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INTRANUCLEAR CASCADE NEWS

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ABSTRACT

We emphasize the main contributions of the INC model to the general understanding of the relativistic heavy ion collisions, especially the quantitative description of off-equilibrium features of the collision process. The importance of two parameters which are expected to control the main off-equilibrium effects is underlined. We point out the possibility of observing in some systems, features of the bulk dynamics, i.e. the dynamics where the off-equilibrium effects are vanishing. We analyse recent data, which indicate that the entropy in high multiplicity events is dominated by the bulk dynamics. Intranuclear cascade calculations of the collective flow are discussed. They seem to be sensitive to the detail of the cascade models. We discuss the relationship between the observed flow and the intrinsic flow, which corresponds to all the particles. This relationship is strongly influenced by the Jacobian effect and the efficiency of the detector. The intrinsic flow angle seems to be sensitive to off-equilibrium effects. The role of the fluctuations is discussed.

1. INTRODUCTION

For the last five years, the intranuclear cascade model (INC) has been applied with an impressive (though not complete as we will see) success to the study of heavy ion collisions with a beam energy per nucleon E/A ranging from 250 MeV to 2 GeV. It has so acquired the status of a competing microscopic theory for the so-called relativistic nucleus-nucleus collisions, along with hydrodynamics and the equations of motion approach. Yet, its connection with general theory is not understood completely. Even, it is not clear yet whether the conditions of applicability are satisfied. The latter may be summarized in the two following inequalities

$$E/A \gg B \qquad (1.1)$$

$$\lambda\!\!\!^{-}_B\ ,\ r_s < d < R\ . \qquad (1.2)$$

In these relations, B is the binding energy per nucleon, $\lambda\!\!\!^{-}_B$ is the de Broglie wavelength for the nucleon-nucleon relative motion, r_s is the range of the nucleon-nucleon forces, d is the average internucleon distance and R is the radius of the nucleus. These conditions are not provided by a general theory, but have rather been derived on purely intuitive arguments.[1,2] The first one ensures that the nucleus-nucleus collision can be described in terms of free space nucleon-nucleon scattering amplitudes. The second one corresponds to a scheme of time-ordered binary collisions, which do not interfere.

To be short, the INC dynamics implies a picture of the collision process as a sequence of space-time well-separated collisions between on-shell hadrons, proceeding as in free space. This is more or less the basic premises of all the INC models, at least those which are popular nowadays.[3-6] Most of them use relativistic kinematics and include pion production (not necessarily in the same manner). In addition, some of them[3,5] incorporate potential and Pauli blocking effects. A relativistic invariant formulation is being constructed.[7-9]

The INC model is often considered as a standard theory, capable of producing the huge background of the observed cross-sections due to the "conventional" physics. Any departure from its predictions could, accordingly, be interpreted as a signal for "unconventional", "exotic" physics, the latter being, in the opinion of some physicists, describable by hydrodynamics. To our opinion, it is time to make these statements more precise. One should replace them by the following ones :

(1) The INC model manages situations where the mean free path λ is not negligible compared to the linear dimension of the system. In fact, this is actually the case in the energy domain under consideration. Furthermore, the INC model keeps track of all the microscopic variations on any scale. On the other hand, the underlying equation of state is more or less the ideal gas equation of state (corrected for some meson degrees of freedom).

(2) In the hydrodynamical model, all the microscopic variations have been averaged out on all scales smaller than the thermalization mean free path.[10] Only is retained the description of "large" scale variations. The evolution of the latter is conditionned by the equilibrium or quasi-equilibrium properties of "large" bodies of the same matter. These properties are usually denoted as the equation of state on the one hand and the transport coefficients on the other hand. The hydrodynamical approach is more flexible in the sense that it can handle various equations of state (and various transport coefficients).

So, the questions which should be raised are, in order : (a) in which conditions do we reach the limit of the bulk dynamics, if any ? (b) Are there equation of state effects, or transport coefficients effects ? It should be realized that the answer to the first question depends very much on the kind of observables one is looking at. It is rather clear that the two particle correlation yield will always show, with a small frequency perhaps, the knock-out process, even in large systems. However, it is difficult to find a good criterion signalling the effect of bulk dynamics. We will however see an example below. Once the bulk dynamics is recognized, it may then be possible to look at quantities which could change with the equation of state.

We will successively look at the past, some of the present and the expected future developments of the INC model in the light of these questions. We will also underline the specific features of the model, which makes it a rather unique tool of theoretical analysis.

2. THE PAST

Very briefly, the important success of the INC model is the overall agreement with many proton, deuteron, pion inclusive cross-sections, as well as with some two proton cross-sections, without the help of any hidden parameter. Furthermore, it has given some enlightments on the connection between observables and the underlying multiple scattering process. Let us point out the main aspects :

(1) It has given a theoretical support to the geometrical separation between participants and spectators, allowing, for deviations, however.[7] This is linked with the smallness of the parameter

$$x = \frac{\Delta p_\perp}{p_0} \qquad (2.1)$$

where $\Delta p_\perp$ is the typical perpendicular momentum transfer in a NN collision, and p_0 is the initial momentum.

(2) The evolution of the participant system is intermediate between a full thermalization and the knock-out (or Knudsen) limit. In other words, the distribution of the number of collisions undergone by the nucleons is very broad, as shown in Fig. 1. Obviously, off-equilibrium effects are linked with the finite value of the parameter

$$y = \frac{\lambda}{R} \qquad (2.2)$$

where λ is the (average) mean free path and R is the nuclear radius. Indicative of the importance of the rapid (knock-out like) non-equilibrium processes are the forward-backward peaking of the c.m. angular distribution of the protons in symmetric systems (which persists even for high multiplicity events) and the two-proton correlation yield for quasi-free kinematics. See ref.[11] for details.

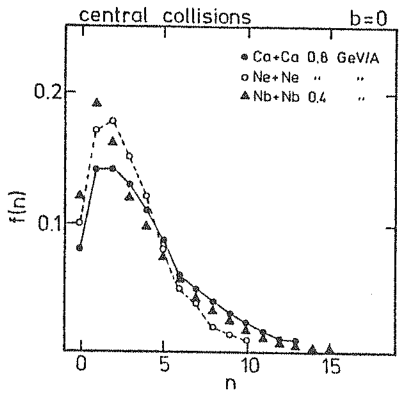

Fig. 1 : Probability distribution for a nucleon to undergo n collisions in central (b=0) collisions for three different systems. They are listed in increasing order for the parameter y (eq. (2.2)).

(3) The compression stage of the collision process is the place of strong non-equilibrium phenomena, whereas the decompression stage ressembles in some sense an isoentropic hydrodynamic expansion.

Aside from these substantial successes, the INC has also presented in the past some failures, the two significant ones are the so-called "entropy puzzle" and the overprediction of the pion multiplicity in central collisions. In the next section, we will speak about the present status of the "entropy puzzle" as well as about the question of the collective flow. The pion multiplicity is covered in Harris's lecture.[12]

3. THE PRESENT

3.1. Entropy

The "entropy puzzle" is a convenient denomination for the observed discrepancy between the INC prediction for the final entropy and the "experimental" value as deduced from ratio "d"/"p" between deuteron-like and proton-like yields through the Siemens-Kapusta relation[13] :

$$\frac{S}{A} = 3.95 - \ln \frac{N_{"d"}}{N_{"p"}} \quad . \qquad (3.1)$$

Many explanations have been proposed, but a new enlightment of the problem is provided by recent measurements[14-15] which give (see Fig. 2) the $N_{"d"}/N_{"p"}$ ratio as a function of the charge multiplicity (previous considerations were based on rather inclusive data). For all the systems shown, there is a steady increase of the produced entropy as charge multiplicity increases and consequently, as the impact parameter decreases. The results can be analysed with Remler's theory[16-18] which assumes that the deuteron-like particles are simply given by the appearance of deuteron-like structures in the final state.

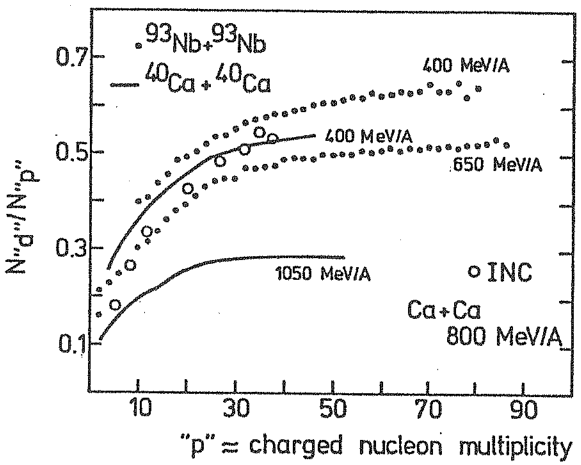

Fig. 2 : Experimental "d"/"p" ratio versus charged multiplicity for four systems. The full dots are data from ref. 16, the lines represent the data of ref. 15 and the open dots are calculations from ref. 20. For the latter the abscissa represents half the average participant number for several impact parameters ranging from zero to 7.15 fm.

If f_2^{np} is the two-body distribution function for neutron-proton pairs, one may write

$$N_{"d"} = \frac{3}{4}\int d^3R\,d^3P \int d^3r\,d^3p\, f_2^{np}(\vec{R}+\vec{r}/2,\vec{R}-\vec{r}/2,\vec{P}/2+\vec{p},\vec{P}/2-\vec{p})\, g_d(\vec{r},\vec{p}) \quad , \qquad (3.2)$$

where $g_d(r,p)$ is the Wigner transform of the deuteron density matrix. We remind that $N_{"p"}$ is given by

$$N_{"p"} = \int d^3r\,d^3p\ f_1^p(\vec{r},\vec{p}) \quad , \qquad (3.3)$$

where f_1^p is the proton one-body distribution functions. Expression (3.2) can be considerably simplified if one assumes the absence of two-body correlations ($f_2^{np} = f_1^n f_1^p$) and an extension of the system in phase space much larger than the

one of a deuteron. Assuming further the neutron and proton distributions to be similar

$$\frac{A}{N}\, f_1^n = \frac{A}{Z}\, f_1^p = f_1 \quad , \qquad (3.4)$$

one has

$$\frac{N_{"d"}}{N_{"p"}} \approx 6\,\frac{N}{A}\,\langle f_1\rangle \quad . \qquad (3.5)$$

In these equations, N, Z and A are the neutron, proton and nucleon numbers of the participant zone, respectively, and $\langle f_1\rangle$ is the average nucleon distribution on the distribution f_1 itself. Finite size (in phase space) effects can be evaluated if both f_1 and g_d are assumed to be gaussian

$$f_1(\vec{r},\vec{p}) = \frac{A}{(R_p \pi\sqrt{2mT})^3}\, \exp\left[-\frac{r^2}{R_p^2} - \frac{p^2}{2mT}\right] \quad , \qquad (3.6)$$

$$g_d(\vec{r},\vec{p}) = \frac{1}{(\pi r_0 p_0)^3}\, \exp\left[-\frac{r^2}{r_0^2} - \frac{p^2}{p_0^2}\right] \quad . \qquad (3.7)$$

In that case, eq. (3.5) becomes

$$R_{dp} = \frac{N_{"d"}}{N_{"p"}} = 6\,\frac{N}{A}\,\langle f_1\rangle X(r_0) Y(p_0) \qquad (3.8a)$$

with

$$X(r_0) = \left(1 + \frac{r_0^2}{2R_p^2}\right)^{-3/2} \quad , \qquad (3.8b)$$

$$Y(p_0) = \left(1 + \frac{p_0^2}{mT}\right)^{-3/2} \quad . \qquad (3.8c)$$

In principle, an additional correction factor may be added to take account of a possible radial flow.[19] The coefficients r_0 and p_0 are not independent ($r_0 p_0 \approx \hbar$) because a ground state orbital occupies at least a natural unit in phase space.

What is responsible for the increase of the R_{dp} ratio ? The temperature is not expected to vary significantly with the number of participants. Therefore $Y(p_0)$ is roughly constant when the multiplicity is varying, and the structure of R_{dp} is due either to $X(r_0)$ or to the quantity $\langle f_1\rangle$.

In the limit of the bulk dynamics, the latter is not expected to vary with the multiplicity since it is an intensive quantity. But, deviations from the bulk dynamics are expected on the basis of a finite mean path (the occupation in momentum space is once again not expected to vary). If a nucleon collides close to the surface of the interaction region, it has a good chance to escape. If on the other hand, it collides deep in the interaction zone, it is confined by the outer layers. This enhanced spreading can be expressed as if the volume was replaced by an effective volume larger by a dimension of the order of a mean free path.[18] Therefore, we conjecture a variation of the type

$$\langle f_1 \rangle = \langle f_1 \rangle^0 \left(1 + \frac{\lambda}{R_p}\right)^3 \quad . \qquad (3.9)$$

It is meaningless to try to fit the data with the help of the above formulae, because they include too many parameters. Additional assumptions are required. The most natural (but not necessarily the most justified) one is the choice of a constant (freeze-out) density : $R_p = r_p A^{1/3}$. Even with such a choice that makes too many parameters. It is interesting however to note that the A-dependence is not the same in the two factors under consideration. We have tried in fig. 3 to see whether the R_{dp} variation is dominated by (3.8b) or by (3.9). We have replotted the data of fig. 2 in such a way to linearize their dependence either to $N_{"p"}^{-1/3}$ (eq. (3.9)) or to $N_{"p"}^{-2/3}$ (eq. (3.8b)). In the range considered, there is no practical difference between the two functional dependences. The important result is however that whatever which factor is the most important, the value of R_{dp} for the largest multiplicities is very close to the bulk dynamics limit.

In the INC model, the R_{dp} ratio and the entropy can be calculated separately, the first one through eqs. (3.2-3.3) and the second one with the help of

$$\frac{S}{A} = 1 - \langle \ln f_1 \rangle + \ln 2 - \frac{N}{A} \ln \frac{N}{A} - \frac{Z}{A} \ln \frac{Z}{A} \qquad (3.10)$$

or if f_1 is not far from a Boltzmann distribution

$$\frac{S}{A} = 1 + \frac{3}{2}(1 - \ln 2) - \ln \langle f_1 \rangle + \ln 2 - \frac{N}{A} \ln \frac{N}{A} - \frac{Z}{A} \ln \frac{Z}{A} \quad . \qquad (3.11)$$

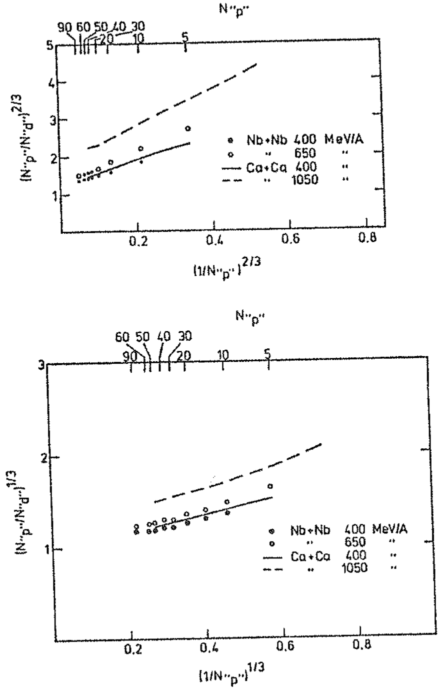

Fig. 3 : Experimental data of fig. 2 plotted in such a way to show the relationship with formula (3.8b) (top) and (3.9) (bottom). See text for detail.

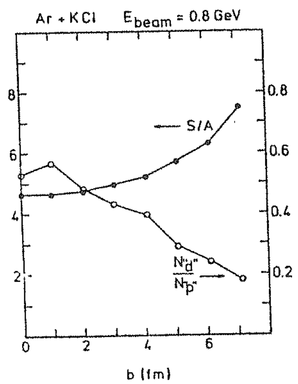

Fig. 4 : INC calculation of the entropy per baryon and the deuteron proton ratio as a function of impact parameter. Adapted from Ref. 20.

Numerical predictions of the INC model[20] are shown in Fig. 4. If one accepts eq. (3.11) as an acceptable formula, one is led to attribute the variation of the calculated entropy to an increase of $\langle f_1 \rangle$ in qualitative agreement with (3.9). Fig. 3 also show that the INC predicts a variation of the R_{dp} ratio in qualitative agreement with experiment. The same results are plotted in Fig. 5 and are compared to the Siemens-Kapusta relation. Deviations for large impact parameters are likely related to the factor $X(r_0)$ (eq. (3.8b)) and possibly to the presence of $\vec{r}.\vec{p}$ correlations.

In conclusion, the bulk dynamics limit seems to be asymptotically reached for large multiplicity events. It is therefore acceptable to consider the limiting values of the extracted entropy as related to bulk properties of nuclear matter. They are actually compared to the INC calculation of ref. [20] in Fig. 6, which shows that, when

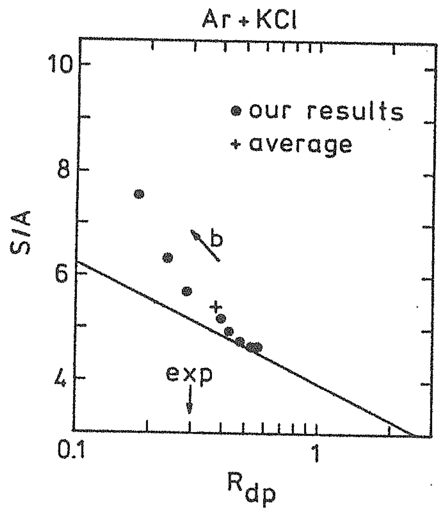

Fig. 5 : INC calculated entropy and "d" over "p" ratio compared with Eq. (3.1) (full line). The dots correspond to regularly spaced impact parameters ranging from 0 to about 7.5 fm. The arrows indicate the experimental value of R_{dp} when integrated cross-sections are used. Adapted from Ref. 20.

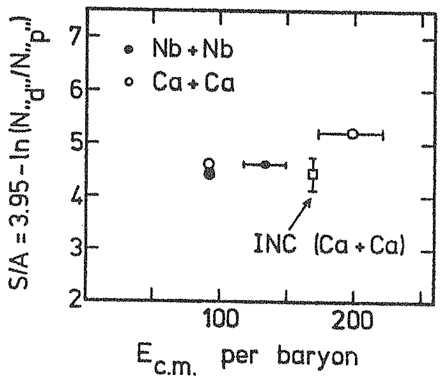

Fig. 6 : Experimental values of the "d"/"p" ratio (or the entropy) for large multiplicities compared to the INC calculations of Ref. 18.

the proper quantities are compared, the entropy puzzle has substantially deflated. A definite discrepancy subsists however. More importantly, with the procedure of extracting the limiting value from high multiplicity events, one is in better position to extract the equation of state under the form of $U(\rho,S)$, when the density ρ can be provided by correlations experiments.

3.2. The Collective Flow

It has been suggested several years ago already[21] that in rather central events, several particles would be emitted more or less collectively, as a result of the pressure built in the participant systems. Such a feature could be indicative of the hydrodynamical behaviour of the colliding nuclei and, more interestingly, could be sensitive to the equation of state. It was recognized that such a feature could hardly be exhibited without ambiguity in inclusive

cross-sections. Therefore, it has been suggested[22-24] to turn to event-by-event analyses, using the global variables. Recently, the question was put forward by the presentation of the first results of this type due to the Plastic Ball group. They observed qualitatively different results for Ca+Ca and Nb+Nb results at 400 MeV/A.[25]

Before entering into the discussion, it may be worthwhile to remind what is actually involved in such an analysis. For any event the sphericity tensor (or ellipsoid) is (usually) constructed as

$$Q_{ij} = \sum_{\nu} \gamma(p^{\nu}) p_i^{\nu} p_j^{\nu} \quad , \qquad (3.12)$$

where $\vec{p}^{(\nu)}$ is the momentum in the c.m. frame of the νth (observed) ejectile, and where γ is a scalar weight, usually taken to be $\gamma(p^{\nu}) = 1/(2\, m_{\nu})$. The tensor is usually described by variables like the aspect ratios $q_1 = \lambda_1/\lambda_3$, $q_2 = \lambda_2/\lambda_3$ and the Euler angles θ , ϕ , χ . It is important here to remind that the λ_i's are the eigenvalues of the tensor in decreasing order, and θ is the angle between the largest axis and the beam axis. The bulk of the experimental results has the form of distributions of some of these variables (or combinations of them). They thus imply average values and fluctuations. It is important to understand what governs the latter. We consider here the flucutations of the elements of the tensor itself, because in this case, it is easy to write down formulae. Let us consider a single impact parameter b and let us assume for simplicity that always the participant system desintegrates into N nucleons (considerations below may be generalized to the case where N varies from event to event). The probability of having the i,jth element of the tensor (3.12) to have a numerical value Q_{ij} is given by

$$\phi(Q_{ij}) = \frac{1}{N!} \int d^3p_1 \ldots d^3p_N f_N(\vec{p}_1, \ldots \vec{p}_N)\, \delta\Big(\sum_{\nu=1}^{N} \gamma(p^{\nu}) p_i^{\nu} p_j^{\nu} - Q_{ij}\Big) \quad , \qquad (3.13)$$

where f_N is the N-body distribution function in momentum space. One readily finds that the average value of Q_{ij} is

$$\langle Q_{ij}\rangle = \int \phi(Q_{ij})Q_{ij}dQ_{ij} = \int d^3p\ \gamma(p)p_ip_jf_1(\vec{p}) \quad . \tag{3.14}$$

The second moment is given by

$$\langle Q_{ij}{}^2\rangle = \int d^3p_1 d^3p_2 f_2(\vec{p}^1,\vec{p}^2)\gamma(p^1)\gamma(p^2)p_i{}^1p_j{}^1p_i{}^2p_j{}^2 + \int d^3p[\gamma(p)p_ip_j]^2 f_1(\vec{p}) \quad . \tag{3.15}$$

Introducing per nucleon average values like

$$\overline{\gamma p_ip_j} = N^{-1}\int \gamma(p)p_ip_jf_1(\vec{p})d^3p \tag{3.16}$$

and the two-body correlation factor

$$X = \frac{\int d^3p_1d^3p_2\gamma(p_1)\gamma(p_2)p_i{}^1p_j{}^1p_i{}^2p_j{}^2f_2(\vec{p}_1,\vec{p}_2)}{[\int d^3p\gamma(p)p_ip_jf_1(\vec{p})]^2} \quad , \tag{3.17}$$

one gets

$$\frac{\sigma_{ij}}{\langle Q_{ij}\rangle} = \frac{(\langle Q_{ij}{}^2\rangle - \langle Q_{ij}\rangle^2)^{1/2}}{\langle Q_{ij}\rangle} = \left[(X-1) + \frac{1}{N}\,\frac{\overline{\gamma^2p_i{}^2p_j{}^2}}{\overline{\gamma p_ip_j}^2}\right]^{1/2} \quad . \tag{3.18}$$

This important result shows that the fluctuations are reduced when the number of the nucleons participating to the sphericity tensor is increasing. The dynamics enters directly through the $(X-1)$ term, which probes (globally) the two-body distribution function, and through the factor multiplying $1/N$, which is a characteristic of the one-body distribution function. This factor is of the order of unity. For a thermal isotropic distribution, it is equal to 5/3. Of course, in practice, one covers at least a certain range of impact parameters. Furthermore, as we said, people are generally interested in shape parameters (like q_1 and q_2) or angles (like θ) which are non-linear functions of the Q_{ij}'s. Therefore, it is not obvious that a clear separation between the mass number effect and the dynamical effects will subsist.

Furthermore, the distribution of the global variables like q_1 and θ is very strongly influenced by the so-

called Jacobian effect.[26] This results from the fact that going from the Q_{ij}'s to q_1, q_2 and the angles, one realizes a mapping of a manifold into another in such a way that an interior point in one case is transformed into a point on the boundary in the other case. In Ref. [26], it is recommended to consider only so-called jacobian-free distributions for the global variables q_1 and θ. We will come back to this point.

The situation concerning the question of the capacity of the INC model to generate a collective flow is presently confused. In Ref. [25], it is saied that the Yariv-Fraenkel cascade is not able to reproduce the Nb+Nb data. Recently,[27] it was shown that the INC code developped in Liège qualitatively reproduces the Plastic Ball data, which seems in contradiction with the claim of Ref. [28]. Also, it seems that the Kitazoe code[29] yields results which are similar to those of Ref. [27]. We are going to present new aspects of the results of Ref. [27] and discuss the associated physics.

Before entering the core of the discussion, we have to say a few words about the comparison with the data. The problem arises from the fact that the detector system misses some ejectiles and misidentifies from time to time the detected particles. Grossly speaking, the target spectators as well as the free neutrons are missed and the particles at very forward angles are misidentified part of the time (see Ref. [34] for detail). The simulation of the detector acceptance within an INC calculation is rendered difficult by the fact that the latter cannot handle the formation of clusters directly. So it is hard to define precisely a free neutron as well as the total charge multiplicity. How it is done is clearly indicated in Ref. [27]. The filter used in this work follows the gross features of the experimental filter. Certainly, there is a need to refine the filter. However, if the results are strongly dependent upon the fine details of the filter, then it is highly questionable whether the experimental results of the Plastic Ball carry any physical meaning.

The main result of Ref. [27] is shown in Fig. 7. It reproduces the general trend of the experimental data. The $dN/d\cos\theta$ distribution displays a peak at zero degree for

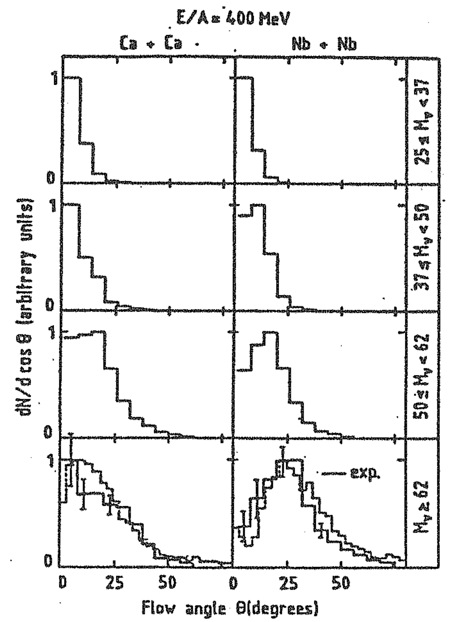

Fig. 7 : Calculated $dN/d\cos\theta$ distributions after application of a filter for several multiplicity bins. The values indicated on the right refer to the Nb+Nb system. For Ca+Ca, the corresponding values are obtained by multiplying by 20/41. The dotted curves give the Plastic Ball data for their highest multiplicity bin. The error bars give the typical uncertainty of the calculation. All the histograms are normalized to unity at their maximum. From Ref. 27.

Ca+Ca and a peak at an angle which increases with the multiplicity in Nb+Nb. The angle of the peak for the largest multiplicity bin (the flow angle in the adopted jargon) may change by a few degrees if the filter or the definition of the multiplicity M_v are changed within reasonable limits, but the presence of a peak in Nb+Nb subsists whatever these modifications.[27]

A deeper insight may be gained when looking at Fig. 8, which shows for different impact parameters, the projection of the extremity of the unit vector $\vec{e}_1$ aligned with the largest axis of the ellipsoid (associated with the sphericity tensor (3.12)) on a plane perpendicular to the beam axis. In this figure, the sphericity tensor includes all the nucleons. For b=0, the ellipsoid points towards the beam axis in the average. When the impact parameter increases, the ellipsoid has a tendancy to point sidewards (in the average) on the side of the impact parameter axis. The angle first increases and later decreases as the collisions are becoming peripheral. The systematic differences between Ca+Ca and

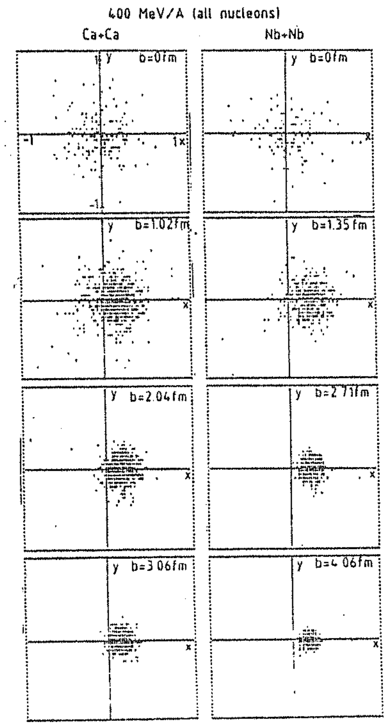

Fig. 8 : Projection on a plane perpendicular to the beam axis of the extremity of a unit vector aligned with the largest axis of the sphericity ellipsoid, as calculated by the INC model of Ref. 27, including all the nucleons. The beam axis points at the center of the crosses. The impact parameter lies, for any event, along the horizontal axis, toward the right of the figure.

Nb+Nb, are : (1) the average angle is larger in Nb+Nb for a given b/b_{max} ratio ; (2) the fluctuations are <u>smaller</u> for Nb+Nb, a feature to be related to the 1/N term in eq. (3.18). From Fig. 8, it is rather easy to understand the interplay of the Jacobian effect, the fluctuations and the filtering, which are coming into play when going from this figure to Fig. 7. The $dN/d\cos\theta$ distribution can roughly be obtained by taking equally spaced circles around the origin in Fig. 7(+). Manifestly for intermediate impact parameters, the $dN/d\cos\theta$ obtained in this way will show a maximum at finite θ for Nb+Nb and essentially a maximum at zero degree for Ca+Ca. This is due to the large fluctuations in Ca+Ca which are masking the tendancy of the ellipsoid to point in the average at finite θ. What happens when the filter is applied ?

(+)The exact relation is given by $d(\cos\theta) = (1-\rho^2)^{-1/2}\,\rho d\rho$ where ρ is the distance from the origin in each graph of Fig. 7.

First consider the removal of the target spectators, particles that are essentially aligned with the beam axis (in the c.m. system). In this case, the average θ angle is expected to increase, as well as the fluctuations, since the relative weight of strongly aligned particles is decreased. With the elimination of the neutrons, one further increases the fluctuations. In Nb+Nb, the fluctuations are small enough and the average θ angle is large enough to preserve the presence of a peak in the final $dN/d\cos\theta$ distributions. It is not the case in the Ca+Ca system. Figure 9 illustrates the above discussion.

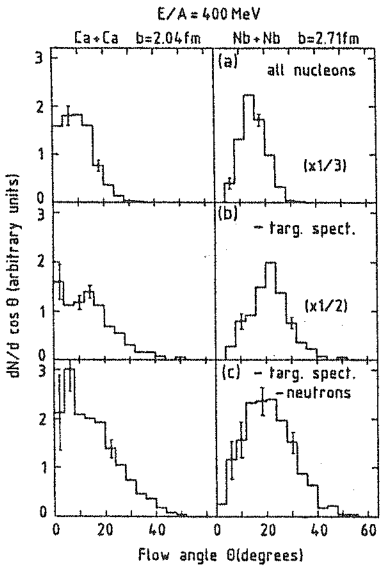

Fig. 9 : Calculation of Ref. 27 showing $dN/d\cos\theta$ histograms for a typical impact parameter : (a) all the nucleons are counted ; (b) undetectable target spectators are removed ; (c) the full filter is applied. Going from (b) to (c) roughly amounts to remove the free neutrons.

One may be surprised by the fact that the small average θ angle, which occurs for central collisions does not show up for large multiplicities. The explanation lies in figure 10, which shows the calculated charged multiplicity (for detail, see Ref. 27) as a function of the impact parameter. The largest multiplicity bin in figure 7 corresponds actually to picking (a fraction of the) events in an impact parameter range which extends up to $b/b_{max} \approx 0.35$. Because of the bdb weight, the largest multiplicity bin is influenced by non-zero impact parameters, with say $\langle b/b_{max}\rangle \approx 0.25$.

The preceeding results induce to believe that the flow, as obtained in Ref. 27, mainly comes from the participants and not from a bounce-off of the spectators. Actually as indicated by Fig. 11, the

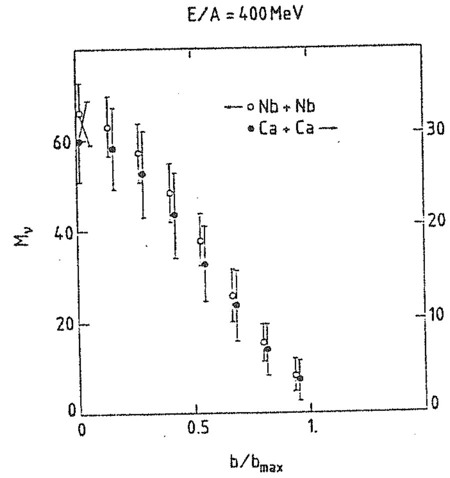

Fig. 10 : INC calculation of Ref. 27. Charge multiplicity as a function of the impact parameter for Ca+Ca (scale on the right) and Nb+Nb (scale on the left) systems. The error bars give the standard event-by-event fluctuations of M_ν .

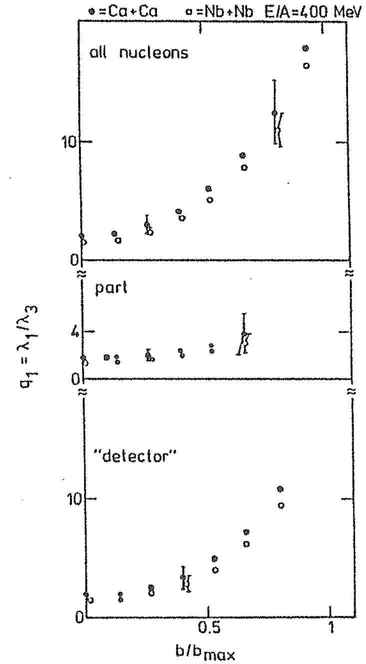

Fig. 11 : INC calculation of the shape parameter of the sphericity tensor. The upper part includes all the nucleons. The middle part refers to participants only. The lower part corresponds to the filtered events. The filter is the same as in Ref. 27. The error bars give the event-by-event fluctuations.

events can typically be described as a superposition of a moderately deformed participants ellipsoid and of a strongly elongated spectators ellipsoid. Here, and in all the subsequent discussion, the participants are those nucleons which have suffered a four-momentum transfer t such that $-t \geq p_F^2$, p_F being the Fermi momentum (= 270 MeV/c here). The ellipsoid of the participants points with large fluctuations at angles which are larger and larger when the impact parameter is smaller and smaller, as displayed by Fig. 12. For intermediate impact parameters, the ellipsoid is (in the average) tilded to larger angles, as compared to Fig. 8, for which all the nucleons are included. Freed from the influence of the spectators, the ellipsoid direction fluctuates much more than in

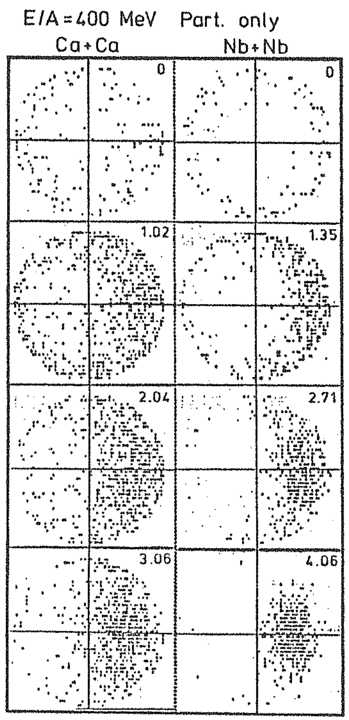

Fig. 12 : Same as Fig. 8 for the participants only.

the previous case. The fluctuations keep to be more important for the Ca+Ca system. There seems to be no effect of the dynamics in the fluctuations, although a refined analysis is necessary to see whether there are deviations from the $1/\sqrt{N}$ law (when X=1 in Eq. (3.18)) or not. It is rather amazing to discover that the b=0 flow pattern (at least in the Nb+Nb system) is very reminescent of an hydrodynamical behaviour, where the matter is flowing at 90° cm. Only the small amount of the spectators, but with large parallel velocity, is sufficient to align the total ellipsoid with the beam axis.

It is our opinion that the transparency at b=0 is linked with the parameters x and y of section 2, and varies in opposite direction to the ratio x/y . As for the flow angle at intermediate impact parameters, its value is very likely increasing when the parameter x increases (increase a deflection) or when y decreases (increase importance of the participants). On this basis, one may make qualitative predictions on the energy and mass-dependences. For a given energy, the flow angle (intrinsic or not, see below) should increase with the mass of the system. For a large enough system, the very central events should be of the hydrodynamical type (pancake with the symmetry axis along the beam axis). For a given system, the flow angle should decrease slightly when going to higher energy, but the narrowing of the angular distribution of the nucleon-nucleon elastic scattering may be compensated by the inelastic processes. This pattern seems to agree with the recent measurements of the Plastic Ball group.[33]

From the theoretical point of view, it is interesting to make a distinction between the "observable" flow angle, which is the flow angle of the ellipsoid after filtering (for the simulation of the detector acceptance) and the <u>intrinsic</u> flow angle, which corresponds to the full ellipsoid. As we have seen, the former is very much influenced by the intricate superposition of the fluctuations and of the filter. The intrinsic flow angle has a smoother dependence on the physical and geometrical parameters of the system as shown by Fig. 13. One also realizes that the filter increases the flow

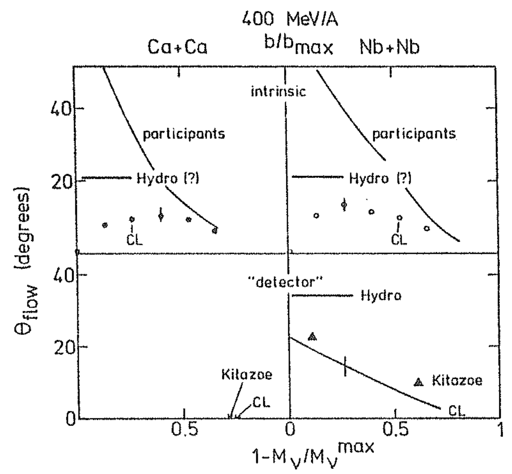

Fig. 13 : The upper part of the figure gives the intrinsic flow angle for all the nucleons (dots) and for the participants (full lines), as a result of an INC calculation using the same model as in Ref. 27. The flow angle is given as a function of the ratio b/b_{max} . The lower part of the figure gives the "observed" flow angle (after filtering) as a function of the quantity mentioned at the bottom. (CL) refers to the same calculation as in the upper part. The error bars give the uncertainty of the calculation. For comparison, we have indicated qualitatively (triangles) the results of the INC calculation of Ref. 29. The mention "Hydro" in the lower part refers to the hydrodynamical calculation of Ref. 28. The mention "Hydro" in the upper part gives gross indications on the intrinsic flow since the detail of the filter applied in Ref. 28 is not kwown.

angle when the fluctuations are sufficiently small.

The difference between the results of Refs. 25, 27 comes, in our opinion, from the general spirit of the respective codes. In Ref. 25, the cascading particles are scattered by a continuous medium, whereas in Ref. 27, the event corresponds to the collision of two collections of particles. The latter is more likely to generate pressure in the contact zone.

The authors of Ref. 29 have made the same choice as in Ref. 27, and get similar results. The situation is not completely clarified anyway, because of the question of the so-called "unfrozen spectators" (see Ref. 30) and because of the difficulty of designing a filter applicable to an INC code which does exactly the same job as the Plastic Ball in a real event.

4. CONCLUSION. THE FUTURE

As we have said, one of the main (and in some people's opinion, unfortunate) features of the heavy ion collisions in the GeV/A range revealed by the INC model is the presence of non-equilibrium aspects in the collision process. The importance of the latter are related to the specific values of some parameters, which are not very well kwown. We have pointed in section 2, two parameters x and y, which could be important parameters for the control of the off-equilibrium effects. The latter may show up differently in various observables. The bulk dynamics, that we have defined as the dynamics which is insensitive to these off-equilibrium effects, may determine the value of some observables of a given system whereas other observables of the same system show departures from bulk dynamics. This is probably the case as we have seen. The limiting value of the d-like to proton-like ratio is probably free of non-bulk effects to a large extent. On the other hand, it seems that the flow pattern is still well influenced by the off-equilibrium effects. This raises some interesting questions. We formulate some of them below.

1. Would it be possible (for larger systems?) to isolate the bulk dynamics limit in the flow pattern ?

2. Is the bulk dynamics in this case sensitive to the equation of state ?

3. The parameters x and y (actually it is λ rather than y) being relevant for the transport properties of the nuclear fluid, would it be possible to disentangle between general off-equilibrium effects and transport properties effects ?

4. Is it possible to isolate transport properties effects in some observables ?

5. Is there any dynamical content in the fluctuations ?

The INC model constitutes a unique tool for the study of the following three aspects (1) the evolution in phase space (2) the off-equilibrium effects (3) the fluctuations. It thus seems to be quite useful for an attempt to answer the questions above. The weakest point of the INC is the unability of handling interaction energy. The introduction of a mean field in the way Malfliet[31], Bertsch et al.[32] or Stocker et al.[30] have done it seems very promising.

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