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## CORRELATIONS IN EQUILIBRIUM AND NON-EQUILIBRIUM SYSTEMS

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### 1. INTRODUCTION

If correlations can be defined as a non factorizability of the two-body distribution function (we will use this terminology here covering classical distribution functions as well as Wigner functions) into a product of two one-body distribution function, there are basically four sources of correlations, namely :

- (1) conservation laws
- (2) dynamics
- (3) quantum statistics
- (4) "spurious" correlations.

The first three sources are rather familiar. We will illustrate the fourth case by a few examples in section 4. Mathematically, correlations are present whenever the following function

$$C(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2) = \frac{f_2(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2)}{f_1(\vec{r}_1, \vec{p}_1) f_1(\vec{r}_2, \vec{p}_2)} - 1 \quad (1.1)$$

is non vanishing. Usually, one turns to simpler functions, obtained by integrating over some of the arguments of the function  $C$  in (1.1). For instance, one often considers

$$C(\vec{r}_1, \vec{r}_2) = \frac{\int d^3 p_1 d^3 p_2 f_2(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2)}{\int d^3 p_1 f_1(\vec{r}_1, \vec{p}_1) \int d^3 p_2 f_1(\vec{r}_2, \vec{p}_2)} - 1 \quad (1.2)$$

or

$$C(\vec{p}_1, \vec{p}_2) = \frac{\int d^3 r_1 d^3 r_2 f_2(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2)}{\int d^3 r_1 f_1(\vec{r}_1, \vec{p}_1) \int d^3 r_2 f_1(\vec{r}_2, \vec{p}_2)} - 1. \quad (1.3)$$

One may also integrate on more variables, but one has then to consider always these variables symmetrically on the indices 1 and 2. Otherwise, the function  $C$  can be different from zero even if  $f_2$  factorizes.

In section 2, we describe the effect of dynamical correlations in Boltzmann systems, both in equilibrium and non-equilibrium situation. In section 3, we do the same thing for bosons and fermion systems and make a review of our present knowledge of dynamical correlations in nuclear matter. In both cases, we critically examine the interferometric methods which are currently used.

## 2. CORRELATIONS FOR BOLTZMANN PARTICLES

### 2.1. Systems at Equilibrium

#### (a) Infinite systems

In this case, the translational and rotational invariances impose that

$$\int d^3 p_1 d^3 p_2 f_2(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2) = n^2 (1 + v(r)) \quad (2.1a)$$

and

$$C(\vec{r}_1, \vec{r}_2) = v(r) \quad (2.1b)$$

where  $n$  is the (uniform) equilibrium density and  $\vec{r} = \vec{r}_1 - \vec{r}_2$ . The function  $v(r)$  is intimately connected to dynamics. It vanishes for independent particles. For a normal fluid, it vanishes at some finite distance  $r$ , which usually allows to define a correlation length. The pair correlation function  $v(r)$  is related to density fluctuations through the relation

$$\frac{\overline{\rho^2} - \rho^2}{\rho} = 1 + n \int d^3 r v(r). \quad (2.2)$$

Density fluctuations are in turn related to the isothermal compressibility :

$$kT \left( \frac{\partial n}{\partial p} \right)_T = 1 + n \int d^3 r v(r), \quad (2.3)$$

where  $p$  is the pressure and  $T$  the temperature.

The calculation of the pair correlation function is quite complicated

and require fancy methods [1]. These are obtained by summing selected terms in the cluster expansion of the joint distribution function  $n_{12} = 1 + v$ . In the Percus-Yevick approach, one ends with the following integral equation

$$n_{12}(r) e^{\beta V(r)} = 1 - n \int d^3 s (e^{\beta V(s)} - 1) n_{12}(\vec{s}) [n_{12}(|\vec{r} - \vec{s}|) - 1], \quad (2.4)$$

where  $V(r)$  is the two-body interaction potential and  $\beta = 1/kT$ . An alternative approach is provided by the hypernetted-chains method, which yields a similar, but somewhat different, integral equation

$$\ln n_{12}(r) = -\beta V(r) + n \int d^3 s [n_{12}(s) - 1 - \ln n_{12}(s) - \beta V(s)] [n_{12}(|\vec{r} - \vec{s}|) - 1]. \quad (2.5)$$

In first order, both methods give the same well-known results :

$$n_{12}(r) = \exp(-\beta V(r)). \quad (2.6)$$

In second order, for a hard sphere gas, one obtains

$$\begin{aligned} n_{12}(r) &= 0, \quad r < d \\ &= 1 + \frac{4}{3} \pi d^3 n \left( 1 - \frac{3}{4} \frac{r}{d} + \frac{1}{16} \left( \frac{r}{d} \right)^3 \right), \quad d < r < 2d \\ &= 1, \quad r > 2d, \end{aligned} \quad (2.7)$$

$d$  being the diameter of the spheres. The non-trivial value for  $d < r < 2d$  is due to the shielding due to a third partner.

#### (b) Finite systems

In this case, the equivalent of  $n_{12}$ , i.e. the pair distribution function, is influenced by the geometry even in the absence of correlations. One has then

$$n_{12}(r) = \int d^3 r_1 \int d^3 p_1 \int d^3 p_2 f_2(\vec{r}_1, \vec{p}_1, \vec{r}_1 + \vec{r}, \vec{p}_2) = \int d^3 r_1 \rho(\vec{r}_1) \rho(\vec{r}_1 + \vec{r}). \quad (2.8)$$

For a spherical system of radius  $R$ , one obtains

$$\begin{aligned} n_{12}(\vec{r}) = h(r) &= 3N(N-1) \left[ 1 - \frac{3}{4} \left( \frac{r}{R} \right) + \frac{1}{16} \left( \frac{r}{R} \right)^3 \right], \quad r < 2R \\ &= 0, \quad r > 2R \end{aligned} \quad (2.9)$$

and

$$h(r) = \frac{N(N-1)}{(\sqrt{2\pi} R)^3} \exp\left(-\frac{r^2}{2R^2}\right) \quad (2.10)$$

for a gaussian shape.

## 2.2. Non Equilibrium Systems

### (a) Generalities

The evolution of the correlations can be handled classically by the BBGKY formalism and quantum mechanically by the density matrices formalism. Using Wigner transforms, the two formalisms have the same structure. The relevant equation can write (in the weak coupling limit)

$$\left(\partial_t - L_1^0 - L_2^0\right) f_2(x_1, x_2) = L_{12} f_2(x_1, x_2) + \int dx_3 (L_{13} + L_{23}) f_3(x_1, x_2, x_3), \quad (2.11a)$$

where  $L_i^0$  is the free Liouvillian for particle  $i$  ( $x_i$  stands for the set of coordinates  $(\vec{r}_i, \vec{p}_i)$ ) and where

$$L_{ij} = \vec{p}_i \cdot \vec{\nabla}_i V_{ij} - \vec{p}_j \cdot \vec{\nabla}_j V_{ij} \quad (2.11b)$$

classically. It has a more complex structure in the quantum case (see ref. [1]) for detail).

One is often interested in the asymptotic ( $t \rightarrow \infty$ , i.e. observable) behaviour of the correlations and more precisely of the correlations between the momenta of the particles. One can see from Eq. (2.11a) that the correlations between two particles are eventually due to their mutual interaction, but also to their interaction with the rest of the system. Eq. (2.11a) is of few practical use. The problem is usually approached by assuming a separation of the interactions : first the interaction involves all the system up to a late time, after which the two particles are assumed to interact independently of the system. This picture is usually denoted by the name final state interaction (FSI).

### (b) Classification of the FSI

In fig. 1, we schematically depict the possible FSI between say particles 2 and 3 for instance. In this picture, "1" can be viewed as an arbitrary number of particles and not necessarily as a single one. Case (i) corresponds to no FSI. Case (iia) is the usual FSI. Case (iib) and (iic) are similar, but are less important for the pair (2,3) of particles. Finally (iia) corresponds to a more complex FSI. It may correspond to just a small perturbation of the FSI

between 2 and 3, if the interaction between 1 and 3 is weak. Case (iiia) has some interest when the interaction between 1 and 3 is strong.

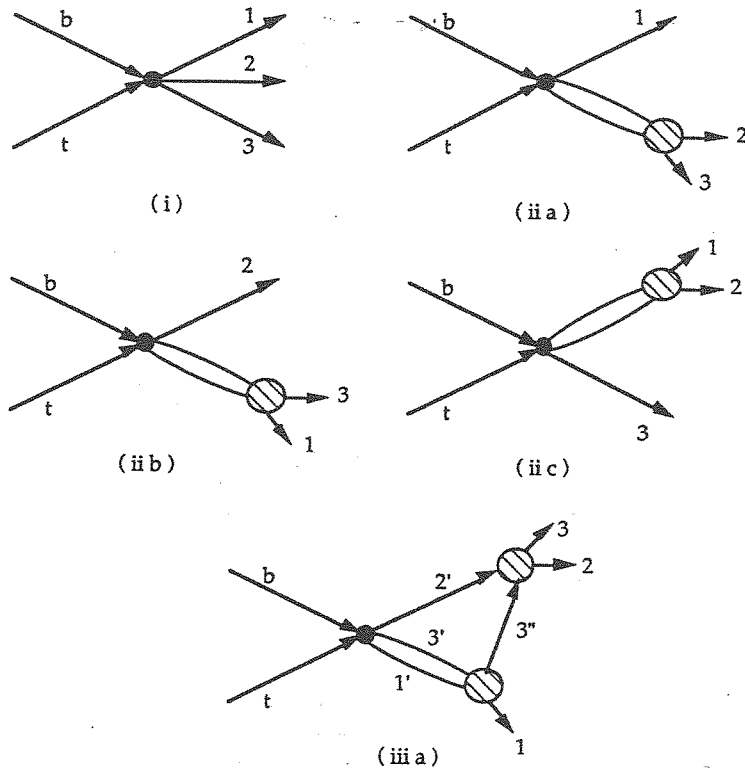


Fig. 1. Classification of three-body final-state interactions processes.

The final state interaction (case (iia)) can be used to study the spatial extension of the system and is then usually treated with semi-classical arguments. However, we think that the scattering theory approach is quite instructive. It is reviewed in the next paragraph.

### (c) Collision theory formalism

We closely follow here ref. [2]. Let us assume that the hamiltonian can be divided as

$$H = H_0 + V + v \quad (2.12)$$

where  $H_0$  is the part which can describe the incoming and outgoing asymptotic state of fig. 1(i),  $V$  is the interaction responsible for the

transition to these states and where  $v$  is the interaction between particles 2 and 3, but which acts only in the final states illustrated by fig. 1(i). It can then be shown that the transition amplitude for process (ii)a writes

$$T_{fi} = \left\langle \phi_f^{(-)} \left| V \left| \psi_i^{0(+)} \right. \right\rangle + \left\langle \psi_f^{(-)} - \phi_f^{(-)} \left| v \left| \psi_i^{0(+)} \right. \right\rangle, \quad (2.13)$$

where

$$\begin{aligned} \psi_i^{0(+)} &= \left( 1 + (E + i\varepsilon - H_0 - V)^{-1} V \right) \left| \chi_i \right\rangle, \quad \phi_f^{(+)} = \left( 1 + (E + i\varepsilon - H_0 - v)^{-1} v \right) \left| \chi_f \right\rangle, \\ \psi_f^{(+)} &= \left( 1 + (E + i\varepsilon - H_0 - V - v)^{-1} (V + v) \right) \left| \chi_f \right\rangle, \end{aligned} \quad (2.14)$$

and where  $\chi_i$  and  $\chi_f$  are unperturbed initial and final states, eigenstates of  $H_0$  with eigenvalue  $E$ . The (+) and (-) signs and the infinitesimal  $\varepsilon$  refer to the usual boundary conditions. The second term in (2.13) is generally negligible since  $v$  cannot efficiently create the final states of the primary interaction. The first term can be specified using plausibility arguments. First, the interaction  $V$  is limited to a finite volume of radius  $R$ . Second, if the coordinates of the system  $\xi$  are divided between the relative distance  $r$  (between particles 2 and 3) and the remaining coordinates  $\xi'$ , the following factorization holds

$$\left\langle \xi \left| V \left| \psi_i^{0(+)} \right. \right\rangle \approx h_f(r) \psi(\xi').$$

The transition matrix element (2.13) can then be written as ( $C$  is a constant)

$$T_{fi} = C \int d^3 r g_q(r) h_f(r), \quad (2.15a)$$

where  $g_q(r)$  is the scattering wavefunction due to the interaction  $v$  with the proper boundary condition. The important thing is the fact that the  $q$ -dependence is solely contained in the integral (2.15). One, of course, so neglects conservation law effects, which is valid if many particles are emitted. Eq. (2.15) yields

$$\left| T_{fi} \right|^2 \propto \int d^3 r \int d^3 r' g_q(r) g_q(r') \rho_2(r, r'), \quad (2.15b)$$

where  $\rho_2(r, r')$  is the density matrix corresponding to the state  $V | \psi_i^{(0)} \rangle$  i.e.,  $\propto h_f(r)h_f(r')$ . One usually approximates by taking the diagonal value of  $\rho_2$  only or by assimilating  $h_f(r)$  to pair distribution function as estimated in Eq. (2.9) or (2.10). One then gets

$$\frac{d\sigma}{dq} \propto q^2 \left| \int d^3r g_q(r) h(r) \right|^2. \quad (2.16)$$

Two cases are of interest:

(1) At small  $q$ , an attractive (repulsive) interaction gives an amplification (depletion) of the wavefunction  $g_q(r)$  for  $r < a$ , the range of the interaction. However, at very low  $q$ , this effect is not important, since the wavefunction have no nodes for  $r \lesssim 2R$ . In that case, the wavefunction is well approximated by

$$g_q(r) = \frac{\sin \delta}{qr} \left[ 1 + \alpha r + \frac{1}{2} q^2 r (r_0 - r) \right], \quad (2.17)$$

where one has made use of effective range theory. Using also the effective range expansion of the phase shift

$$k \cot \delta = \alpha + \frac{1}{2} r_0 q + \dots, \quad (2.18)$$

one obtains in first order for

$$C(q) = C(\vec{p}_1, \vec{p}_1 + \vec{q}) = \frac{\left( \frac{d\sigma}{dq} \right)}{\left( \frac{d\sigma}{dq} \right)_0} - 1, \quad (2.19)$$

where  $\left( \frac{d\sigma}{dq} \right)_0$  is the cross-section without FSI, the following expression

$$C(q) = \frac{1}{(\alpha^2 + q^2) R^2}. \quad (2.20)$$

In conclusion, the  $q$ -dependence is simply related to the scattering length  $\alpha^{-1}$ . Note that this is only true if the third term in (2.17) is neglected, which is only valid if  $\alpha R \ll 1$ . In other words, if the scattering length is large, the



correlation is not sensitive to the other typical lengths as  $r_0$  and  $R$ . Note however that if  $q$  is not so small, Eq. (2.20) is no longer valid and the correlation will be sensitive to the sign of  $\alpha$  and therefore to the attractive or repulsive nature of the interaction.

Small  $q$  correlations dominated by the behaviour expressed by Eq. (2.20) has been observed in n-n [3] and in n-p correlations [4]. For n-n correlations, (2.20) should be corrected for quantum statistics, see section 3, but this does destroy the enhancement, but just reduces it. For p-p, the Coulomb interaction makes this enhancement since then  $C(q) \rightarrow 0$  for  $q \rightarrow 0$ .

(2) At large  $q$ , the interaction  $v$  may generate resonance states. In this case, using the simplest approximation, one gets

$$\frac{d\sigma}{dq} \propto \frac{\pi(2\ell + 1)}{q^2} \frac{\Gamma \Gamma_i}{(E - E_0)^2 + \frac{\Gamma^2}{4}}, \quad (2.21)$$

where  $\Gamma_i$  is proportional to the probability of forming the resonant state by the primary interaction. The physical link between Eq. (2.15) and (2.19) is due to the fact that the scattering wavefunction is considerably enhanced in the inner region with a Breit-Wigner coefficient. Eq. (2.21) however neglects the structure of  $h(r)$ , i.e. finite size effects. The effects have been studied in ref. [5], where the authors used the structure of the wavefunction and the statistical modification of the density of states. They show that both approaches lead to the same conclusion which is not surprising, since the density of states is determined by the number of nodes of the wavefunction. They arrive at

$$C(q) = \frac{3}{2} \frac{1}{R^3 q^2} (2\ell + 1) \frac{d\delta_\ell(q)}{dq}. \quad (2.22)$$

This however neglects the fact the resonant state wavefunction is formed in a finite volume. The influence of this fact has been worked out at long time ago [6] and leads to the following modification

$$C(q) = \frac{3}{2} \frac{1}{R^3 q^2} (2\ell + 1) \left( \frac{d\delta_\ell(q)}{dq} + \frac{d\varphi_\ell(q)}{dq} \right) \quad (2.23)$$

where

$$\varphi_\ell(q) = \tan^{-1} \frac{j_\ell(qR)}{n_\ell(qR)} \quad (2.24)$$

In most practical cases, this additional term is however small.

It should be noticed that the range  $a$  of the interaction does not enter Eq. (2.22), but is rather replaced by a length  $\Lambda = d\delta/dq$ , which at the resonances reaches  $\Lambda_{\max} = q^{-1} E^*/\Gamma$ , where  $E^*$  is the energy of one of the particles in the c.m. system. For narrow resonances,  $\Lambda_{\max}$  may reach several fm's and be as large as  $R$ . The physical meaning of  $\Lambda$  is the range within which the incoming and outgoing waves interfere constructively (even if this interference builds up within  $a$  only). In other words if particles 2 and 3 are within  $\Lambda$  at the right energy ( $E_0$ ), they will interfere strongly.

This leads naturally to the question to know whether the resonance is formed at once or whether particles 2 and 3 are first formed and interact later. This question is irrelevant if  $\Lambda$  is larger or comparable to  $R$ . As stressed in refs. [6, 7], this is also reflected by the fact that taking interaction into account appears as modifying the density of states of the compound (2+3) system.

The case of broad resonances (like the  $\Delta$ 's) appears as the opposite case. Then  $\Lambda \sim 1-2$  fm and is much smaller than the size of the system. This is why the effect of correlated pairs is small compared to the background in the analysis of ref. [8]. Here, the question of whether the  $\Delta$  is formed or not is pertinent. There is however not so clear a separation between primordial and final state interaction in the reactions involving  $\Delta$ 's.

#### (d) Proximity scattering

The relevant graph (iiia) of fig. 1 has been evaluated by Aitchison and Kacser [9], who arrive at the following expression for low  $q$  (for particles 2 and 3)

$$T(q) = \frac{C}{\alpha - iq} \left\{ K + \frac{i\gamma}{mp} \ln \left[ \frac{(q - q_s) + \left(\frac{m}{M}\right)(p + p_s)}{(q - q_s) - \left(\frac{m}{M}\right)(p - p_s)} \right] \right\}, \quad (2.25)$$

where  $C$ ,  $K$  are constants,  $\alpha$  is the inverse scattering length,  $q_s$  and  $p_s$  are quantities fixed by the kinematics. When the second term is neglected, the Watson formula (2.11a) is recovered. Apparently, the application of formula (2.25) reduces the correlation at low  $q$  [10].

### 3. CORRELATIONS FOR FERMION (BOSON) PARTICLES

#### 3.1. At Equilibrium

##### (a) Infinite homogeneous systems

Just like for Boltzmann particles, one can write Eq. (2.1). But for

quantum statistics,  $v$  is different from zero, even for independent particles, at least for parallel spin directions. In nuclear matter, summing over spin states, one obtains

$$n_{12} = 1 - \frac{9}{2k_F^6} \left( \frac{\sin k_F r}{r^3} - \frac{k_F \cos k_F r}{r^2} \right)^2. \quad (3.1)$$

This behaviour is illustrated in fig. 2. It should be stressed that correlation

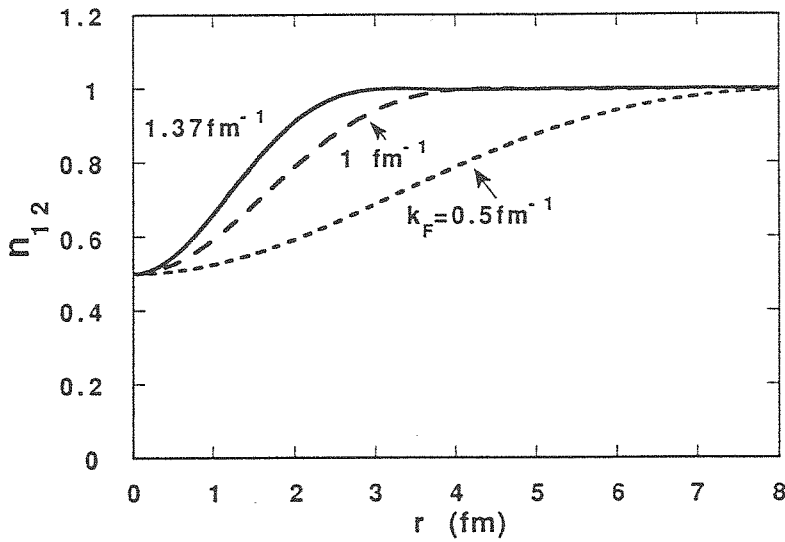


Fig. 2. Correlated pair distribution function in a Fermi gas of nucleons (Eq. (3.1)).

effects increase when the density decreases. This is however in keeping with the fact that the particles are restricted to smaller and smaller momentum and thus to larger and larger de Broglie wave lengths.

For interacting fermions, the function  $n_{12}(r)$  is given by

$$n_{12}(r) = \sum_{k < k_F} \sum_{j < k_F} |\psi_{kj}(\vec{r})|^2, \quad (3.2)$$

where  $\psi_{kj}$  is the correlated wavefunction for particles  $\vec{k}$  and  $\vec{j}$ . Calculations [11] of  $n_{12}$  in the Brueckner-Hartree-Fock approximation for the Paris potential are shown in fig. 3. The remarkable fact is that the attractive interaction reduces considerably the correlation length.

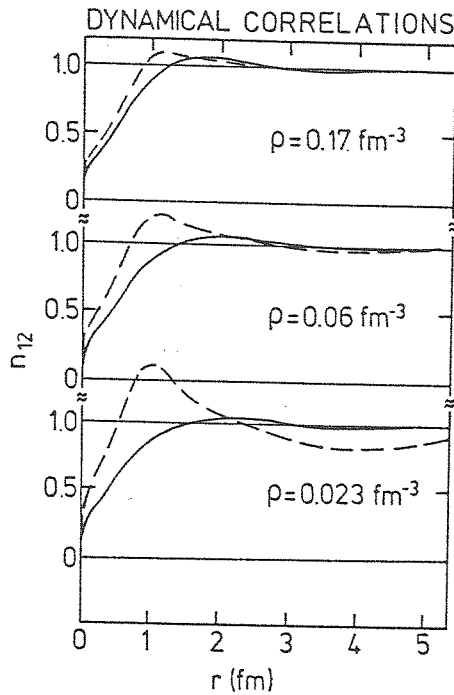


Fig. 3. Correlated pair distribution function in nuclear matter as calculated in ref. [11], for  $T=0$  (full lines) and  $T=10$  MeV (dashed lines).

Interactions are also responsible for modifying the momentum distribution, giving rise to a depletion of the Fermi sphere and a consequent population above the Fermi level. A calculation [12] of the momentum distribution  $n(k)$  at the same level of approximation as  $n_{12}(r)$  is given in fig. 4. It should be noted that the depletion is quite large ( $\sim 20\%$ ). The recently measured occupation probability in finite nuclei supports this strong effect [13]. However, it is not clear that finite size effects are not cooperating to this depletion. The relation between  $n(k)$  and  $n_{12}$  (in second order) is given by ( $k < k_F$ )

$$n(k) = \sum_j \frac{\delta}{\delta n(j)} \int d^3 r (n_{12}(\vec{r}) - 1). \quad (3.3)$$

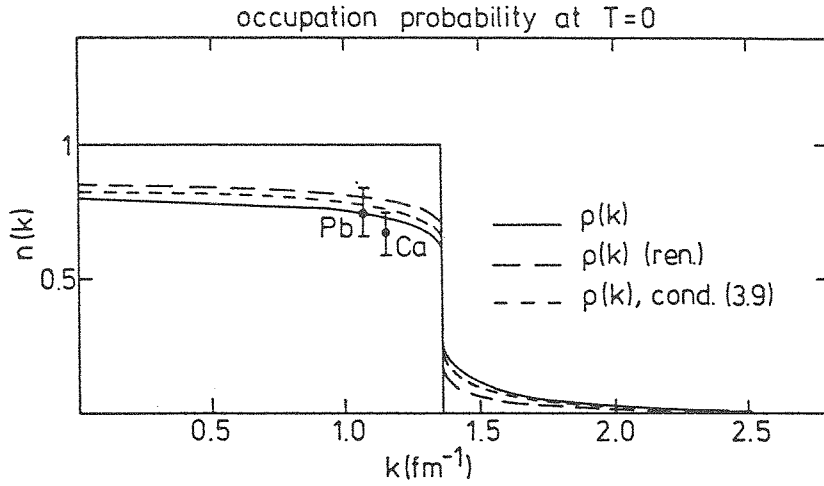


Fig. 4. Occupation probability in cold nuclear matter, as calculated in ref. [12] (full curves). Slashed curves correspond to less accurate approximations. The experimental data of ref. [13] for Pb and Ca nuclei are schematically given.

The correlations are also responsible for corrections to the nucleon mean field beyond the (Brueckner)Hartree-Fock approximation (see ref. [11] for detail). For this correction  $V_{CO}$ , one has the following approximate relation

$$V_{CO}(k) = -\kappa V_{PO}(k), \quad \kappa = \int_{k < k_F} n(k), \quad (3.4)$$

where  $V_{PO}$  is the lowest order Brueckner-Hartree-Fock contribution.

#### (b) Finite systems

The correlations at small  $\vec{p}$  are connected to the spatial extension of the system, as the correlations at small  $\vec{r}$  are connected to the momentum extension of the system. Indeed, for independent particles, the two-body Wigner function reads

$$\begin{aligned}
f_2(\vec{k}_1\vec{p}_1, \vec{k}_2\vec{p}_2) &= f_1(\vec{k}_1\vec{p}_1) f_1(\vec{k}_2\vec{p}_2) \\
&\pm f_1\left[\frac{1}{2}(\vec{k}_1 + \vec{k}_2) - \frac{1}{\hbar}(\vec{p}_1 - \vec{p}_2), \frac{1}{2}(\vec{p}_1 + \vec{p}_2) - \frac{\hbar}{4}(\vec{k}_1 - \vec{k}_2)\right] \\
&f_1\left[\frac{1}{2}(\vec{k}_1 + \vec{k}_2) + \frac{1}{\hbar}(\vec{p}_1 - \vec{p}_2), \frac{1}{2}(\vec{p}_1 + \vec{p}_2) + \frac{\hbar}{4}(\vec{k}_1 - \vec{k}_2)\right]. \quad (3.5)
\end{aligned}$$

Actually, these are rather Fourier transforms (on the spatial variables) of the Wigner function. One then has, assuming  $f_1(\vec{r}, \vec{p}) = \psi(\vec{r})\varphi(\vec{p})$

$$\begin{aligned}
\int d^3k_1 \int d^3k_2 f_2(\vec{k}_1\vec{p}_1, \vec{k}_2\vec{p}_2) &= \int d^3r_1 \int d^3r_2 f_2(\vec{r}_1\vec{p}_1, \vec{r}_2\vec{p}_2) = \\
\varphi_1(\vec{p}_1) \varphi_2(\vec{p}_2) \pm \int d^3K \psi\left(\vec{K} - \frac{1}{\hbar}(\vec{p}_1 - \vec{p}_2)\right) \psi\left(\vec{K} + \frac{1}{\hbar}(\vec{p}_1 - \vec{p}_2)\right) \\
\int d^3k \varphi\left(\frac{\vec{p}_1 + \vec{p}_2}{2} - \frac{\hbar}{4}\vec{k}\right) \varphi\left(\frac{\vec{p}_1 + \vec{p}_2}{2} - \frac{\hbar}{4}\vec{k}\right). \quad (3.6)
\end{aligned}$$

Assuming  $\psi$  and  $\varphi$  to be gaussian functions (with a radius  $R$  for  $\psi$ ), one obtains

$$\begin{aligned}
\int d^3k_1 \int d^3k_2 f_2(\vec{k}_1\vec{p}_1, \vec{k}_2\vec{p}_2) &= \varphi(\vec{p}_1) \varphi(\vec{p}_2) \pm \left[\varphi\left(\frac{\vec{p}_1 + \vec{p}_2}{2}\right)\right]^2 e^{-\frac{2(\vec{p}_1 - \vec{p}_2)^2}{R^2\hbar^2}} \\
&\approx \varphi(\vec{p}_1) \varphi(\vec{p}_2) \left[1 \pm e^{-\frac{2(\vec{p}_1 - \vec{p}_2)^2}{R^2\hbar^2}}\right]. \quad (3.7)
\end{aligned}$$

This is at the basis of the Hanbury-Brown/Twiss interferometry [14,19].

### 3.2. Non Equilibrium Situations

The dynamical evolution of the correlations can be handled in principle with the help of Eq. (2.11) (or similar ones obtained with Green functions), with however a more complicated definition of  $L'_{ij}$  (see ref. [1] for detail). However, these equations are not tractable. We just discuss two physical aspects of the dynamical evolution below.

(a) Decay rate of correlations

Instead of single-time correlation function, one can consider two times correlations. The simplest quantity in this perspective is

$$S(\vec{r}, \tau) = \int d^3 p_1 d^3 p_2 f_2(\vec{r}_2 + \vec{r}, \vec{p}_1, t_2 + \tau, \vec{r}_2, \vec{p}_2, t_2), \quad (3.8)$$

where we disregard the  $\vec{r}_2$  and  $t_2$  dependence (correct for a large system).  $S(\vec{r}, \tau)$  can then be considered as

$$S(\vec{r}, \tau) = \overline{\rho(\vec{r}_2 + \vec{r}, t_2 + \tau) \rho(\vec{r}_2, t_2)}, \quad (3.9)$$

i.e. the correlation between densities at some point in space-time and the density in a near-by point. The bar indicates a quantum average over the state of the system. This quantity also enters in the description of the evolution of density fluctuations [20]. In a classical (boltzmannian) fluid, it can be shown that the time evolution of the quantity  $S(\vec{r}, \tau)$  (for long distance  $r$  or rather, for a small wave vector) behaves like

$$S(\vec{r}, \tau) \propto \left( \frac{d^2 - 2}{(2\eta')^{d/2}} + \frac{1}{\Gamma^{d/2}} \right) \tau^{-d/2}, \quad (3.10)$$

where  $\eta'$  is the viscosity coefficient,  $\Gamma$  the sound attenuation coefficient and  $d$  is the dimensionality of the system. It is natural to find that correlations decay with the viscosity coefficient (i.e. they are destroyed by collisions in heavy ion collisions). The surprising aspect is the power law involving the dimension of the system. Typical values of  $\Gamma$  and  $\eta'$  are of the order of 2-5 (fm/c)<sup>-1</sup>. Relation (3.10) is valid for Boltzmann systems. For Fermi systems, a smaller damping of sound waves is expected. Therefore, a typical survival time could be of the order of 2-3 fm/c. Initial correlations are thus not so much important. This does not mean that correlations are vanishing. They are rebuilt regularly by the collisions themselves.

(b) Remarks on interferometry

For fermions, interferometry is usually based on Koonin's formula [18], which writes

$$C(\vec{q}) = N^{-1} \int d^3 r d^3 p_1 d^3 r_1 |\psi(\vec{r}, \vec{q})|^2 \phi(\vec{r}_1, \vec{p}_1) \phi(\vec{r}_1 + \vec{r}, \vec{p}_1 + \vec{q}) - 1 \quad (3.11)$$

where  $N$  is the uncorrelated integral. Once again, this formula assumes a decoupling between FSI and primordial interaction. As we indicate in

section 2, the basic aspect is the fact that the distribution of the relative distance  $r$  is a function of the radial extension of the system, even for independent particles. Formula (3.11) neglects Fermi statistics. At very low density, as it is often referred to, and for two proton interferometry, say  $\rho \approx \rho_0/2$  ( $k_F \approx 0.9 \text{ fm}^{-1}$ ), the Fermi correlations will extend up a few fm's. This would have introduced a sizeable error in relation (3.11). Fortunately, as we showed, dynamics reduces the correlation up to say  $r \sim 2 \text{ fm}$ . The wavefunction in (3.11) is more extended and the net effect is rather small.

#### 4. SPURIOUS CORRELATIONS

We discuss a few examples of situations where correlations (which should be redefined in any case) are observed even if there is a priori no knowledge of the two-body distribution function.

##### (a) Discretization of hydrodynamics

In hydrodynamical calculations, it is customary at the end of the calculation to shift to a (quasi)-particle description which mocks up the density and velocity profile  $(\rho(\vec{r}), \vec{v}(\vec{r}))$  in order to make more specific predictions. Generally, this is done by dividing the space in cells and filling the cells with particles at random, but in accordance with  $\rho(\vec{r})$  and  $\vec{v}(\vec{r})$ . Let us recall that these quantities are only the zeroth and first moment in  $\vec{v}$  of the  $f_1(\vec{r}, \vec{p}, t)$ . The procedure amounts to consider a N-body distribution function of the following type ( $x = (\vec{r}, \vec{p})$ )

$$F_N(x_1, \dots, x_N) = \sum_{\text{cells } \mu} \sum_{i=1}^{N(\mu)} \Theta_{\mu}(r_i) e^{-\left(\vec{p}_i - m \vec{v}_{\mu}\right)^2 / 2\tau(\mu)} \quad (4.1)$$

where  $N(\mu)$  is the number of particles in cell  $\mu$  ( $N(\mu) = \rho(\mu) \times \Delta v(\mu)$ ),  $\Delta v$  being the volume of the cell and where  $\Theta_{\mu}(r)$  is a function which is roughly equal to one inside cell  $\mu$  and vanishes outside. Of course,  $F_N$  is consistent with the following  $f_1$  distribution function

$$f_1(x_1) = \sum_{\mu} N(\mu) \Delta(r_1 - \mu) e^{-\left(\vec{p}_1 - m \vec{v}_{\mu}\right)^2 / 2\tau_1(\mu)}, \quad (4.2)$$

where the  $\Delta$  function means that  $r_1$  should be inside cell  $\mu$ , and with the  $\rho(\vec{r})$  and  $\vec{v}(\vec{r})$  profile. The two-body distribution function issued from (4.1) reads



$$f_2(x_1, x_2) = \sum_{\mu_1} \sum_{\mu_2} N(\mu_1) e^{-\frac{(\vec{p}_1 - m \vec{v}_{\mu_1})^2}{2\tau(\mu_1)}} N(\mu_2) e^{-\frac{(\vec{p}_2 - m \vec{v}_{\mu_2})^2}{2\tau(\mu_2)}}, \quad (4.3)$$

which is obtained by integrating over  $x_3, \dots, x_N$ . Obviously  $f_2$  factorizes into two  $f_1$  factors and therefore no correlation should be observed from this procedure, provided  $f_1$  is constructed as  $f_2$ ! The only correlations can come from conservation laws. For instance, conservation of current improves  $\int \rho(\vec{r}) \vec{v}(\vec{r}) d^3r$  to be a constant (in the c.m. system). Therefore  $\vec{v}(r_1)$  and  $\vec{v}(r_2)$  are not completely independent and this would indicate a relationship upon  $\vec{v}_{\mu_1}$  and  $\vec{v}_{\mu_2}$  above. In the extreme jet picture one would have  $\vec{v}_{\mu_1} = -\vec{v}_{\mu_2}$  and factorization is badly broken down. The correlation mentioned in the hydrodynamical calculation of ref. [21] is a good example of these "spurious" correlations.

#### (b) Mean field correlations

The same considerations hold when one looks at two particles, imposing some constraint on the observables like on the relative angle between the directions of these outgoing particles. Then the relevant quantity is

$$R(p_1\Theta_1, p_2\Theta_2)_{\Delta\phi} = \int d^3r_1 d^3r_2 \int d\phi_1 f_2(\vec{r}_1 p_1\Theta_1, \phi_1, \vec{r}_2 p_2\Theta_2, \phi_1 + \Delta\phi). \quad (4.4)$$

If  $f_2$  factorizes (which one has to assume in theories like Landau-Vlassov, predicting  $f_1$  only), one has

$$R(p_1\Theta_1, p_2\Theta_2)_{\Delta\phi} = \int d^3r_1 d^3r_2 \int d\phi_1 f_1(\vec{r}_1, p_1, \Theta_1, \phi_1) f_1(\vec{r}_2, p_2, \Theta_2, \phi_1 + \Delta\phi). \quad (4.5)$$

Now, even if there is a decoupling between  $\vec{r}$  and  $\vec{p}$  coordinates  $f_1 = \psi(\vec{r}) \phi(\vec{p})$ , the quantity (4.5) does not factorize into a function of  $p_1$  times a function of  $p_2$

$$R(p_1\Theta_1, p_2\Theta_2)_{\Delta\phi} = \int d\phi_1 \phi(\vec{p}_1) \phi(\vec{p}_2) \neq F(p_1\Theta_1) F(p_2\Theta_2). \quad (4.6)$$

Therefore the quantity  $R$  bears some information about the function  $\phi(\vec{p})$ . Sometimes [22], property (4.6) is called improperly mean field correlations and this was partly justified by the fact that in these particular cases, the main structures of  $f_1$  came from the mean field dynamics.

(c) Reaction plane correlations

This refers to a situation similar to (a), which imposes that there exists a vector  $\vec{n}$  which is such that

$$\int \vec{n} \cdot \vec{p} f_1(\vec{r}, \vec{p}) d^3p = 0. \quad (4.7)$$

The latter correlation is the largest for quasi-free scattering, as it is well known from two proton correlation experiment at large momentum [23]. See also refs. [24,25] for other examples.

(d) Impact parameter correlations

Quite often, experiments imply a summation over a certain range of impact parameter. This implies that the relevant quantity is

$$\sum_{b \in \Delta b} f_2(r_1 p_1, r_2 p_2, b) \neq \sum_{b \in \Delta b} f_1(r_1 p_1, b) \sum_{b \in \Delta b} f_1(r_2 p_2, b), \quad (4.8)$$

even if  $f_2$  factorizes in  $f_1 f_1$  for any  $b$ . The latter correlation will reflect the  $b$ -dependence of the one-body distribution function.

## 5. CONCLUSION

We have reviewed the main aspects of the two-body correlations for both equilibrium and non-equilibrium situations. We have clarified the physical meaning of low-energy and resonant interferences. We also presented detailed calculations of correlations and related quantities for both equilibrium nuclear matter.

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