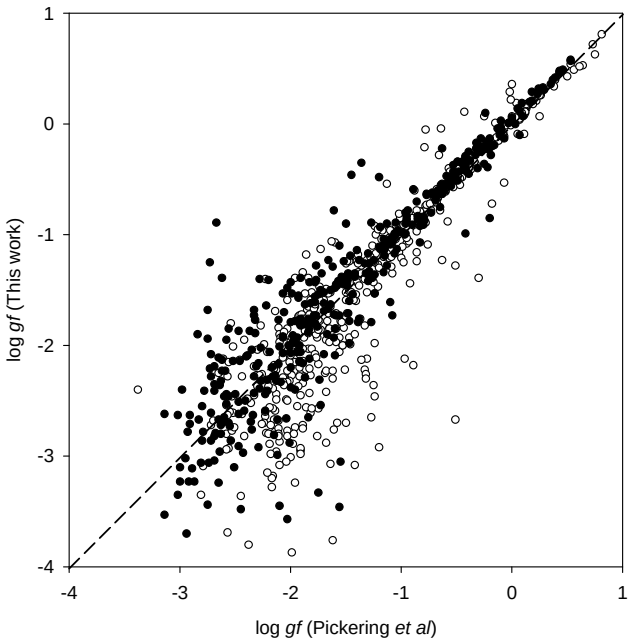

Figure 4. Comparison between the oscillator strengths (log gf) calculated in the present work and those published by Pickering et al. (2001). **The circles correspond to the combination of measured branching fractions with experimental (black) and theoretical (white) lifetimes in Pickering et al. (2001).**

detailed comparison between our calculated log gf values and those obtained by combining the new experimental lifetimes in Table 3 with the branching fractions measured by Pickering et al (2001). A very good agreement is observed, in particular for the strongest transitions for which both sets of results agree within 10-20%. The average deviation in the log gf values is 0.1 with a standard deviation of 0.3.

In the analysis of Wood et al. (2013), atomic transition probability measurements for 364 lines in the UV to near-IR are reported. They were obtained from branching fraction measurements using a Fourier transform spectrometer and an echelle spectrometer combined with published radiative lifetimes. A comparison between these values and our calculated log gf values is shown in Fig. 5, and we note again the consistency between the calculated and observed values.

Kurucz (2011) used a similar semi-empirical HFR model as the one considered in our work but without inclusion of core-polarization effects. More recently, Ruczkowski et al. (2014) used a semi-empirical oscillator strength parametrization method to compute 1340 log gf values for spectral lines in Ti II. As seen in Table 7, our new gf-values are in rather good agreement with these last results, in particular for the strongest lines (log gf > -1), for which the typical average deviations are found to be within 20%.

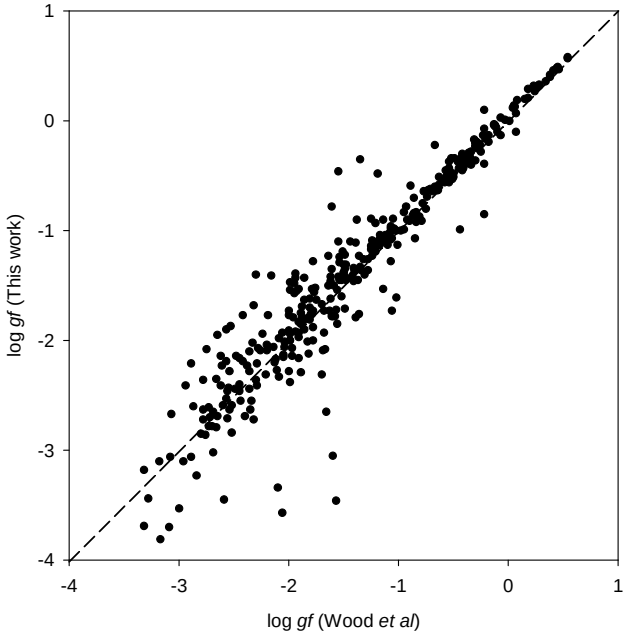

Figure 5. Comparison between the oscillator strengths (log gf) calculated in the present work and those published by Wood et al. (2013).

5 CONCLUSIONS

We report new experimental radiative lifetimes of ten 4p and 5s levels in singly ionized titanium. The measurements are performed using time-resolved laser-induced fluorescence on ions produced by laser ablation. One- and two-step photon excitation is employed to reach the 4p and 5s levels, respectively. **For 5 of the 6 measured 5s levels we have combined our lifetimes with the experimental branching fractions measured previously by Pickering et al. (2001) to obtain 57 experimental absolut transition probabilities and log gf values.** In addition, we report calculated transition probabilities for 3336 Ti II spectral lines from 138 to 9966 nm. Where possible to compare, we find a good agreement with previous experiments and calculations. **The transition probabilities are needed for accurate abundance determinations in astronomical objects.**

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Table 4. Transition probabilities and oscillator strengths for lines from the 5s levels measured in this work.

Upper level	Lower level	λ[a]/nm	BF[a]	A[b] / 10^6 s^{-1}	log gf_{exp}[b]	log gf_{calc}[b]
$3d^2(^3F)5s\ e^4F_{5/2}$	$3d^2(^3F)4p\ z^2G_{5/2}$	360.53046	0.35±0.03	1.15±0.12	−1.872±0.045	-1.80
$\tau = 3.05 \pm 0.20$ ns	$3d^2(^3F)4p\ z^4D_{7/2}$	338.82630	0.15±0.02	0.49±0.07	−2.294±0.060	-2.19
	$3d^2(^3F)4p\ z^4D_{5/2}$	338.03449	5.12±0.16	16.8±1.2	−0.763±0.030	-0.84
	$3d^2(^3F)4p\ z^4D_{3/2}$	336.94669	15.32±0.49	50.2±3.7	−0.290±0.031	-0.34
	$3d^2(^3F)4p\ z^2D_{5/2}$	330.51839	1.25±0.05	4.1±0.3	−1.395±0.032	-1.54
	$3d^2(^3F)4p\ z^2D_{3/2}$	327.60534	0.53±0.03	1.74±0.15	−1.775±0.036	-1.59
	$3d^2(^3F)4p\ z^2F_{7/2}$	324.77703	0.28±0.03	0.92±0.12	−2.060±0.051	-2.68
	$3d^2(^3F)4p\ z^4F_{7/2}$	320.84482	7.59±0.24	24.9±1.8	−0.638±0.031	-0.71
	$3d^2(^3F)4p\ z^4F_{5/2}$	319.25568	21.68±0.69	71.1±5.2	−0.186±0.031	-0.18
	$3d^2(^3F)4p\ z^4F_{3/2}$	318.01500	5.29±0.17	17.3±1.3	−0.802±0.031	-0.83
	$3d^2(^3F)4p\ z^4G_{5/2}$	307.24588	37.27±1.45	122.2±9.3	0.016±0.032	0.01
	$3d^2(^3F)4p\ z^4G_{5/2}$	305.46055	5.15±0.17	16.9±1.2	−0.849±0.031	-0.85
$3d^2(^3F)5s\ e^4F_{7/2}$	$3d^2(^3P)4p\ y^4D_{5/2}$	457.97106	0.78±0.16	2.6±0.6	−1.187±0.085	-1.98
$\tau = 3.02 \pm 0.20$ ns	$3d^2(^3F)4p\ z^2G_{9/2}$	361.39766	0.28±0.02	0.93±0.09	−1.838±0.040	-1.76
	$3d^2(^3F)4p\ z^4D_{7/2}$	337.24219	3.85±0.13	12.7±0.9	−0.760±0.031	-0.88
	$3d^2(^3F)4p\ z^4D_{5/2}$	336.45792	16.28±0.54	53.9±4.0	−0.136±0.031	-0.18
	$3d^2(^3F)4p\ z^2D_{5/2}$	329.01096	1.42±0.05	4.70±0.35	−1.214±0.031	-1.10
	$3d^2(^3F)4p\ z^4F_{9/2}$	321.34827	4.50±0.15	14.9±1.1	−0.734±0.031	-0.82
	$3d^2(^3F)4p\ z^4F_{7/2}$	319.42417	27.02±1.05	89.5±6.9	0.039±0.032	-0.01
	$3d^2(^3F)4p\ z^4F_{5/2}$	317.84902	5.12±0.17	17.0±1.3	−0.687±0.031	-0.72
	$3d^2(^3F)4p\ z^4G_{9/2}$	308.14689	35.28±1.38	117±9	0.124±0.032	0.14
	$3d^2(^3F)4p\ z^4G_{5/2}$	305.94286	5.12±0.17	17.0±1.3	−0.721±0.031	-0.73
	$3d^2(^3F)4p\ z^4G_{5/2}$	304.17252	0.17±0.02	0.56±0.08	−2.204±0.055	-2.26
$3d^2(^3F)5s\ e^4F_{9/2}$	$3d^2(^3P)4p\ y^4D_{7/2}$	458.65599	0.10±0.02	0.32±0.07	−1.998±0.083	-1.85
$\tau = 3.14 \pm 0.20$ ns	$3d^2(^3F)4p\ z^4D_{7/2}$	335.15947	20.00±0.80	63.7±4.8	0.030±0.031	0.04
	$3d^2(^3F)4p\ z^2F_{7/2}$	321.40667	0.48±0.02	1.53±0.12	−1.626±0.032	-1.71
	$3d^2(^3F)4p\ z^4F_{9/2}$	319.45671	30.05±1.20	95.7±7.2	0.166±0.031	0.18
	$3d^2(^3F)4p\ z^4F_{7/2}$	317.55511	3.18±0.11	10.1±0.7	−0.815±0.030	-0.85
	$3d^2(^3F)4p\ z^4G_{11/2}$	308.98892	42.53±1.70	135±10	0.287±0.031	0.26
	$3d^2(^3F)4p\ z^4G_{9/2}$	306.40711	3.50±0.12	11.1±0.8	−0.804±0.030	-0.85
	$3d^2(^3F)4p\ z^4G_{5/2}$	304.22786	0.16±0.02	0.51±0.07	−2.151±0.057	-2.37
$3d^2(^3F)5s\ e^2F_{5/2}$	$3d^2(^3P)4p\ x^2D_{3/2}$	547.66664	0.65±0.21	2.1±0.7	−1.239±0.123	-1.40
$\tau = 3.04 \pm 0.15$ ns	$3d^2(^1G)4p\ y^2G_{5/2}$	514.56899	1.02±0.08	3.4±0.3	−1.097±0.038	-1.04
	$3d^2(^1D)4p\ y^2F_{5/2}$	430.12959	1.82±0.11	6.0±0.5	−1.002±0.033	-1.29
	$3d^2(^1D)4p\ y^2D_{3/2}$	424.21659	0.31±0.06	1.02±0.20	−1.782±0.079	-1.53
	$3d^2(^1D)4p\ y^2D_{5/2}$	421.96168	0.43±0.05	1.41±0.18	−1.645±0.052	-1.13
	$3d^2(^1D)4p\ z^2P_{3/2}$	417.66855	1.03±0.09	3.39±0.34	−1.274±0.042	-2.13
	$3d^2(^3F)4p\ z^2G_{5/2}$	349.23637	41.04±1.30	135.0±7.9	0.171±0.025	0.10
	$3d^2(^3F)4p\ z^4D_{5/2}$	328.08638	0.40±0.06	1.32±0.21	−1.895±0.064	-2.00
	$3d^2(^3F)4p\ z^4D_{3/2}$	327.06165	1.30±0.07	4.28±0.31	−1.386±0.031	-1.39
	$3d^2(^3F)4p\ z^2D_{3/2}$	318.25307	17.15±0.54	56.4±3.3	−0.289±0.025	-0.35
	$3d^2(^3F)4p\ z^2F_{7/2}$	315.58332	1.47±0.07	4.84±0.33	−1.363±0.029	-1.37
	$3d^2(^3F)4p\ z^2F_{5/2}$	312.78498	32.11±1.01	105.6±6.2	−0.032±0.025	-0.05
	$3d^2(^3F)4p\ z^4F_{5/2}$	310.36747	0.18±0.06	0.59±0.20	−2.290±0.126	-3.80
	$3d^2(^3F)4p\ z^4F_{3/2}$	309.19494	0.26±0.05	0.86±0.17	−2.133±0.079	-2.79
	$3d^2(^3F)4p\ z^4G_{5/2}$	297.31417	0.20±0.07	0.66±0.23	−2.281±0.131	-2.52
$3d^2(^3F)5s\ e^2F_{7/2}$	$3d^2(^3P)4p\ x^2D_{5/2}$	539.12020	0.56±0.08	1.85±0.28	−1.190±0.061	-1.20
$\tau = 3.02 \pm 0.15$ ns	$3d^2(^1G)4p\ y^2G_{9/2}$	508.37407	1.26±0.09	4.17±0.36	−0.888±0.036	-0.91
	$3d^2(^1D)4p\ y^2F_{5/2}$	425.06449	0.25±0.03	0.83±0.11	−1.746±0.053	-1.39
	$3d^2(^1D)4p\ y^2D_{5/2}$	417.08631	0.83±0.06	2.75±0.24	−1.242±0.037	-1.82
	$3d^2(^3F)4p\ z^2G_{9/2}$	348.36246	39.53±1.27	130.9±7.7	0.280±0.025	0.21
	$3d^2(^3F)4p\ z^2G_{5/2}$	345.88996	1.12±0.05	3.71±0.25	−1.274±0.028	-1.35
	$3d^2(^3F)4p\ z^4D_{7/2}$	325.86364	0.62±0.05	2.05±0.19	−1.583±0.039	-1.78
	$3d^2(^3F)4p\ z^4D_{5/2}$	325.13143	2.55±0.10	8.44±0.53	−0.970±0.027	-1.02
	$3d^2(^3F)4p\ z^2D_{5/2}$	318.17219	18.96±0.61	62.8±3.7	−0.118±0.025	-0.19
	$3d^2(^3F)4p\ z^2F_{7/2}$	312.84833	31.27±1.00	103.5±6.1	0.085±0.025	0.07
	$3d^2(^3F)4p\ z^4G_{9/2}$	298.61916	0.44±0.05	1.46±0.18	−1.807±0.051	-1.72

[a]Pickering et al. (2001)
[b]This work

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Table 5. Radial parameters adopted in the HFR+CPOL calculations for the $3d^3$, $3d^24s$, $3d^25s$, $3d^24d$ and $3d4s^2$ even-parity configurations of Ti II.

Configuration	Parameter	Ab initio E / cm^{-1}	Fitted E / cm^{-1}	Ratio	Note[a]
$3d^3$	E_{av}	15742	12262		
	$F^2(3d,3d)$	56469	44972	0.796	
	$F^4(3d,3d)$	34782	24920	0.716	
	α	0	-22		
	β	0	486		
	ζ_{3d}	102	102	1.000	F
$3d^24s$	E_{av}	8511	8295		
	$F^2(3d,3d)$	65280	50087	0.767	
	$F^4(3d,3d)$	40580	31161	0.768	
	α	0	37		
	β	0	153		
	ζ_{3d}	125	125	1.000	F
	$G^2(3d,4s)$	10847	8084	0.745	R1
$3d^25s$	E_{av}	69222	69392		
	$F^2(3d,3d)$	67340	51820	0.770	
	$F^4(3d,3d)$	41960	30640	0.730	
	α	0	35		F
	β	0	150		F
	ζ_{3d}	129	129	1.000	F
	$G^2(3d,5s)$	2009	1497	0.745	R1
$3d^24d$	E_{av}	73261	74068		
	$F^2(3d,3d)$	67349	53704	0.797	
	$F^4(3d,3d)$	41970	32667	0.778	
	α	0	35		F
	β	0	150		F
	ζ_{3d}	129	129	1.000	F
	ζ_{4d}	9	9	1.000	F
	$F^2(3d,4d)$	6812	3961	0.581	
	$F^4(3d,4d)$	3087	1999	0.647	
	$G^0(3d,4d)$	5311	2374	0.447	R2
	$G^2(3d,4d)$	3562	1593	0.447	R2
	$G^4(3d,4d)$	2417	1081	0.447	R2
$3d4s^2$	E_{av}	30013	30791		
	ζ_{3d}	149	140	0.945	
$3d^3$–$3d^24s$	$R^2(3d,3d;3d,4s)$	-9870	-7356	0.745	

[a] F: fixed parameter value; Rn : Fixed ratio between these parameters.

Table 6. Radial parameters adopted in the HFR+CPOL calculations for the $3d^24p$ and 3d4s4p odd-parity configurations of Ti II.

Configuration	Parameter	Ab initio E / cm^{-1}	Fitted E / cm^{-1}	Ratio	Note[a]
$3d^24p$	E_{av}	37888	38418		
	F^2(3d,3d)	66208	50859	0.768	
	F^4(3d,3d)	41202	29899	0.726	
	α	0	49		
	β	0	85		
	ζ_{3d}	127	127	1.000	F
	ζ_{4p}	176	176	1.000	F
	F^2(3d,4p)	14522	11668	0.803	
	G^1(3d,4p)	6343	5603	0.883	
	G^3(3d,4p)	5111	3249	0.636	
3d4s4p	E_{av}	56603	59030		
	ζ_{3d}	150	150	1.000	F
	ζ_{4p}	236	236	1.000	F
	F^2(3d,4p)	15975	14378	0.900	
	G^2(3d,4s)	10055	8363	0.832	
	G^1(3d,4p)	9785	6346	0.649	
	G^3(3d,4p)	5244	3136	0.598	
	G^1(4s,4p)	38957	25804	0.662	
$3d^24p$–3d4s4p	R^2(3d,3d;3d,4s)	-6658	-4420	0.664	R
	R^2(3d,4p;4s,4p)	-13417	-8906	0.664	R
	R^1(3d,4p;4s,4p)	-13786	-9152	0.664	R

[a] F: fixed parameter value; R: Fixed ratio between these parameters.

Table 7. Radiative transition rates for Ti II spectral lines. The full table is available online.

	Lower level[b]			Upper level[b]			Previous works				This work		
λ/nm[a]	E/cm^{-1}	Parity	J	E/cm^{-1}	Parity	J	log gf[c]	log gf[d]	log gf[e]	log gf[f]	log gf	gA/s^{-1}	CF
138.1535	226	(e)	3.5	72609	(o)	3.5		-2.72			-3.21	2.16E+06	0.092
138.2025	94	(e)	2.5	72452	(o)	2.5		-2.61			-3.11	2.71E+06	0.096
138.2395	0	(e)	1.5	72338	(o)	1.5		-2.73			-3.23	2.03E+06	0.101
138.3689	0	(e)	1.5	72271	(o)	0.5		-2.32			-2.83	5.10E+06	0.119
138.4196	94	(e)	2.5	72338	(o)	1.5		-2.11			-2.62	8.26E+06	0.114
138.4543	226	(e)	3.5	72452	(o)	2.5		-1.93			-2.44	1.25E+07	0.112
138.4744	393	(e)	4.5	72609	(o)	3.5		-1.77			-2.29	1.77E+07	0.111
139.0478	1216	(e)	4.5	73134	(o)	4.5		-3.76			-3.71	6.69E+05	0.111
139.6159	984	(e)	2.5	72609	(o)	3.5		-3.77			-3.43	1.28E+06	0.294
139.7573	393	(e)	4.5	71946	(o)	5.5		-2.88			-2.37	1.47E+07	0.077
...	...	...	...	...	...	...	...	...	...	...	...	...	

[a] Wavelengths (in vacuum/air below/above 200 nm) deduced from experimental energy levels.
[b] Experimental energy levels taken from Huldt et al. (1982) and Saloman (2012). (e) and (o) stand for 'even' and 'odd', respectively.
[c] Pickering et al. (2001)
[d] Kurucz (2011)
[e] Wood et al. (2013)
[f] Ruczkowski et al. (2014)