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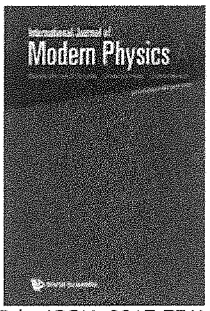

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STRONG COULOMB COUPLING IN RELATIVISTIC QUANTUM CONSTRA

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STRONG COULOMB COUPLING IN CONSTRAINT DYNAMICS†

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ABSTRACT

We study, in the framework of Relativistic Quantum Constraint Dynamics, the bound state problem of two oppositely charged spin-1/2 particles, with masses m_1 and m_2, in mutual electromagnetic interaction. We search for the critical value of the coupling constant α for which the bound state energy reaches the lower continuum, thus indicating the instability of the heavier particle or of the stongly coupled QED vacuum in the equal mass case. Two different choices of the electromagnetic potential are considered, corresponding to different extensions of the substitution rule into the nonperturbative region of α .

1. Introduction

The instability, due to spontaneous pair creation, of external strong Coulomb fields[1], raises the question of whether this phenomenon might be a preliminary signal of an eventual phase transition in QED in its strong coupling regime. If the instability phenomenon persists in the case of a two-particle system, made of two fermions with opposite charges and with masses m_1 and m_2 , say, then this would mean that the ground state of the two-particle system has a mass of $|m_2 - m_1|$. If the phenomenon still persists in the limit $m_2 = m_1$, then one would obtain a zero mass bound state, having the same quantum numbers as a pseudoscalar boson, which might lead to a spontaneous breakdown of chiral symmetry in QED.

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Therefore, the study of the bound state spectrum of two oppositely charged particles, with the search for a massless bound state in the equal mass case, is a key probe of the existence of a phase transition in QED.

On the other hand, lattice calculations of quenched QED seem to predict a new phase of QED for $\alpha > \alpha_c$ ($\alpha_c \sim 0.3$) , with spontaneous chiral symmetry breaking[2]. This conclusion is also supported by the solution of the Bethe-Salpeter equation in the ladder approximation[3,4]. The latter results would imply an instability of the QED vacuum itself accompanied by the occurrence of a zero mass bound state in the two-body equal mass case.

Our purpose is to investigate, in the framework of Relativistic Quantum Constraint Dynamics[5-8] the stability properties, with respect to spontaneous pair creation, of charged particles heavier than the electron as well as of the QED vacuum. This is done by studying the bound state problem of two oppositely charged spin 1/2 particles with masses m_1 and m_2 in mutual electromagnetic interaction and searching for the possibility that the bound state mass reaches the lower continuum.

One of the main advantages of Relativistic Quantum Constraint Dynamics is that it provides a three-dimensional manifestly covariant description of the internal motion of the two-body system, once the redundant relative energy variable is eliminated by means of the constraint. These equations correctly reproduce the muonium and positronium spectra[8] up to terms of order α^4. Furthermore, one can establish a correspondence between the Feynman diagrams of the kernel of the Bethe-Salpeter equation and the interaction potential of the Constraint Theory wave equations[9]. When one of the particles becomes infinitely massive, the two-body wave equations reduce to the one-body Dirac equation in an external Coulomb field.

The electromagnetic interaction (in local approximation) can be introduced in the Constraint Theory wave equations by means of simple substitution rules. However, extension of the substitution rule to higher orders depends on the stage at which the latter is applied. In particular, two diffrent expressions of the interaction potential are found, depending on whether one applies the substitution rule in the final eigenvalue equation of the relative motion, or in the initial individual wave equations. In the first case, one finds the Todorov potential, already introduced in the Quasipotential approach[10] and later used by Crater and Van Alstine and collaborators in Constraint Dynamics[5,8]. The main feature of the Todorov potential is that it is dominated by the one photon exchange contribution even for large values of the coupling constant and necessitates in this domain the use of a short-distance regularization cut-off. In the second case, another potential is found, the main feature of which is to be less dominated by the one photon exchange contribution in the large coupling regime, exhibiting a regular behaviour at short distances. This potential will henceforth be designated as "potential II".

2. Wave Equations of Constraint Dynamics

The relativistic wave equations of Constraint Dynamics for two particles of masses m_1 (fermion) and m_2 (antifermion) in mutual interaction are given by[7] :

$$(\gamma_1.p_1 - m_1)\ \widetilde{\Psi} = -\ (\gamma_2.p_2 - m_2)\ \widetilde{V}\ \widetilde{\Psi}\ , \tag{1a}$$

$$(\gamma_2.p_2 + m_2)\ \widetilde{\Psi} = -\ (\gamma_1.p_1 + m_1)\ \widetilde{V}\ \widetilde{\Psi}\ , \tag{1b}$$

where the indices 1 and 2 refer to particles 1 and 2, respectively; p_1 and p_2 are the four-momentum operators and γ_1 and γ_2 are the Dirac matrices relative to particles 1 and 2. $\widetilde{V}$ is a Poincaré invariant interaction potential. The wave function $\widetilde{\Psi}$ is a 4×4 matrix function and the matrices γ_2, acting on the antifermion indices, act on $\widetilde{\Psi}$ from the right.

Following an observation of Crater and Van Alstine[11], the potential $\widetilde{V}$ is parametrized by means of a hyperbolic function :

$$\widetilde{V} = \tanh\left[\frac{1}{2}\,C\ (\ \gamma_1.\gamma_2\ )\right]\ , \tag{2}$$

where the Feynman gauge has been chosen in lowest order of the interaction. To this order, the new potential C is related to the photon propagator[9].

Equations (1) can be solved by decomposing the wave function $\widetilde{\Psi}$ along 2×2 matrix components and eliminating these with respect to one of them. For the sector of solutions with quantum numbers $j = \ell = s = 0$ the reduced wave function φ satisfies in the center-of-mass (c.m.) frame the eigenvalue equation :

$$\left(\frac{E^2}{4}\,e^{4C} - \frac{1}{2}\,(m_1^2 + m_2^2)\ e^{2C} + \frac{(m_1^2 - m_2^2)^2}{4E^2} + \nabla^2 \right.$$
$$\left. -\ 4r^2 h'^{\,2} + 6h' + 4r^2 h'' \right)\varphi = 0\ , \tag{3}$$

where one has :

$$h = \ln \left[1 - \frac{(m_1 - m_2)^2}{E^2} e^{-2C} \right]^{\frac{1}{2}} , \tag{4}$$

E being the c.m. value of the total energy :

$$E^2 = (p_1 + p_2)^2 = P^2 , \quad P = (p_1 + p_2) , \tag{5}$$

while r is the invariant c.m. distance (or transverse relative coordinate):

$$r^2 = -(x^T)^2 , \tag{6}$$

$$x^T_\mu = x_\mu - \frac{P.x}{P^2} P_\mu , \quad x = x_1 - x_2 , \tag{7}$$

and where

$$h' \equiv \frac{\partial h}{\partial (r^2)} . \tag{8}$$

(Details on the resolution of Eqs. (1) are presented in the Appendix of Ref. 12.)

The expression of potential C can be fixed with recourse to the substitution rule. The result, however, depends on the stage at which this rule is applied. Two possibilities can be distinguished.

In the first place, one can apply the substitution rule in the final eigenvalue equation (3) by identifying it with the Klein-Gordon equation of a fictitious particle in the presence of an external electromagnetic field. This method was used by Todorov in the Quasipotential approach for two spin-0 particles[10]; by identifying the electric part of the electromagnetic field with the Coulomb potential, one finds for C the expression (henceforth designated as the Todorov potential, or potential I) :

$$C = \frac{1}{2} \ln \left(1 - \frac{2V}{E} \right) , \tag{9}$$

where V is the Coulomb potential :

$$V = -\frac{\alpha}{r} . \tag{10}$$

In the second place, the substitution rule can also be used in the individual wave equations (1). To lowest order of perturbation theory, the expression of C can be determined from the Bethe-Salpeter kernel[9] (in the Feynman gauge). One finds

: $C = -V/E$, V being the Coulomb potential [Eq. (10)]. It is seen in Eqs. (1) that the total energy E undergoes, to lowest order in V, the modification $E \to (E - V)$. By extending this substitution to higher orders, one obtains for C the following expression (henceforth designated as potential II) :

$$C = -\frac{V}{E - V} , \tag{11}$$

V being defined in Eq. (10).

We notice that both potentials [Eqs. (9) and (11)] formally coincide up to $O(V^2)$, and hence lead to the same $O(\alpha^4)$ effects in perturbation theory. For both of them, Eq. (3) reproduces the correct $O(\alpha^4)$ muonium and positronium spectra for the $^1S_0$ sector and reduces to the Dirac equation for the (properly normalized) ground state radial wave function and its radial excitations when one of the masses becomes infinite[8].

In the following, we first consider the case of potential II.

3. Potential II

Potential II corresponds to the choice of C given by Eq. (11). We have solved Eq. (3) with this potential. A particular feature of potential II is that it does not need any short-distance regularization : no V-dependent term in Eq. (3) is singular. Figure 1 shows the variation of the lowest $^1S_0$ eigenvalue E with respect to α in the equal mass case. The quantity E approaches a constant value for large α values and remains positive for any α. This feature can be related to the fact that for large α one has $C \sim 1$, $C', C'' \sim 0$. One then obtains from Eq. (3) the lower bound for E in the equal mass case[12] :

$$E > \frac{2m}{e} , \tag{12}$$

which represents the asymptotic value of E in Fig. 1 .

Figure 2 shows how E varies with α in the unequal mass case for various values of m_2 , while m_1 $(m_2 \geq m_1)$ is kept fixed and equal to the electron mass. The function which is plotted there (and also in Fig. 3) is the quantity

$$\frac{W}{m_1} = \frac{E - (m_2 - m_1)}{m_1} , \tag{13}$$

which is always contained in the interval [0,2] for a bound state.

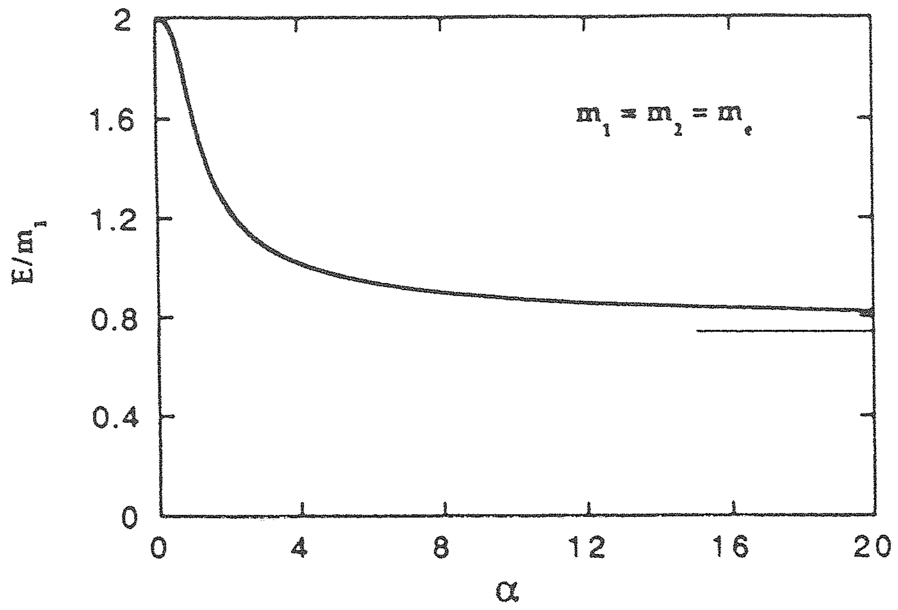

Fig. 1. The ground state energy E in the equal mass case, as a function of the coupling constant, for potential II. The horizontal line indicates the lower bound in inequality (12).

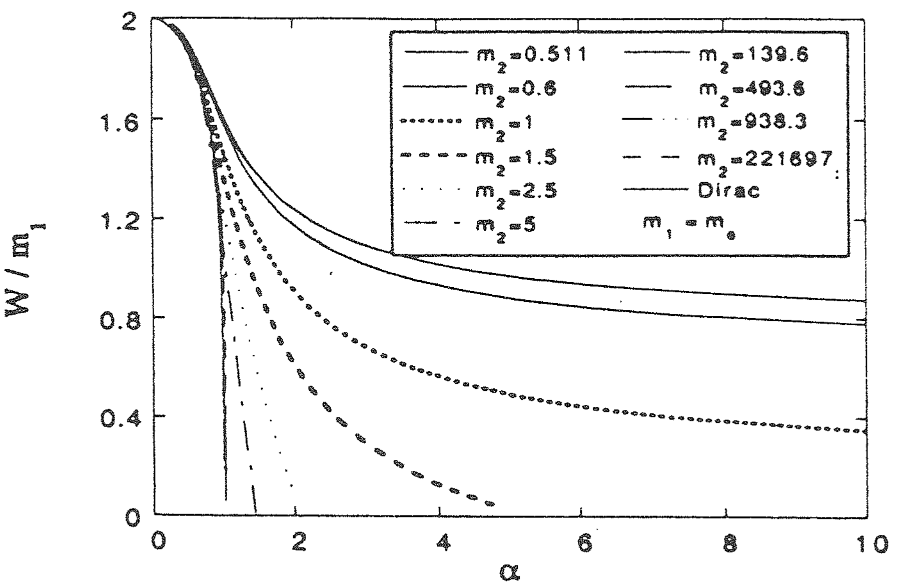

Fig. 2. The ground state energy in the unequal mass case. The mass m_1 is set equal to the electron mass. The quantity in ordinate is equal to 2, when $E = m_2 + m_1$, and to zero, when $E = m_2 - m_1$.

As long as m_2 is greater than a lower bound m_{20} (that will be specified below), there exists a critical value α_c of α , for which $E = m_2 - m_1$. This value of α is critical in the sense that when $\alpha = \alpha_c$, one can have the spontaneous decay

$$\mu^{\pm} \rightarrow (e^{\mp} \mu^{\pm}) + e^{\pm} , \tag{14}$$

where μ is the heavier particle (of mass m_2) and e is the lighter particle (of mass m_1), since the bound system ($e^- \mu^+$) has energy $(m_2 - m_1)$. In the equal mass case, a zero energy state for the ($e^+ e^-$) system would imply the instability of the vacuum itself, according to the same argument.

The lower bound m_{20} of m_2 for the existence of the instability phenomenon can be obtained by analyzing the behavior of the effective potential of Eq. (3) for large values of α . When m_2 is smaller than m_{20} , the instability phenomenon disappears (as in the equal mass case) and one finds for the ground state energy the lower bound

$$E > \frac{m_1 + m_2}{e} , \tag{15}$$

which generalizes Eq. (12) for the unequal mass case. The lower bound m_{20} of m_2 is found by equating E to $m_2 - m_1$. This gives :

$$m_{20} = m_1 \left(\frac{e+1}{e-1} \right) = 2.16 m_1 . \tag{16}$$

This particular value $(E = m_2 - m_1)$ is obtained at $\alpha_c = \infty$ only. For larger values of m_2 , α_c is finite and smaller. As m_2 increases, the critical value α_c decreases smoothly down to $\alpha_c = 1$, which is the limiting value for $m_2 \rightarrow \infty$, as expected from the static limit.

In summary, potential II does not predict any instability of the QED vacuum (see the equal mass case), although instability occurs for heavy charged particles when $m_2 > m_{20} = 2.16 m_1$ and $\alpha > \alpha_c(m_2/m_1)$.

4. The Todorov Potential

The Todorov potential corresponds to the choice of C given by Eq. (9). Because of the singularity in $1/r^2$, a cut-off radius is needed to solve Eq. (3) with this potential for values of α greater than 1/2. We adopt the following cut-off :

$$\begin{aligned} V(r) &= -\frac{\alpha}{r} , \quad r > r_0 , \\ &= -\frac{\alpha}{r_0} , \quad r \leq r_0 . \end{aligned} \tag{17}$$

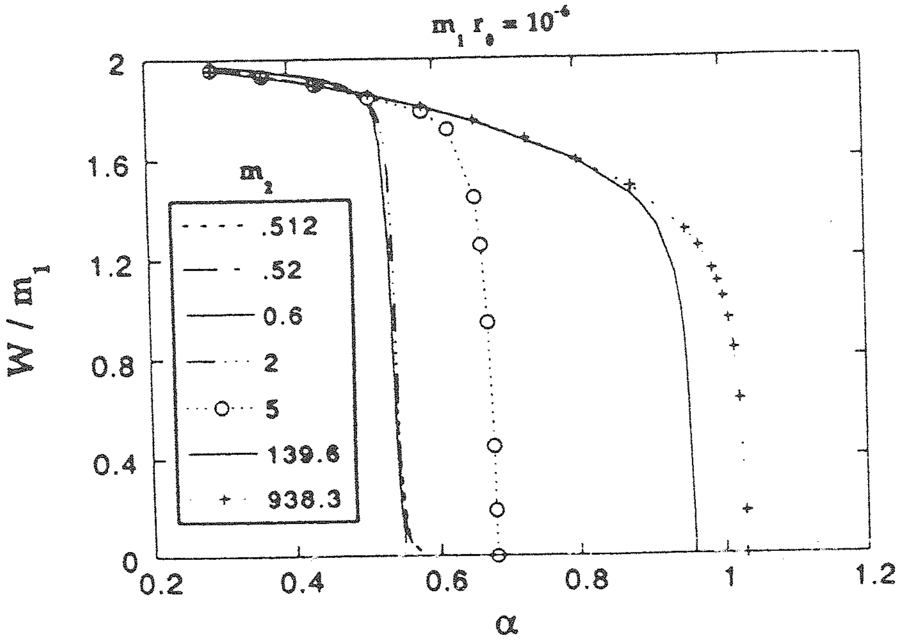

Fig. 3. Same as Fig. 2 for the Todorov potential. A cut-off radius satisfying $m_1 r_0 = 10^{-6}$ has been chosen.

For $m_2 \neq m_1$, Eq. (3) leads to a critical value $\alpha = \alpha_c$, depending on m_2/m_1 , for any value of m_2 $(> m_1)$. This is mainly due to the presence of the singular interaction term $(\sim \alpha^2/r^2)$. It is well known that such a singular term implies an instability of the bound state equation[1,13] for $\alpha > 1/2$ (the instability is not removed when a cut-off of the type of Eq. (17) is introduced[13]). When $m_2 \to \infty$, $\alpha_c \to 1$ (the critical value of α in the Dirac equation) and when $m_2 \to m_1$ (but $m_2 > m_1$), $\alpha_c \to 1/2$ (the critical value of α in the Klein-Gordon equation). (See Fig. 3 .)

The equal mass case is more difficult to analyze, because of the presence of the term $V(m_1^2 + m_2^2)/(m_2 - m_1)$ in Eq. (3), when E is replaced by $(m_2 - m_1)$. This case was studied in detail in Ref. 14 . As long as r_0 is different from zero, there is no critical value of α for $m_2 = m_1$ and E remains positive. However, when the cut-off radius vanishes, one finds zero energy solutions for $\alpha > 1/2$, which can be interpreted as a signal of vacuum instability, starting at $\alpha_c = 1/2$.

In summary, the Todorov potential leads to an instability of the heavy charged particle for any value of its mass $(m_2 > m_1)$ and with a critical value of $\alpha = \alpha_c(m_2/m_1)$ varying from 1/2 to 1, when m_2 varies from m_1 to infinity. In the equal mass case, instability occurs when the cut-off radius vanishes.

5. Conclusion

We found that the instability, due to spontaneous pair creation, in electromagnetically bound systems, depends crucially on the way electromagnetic interaction is extended to the strong coupling regime. The two potentials we considered coincide up to $O(\alpha^4)$ effects, but drastically differ in the nonperturbative region of the coupling constant α . The Todorov potential continues to be dominated for large α by the one photon exchange contribution and hence displays short distance singularities that are typical of the relativistic Coulomb potential. In potential II, the multiphoton exchange contributions add up in such a way that they regularize the potential at the origin.

The implications of these two potentials go in two different directions. The Todorov potential leads to instability for all values of the ratio m_2/m_1 and with $1/2 \leq \alpha_c \leq 1$, the upper bound being reached for $m_2/m_1 = \infty$ and the lower bound for $m_2 = m_1$; in the equal mass case, instability occurs only for a vanishing value of the cut-off radius r_0 . Qualitatively, these results agree with those obtained from lattice calculations of quenched QED and from the Bethe-Salpeter equation in the ladder approximation.

Potential II, on the other hand, does not lead to an instability of the QED vacuum, although it predicts an instability of the heavy charged particle for $2.16 < m_2/m_1 \leq \infty$, with $1 \leq \alpha_c < \infty$, the lower bound of α_c corresponding to $m_2/m_1 = \infty$ and the upper bound to $m_2/m_1 = 2.16$.

The analysis of the above potentials in terms of Feynman diagrams might help us to better understand their origin at the field theoretic level.

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