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CHAPTER V

DISSIPATIVE PROCESSES

NUCLEAR MEAN FIELD AND RESIDUAL TWO-BODY COLLISIONS IN THE LANDAU-VLASSOV DESCRIPTION OF MEDIUM ENERGY HEAVY ION REACTIONS*

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ABSTRACT : We show that the study of the optical-model potential in a local and thermal approximation in the nuclear medium gives a consistent link between the nuclear mean field and residual two-body collisions in the Landau-Vlassov transport theory. We calculate medium correction to the free nucleon-nucleon cross section and present them so that they can be readily implemented in existing Landau-Vlassov calculations.

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Energetic heavy ion collisions are currently performed to investigate nuclear matter properties away from normal conditions of pressure, density and temperature. As the data accumulate (momentum transfer, particle multiplicities, mass distribution, etc...), the theoretical framework for their description try to bridge the gap between the time dependent Hartree-Fock (TDHF) theory successful at low incident energy ($\lesssim$ 5-10 MeV/A) and the intranuclear cascade model (INC) used at high energies ($\gtrsim$ 100 MeV/A). In this intermediate energy domain the theoretical description must certainly account for the continuous exchange of energy between the collective motion of the mean field and the individual nucleonic excitations, in a manner consistent with usual conservation laws. It is a non trivial problem, not yet solved in practical terms and which rises important conceptual questions. To mention one, and as we shall see below, the nuclear mean field is built up from the two-body effective interaction felt by the nucleons in the nuclear medium ; it is in turn the same effective interaction which governs the residual two-body collisions. Thus it is conceivable that the lack of consistency between the mean field itself and the two-body collision process may lead to erroneous conclusions from the analysis of the data on such an important question as the nuclear equation of state (EOS). Recently G. Bertsch et al.[1] examined the sensitivity of nuclear transverse momentum distributions to the nucleon-nucleon collisions cross sections and to the nuclear EOS in the framework of the classical limit of TDHF with a Uehling-Uhlenbeck collision term. They found indeed that the mean transverse momentum is as sensitive to uncertainties in the nucleon-nucleon cross section as it is to the nuclear equation of state.

Nevertheless, letting aside formal questions, a rather large consensus has emerged among practitionners that a transport equation suitable to describe heavy ion collisions at intermediate energy has a Landau-Vlassov (LV) form :

$$\left(\frac{\partial}{\partial t} + \vec{p}.\vec{\nabla} - (\vec{\nabla}U).\vec{\nabla}_{\vec{p}}\right) f(\vec{r},\vec{p},t) = G - L \qquad (1)$$

with ($\hbar$ = 1)

$$G = \int \frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}\, \omega(pp_2 \to p_3p_4) f_3 f_4 (1-f)(1-f_2) \times$$

$$\delta^3(\vec{p}+\vec{p}_2-\vec{p}_3-\vec{p}_4)\,\delta(e(p)+e(p_2)-e(p_3)-e(p_4)) \qquad (2)$$

and

$$L = \int \frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}\, \omega(pp_2 \to p_3p_4) f f_2 (1-f_3)(1-f_4) \times$$

$$\delta^3(\vec{p}+\vec{p}_2-\vec{p}_3-\vec{p}_4)\,\delta(e(p)+e(p_2)-e(p_3)-e(p_4)) \quad . \qquad (3)$$

In this equation, $f(\vec{r},\vec{p},t)$ can be considered as the probability of finding a particle with momentum $\vec{p}$ at position $\vec{r}$ (or at least as the usual quantal Wigner function) normalized as

$$\iint \frac{d^3\vec{p}\; d^3\vec{r}}{(2\pi)^3} f(\vec{r},\vec{p},t) = A \quad , \qquad (4)$$

A being the mass number of the system, U is the average single particle field, $\omega(\vec{p}\vec{p}_2 \to \vec{p}_3\vec{p}_4)$ is the transition probability for a collision between two particles of momenta $\vec{p}$ and $\vec{p}_2$ to give a final state with momenta $\vec{p}_3$ and $\vec{p}_4$, and f_i stands symbolically for $f(\vec{r},\vec{p}_i,t)$. Finally, $e(p)$ is the single-particle energy.

The situation is however confuse, as for knowing what is the exact meaning of the "input" quantities of eq. (1), and, especially of the transition matrix ω . The theoretical basis for the LV equation has been studied by several authors[2-6]. In general, the LV equation is obtained in the weak coupling limit, for fermions interacting through a potential v . In that case (all the matrix elements below are antisymmetrized) :

$$\omega(pp_2 \to p_3p_4) = |\langle \vec{p}\vec{p}_2 | v | \vec{p}_3\vec{p}_4 \rangle|^2 \quad . \qquad (5)$$

In nuclear physics, the interaction is too strong for this limit to be valid. Rather, it is considered that the free transition matrix

$$\omega(pp_2 \to p_3p_4) = |\langle \vec{p}\vec{p}_2 | T | \vec{p}_3\vec{p}_4 \rangle|^2 \qquad (6)$$

would be a more appropriate choice. The state vectors in eq. (6) are referring to free plane waves. But, medium corrections which are well studied in equilibrium situations[7] are also expected in not too far from equilibrium situations, like in medium-energy heavy ion collisions. Recently, Botermans and Malfliet[8] have shown that the usual Brueckner resummation of ladder diagrams can be made in very much the same way as in equilibrium situations. Therefore, the most reasonable choice seems to be

$$\omega(pp_2 \to p_3p_4) = |\langle \vec{p}\vec{p}_2 | g | \vec{p}_3\vec{p}_4 \rangle|^2 \quad , \qquad (7)$$

where g is the usual Brueckner reaction matrix, describing the scattering of two nucleons in a nuclear medium. In principle, g should be recalculated for the instantaneous occupation in phase space, described by $f(\vec{r},\vec{p},t)$. However, this is a tremendeous task which can be avoided if the momentum distribution can be approximated by a Fermi-Dirac distribution at temperature T, which seems reasonable for not too far from equilibrium situations. Indeed, for these conditions, the g-matrix has been calculated for several densities and temperatures, typical of the medium energy domain[9].

In Brueckner approach, one usually recasts the perturbation series into an expansion in term of the number of hole lines in Goldstone diagrams. For the mass operator which enters in the equation obeyed by the one-body density[3] (e.g. the distribution $f(\vec{r},\vec{p};t)$) and consequently defines the mean field[7], its expansion may be written :

$$M(p,E) = U(p,E) + iW(p,E) = M_1(p,E) + M_2(p,E) + \ldots \qquad (8)$$

with

$$M_1(p,E) = U_1(p,E) + iW_1(p,E) =$$
$$\int \frac{d^3p_2}{(2\pi)^3} f_2 \langle \vec{p}\vec{p}_2 | g(E+e(p_2)) | \vec{p}\vec{p}_2 \rangle \quad , \qquad (9)$$

and

$$M_2(p,E) = U_2(p,E) + iW_2(p,E) =$$
$$\frac{1}{2}\int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} f_3 f_4 (1-f_2) \times$$
$$\frac{|\langle \vec{p}_3\vec{p}_4 | g(e(p_3)+e(p_4)) | \vec{p}\vec{p}_2 \rangle|^2}{E + e(p_2) - e(p_3) - e(p_4) - i\epsilon} \quad . \qquad (10)$$

To this order the single particle energy $e(p)$ reads :

$$e(p) = \frac{p^2}{2m} + \mathrm{Re}[M_1(p,e(p)) + M_2(p,e(p))] \quad . \qquad (11)$$

Using the properties[7] of the g-matrix one finds from (9) :

$$L = \frac{2}{\pi} f\, W_1(p,e(p)) \quad , \qquad (12)$$

and from (10)[10] :

$$G = \frac{2}{\pi} (1-f)\, W_2(p,e(p)) \quad . \qquad (13)$$

Eqs. (8) to (13) show the intricate self-consistency link between the mean field U entering eq. (1) and the residual two-body collisions. Hence, the study of the energy and temperature dependence of the optical-model potential is of central importance for nuclear dynamics. However, as discussed extensively in refs.[11,12], the effectiveness of the collision term in a finite system is also closely linked to the time dependence of the mean field. Clearly this important mechanism of energy transfer between the ordered motion of the mean field into disordered, random nucleonic motion is severely constrained in eq. (1) as only collisions on the energy shell can take place due to the presence of the δ-function in energies. This occurs due to simplifying assumption on the time dependence of the single particle energies, and cannot be replaced simply[12] by any Gaussian-type approximation without violating energy conservation.

The attitude of practitionners of eq. (1) consists in solving it in the most reliable way for the collision of two heavy ions, a difficult task which involves many successive approximations. In this way it is the failures and/or the ambiguities introduced by these approximations in the reproduction of experimental data which will dictate the needs for improvment either at the technical level of the method of solution or at a deeper level such as the mechanism of energy transfer mentioned above. Here we simply want to show that the study of the optical-model potential in the nuclear medium at finite temperature[9,13] helps to eliminate uncertainties related to input assumptions. Thereby practitionners will hopefully be able to pinpoint with greater confidence failures due to theoretical limitations of the model from avoidable input simplifications.

The collisional cross section in the medium is usually[14,15] arbitrarily scaled from the free scattering one and forms the basic input to eq. (1). Let us denote by α the ratio

$$\alpha = \frac{W_1(p,\rho,T)}{\tilde{W}_1(p,\rho,T)} \qquad (14)$$

where

$$\tilde{W}_1(p,\rho,T) = \int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} f_2(1-f_3)(1-f_4)|\langle \vec{p}\vec{p}_2|T|\vec{p}_3\vec{p}_4\rangle|^2 \times \delta^3(\vec{p})\delta(\frac{p^2}{2m} + \frac{p_2^2}{2m} - \frac{p_3^2}{2m} - \frac{p_4^2}{2m}) \quad . \qquad (15)$$

In eqs. (14) and (15) we have indicated that the quasi particle of momentum $\vec{p}$ is travelling in a medium of density ρ at temperature T . $\tilde{W}_1$ is the value the imaginary part would take if the interaction were not influenced by the medium. It is clear that medium effects are operating in two places : (i) the phase-space is distorted because the particles are feeling a mean field, which is embodied by the energy-conserving delta function in eqs. (2) and (3) ; (ii) the interaction itself is modified, which is responsible for the replacement of the T-matrix by the g-matrix. This modification comes in turn from two effects, as is well known : 1) the Pauli principle forbids to have intermediate states that are already occupied ; 2) the particles are feeling the mean field in the intermediate states.

We see from eq. (14) that α is the required multiplicative factor that accounts for medium corrections in the loss term (in the conditions mentioned above) compared to the commonly used eq. (15). Details of the calculation of α are reported in ref. [16] and our results are shown in fig. 1, taken from this work. Therein a convenient parametrization of α is also given in terms of the variables p, ρ, T .

The medium effect on the single-particle energies can be incorporated readily in the collision term. The "free scattering" loss term would then be given by eq. (12) with the following expression of the optical-model potential :

$$\bar{W}_1(p,\rho,T) = \int \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} f_2(1-f_3)(1-f_4) |\langle \vec{p}\vec{p}_2|T|\vec{p}_3\vec{p}_4\rangle|^2 \delta^3(\vec{p})\delta(e(p)+e(p_2)-e(p_3)-e(p_4)), \qquad (16)$$

instead of eq. (15). In the simplest case, the single-particle energies can be taken in the constant effective mass approximation

$$e(p) = \frac{p^2}{2m^*} + U_0 \quad , \qquad (17)$$

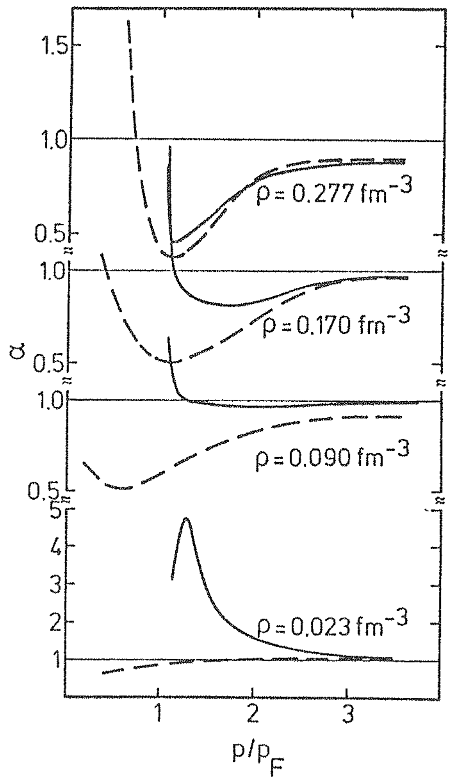

Fig. 1 : the ratio α eq. (14), as a function of the relative momentum p/p_F and for various densities. Full curve : T = 0 MeV ; dashed curves : T = 10 MeV.

where U_0 is a constant. The quantity m^* is extracted from ref. [9] for the same conditions of density and temperature. Its value is given in table 1 of ref. [16]. For the conditions considered in the calculation of α, i.e. in the thermal equilibrium limit, the gain term is equal to the loss term. Since this is independent of the transition probability, the same renormalization has to be applied to the gain term. This renormalization is presumably not very much different if one departs from equilibrium[10,13].

Thus in the framework of eq. (1) we see that the study of the optical-model potential in the nuclear medium provides a consistent picture between the underlying EOS embbeded in the self-consistent field U and the residual two-body collision process. This is very

much desirable as one wishes ultimately to distinguish failures due to theoretical limitation of the LV model itself from those related to input assumptions. However, on the one hand, they are indications from relativistic nuclear matter calculations that the momentum dependence and the Lorentz structure of the mean field substantially changes the features of the EOS with respect to its nonrelativistic counterpart[17,18]. On the other hand, it is well known from two-body scattering data that the delta governs the inelasticity channel from the pion production threshold up to 1 GeV and must therefore be taken into account in a formal treatment aiming at analyzing the pion yield. Thus a relativistic quantum transport theory is probably unavoidable and its development[19] is a major future challenge to theoricians.

REFERENCES

1. BERTSCH G.F., LYNCH W.G. and TSANG M.B., Preprint MSUCL 586, December 1986.
2. LUTTINGER J.M. and NOZIERES P., Phys.Rev. **127**(1962)1431.
3. KADANOFF L.P. and BAYM G., "Quantum Statistical Mechanics", W.A. Benjamin, New York, 1962.
4. WOLFE P., Z.Phys. **232**(1970)38.
5. DANIELEWICZ P., Ann.Phys. **152**(1984)239, 305.
6. TANG H.H.K. and SIEMENS P.J., Nucl.Phys. **A453**(1986)251.
7. JEUKENNE J.-P., LEJEUNE A. and MAHAUX C., Phys.Rep. **25C**(1976)85.
8. BOTERMANS W. and MALFLIET R., Phys.Lett. **B171**(1986)22.
9. LEJEUNE A., GRANGE P., MARTZOLFF M. and CUGNON J., Nucl.Phys. **A453**(1986)189.
10. CUGNON J., GRANGE P. and LEJEUNE A., "International Workshop on Semi-Classical and Phase-Space Approaches to the Dynamics of the Nucleus", Aussois, March 1987.
11. BALIAN R. and VENERONI M., Ann.Phys. **135**(1981)270.
12. GRANGE P., WEIDENMÜLLER H. and WOLSCHIN G., Ann.Phys. **136**(1981)190.
13. GRANGE P., CUGNON J. and LEJEUNE A., Preprint, March 1986.
14. AICHELIN J. and BERTSCH G., Phys.Rev. **C31**(1985)730.
15. GREGOIRE C., REMAUD B., SCHEUTER F. and SEBILLE F., Nucl.Phys. **A436**(1985)365.
16. CUGNON J., LEJEUNE A. and GRANGE P., Phys.Rev. **C35**(1987)861.
17. AINSWORTH T.L., BROWN G.E., PRAKASH M. and WOLTER H.H., Contribution to the XV International Workshop of "Gross Properties of Nuclei and Nuclear Excitations", Hirschegg (Austria), January 12-17, 1987.
18. GALE C., BERTSCH G. and DAS GUPTA S., Preprint MSUCL, October 1986.
19. ELZE H.T., GYULASSY M., VASAK D., HEINZ H., STÜCKER H. and GREINER W., IFTP Preprint Frankfurt, January 1987.