How to count leaves in the trees from Tetris?

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Colloque panquébécois des étudiants de l'ISM
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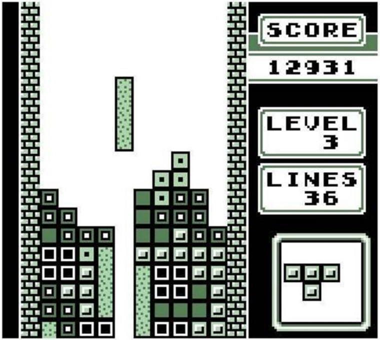
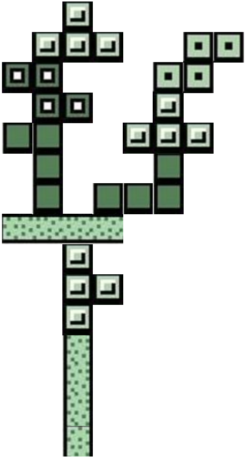
TETRIS™
▸1PLAYER 2PLAYER
©1989 Nintendo

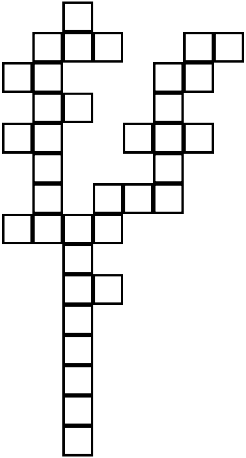

SCORE
12931
LEVEL
3
LINES
36

Trees in Tetris

Trees in Tetris

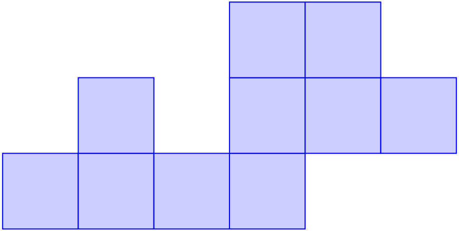


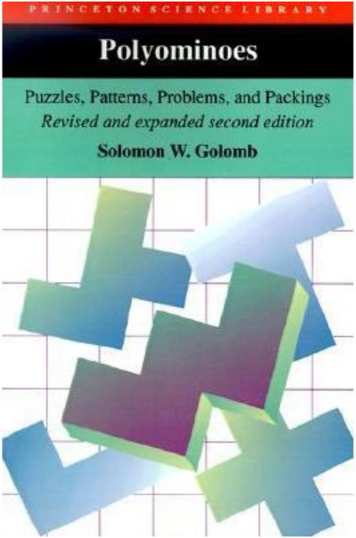
Trees in Tetris



Polyomino

A polyomino is a connected union of unit squares, called cells.





Solomon W. Golomb (1932 - 2016)

Combinatorial Problems

- Monomino: 

Combinatorial Problems

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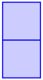
- Domino: 

Combinatorial Problems

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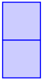
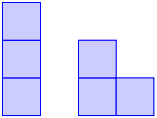
- Domino: 
- Tromino: 

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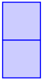
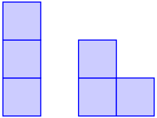
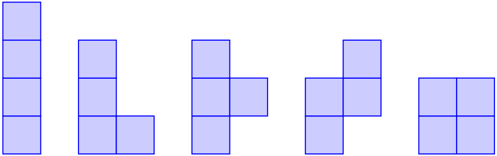
- Domino: 
- Tromino: 
- Tetromino: 

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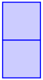
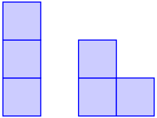
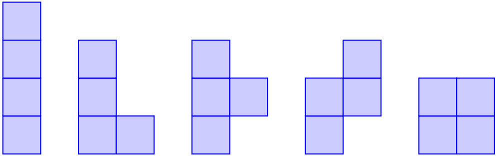
- Domino: 
- Tromino: 
- Tetromino: 

How many polyominoes with n cells?

Combinatorial Problems

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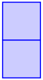
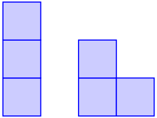
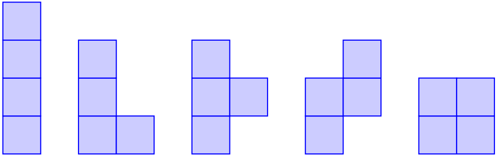
- Domino: 
- Tromino: 
- Tetromino: 

How many polyominoes with n cells?

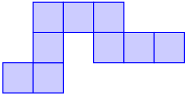
Unknown for $n \geq 57$

Combinatorial Problems

Can we pave the plane with a finite set of polyominoes?

Examples:

- Polyominoes:



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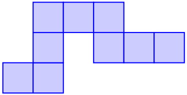
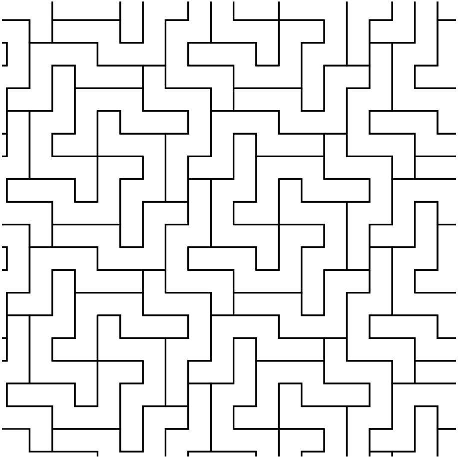


Yes, we can!



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  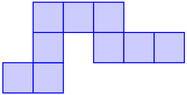

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  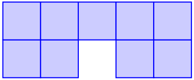

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  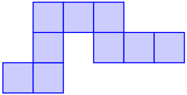

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  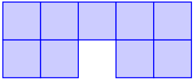

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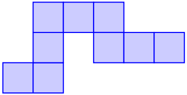

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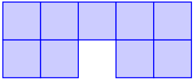


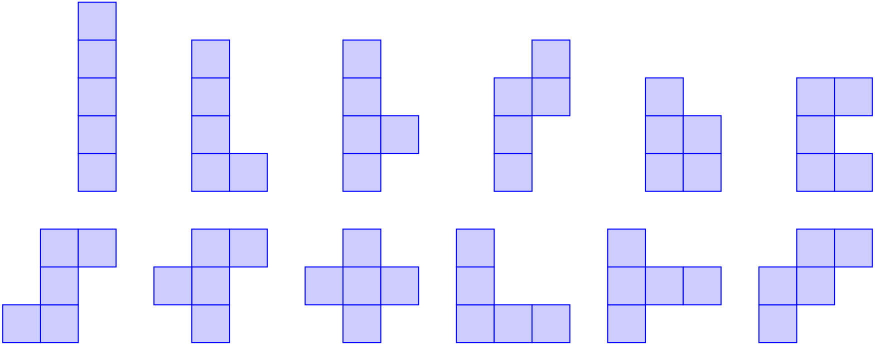
No, we can't!

In general, the problem is undecidable.

Combinatorial Problems

Can we pave a finite area with a finite set of polyominoes?
Example:

- Area: 45×36 rectangle
- Polyominoes: all pentominoes



Combinatorial Problems

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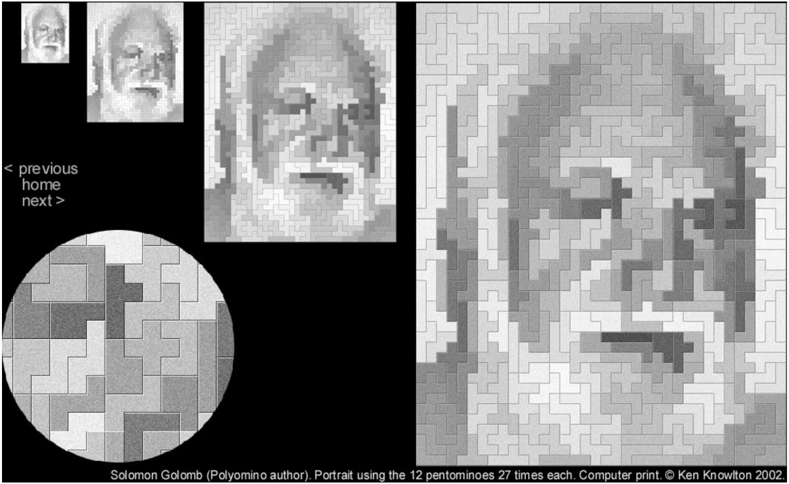
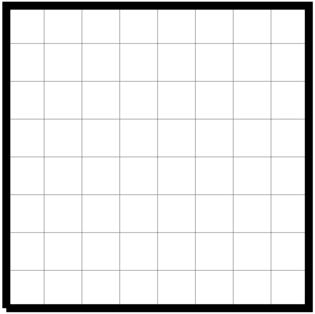


http://www.knowltonmosaics.com

Combinatorial Problems

Can we pave a finite area with a finite set of polyominoes?
Example:

- Area: a checkerboard

- Polyominoes: a domino 

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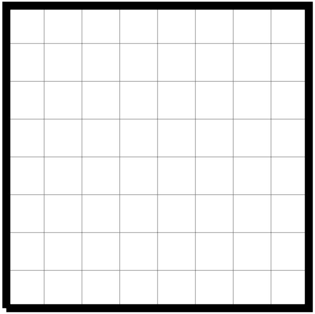

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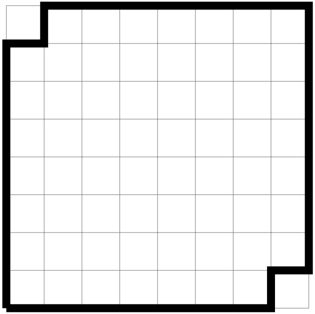
Yes, we can!

Combinatorial Problems

Can we pave a finite area with a finite set of polyominoes?
Example:

- Area: a checkerboard without two opposite corners



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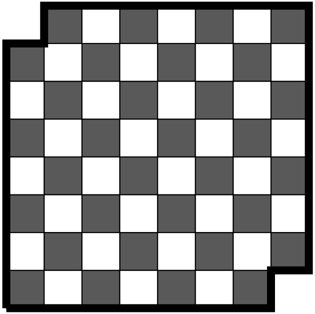

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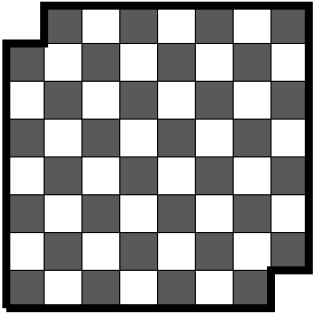

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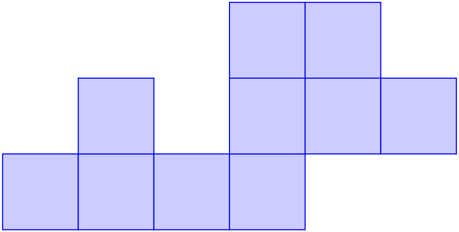
No, we can't!

In general, the problem is NP-complete.

Behind each polyomino hides a graph



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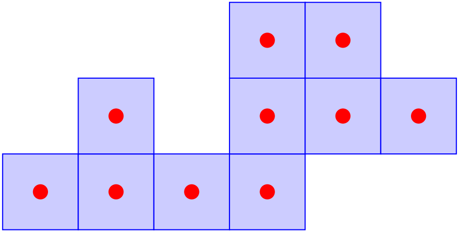
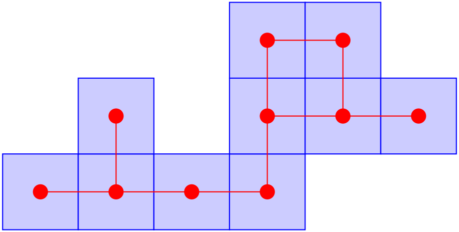


- Vertices are the cells.

Behind each polyomino hides a graph



- Vertices are the cells.
- Two vertices are adjacent if their corresponding cells have a common side.

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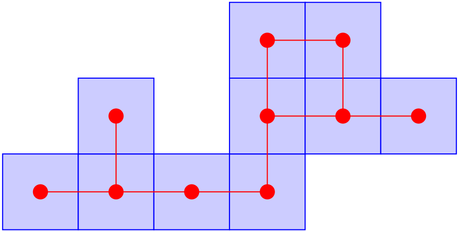
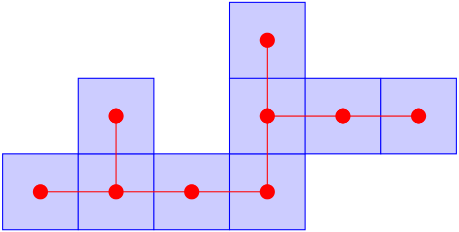


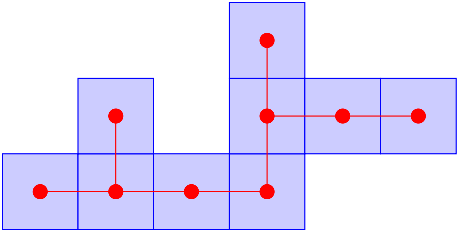
- Vertices are the cells.
- Two vertices are adjacent if their corresponding cells have a common side.
- Such graph can contain cycles.

Tree-like Polyominoes



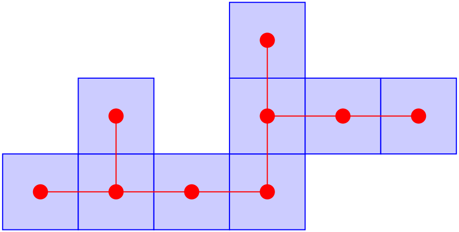
- If there are no cycles, we call it a tree-like polyomino.

Tree-like Polyominoes



- If there are no cycles, we call it a tree-like polyomino.
- This tree-like polyomino has 9 cells and 4 leaves.

Tree-like Polyominoes



- If there are no cycles, we call it a tree-like polyomino.
- This tree-like polyomino has 9 cells and 4 leaves.
- For a given n, what is the maximal number of leaves realized by a tree-like polyomino with n cells?

Maximal number of leaves?

Theorem (Blondin Massé, de Carufel, Goupil, Samson 2017)

For $n \geq 1$, the maximal number of leaves realized by a tree-like polyomino with n cells is given by

$$\ell(n) = \begin{cases} 0 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ n - 1 & \text{if } n = 3, 4, 5 \\ \ell(n-4) + 2 & \text{if } n \geq 6 \end{cases}$$

Trees in Minecraft

polyominoes ⇒ polycubes

Trees in Minecraft

tree-like polyominoes ⇒ tree-like polycubes

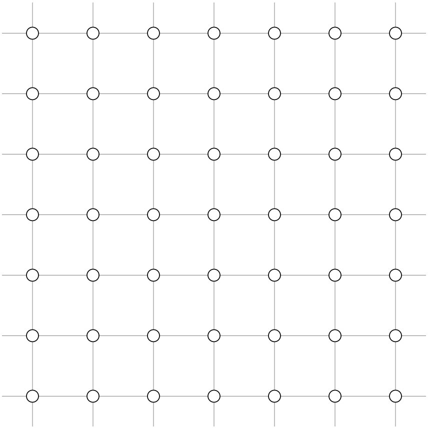
Trees in Minecraft

tree-like polyominoes ⇒ tree-like polycubes

Theorem (Blondin Massé, de Carufel, Goupil, Samson 2017)

The maximal number of leaves realized by a tree-like polycube satisfies a linear recurrence.

- Tree-like polyominoes are induced subtrees of $\mathbb{Z}^2$.



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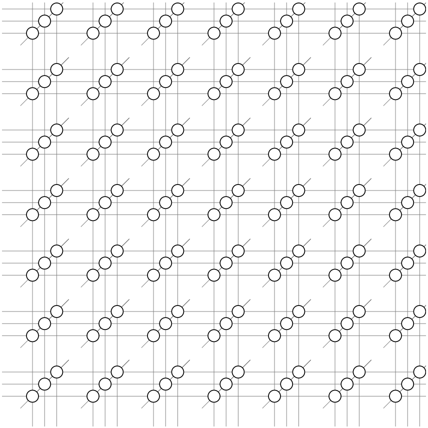
- Tree-like polycubes are induced subtrees of $\mathbb{Z}^3$.



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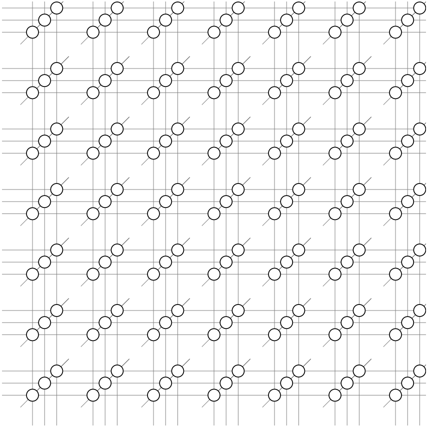
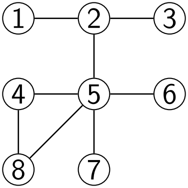


Can we study this problem in other graphs?

Definition

For a graph $G = (V, E)$ and $n \geq 2$

- $\mathcal{T}_n$ = set of induced subtrees with n vertices
- $L_G(n) = \max\{\#\text{ leaves in } T | T \in \mathcal{T}_n\}$
- Leafed sequence of G : $L_G(n)_{n \in \{2,\ldots,|V|\}}$



n	2	3	4	5	6	7	8
$L_G(n)$	2						

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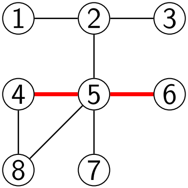

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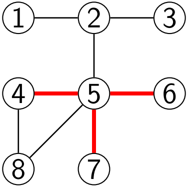

n	2	3	4	5	6	7	8
$L_G(n)$	2	2	3				

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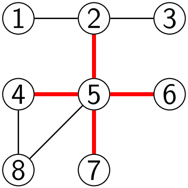

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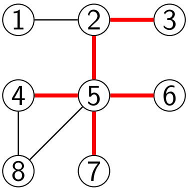

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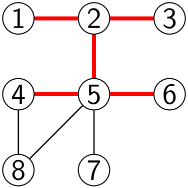

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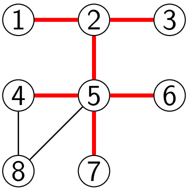

n	2	3	4	5	6	7	8
$L_G(n)$	2	2	3	4	4	5	

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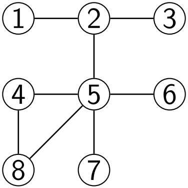
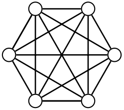


n	2	3	4	5	6	7	8
$L_G(n)$	2	2	3	4	4	5	$-\infty$

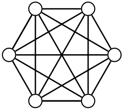
Particular cases

K_6 

$$L_{K_p}(n) = \begin{cases} 2 & \text{if } n = 2 \text{ et } p \geq 2 \\ -\infty & \text{if } 3 \leq n \leq p \end{cases}$$

Particular cases

K_6



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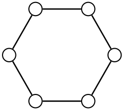

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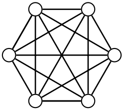
K_6 

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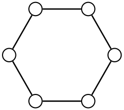
$\mathcal{C}_6$ 

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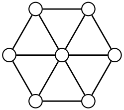
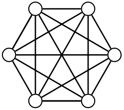
R_6 

$$L_{R_p}(n) = \begin{cases} 2 & \text{if } 2 = n \text{ ou } \lfloor \frac{p}{2} \rfloor + 1 < n < p \\ n-1 & \text{if } 3 \leq n \leq \lfloor \frac{p}{2} \rfloor + 1 \\ -\infty & \text{if } p \leq n \leq p+1 \end{cases}$$

Particular cases

K_6 

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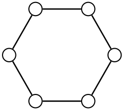
$\mathcal{C}_6$ 

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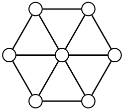
R_6 

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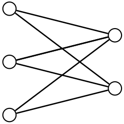
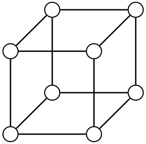
$K_{3,2}$ 

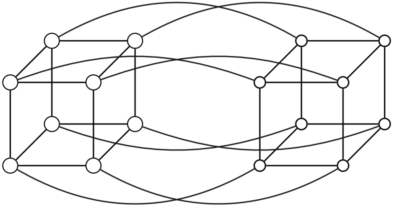
$$L_{K_{p,q}}(n) = \begin{cases} 2 & \text{if } n = 2 \\ n-1 & \text{if } 3 \leq n \leq \max(p,q)+1 \\ -\infty & \text{if } \max(p,q)+1 < n \leq p+q \end{cases}$$

Particular cases: hypercubes

Only partial results:

$$L_{H_3}(n) = \begin{cases} 2 & \text{if } 2 \leq n \leq 3 \\ 3 & \text{if } n = 4 \\ -\infty & \text{if } 5 \leq n \leq 8 \end{cases} \qquad L_{H_4}(n) = \begin{cases} 2 & \text{if } n \in \{2,3\} \\ 3 & \text{if } n \in \{4,6,8\} \\ 4 & \text{if } n \in \{5,7,9\} \\ -\infty & \text{if } 10 \leq n \leq 16 \end{cases}$$





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Observations:

- The leafed sequence can increase by at most 1 each step.
- The leafed sequence is not always non-decreasing.
- The leafed sequence $L_G(n)_{n\in\{2,\ldots,|V|\}}$ is non-decreasing iff G is a tree.

Complexity

Problem LIS

- Instance: a graph G and two integers $n, k \geq 1$
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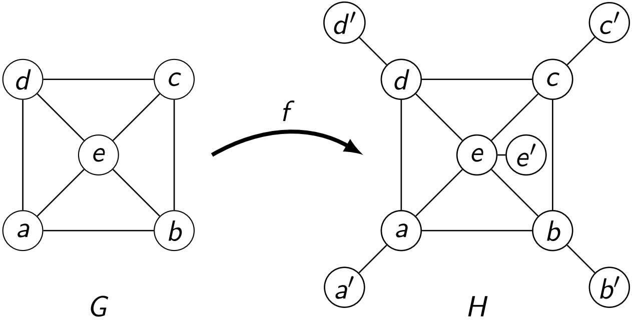
Reduction to the problem:

- Instance: a graph G and an integer $n \geq 1$
- Question: Is there an induced subtree of G with more than n vertices?

which is NP-complet. [Erdös, Saks, Sós 1986]

LIS is NP-complete

- Polynomial transformation $f : (G, n) \mapsto (H, 2(n+1), n+1)$



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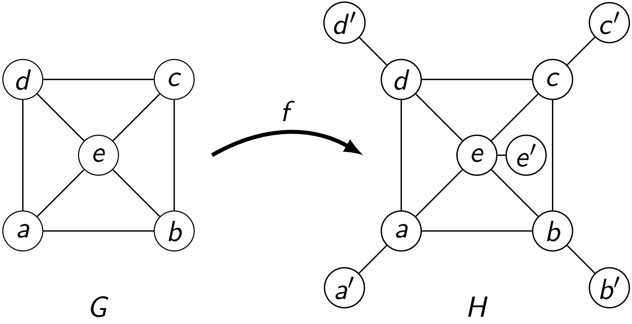

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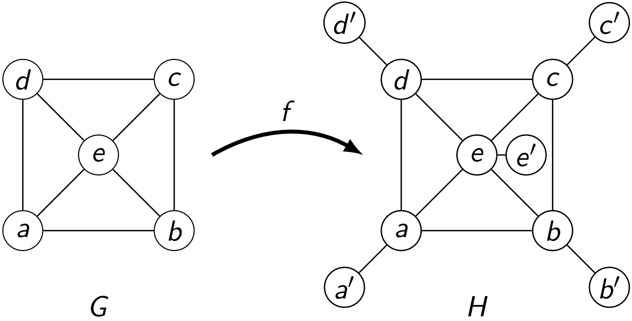

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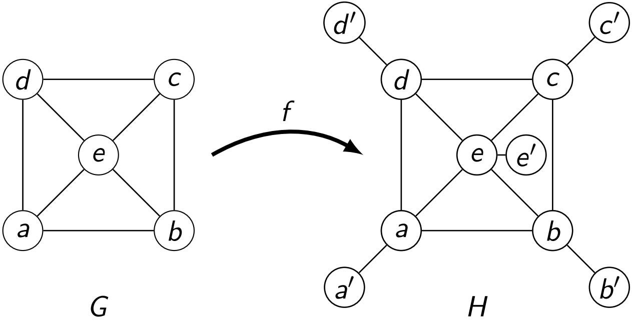

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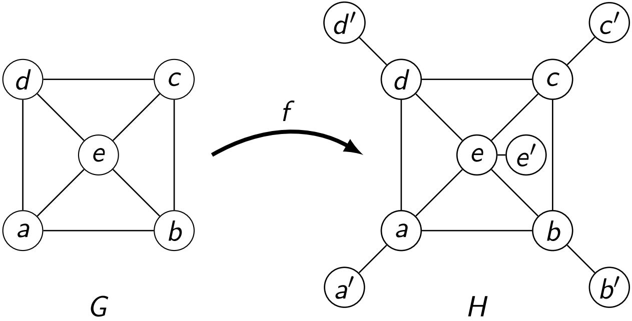

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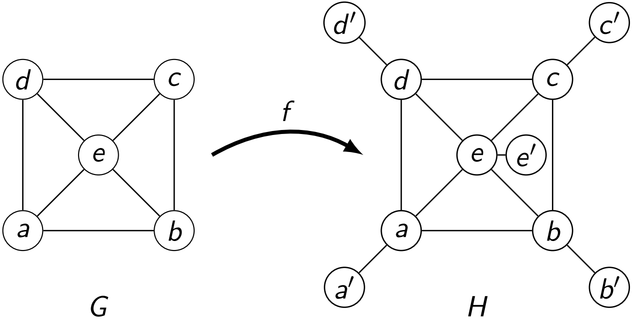

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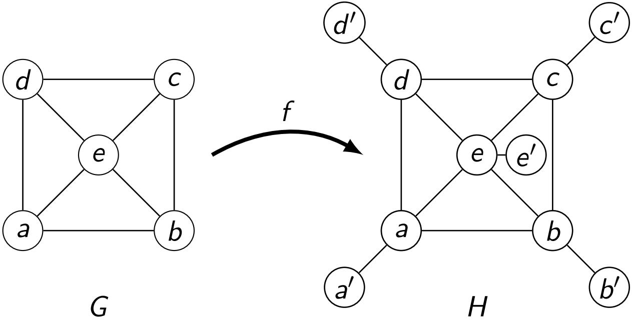

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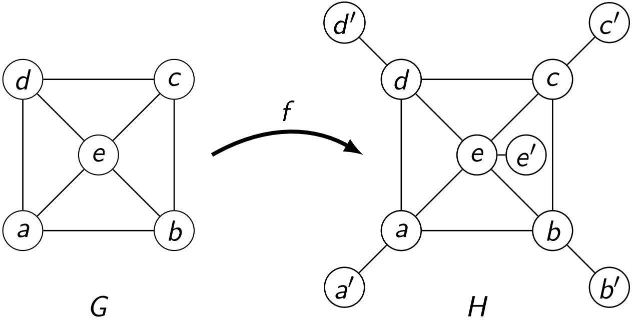


- If H has an induced subtree T with $2(n+1)$ vertices and $n+1$ leaves

$\Rightarrow$ T has $\geq n+1$ vertices in V

$\Rightarrow$ G has an induced subtree with $> n$ vertices

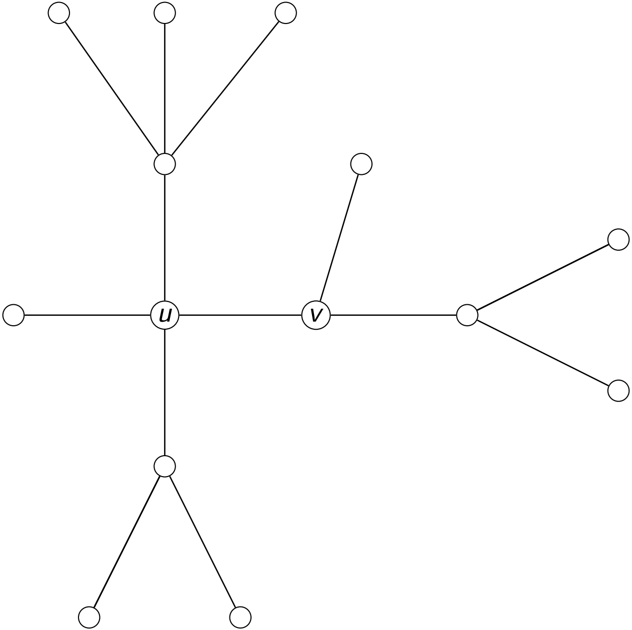
What about trees?

Theorem (Blondin Massé, de Carufel, Goupil, V. 2017)

For a tree T with m vertices, the leafed sequence $L_T(n)$ is computed in polynomial time and space.

Algorithm based on the dynamic programming paradigm

Idea of the algorithm



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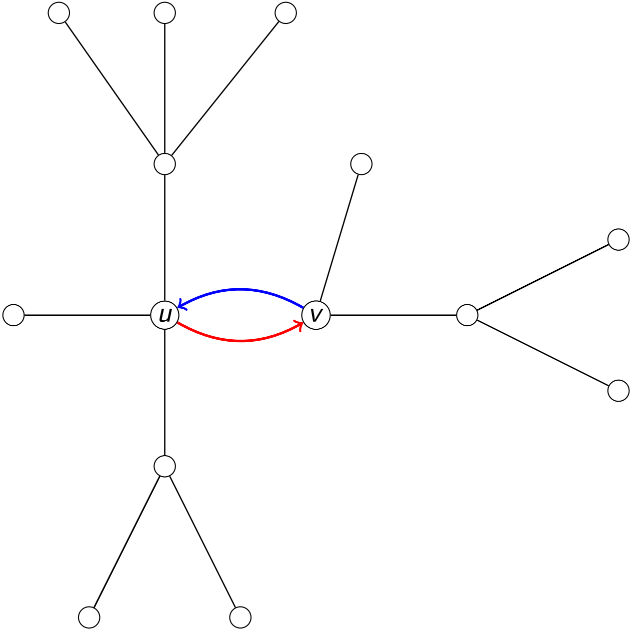

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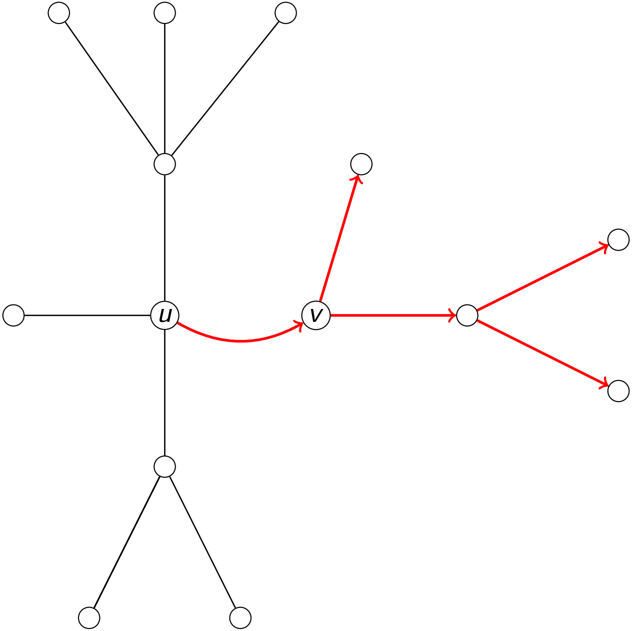

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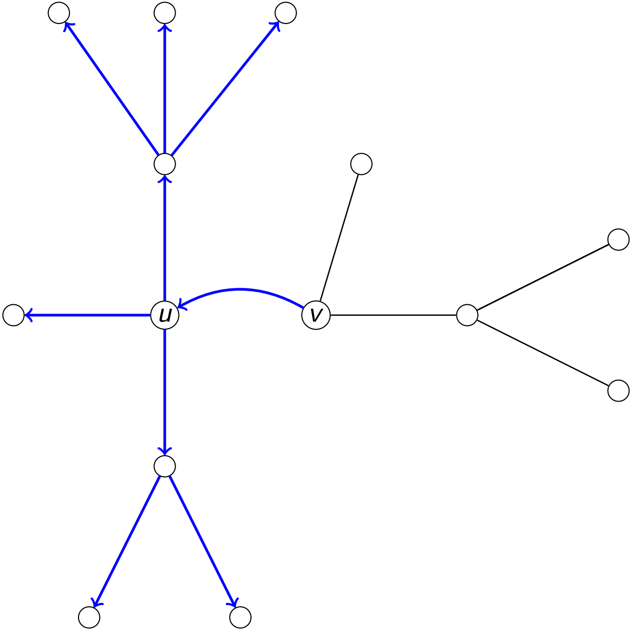

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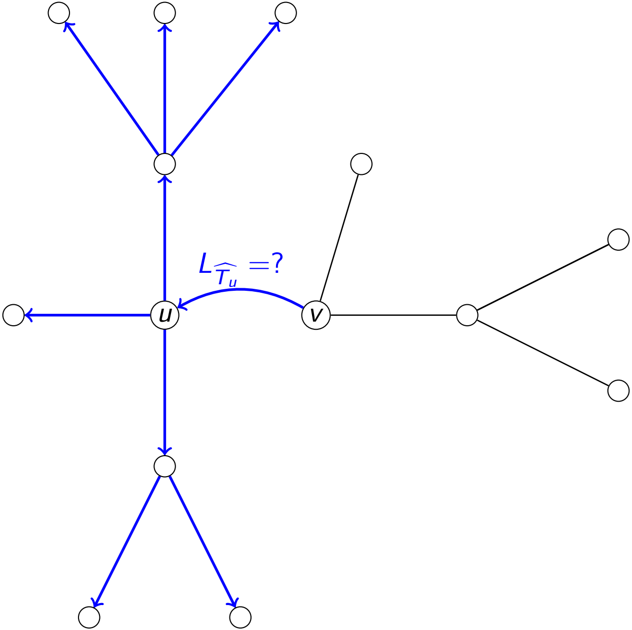


Idea of the algorithm

For a directed tree $\widehat{T_u}$ rooted in u

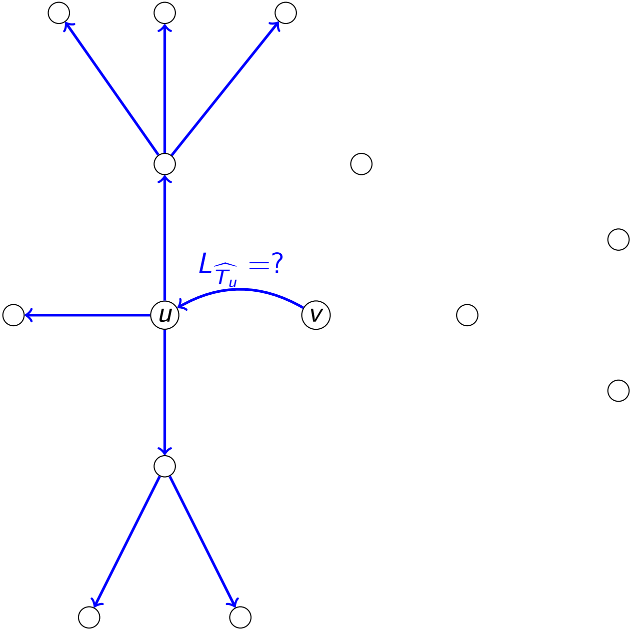
- $f(\widehat{T_u}) = \#\{x \in \widehat{T_u} \mid \deg^+(x) = 0\}$
- $L_{\widehat{T_u}}(n) = \max\{f(\widehat{T'_u}) : \widehat{T'_u} \subseteq \widehat{T}_u, |\widehat{T'_u}| = n\}$
- Generalization to a directed forest $\widehat{F}$ with k connected components $\widehat{F}_i$:

$$L_{\widehat{F}}(n) = \max \left\{ \sum_{i=1}^{k} L_{\widehat{F}_i}(\lambda(i)) \;\middle|\; \lambda \in C(n, k) \right\}$$

- If $\widehat{F}$ is the forest of the subtrees rooted in the children of u,

$$L_{\widehat{T_u}}(n) = \begin{cases} n & \text{if } n = 0, 1 \\ L_{\widehat{F}}(n-1) & \text{otherwise} \end{cases}$$

Idea of the algorithm



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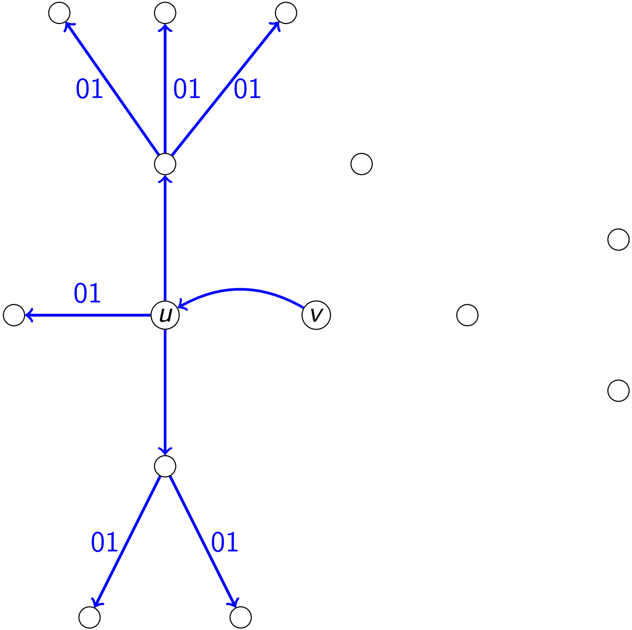

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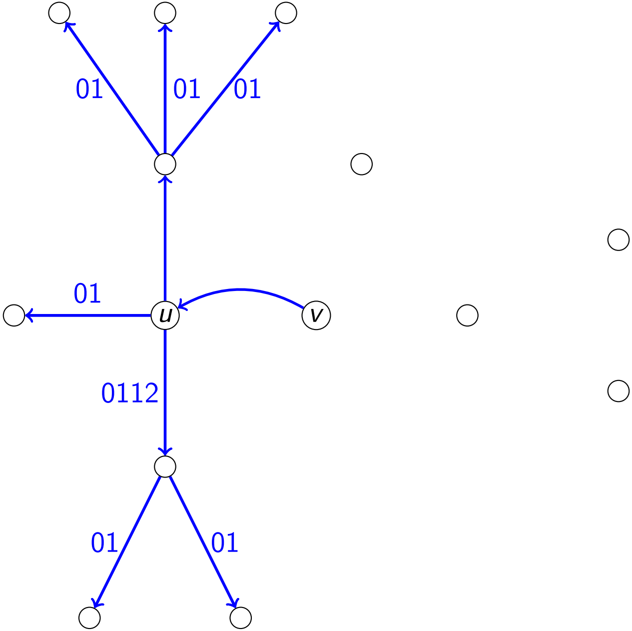

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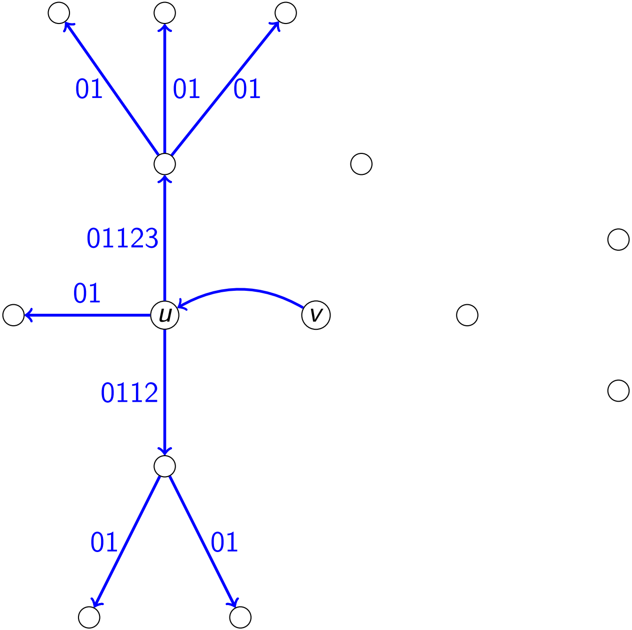

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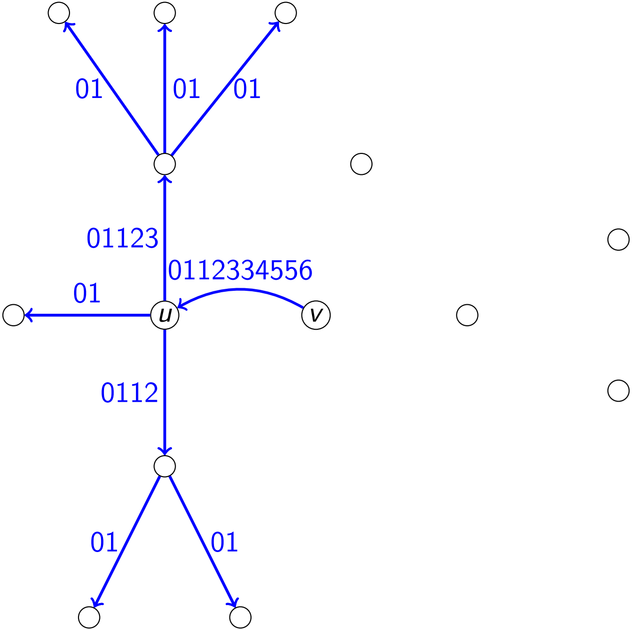

Idea of the algorithm

Assuming $L_{\widehat{F}_i}$ are known,

- Naive computation of $L_{\widehat{F}}$ via

$$L_{\widehat{F}}(n) = \max\left\{ \sum_{i=1}^{k} L_{\widehat{F}_i}(\lambda(i)) \;\middle|\; \lambda \in C(n, k) \right\}$$

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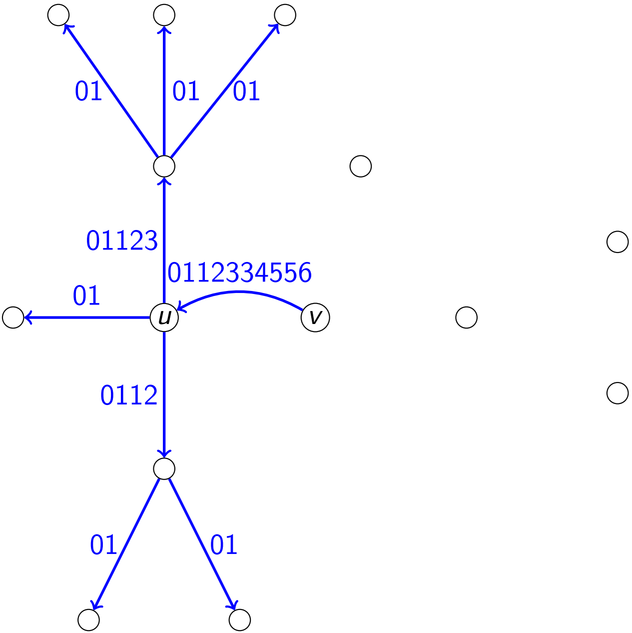
$$L_{\widehat{F}}(n) = \max\{L_{\widehat{F-F_1}}(i) + L_{\widehat{F_1}}(n-i) \mid 0 \leq i \leq n\}$$

  in time $\Theta(k|F|^2)$

Idea of the algorithm



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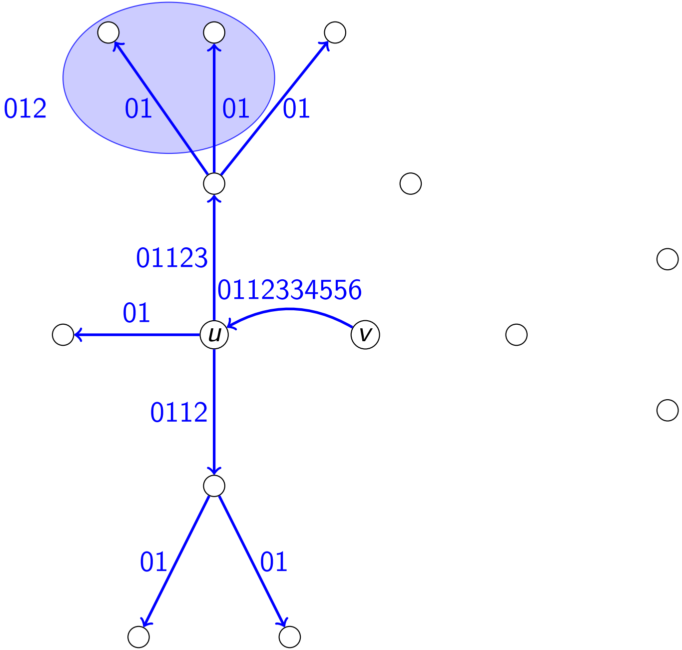

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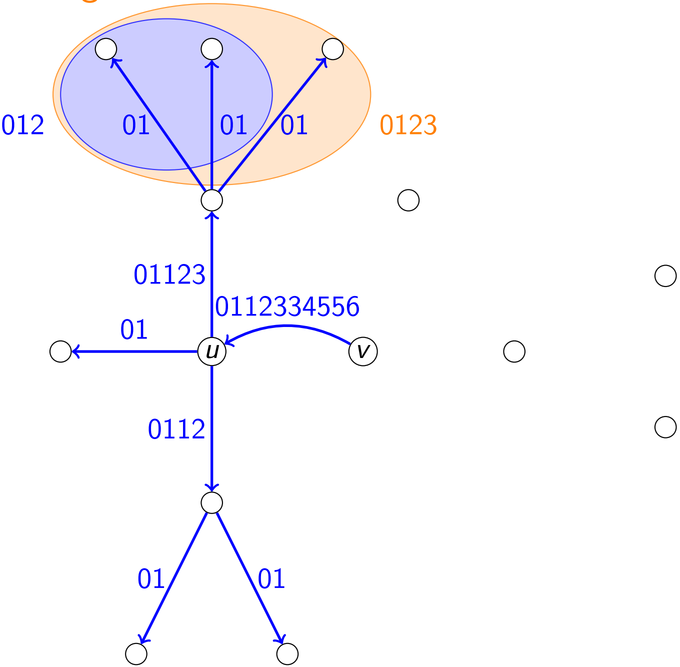

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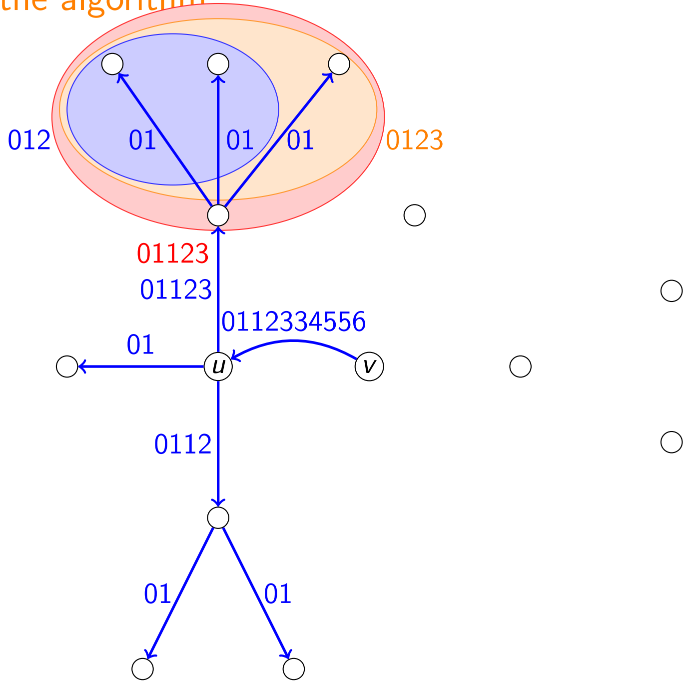

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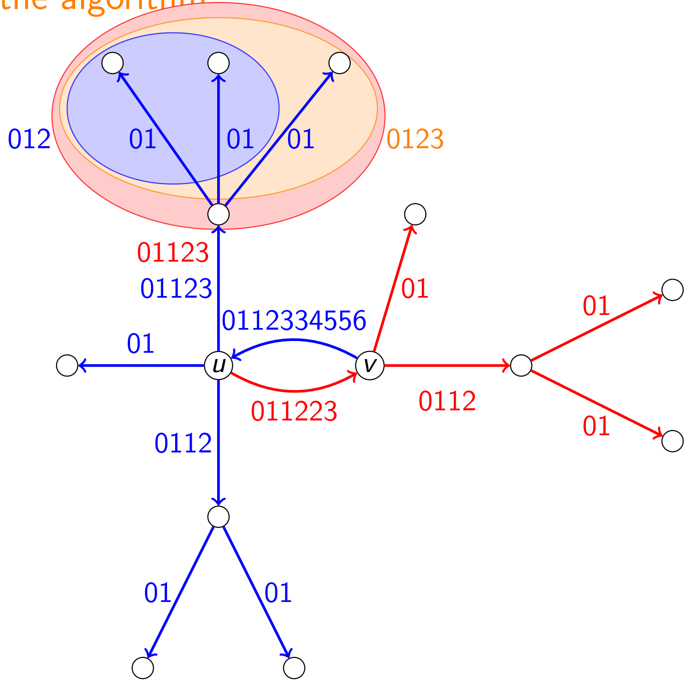
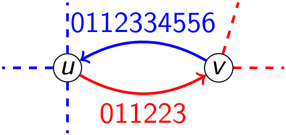


Idea of the algorithm

- Maximal number of leaves in the subtrees with n vertices, containing the edge $\{u, v\}$:

$$L_{\{u,v\}}(n) = \max\left\{ L_{\widehat{T_u}}(i) + L_{\widehat{T_v}}(n-i) \;\middle|\; 1 \leq i \leq n-1 \right\}$$



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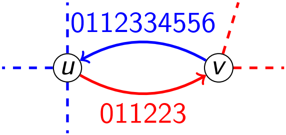

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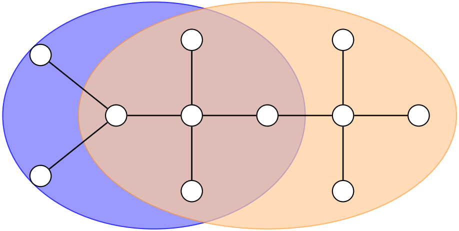
- $L_T(n) = \max\left\{L_{\{u,v\}}(n) \;\middle|\; \{u,v\} \in E\right\}$

Could we improve the time and space complexity?

- We can not hope to obtain a procedure computing $L_T(n)$ which deletes leaves successively.
- Counter-example :



$L_T(7) = 5$ et $L_T(9) = 6$

From graph theory to combinatorics on words

- Each tree has a non-decreasing leaf sequence $L = (L_i)_{i \geq 2}$.
- For any graph, $L_{i+1} - L_i \leq 1$.

The difference word $\Delta L := (L_{i+1} - L_i)_{i \geq 3}$ is a binary word.

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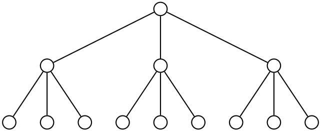

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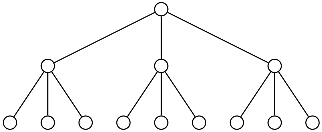

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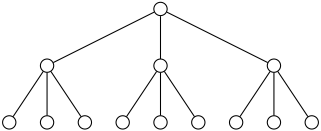

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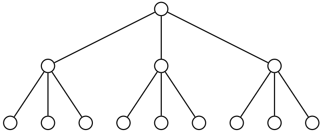

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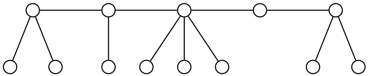
Language of Trees =???

Caterpillars

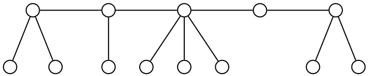
Caterpillars



$L = 223455667778$ and $\Delta L = 1110101001$

What is the language of caterpillars ?

Caterpillars



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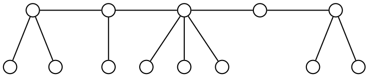

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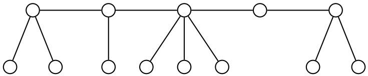

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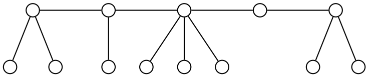

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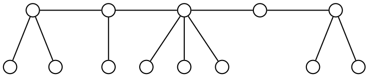

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n	0	1	2	3	4	5	6	7	8	9
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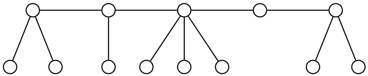

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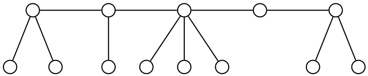

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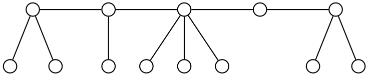
Prefix normal word

A prefix normal word $w_1 \cdots w_n$ is a binary word such that $\forall \ell \leq n, i \leq n - \ell,$

$$|w_1 \cdots w_\ell|_1 \geq |w_i \cdots w_{i+\ell}|_1.$$



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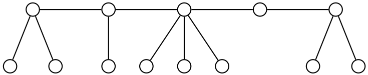

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Conjecture:

$$\text{Language of Caterpillars} = \{\text{prefix normal words}\}$$

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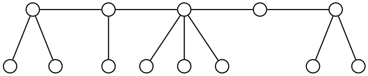

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Proposition (Blondin Massé, de Carufel, Goupil, Lapointe, V.)

$$\text{Language of Caterpillars} \supseteq \{\text{prefix normal words}\}$$

Perspectives

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- Extend the results obtained in $\mathbb{Z}^2$ to the infinite triangular grid (polyiamonds) and to the infinite hexagonal grid (polyhexes)