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Optical Model for the Width of Heavy-Quarkonium States.

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Abstract. – A simple optical-model description of the width of heavy-quarkonium states is proposed. It is based on the string fragmentation picture. Experimentally known widths are fairly well reproduced. Predictions are made for other states.

It is well known that charmonium and bottonium states can be calculated by using a nonrelativistic Schrödinger equation [1-6]. The basic reasons are: 1) the mass of the charm and bottom quarks is much larger than the QCD scale, which makes this system free of strong renormalization effects and 2) the binding energy is small comparatively to the mass energy. One may wonder whether the widths of the high-lying states (above the D-$\overline{\text{D}}$ threshold in charmonium and above the B-$\overline{\text{B}}$ threshold in bottonium) can be described by a nonrelativistic approach. In such an approach, the calculation of the widths may be tackled by explicitly introducing the D-$\overline{\text{D}}$ (or B-$\overline{\text{B}}$) channels (and the appropriate couplings) or by introducing an optical model phenomenologically. In this note, we adopt the second choice and show that the widths can be reproduced by a simple-complex optical-model potential, with a physically plausible imaginary part.

The model corresponds to the following Schrödinger equation:

$$\left(-\frac{h^2}{2\mu}\Delta + V(r) - iW(r) \right)\psi = E\psi, \tag{1}$$

where μ is the reduced mass of the system, $V(r)$ is the usual charmonium (or bottonium) potential (including spin-orbit, spin-spin and tensor parts) and $W(r)$ accounts for the possible loss of flux, due to the coupling of Q-$\overline{\text{Q}}$ relative motion to the D-$\overline{\text{D}}$ or B-$\overline{\text{B}}$ channels. Although a microscopic derivation would very likely yield a nonlocal and energy-dependent imaginary potential, it is expected that in a limited range of energy a local, energy-independent imaginary part will give a good description of the loss of probability, similarly with the hadron-nucleus case where this situation holds and is well documented [7]. The r-dependence comes naturally in the string or colour flux tube model. If the string is stretched too much, it

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is expected to break up and lead to the formation of a D-$\overline{\text{D}}$ or B-$\overline{\text{B}}$ system. Since the breaking requires some excitation energy, it is natural to consider that it will break if its length exceeds some critical value. These simple considerations are embodied in the following form of $W(r)$:

$$W(r) = W_0\,\theta(r - L)\,, \tag{2}$$

where $\theta(x)$ is equal to unity if $x > 0$ and zero otherwise. In many (classical) fragmentation models [8], the breaking probability increases with the length of the string, an effect which will appear here as a result of the quantum average. Indeed, the width Γ_i of a state ψ_i can be written, in first order, as

$$\frac{\Gamma_i}{2} = \langle \psi_i \,|\, W(r) \,|\, \psi_i \rangle\,. \tag{3}$$

Although the two experimentally known widths of the S states of charmonium can be fairly well reproduced with a single value of W_0 and L, namely 38 MeV and 1.1 fm, respectively, we prefer to incorporate in our description angular-momentum effects and mass effects in the final states, in order to provide a more complete picture as well as a better quantitative agreement.

A rotating string of a given length contains more excitation energy than a $l = 0$ one. Furthermore, the semi-classical phase space, *i.e.* the number of possible final relative angular-momentum states of the final pieces issued from the breaking of the original system in a given orbital angular momentum l, is increasing with l, if l is smaller than twice the spin of the final fragments, and is independent of l otherwise. In brief, there are good reasons to consider that the quantity W_0 should increase with l. We found that the latter form is the most adequate,

$$W_0 = W_1\left(1 + \frac{l}{4}\right), \tag{4}$$

if l is the orbital angular momentum.

The string tension is fairly independent of the flavour of the quarks joined by the string [1, 9]. This is reflected by the fact that the spectrum of charmonium and bottonium is obtained with the same potential. (Here we used the Richardson potential [6].) However, it is expected that the widths of the states of the two systems are influenced by the parameters of the D-$\overline{\text{D}}$ and B-$\overline{\text{B}}$ channels, respectively, by the masses of the respective mesons, at least. One of the indications is provided by the abscissa r_t of the classical turning points at the opening of the D-$\overline{\text{D}}$ or B-$\overline{\text{B}}$ channels. Despite the fact that the Q-$\overline{\text{Q}}$ potential is the same in charmonium and bottonium, the turning points are not the same: for S-states, $r_t = 1.21$ fm in charmonium and 1.26 fm in bottonium; for D-states, the figures are 1.04 fm and 1.22 fm, respectively. This suggests that the minimum length L for break-up is state dependent. To keep in with this observation, we adopt to take, for any state,

$$L = r_t\,. \tag{5}$$

If we had used a more general form, with $L = r_t + a$, the best fit would occur for $a = -0.03$ fm (see fig. 1).

Furthermore, the mass of the final mesons may have another effect, as the phase space depends upon the mass of the fragments, for a given available energy above the threshold. In our fitting procedure, we allowed a different value of W_1 for charmonium and bottonium. It turned out that the best values are $W_1 = 40$ MeV for charmonium and $W_1 = 72$ MeV for bottonium. It is interesting to note that the ratio of these values is very close to the square

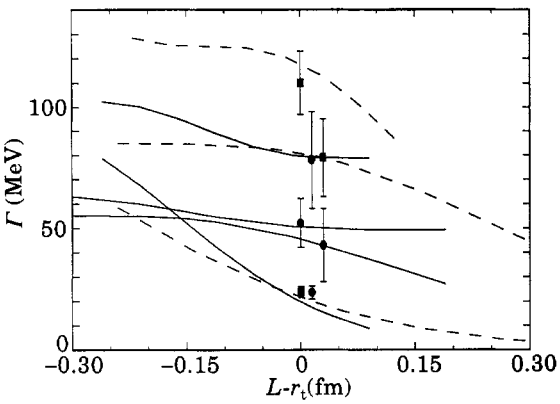

Fig. 1.

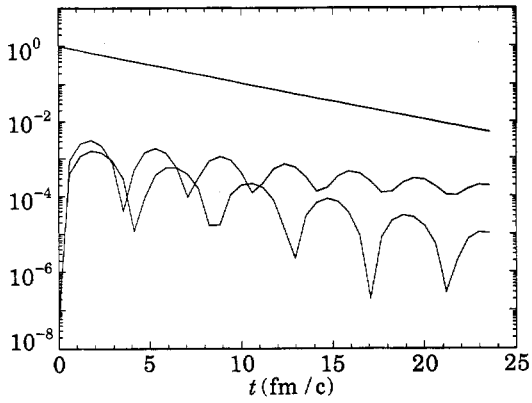

Fig. 2.

Fig. 1. – Variation of the widths, as calculated from eq. (3), with the quantity L, for four states in charmonium and three states in bottonium. The comparison is made with the experimental values [10] (squares for bottonium states and dots for charmonium states; see table I for identification of the states). Our predictions correspond to $L = r_t$ (eq. (5)).

Fig. 2. – Quantities $|c_j|^2$ (eq. (8)) for the time-dependent problem (eq. (6)). The upper curve corresponds to the $3S$ (4040) state of charmonium. The two lower curves correspond to the $4S$ (4415) and $2S$ (3685) states.

root of the ratio of the masses of the respective mesons. In summary, we fitted the experimentally known widths with the parameters W_1 (and the l-dependence contained in (4)). The results are shown in table I along with our predictions for some yet unknown widths. Figure 1 shows the rather large sensitivity of the widths to the parameter L.

Several comments are in order. The width of the $1D$ (3770) state of charmonium is very small, because of the low energy of this state: almost all the wave function lies within 1 fm

TABLE I. – *Comparison of the predictions of our model with the experimental values of the widths of several charmonium and bottonium states (from ref.* [10]). *Whenever the L value and the energy is not known experimentally, the assignment is based on our calculation of the spectrum. See text for details.*

State	Width (exp) (MeV)	Width (model) (MeV)
charmonium		
$1D$ (3770)	23.6 ± 2.7	22
$3S$ (4040)	52 ± 10	45
$2D$ (4160)	78 ± 20	81
$4S$ (4415)	43 ± 15	51
bottonium		
$4S$ (10580)	23.8 ± 2.2	
$2D$ (10644)	—	19
$5S$ (10820)	—	46
$3D$ (10860)	110 ± 13	73
$6S$ (11020)	79 ± 16	118
$4D$ (11050)	—	79
		121

and thus contributes little to the width. The surprisingly large width of the $2D$ (4160) state compared to the one of the $3S$ (4040) state of about the same energy is simply accounted for by the l-dependence indicated in our model by eq. (4). The near equality of the $3S$ (4040) and $4S$ (4415) states comes out very simply in our model: the $4S$ wave function extends farther inside the absorption ($r > L$) region. However, its absolute value is slightly smaller (because the overall normalization is unity) and the most external node of the $4S$ wave function has penetrated the absorption region, making the average value of its modulus inside the absorption region less important. We assigned the (4415) resonance as mainly a S-state, although this is not totally confirmed experimentally, because it appears like that in our computation of the spectrum. In the light of our analysis, it would not easily be another orbital angular-momentum state.

Our local, energy-independent imaginary part provides a nonvanishing width for the $1S$ and $2S$ states. This defect is however really minor, as the theoretical widths are very small: 0.05 MeV for the $1S$ state. In fact, the ability of the model to produce large widths above 4000 MeV and very small widths below reinforces the well-foundedness of the r-dependence of eq. (2).

In bottonium, the width of the $6S$ state is larger than the $4S$ one, because of the larger extension of the wave function of the former state in the absorption region. Our predictions for the $5S$ and $6S$ state reveal the same behaviour as for the $3S$ and $4S$ states in charmonium and as the $3D$ and $4D$ states in bottonium. The same explanation holds for all these cases. The large width of the $3D$ state in bottonium is due to the l-dependence.

We have to note that we attribute the whole of the widths to the D-$\overline{\text{D}}$ or B-$\overline{\text{B}}$ decay. There is no experimental indication on this matter except for the (3770) state only, for which this is largely true.

In spite of the large widths, it can be shown that the approximate formula (3) is quite accurate in the physical situations encountered here. The reason is that the widths ($\lesssim 120$ MeV) are smaller than the separation distance between the states of same quantum numbers ($\gtrsim 400$ MeV). In other words, the time scale associated with the decay is generally much shorter than the time necessary for two neighbouring states (of the real Hamiltonian) to be considerably mixed. We have verified this point by solving the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \left(- \frac{\hbar^2}{2m} \Delta + V(r) - iW(r) \right) \psi , \tag{6}$$

starting with $\psi(\boldsymbol{r}, 0) = \psi_i$, the wave function of the $1D$ (3770) eigenstate of the real Hamiltonian (see ref. [9] for detail). The various curves in fig. 2 show the quantities

$$c_j = |\langle \psi_j \, | \psi(\boldsymbol{r}, t) \rangle|^2 , \tag{7}$$

where the ψ_j's are eigenstates of the real Hamiltonian. One can see that c_i is decreasing by two orders of magnitude in an almost perfectly exponential manner. The decay constant of c_j is within less than one percent equal to $\hbar/\Gamma_i$, as calculated from eq. (3). Several states are weakly populated because $W(r)$ introduces a nondiagonal coupling between the various states ψ_j.

In conclusion, we have described an optical-model potential, with two free parameters, which provides a good description of the known widths in heavy quarkonia. The real success of such a simple model gives some credit to the underlying string picture, which should carry the main aspects of the physics. Of course, such a model can describe gross properties only. It cannot make any prediction for some specific inelastic channels, which may exhibit subtle effects. This situation occurs also in proton-nucleus scattering [7]: some collective states can

be strongly excited. Nevertheless a simple imaginary part can account for the *global* loss of flux.

A possible application of the optical-model picture deals with the evolution of a J/ψ when it propagates through (excited or nonexcited) matter. This propagation may be described by solving the time-dependent Schrödinger equation for the $c\text{-}\bar{c}$ relative motion [11]. The effect of the medium can be accounted for by using a time-dependent $c\text{-}\bar{c}$ potential. The latter will produce excitations of the $c\text{-}\bar{c}$ system. One has, however, to include the coupling to the inelastic channels. An imaginary part of the type described above is very suitable for this purpose.

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