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CASCADE ANALYSES OF THE NUCLEAR FLOW EFFECT

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The properties of the collective flow are studied in the frame of the intranuclear cascade model. It is shown that the latter produces an intrinsic flow, which seems to be too small, when compared with experiment. The dynamics of the flow is investigated in detail.

1. INTRODUCTION

The collective flow, as revealed by the event by event analysis of the sphericity tensor, is generally considered as a manifestation of a typical hydrodynamical behaviour, reflecting in some way the equation of state. The intranuclear cascade model (INC), which has no collectivity explicitly built into it, was considered as unable to produce a collective flow[1]. However, last year, it was claimed[2] by the present authors, that the INC model of reference 3 is able to produce an intrinsic flow and that, when the acceptance of the detector is taken into account, the numerical results are in qualitative agreement with experiment[1]. These predictions are weakened by the fact that, as observed by Stöcker[4], a spurious expansion of the spectators increases the flow. In the meantime, this defect has been corrected[5] and the results are outlined here. We also briefly discuss the comparison with experiment and the mass and energy dependence of the flow. We investigate the sources of the flow in detail.

2. INTRINSIC FLOW INSIDE THE INC MODEL

We concentrate on the sphericity tensor

$$Q_{ij} = \sum_{\nu} (2m_{\nu})^{-1}\, p_i^{\nu} p_j^{\nu} \quad , \qquad (2.1)$$

where $\vec{p}^{\nu}$ is the c.m. momentum of the ν^{th} ejectile. We denote by $\lambda_1 \geq \lambda_2 \geq \lambda_3$ and $\vec{e}_i$ the eigenvalues and unit eigenvectors of this ellipsoid tensor. We will focus on the polar angle ϑ of e_1 with respect to the beam axis.

Figure 1 gives one of our basic results, namely the by now familiar $dN/d\cos\vartheta$ distribution. This clearly shows that there is an intrinsic (i.e. when no filter of any sort is applied) flow inside the cascade. Compared to reference 2, the peaks are displaced toward smaller angles. The properties of the flow are extensively studied in reference 5. We summarize the main points :

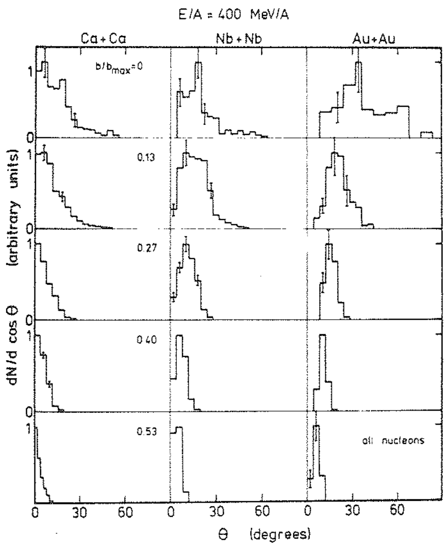

Figure 1
INC calculations[5] of the flow for several systems. The sphericity tensor contains the contribution of all nucleons.

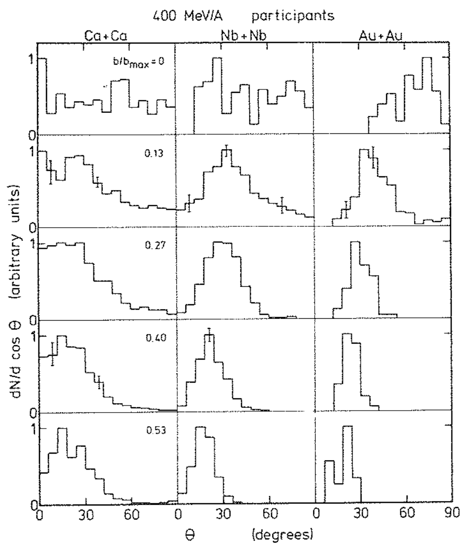

Figure 2
Same as figure 1, when only participants are retained.

(a) In Ca + Ca, the sphericity tensor points on the average toward some finite ϑ ($\langle\vec{e}_{1x}\rangle \neq 0$). This feature does not show up in the dN/d cos ϑ distribution because of the fluctuations. (b) When the mass of the system increases, the flow increases and the fluctuations decrease. (c) The ellipsoid for the participants only is less deformed than the one for the whole system and points toward larger angles, as shown by figure 2. The spectators stretch the ellipsoid and tend to "reduce the flow".

3. COMPARISON WITH EXPERIMENT

The comparison with experiment is a delicate question. The real events are subject to the experiment filter coming from the acceptance of the apparatus and from the analysis procedure[1] which f.i. removes the "double hits" in a single telescope. The experimental filter cannot be applied directly to the cascade events, basically because the INC cannot predict the clusterisation. Furthermore, the experimental events are classified according to the charged particle multiplicity m_c, which is a characteristic of the filtered real events. What has been done in reference 5 is to use a simplified filter which accounts for the gross features of the acceptance of the Plastic Ball and to classify the events according to the charge of the participants M_p. It is hoped that the impact parameter dependence of M_p and m_c is the same up to an overall scale factor. The results are shown in figure 3. The distribution for the highest multiplicity bin is compared to the highest m_c experimental data for Ca + Ca and Nb + Nb. For the last system, the theoretical distribution shows a peak at too small an angle (~ 17°) compared to experiment (~ 23°). The comparison is somewhat uncertain, mainly because of the arbitrary choice of $M_{\nu L}$, the threshold defining the largest multiplicities M_ν. It seems that the adopted value ($M_{\nu L}$ = 68) is too large[6], if one requires the same cross-section in the highest M_ν and highest m_c multiplicity bin. This experimental quantity is not precisely known. The theoretical cross-section is about 5% of the geometrical cross-section. In conclusion, the INC model of reference 5 generates a too small flow in all likelihood, although it is not easy to accurately estimate the lack of flow. Reference 7 addresses to this specific question.

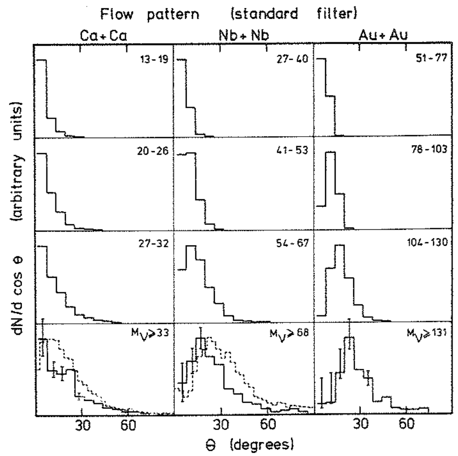

Figure 3
Same calculation as in figure 1, after application of the "standard filter"[2]. The dotted lines give the experimental data[1] for the highest multiplicity bin.

4. MASS AND ENERGY DEPENDENCE

From figures 1-3, it is already clear that the flow increases with the mass of the (symmetric) systems. In studying Nb + Nb at 650 MeV/A and Au + Au at 800 MeV/A, it can be shown that the intrinsic flow both for the participants and the whole system decreases with increasing energy. The same behaviour is expected for the filtered events. The dependence of the flow is summarized in figure 4, where the average of the flow angle corresponding to the *participants* is shown. More precisely, what is plotted is the average component of $\vec{e}_1$ along the impact parameter axis $\vec{e}_x$. This quantity is very close to the sine of the angle $\bar{\vartheta}$, corresponding to the largest axis of the sphericity tensor (2.1) calculated by summing over all events and close to over events average of sin ϑ, when $\bar{\vartheta} \neq 0$, i.e. for $b \gtrsim 0.1\, b_{max}$ [5].

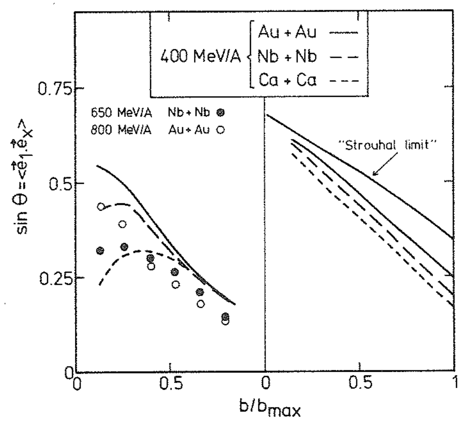

Figure 4
Left part : flow angle ϑ (see text, for definition) for several systems, as calculated in reference 5. The calculation includes participants only. The right part gives the simple estimate provided by eq. (5.16).

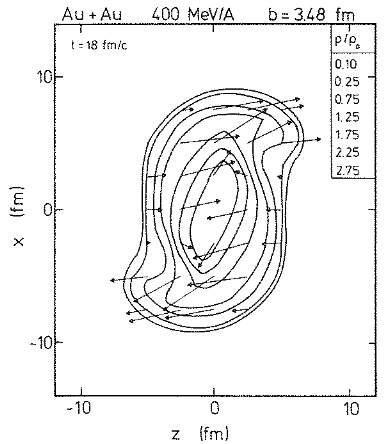

Figure 5
Mass flow density at about time of maximum compression, as calculated in the INC model[5]. The beam axis is along the z direction.

5. DYNAMICS OF THE FLOW

It is shown in reference 5 that the value of the flow angle is on the average (over events) fixed at the end of the compression stage. This suggests that the flow comes from the pressure developed in the compression zone. As a result, the spectator "caps" (i.e. the part of the projectile (target) which in the initial state does not intercept the target (projectile)), which are flying by during the compression state are pushed apart. This clearly appears on figure 5, which shows the mass current about the time of maximum compression. The momentum is not transferred coherently on the caps, but through nucleon-nucleon collisions which, however, leave behind real spectators, i.e. nucleons which make no collisions.

Is compression sufficient to generate a flow, or do we need interaction energy, something not contained in the cascade, as claimed in reference 8 ? To investigate this important question, we turn to the Landau-Vlassov equation

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}.\vec{\nabla} - (\vec{\nabla}U).\vec{\nabla}_p\right) f_1(\vec{r},\vec{p},t) = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} W(pp_1 \rightarrow p_2p_3)$$

$$[f_1(\vec{r},\vec{p}_2,t)f_1(\vec{r},\vec{p}_3,t) - f_1(\vec{r},\vec{p},t)f_1(\vec{r},\vec{p},t)] \quad , \qquad (5.1)$$

with standard notation, which basically embodies the same physics as the INC plus a self-consistent Hartree type mean field ($U = U(\rho)$). The physical content is more transparent if we turn to the equations for the first moments

$$\left\{ \begin{matrix} \rho \\ \rho\vec{u} \\ \tau_{ij} \end{matrix} \right\} = \int d^3p \left\{ \begin{matrix} 1 \\ \vec{p} \\ p_i p_j \end{matrix} \right\} f_1(\vec{r},\vec{p},t) \quad . \tag{5.2}$$

The momentum flux tensor τ_{ij} can be split into a collective and an internal parts by writing $\vec{p} = m\vec{u} + \delta\vec{p}$:

$$\tau_{ij} = R_{ij} + S_{ij} \quad , \tag{5.3}$$

$$R_{ij} = \rho\, u_i u_j \quad , \quad S_{ij} = \int d^3p\, \delta p_i\, \delta p_j\, f_1(\vec{r},\vec{p},t) \quad , \tag{5.4}$$

the last quantity being the usual stress tensor. The sphericity tensor (eq. (2.1)) is related to τ_{ij} by

$$Q_{ij} = \lim_{t\to\infty} \frac{1}{2m} \int d^3\vec{r}\, (R_{ij} + S_{ij}) = Q^R_{ij} + Q^S_{ij} \quad . \tag{5.5}$$

Using eqs. (5.3)-(5.4), the second moment of eq. (5.1) can be split into

$$[\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}]R_{ij} = - u_i \sum_k \nabla_k \Pi_{kj} - u_j \sum_k \nabla_k \Pi_{ki} \tag{5.6}$$

and an equation for S_{ij}

$$[\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}]S_{ij} = - \sum_k (S_{ik}\nabla_k u_j + S_{jk}\nabla_k u_i) - S_{ij}\, \vec{\nabla}.\vec{u} + \int d^3p\, \delta p_i\, \delta p_j\, I \quad , \tag{5.7}$$

where I stands for the collision term (r.h.s. of (5.1)) and where we have left out a term in $(\delta p)^3$. The tensor Π_{ij} is equal to

$$\Pi_{ij} = S_{ij} + \delta_{ij}\, (\rho U(\rho) - \int_0^\rho U(\rho')d\rho') \quad . \tag{5.8}$$

Writing

$$p_{th} = \frac{1}{3} \operatorname{tr} S_{ij} \quad , \quad p_{int} = \rho U(\rho) - \int_0^\rho U(\rho')d\rho' \quad , \quad S_{ij} = p_{th}\, \delta_{ij} + \tilde{S}_{ij} \tag{5.9}$$

where $\tilde{S}_{ij}$ is called the deviator, eqs. (5.7)-(5.8) can be rewritten as

$$[\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}]R_{ij} = - (u_i\nabla_j + u_j\nabla_i)(p_{th} + p_{int}) - u_i \sum_k \nabla_k \tilde{S}_{kj} - u_j \sum_k \nabla_k \tilde{S}_{kj} \tag{5.10}$$

$$[\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}]p_{th} = - p_{th}\, \vec{\nabla}.\vec{u} - \sum_i \sum_k \tilde{S}_{ik}(\nabla_k u_i + \nabla_i u_k - \frac{1}{3} \vec{\nabla}.\vec{u}\, \delta_{ik}) \quad , \tag{5.11}$$

$$[\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}]\tilde{S}_{ij} = - \sum_k (\tilde{S}_{ik} \nabla_k u_j + \tilde{S}_{jk}\nabla_k u_i - \tilde{S}_{ij} \nabla_k u_k)$$
$$+ \int d^3p \{\delta p_i \; \delta p_j - \frac{1}{3}\sum_i (\delta p_i)^2\} \; I \quad . \tag{5.12}$$

Although these equations look very complicated, the physics is rather transparent. The rate of collision controls the evolution of the deviator, which describes the deviation from local equilibrium. The variation of p_{th}, which in the limit of local equilibrium is the thermal pressure, is governed by the compression ($\vec{\nabla}.\vec{u}$ term) and also by off-equilibrium effects. As eq. (5.10) clearly demonstrates, the collective flow is influenced by *three* agents : *the thermal pressure* p_{th}, *the interaction pressure* p_{int} and *off-equilibrium effects*. Within the cascade, the second agent is missing, but the cascade can support a flow because there is ***no*** particular status for the interaction pressure compared to the thermal pressure.

Important off-equilibrium effects are expected in the cascade because of the very presence of spectator nucleons. One may wonder whether the removal of these nucleons brings the cascade close to the local equilibrium limit or to the viscous hydrodynamical limit. In the latter, eq. (5.10) reads, with $p = p_{th} + p_{int}$:

$$(\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla})R_{ij} = - u_i \nabla_j p + \eta[u_i \Delta u_j + \frac{1}{3} u_i \nabla_j \; \vec{\nabla}.\vec{u}] + \text{symm. in } i,j \quad . \tag{5.13}$$

If this limit is taken strictly, the sphericity tensor reduces to the first term in eq. (5.5), since the internal stresses vanish as the system is expanding. It is enlightning to look at the properties of the flow in such a limit. Because of the law of similarity[9], the dimensionless quantity $\langle Q_{xz}\rangle/\langle Q_{zz}\rangle$ which is closely related to the flow angle should be a function of all the dimensionless parameters characterizing the system : the ratio b/b_{max} (b_{max} = twice the nuclear radius R in symmetric systems), the Strouhal and the Reynolds numbers

$$S = \frac{u \; \tau}{R} \quad , \qquad \mathcal{R} = \frac{u \; \rho \; R}{\eta} \quad . \tag{5.14}$$

Here, u is the incident c.m. velocity, τ is the collision time and ρ is the nuclear mass density. We thus have

$$\sin \vartheta = f \left(\frac{b}{b_{max}} , S, \mathcal{R}\right) \quad . \tag{5.15}$$

In the absence of viscosity, this implies the flow to be the *same* for all symmetric systems at a given energy, provided the same b/b_{max} ratio is considered. Indeed, such systems have the same Strouhal number. This arises from the fact that, for $\eta = 0$, eq. (5.13) (and the continuity and Euler equations) are invariant under a multiplication of $\vec{r}$ and t by a common scaling factor Λ. (Actually, the cascade agrees with a proportionality of τ and R).

In order to exhibit the gross properties of the function f, a first crude estimate of Q_{xz} and Q_{zz} for the participants is made. We only give the results here :

$$\frac{\langle Q_{xz}\rangle}{\langle Q_{zz}\rangle} \approx \{ S \frac{R}{L(b)} - \frac{3}{R} \} \frac{\bar{\varepsilon}}{3\bar{\varepsilon}_z} \quad , \tag{5.16}$$

where L(b) is roughly the linear size of the system in the x direction, and where $\bar{\varepsilon}(\bar{\varepsilon}_z)$ is the average nucleon energy (in the z direction). At 400 MeV/A, the Strouhal number is about 2.2 and the Reynolds number for Nb + Nb (using the perfect gas expression of the viscosity) lies between ~ 9 and ~ 15. We here choose $R = 12$ for illustration, but, in any case, the nuclear Reynolds number is much lower than the critical value (~ 7000) for the onset of turbulence (same conclusion as in reference 10). In figure 4, we show the prediction of eq. (5.16) for sin ϑ, taking estimates of the last factor from cascade calculations[5]. One can see that already the simple picture adopted, which surely provides a rather crude representation of the function f in eq. (5.15), explains the qualitative mass dependence of the flow pattern. Equation (5.16) has also the right qualitative energy dependence. Indeed, S does not change significantly with energy, but R decreases with energy, since the viscosity parameter η increases with energy.

It is very likely that the flow in the cascade is influenced by more general off-equilibrium effects, even for the participants. To exhibit the gross properties of the latter, we focus our attention on eq. (5.12). If the time variation of $\tilde{S}_{ij}$ is dominated by the collision term and if the relaxation time hypothesis is made, one can write

$$\tilde{S}_{ij}(t) = \tilde{S}_{ij}(t=0) \exp\left(-\frac{t}{\tau_{rel}}\right) \quad , \tag{5.17}$$

where $\tilde{S}_{ij}(t=0)$ is the deviator at the beginning, when nuclei start to interpenetrate each other. It is then a strongly aligned tensor. One sees that the important parameter is τ/τ_{rel} or equivalently $y = \lambda_{th}/R$, where λ_{th} is the thermalization mean free path. This indicates that off-equilibrium effects should influence the flow in the way indicated by experiment when energy and mass are varying.

6. DISCUSSION

We have seen that there is a definite intrinsic flow inside the cascade, coming mainly from the participants. When a filter is applied to account for the acceptance apparatus, the cascade is able to produce a maximum in the Nb + Nb dN/d cos ϑ distribution at non zero angle for large multiplicities only. Despite of the uncertainty of the filtering procedure, one can say that the results of reference 5 predict too small a flow in comparison with experiment. However, it reproduces correctly the gross features of the mass and energy dependence of the flow[11,12].

There seems to be intriguing differences between existing cascade calculations[1,5,13]. It is however very difficult to make a comparison, since the results of these calculations are generally presented after application of a filtering routine which is not the same in the three cases. A comparison between the three approaches on the intrinsic flow would be welcomed. See reference 7.

We have investigated the causes of the flow in a very general framework. The flow is originating from the pressure built into the system during the compression stages. Under the work of this pressure, the spectator "caps" are deviated. The flow is however reduced by off-equilibrium effects, which reduce to viscosity effects in the limit of local equilibrium. We have shown that off-equilibrium effects account for the mass and energy dependence of the flow, at least qualitatively.

The pressure may be of thermal origin, as in the cascade, or due to the interactions. The lack of flow in reference 5 would indicate a need for extra pressure, i.e. for a stiff equation of state[4]. However, one should keep in mind that, in the light of our discussion, a lack of flow may very well come from an unappropriate treatment of the off-equilibrium effects, as well.

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