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RELAXATION AND THERMALISATION

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Abstract - The Landau-Vlassov equation is investigated as a general framework to study the thermalisation in nucleus-nucleus collisions. The relaxation time is studied numerically both at high and intermediate energy.

Résumé - L'équation de Landau-Vlassov est utilisée comme un cadre général pour l'étude des phénomènes de thermalisation apparaissant au cours des collisions noyau-noyau. Le temps de relaxation est étudié numériquement à haute énergie ainsi qu'aux énergies intermédiaires.

I - INTRODUCTION

A nuclear system in collision is certainly not in equilibrium. The question is whether it can be considered as in local equilibrium. This question is far from academic, since the implicit goal of the study of nucleus-nucleus collisions is the determination of the equation of state. If local equilibrium is not satisfied, this goal would be much more difficult to reach, since the interpretation of the collision features would then go further than using thermodynamical concepts.

The question will be investigated in the frame of relativistic nucleus-nucleus collisions, because in this case the long range Coulomb force is much less important in regard with available energy and also because the theoretical investigation is more advanced in this case.

II - GENERAL THEORETICAL FRAME

The basic quantities describing the system (supposed to be composed of point-like nucleons interaction through potential forces) are the one-body function $f_1(\vec{r},\vec{p},t)$, which, classically, gives the probability of finding a nucleon with momentum $\vec{p}$ at position $\vec{r}$, the two-body distribution function $f_2(\vec{r}_1,\vec{p}_1,\vec{r}_2,\vec{p}_2)$, etc.... We disregard here for simplicity, pion production. In such a microscopic description, a local equilibrium is characterized by ($k = 1$)

$$f_1(\vec{r},\vec{p},t) = \rho(\vec{r},t) \exp\left(-\frac{e(\vec{p})}{T(\vec{r},t)}\right) \quad , \qquad (2.1)$$

where $e(\vec{p})$ is the single-particle energy, which of course could depend on ρ and T. For a perfect gas, $e(\vec{p}) = p^2/2m$. The local equilibrium is characterized by some properties of f_2 (absence of correlations in a perfect gas, f.i.) that we do not investigate here.

A theory, suitable to the 0.25 - 2 GeV/A collision domain, which deals

with the evolution of f_1 is the Landau-Vlassov (LV) equation. It writes ($m = 1$, $\hbar = 1$)

$$\left(\frac{\partial}{\partial t} + \vec{p}.\vec{\nabla} - (\vec{\nabla}U).\vec{\nabla}_p\right) f_1(\vec{r},\vec{p},t) = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3} W(pp'_1 \to p_2p_3)$$

$$\delta^3(\vec{p})\delta(E)\{\tilde{f}_3\tilde{f}_4\,(1-\tilde{f})(1-\tilde{f}_1) - \tilde{f}\tilde{f}_1(1-\tilde{f}_2)(1-\tilde{f}_3)\} \quad . \tag{2.2}$$

The last two terms in the first member are due to the displacement of the matter and the second number is due to the collisions between the particles. It contains a gain term and a loss term, corresponding to population and depopulation of particles with momentum $\vec{p}$. The factors $(1-\tilde{f}_i)$, where

$$\tilde{f}_i = (2\pi)^3 f_1(\vec{r},\vec{p}_i,t) \quad , \tag{2.3}$$

account for the possible occupation of final states (Pauli principle). The quantity $W(pp_1 \to p_2p_3)$ is the probability of a transition from $\vec{p}\vec{p}_1$ to $\vec{p}_2\vec{p}_3$. Usually, W is taken to come from two-body collisions. Then it writes

$$W = (2\pi)^3\,\sigma(E)\,v \quad , \tag{2.4}$$

where v is the relative velocity and E the c.m. energy. The quantity U(r) accounts for interactions. It is taken as a Hartree-type mean field $U(\vec{r}) = U(\rho(\vec{r}))$. Equation (2.2) can be derived from the Schrödinger equation [1]. It embodies, except for the mean field, the same physics as the intranuclear cascade [2].

To discuss the relation with local equilibrium, it is interesting to look at the first moments (in $\vec{p}$) of f_1 [3] :

$$\begin{Bmatrix} \rho \\ \rho\vec{u} \\ \tau_{ij} \end{Bmatrix} = \int d^3p \begin{Bmatrix} 1 \\ \vec{p} \\ p_i\,p_j \end{Bmatrix} f_1(\vec{r},\vec{p},t) \quad . \tag{2.5}$$

We now take the first moments of the LV equation, successively. The o^{th} moment is the continuity equation

$$\frac{\partial\rho}{\partial t} = \vec{\nabla}.(\rho\vec{u}) = 0 \quad . \tag{2.6}$$

Before, proceeding further, using the decomposition $\vec{p} = \vec{u} + \delta\vec{p}$ we write

$$\tau_{ij} = R_{ij} + S_{ij} \quad , \tag{2.7}$$

with

$$R_{ij} = \rho\,u_iu_j \quad , \qquad S_{ij} = \int d^3p\;\delta p_i\;\delta p_j\;f_1(\vec{r},\vec{p},t) \quad . \tag{2.8}$$

The momentum flux tensor is thus split into a collective flow tensor R_{ij} and an internal flux tensor, called the stress tensor. We further write

$$S_{ij} = \left(\frac{1}{3}\,\mathrm{tr}\,S\right)\delta_{ij} + S^o_{ij} = \tilde{p}_t\,\delta_{ij} + S^o_{ij} \quad . \tag{2.9}$$

The tensor S^o_{ij} is called the deviator. The second moment equation is equivalent to the following equations.

$$\frac{\partial R_{ij}}{\partial t} + \vec{\nabla}.(R_{ij}\,\vec{u}) = -u_i \nabla_j \tilde{p} - u_j \nabla_i \tilde{p} - u_i \sum_k \nabla_k S^o_{kj} - u_j \sum_k \nabla_k S^o_{ki} \qquad (2.10)$$

$$\frac{\partial \tilde{\epsilon}}{\partial t} + \tilde{\nabla}.(\tilde{\epsilon}\vec{u}) = -\tilde{p}\,\vec{\nabla}.\vec{u} - \frac{1}{2}\sum_i \sum_k S^o_{ik}(\vec{\nabla}\vec{u})^S_{ki} - \vec{\nabla}.\vec{J} \quad , \qquad (2.11)$$

$$\frac{\partial S^o_{ij}}{\partial t} + \vec{\nabla}.(S^o_{ij}\,\vec{u}) = -\frac{2}{3} S^o_{ij}\,\vec{\nabla}.\vec{u} - \vec{\nabla}.\int d^3p[\delta p_i \delta p_j - \frac{1}{3}(\delta p)^2\delta_{ij}]\vec{p}\,f_1$$
$$+ \tilde{p}_t(\vec{\nabla}.\vec{u})^S_{ij} - \frac{1}{2}[S^o:(\vec{\nabla}u)^S]^S_{ij} - \frac{1}{2}\sum_k [S^o_{jk}(\vec{\nabla}\vec{u})^A_{ki} + S^o_{ik}(\vec{\nabla}\vec{u})^A_{kj}]$$
$$+ \int [\delta p_i \delta p_j - \frac{1}{3}(\delta\vec{p})^2\,\delta_{ij}]\,I \quad . \qquad (2.12)$$

The first equation is equivalent to the first moment equation. The quantity $\tilde{p}$ is

$$\tilde{p} = \tilde{p}_t + \rho U - V \quad , \qquad V = \int_o^\rho U(\rho')d\rho' \quad . \qquad (2.13)$$

In equation (2.12) the indices S and A mean the symmetric and the antisymmetric parts of a tensor, the symbol (:) is for the contraction of two tensors and I stands for the collision term of eq. (2.2). In equation (2.11), the quantity $\tilde{u}$ is given by

$$\tilde{u} = \frac{3}{2}\tilde{p}_t + V \quad . \qquad (2.14)$$

Obviously, the dynamics will restrict to local quantities usually encountered in thermodynamics and hydrodynamics, if $S^o_{ij} \approx 0$, a condition much less restrictive than (2.1). In that case, eqs. (2.6), (2.10) and (2.11) are the usual hydrodynamical equations, where $\tilde{u}$, $\tilde{p}_t$ and $\tilde{p}$ play the role of the energy density, of the thermal pressure and of the pressure, respectively.

III. RELAXATION

In practical cases, there is no need for S^o_{ij} to be exactly zero. Indeed, in most cases, the variation of S^o_{ij} is controlled by the collision term. If we make a relaxation time hypothesis, we have

$$\int [\delta p_i\,\delta p_j - \frac{1}{3}(\delta\vec{p})^2\,\delta_{ij}]\,I \approx \frac{-S^o_{ij}}{\tau} \quad . \qquad (2.15)$$

The value of τ can be roughly estimated using the initial conditions of the collision. One has

$$\tau = \frac{1}{\rho\sigma\bar{v}}\,\frac{\langle\Delta p_z^2\rangle}{\langle p_z^2\rangle} = \frac{\lambda}{\bar{v}}\,f \quad , \qquad (2.16)$$

where σ and v are the average cross-section and relative velocity entering in the collision term, λ is the mean free path to make a collision. Finally f represents the efficiency of the collision. The more peaked the differential cross-section is, the smaller f is. If at some

if we average over $\lambda_{th} = \lambda f$. At high energy, these averages are marginally possible, since $\lambda_{th} \approx 2 - 5$ fm, smaller but not terribly smaller than the size of the system. As an illustration, we give in Fig. 1, the values of

$$X = \sum_i \frac{|S^o_{ii} - \tilde{p}_t|}{3\,\tilde{p}_t} \quad , \tag{2.17}$$

which roughly measures the importance of S^o_{ij} in a particular case. Local equilibrium would require $X = 0$.

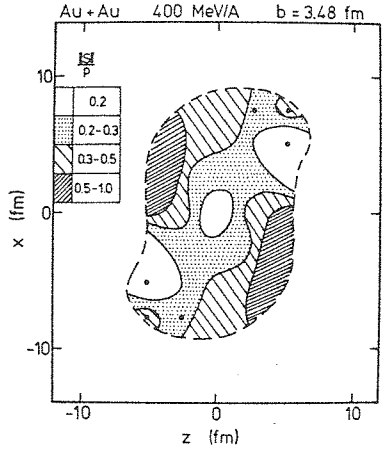

Fig. 1 - Distribution of the quantity (2.17) in the reaction plane of Au + Au collision at the time of maximum compression. In the initial state the two nuclei are running against each other along the horizontal direction. Taken from Ref. [4].

At low energy, the situation is comparable. Estimates of τ are shown in Fig. 2, in comparison with the collision time. The curve labelled CL results from a recent microscopic calculation of the nucleon propagator using Brueckner theory. It shows that at 10 MeV the thermalisation is not very much more effective than for cold nuclei. Note, however, that at low energy the relation between τ and the single-particle mean free path is not as simple as in (2.16).

The connection between the observables and the equilibrium quantities during the collision is not obvious. It is well-known, for instance, that the temperature extracted from the particle production spectra should not be identified with the initial average temperature of the compressed state [6].

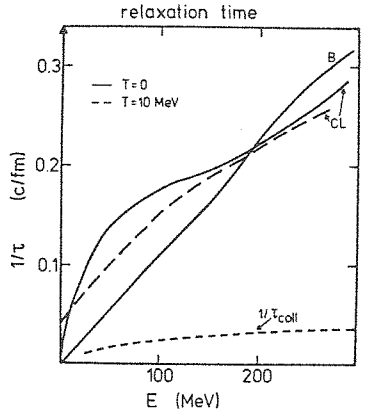

Fig. 2 - Estimates of the relaxation time τ. The curve labelled B is based on an approximate calculation of the collision term (from Ref. [3]). The other curves are based on eq. (2.16) and on a microscopic calculation of the nucleon propagator in nuclear matter at two energies (from Ref. [5]). The quantity τ_{coll} is the estimate of the collision duration time.

IV. CONCLUSION

We have analyzed the problem of the thermalisation in heavy ion collisions. We have seen that the situation is not fully favourable, but is not bad enough for preventing the understanding of the dynamics in terms of thermodynamical of hydrodynamical concepts as a first approach. As an example, the analysis of the dynamics in terms of entropy has revealed itself to be very fruitful at high energy [7-8]. An extension of analyses of the same kind should be undertaken at low energy.

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