DO NUCLEI FLOW INSIDE THE INTRANUCLEAR CASCADE MODEL?

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An intranuclear cascade calculation of Ca + Ca and Nb + Nb at 400 MeV/A is performed and analysed in terms of giot variables. Our results are in qualitative agreement with the recent plastic ball data. The ability of the cascade dynamics to generate a collective flow and the effects of the fluctuations are underlined.

It is a long-standing problem to know whether the dynamics of the collision process between nuclei in the energy range between ~200 MeV/A and 2 GeV/A proceeds from hydrodynamics or from the intranuclear cascade model (INC) dynamics. The former, which can be considered as the short mean free path limit, may also generate equation of state effects, whereas the INC embodies a rather long mean free path situation and corresponds more or less to an ideal gas equation of state. The inclusive measurements are not very helpful in distinguishing between the two approaches, although better agreement is generally achieved with the INC model, at least around 1 GeV/A. So the attention has turned to exclusive measurements, and the hope was that global variables would help to discriminate [1–5]. In particular, if a collective flow of the matter arises after the collision process, it will expectedly show up in the orientation of the sphericity ellipsoid, one of the common variables built with the momenta of all the ejectiles (see below). However, as nicely shown by Gyulassy and Danielewicz [6], and also indicated in refs. [3,7] the flow angle may be completely masked by fluctuations due to the finite number of ejectiles assumed to flow in the average in a given direction. It is true that the Ca + Ca data at 400 MeV/A [8] produced by the plastic ball group did not show any peak in the so-called dN/d cos θ plot. The latter is especially designed to get rid of the distortions due to the jacobian. Recently [9], the same group published the Nb + Nb data, which, for high multiplicity events, show a clear peak in the same plot around θ ~ 23°. In the same ref. [9], it is indicated that calculations based on the INC model of Yariv and Fraenkel [10] were unable to show any sideward peaking in the dN/d cos θ plot, in contradistinction with the results of the hydrodynamical calculation of the Frankfurt group [11], and of the Los Alamos group [12]. We have reexamined here the problem in the frame of the INC code developed in Liège for several years. In particular, we want to answer the following questions: is it possible to reproduce the Nb + Nb data with this code? How sensitive are the results on the experimental cuts? What is the nature of the fluctuations? Is there a flow inside our INC model?

The cascade model that we used is described in ref [13,14]. For each event, we calculate the sphericity tensor

\[ Q_{ij} = \sum \nu w(\nu) p_i^{(\nu)} p_j^{(\nu)}, \]

where \( p_i^{(\nu)} \) is the \( i \)-th cartesian coordinate of the \( p \)-th nucleon in the cm frame, and where the weight factor \( w(\nu) \) has been chosen equal to \( 1/[2m(\nu)] \), which makes the sphericity tensor coalescence invariant [4]. In order to be compared with experiment, the calculated events must be filtered in such a way as to follow the acceptance of the detector and be classified along the resulting charged multiplicity. Because the INC model cannot handle cluster production directly, the

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is not a trivial question at all. The fact that our model neglects isospin effects (in its present form), adds an additional uncertainty, although it is not expected that the flow is very much charge-dependent. Just to give an idea of the difficulty, it is worth remembering that a proton emitted by the target would not be identified in the plastic ball if its energy in the laboratory frame were less than \(\sim 25 \text{ MeV}\). On the other hand, if such a proton appears in a triton particle, it will be detected if its energy is larger than \(\sim 16 \text{ MeV}\). Therefore we have adopted the following procedure.

We have tried to determine for fitting our results a filter which, in our opinion at least, is as close as possible to the experimental filter. In addition, we have looked to several other filters, more or less stringent, in order to see the sensitivity of the results upon the definition of the filter.

The standard filter is defined as follows: (1) all the nucleons of the target which have suffered a momentum transfer \(\Delta p\) smaller than 194 MeV/c (=0.7 \(p_F\)) are rejected (this corresponds to an energy transfer of \(\sim 20 \text{ MeV}\)); (2) all the remaining nucleons with an energy \(E_{\text{lab}} < 20 \text{ MeV}\) are rejected; (3) all nucleons appearing at \(\theta_{\text{lab}} > 160^\circ\) are excluded; (4) a fraction of the kept nucleons are rejected randomly to simulate the free neutrons. The fraction is determined by assuming the same \(N/Z\) ratio as in the original nuclei and by estimating the degree of clustering from the \(\tilde{d}/\tilde{p}\) ratio [15] (only the free neutrons are not measured in the detectors). Our estimates for the free neutral fraction of the nucleons are \(\sim 28\%\) for \(\text{Ca} + \text{Ca}\) and \(\sim 37\%\) for \(\text{Nb} + \text{Nb}\). No special prescription has been applied to account for the (small) undeterminacy arising from the plastic wall and from the trigger. (Actually the trigger rejects a very small amount of events, especially of the high multiplicity ones, which are the most important for our purpose).

We can classify the calculated events according to the following multiplicity \(M_\nu\). The latter quantity is chosen as the number of nucleons having suffered a momentum transfer larger than \(p_F\), diminished by a percentage of \(N/A\) to remove the neutrons, using the same procedure as described above. This multiplicity is probably different from the true observed charged multiplicity. The difficulty of comparing the two comes from the degree of clusterization. But this is not really a problem since the multiplicity serves principally as a relative measure of the impact param-

![Figure 1](image)

**Fig. 1.** Multiplicity \(M_\nu\) (see text) as a function of the impact parameter for \(\text{Ca} + \text{Ca}\) (scale on the right) and \(\text{Nb} + \text{Nb}\) (scale on the left) systems. The error bars give the event per event fluctuations of \(M_\nu\).
are given in the top of fig. 3, for the last multiplicity bin displayed in fig. 2. To give an idea of the uncertainty of our results, we also show in fig. 3 the high $M_\nu$ events, but using another lower limit ($M_\nu \geq 50$), as compared to the bottom of fig. 2. As one can see, the main properties of the results are preserved, although some details (position of the peak in Nb + Nb) may be altered.

Now, we would like to analyze our results a little bit. We first want to mention that, contrary to what is often stated [5,9], a collective behaviour may very well arise from a cascade dynamics. This was already mentioned in ref. [13], and also in ref. [16], where the space–time evolution of a colliding system was studied in detail. Furthermore, this was also reasserted in ref. [7], in connection with the flow analysis. We show in fig. 4 the calculated flow in the two systems. Actually, what is plotted is the projection of the extremity of the unit vector along the large axis of the sphericity ellipsoid on a plane perpendicular to the beam axis. It is clear that for intermediate impact parameters, the flow is definitely sidewards. The deviation is larger for Nb + Nb because the system is larger, compared to the mean free path. The pressure in the contact zone builds up more easily. Fig. 4 also shows that the fluctuations are larger in Ca + Ca, which is very likely due to the finite number effect as discussed in ref. [7].

The difference between the Ca + Ca and Nb + Nb systems in the dN/d(cos $\theta$) can be understood on the basis of fig. 5. Let us consider a typical impact parameter ($b \approx 2$ fm for Ca + Ca and $b \approx 2.7$ fm for Nb + Nb to have the same scale). The dN/d(cos $\theta$) for this impact parameter and for counting all the nucleons is displayed in fig. 5a: it exhibits a peak at $\sim 15^\circ$ for Nb + Nb. The peak is narrower in Nb + Nb
than in Ca + Ca because of the larger number of particles. In fig. 5b, the particles of the target rejected by the filter have been discarded. The latter having few fluctuations (they are essentially spectators), that peak broadens because of the large importance of the participants. In addition, the peak shifts to larger $\theta$ (at least in Nb + Nb). This is mainly due to the fact that the participant nucleons are flowing in the average around $\theta \approx 35^\circ$, although their corresponding ellipsoid is rather spherical. When the neutrons are removed (fig. 5c), the fluctuations are larger because less particles are contributing to the sphericity tensor.

They are however not sufficient to wash out the peak in Nb + Nb because, once more, of the large number of particles contributing to the flow and because the peak stands at large angles. Due to the contribution of various impact parameters, these features are smoothened in a given multiplicity bin. Nevertheless, the results of fig. 2 can be understood as a competition between the real flow and the fluctuations arising from the contributing number of particles.

We have also to mention that the first version of our code (which assumes long-lived deltas) [17] gives a somewhat different result for Nb + Nb at 2.7 fm.
Fig. 4. Projection on a plane perpendicular to the beam axis of the extremity of a unit vector aligned with the large axis of the sphericity ellipsoid [eq. (1)], as calculated in our INC model, including all the nucleons. The beam axis points at the center of the crosses. The impact parameter lies along the horizontal axis and towards the right part of the figures.

(using all the nucleons): the maximum of $dN/d\cos \theta$ arises at $\sim 9^\circ$ instead of the value $\sim 15^\circ$ shown in fig. 5a.

In conclusion, we have shown that the INC model we have used, is capable of producing a sideward collective flow subject to fluctuations which tend to smear it. The interplay between the two features builds up the qualitative properties of the experimental data of ref. [9]. This result is at variance with the one quoted in ref. [9], which uses the Yariv and Fraenkel code. It is very hard to tell where the different result given by the two codes come from. Our suspicion is that our code, which views the two-partner nuclei as collections of nucleons, generates more pressure than the INC model of ref. [10].

Our results are subject to some uncertainty, due to the difficulty of defining, in the frame of our INC model, a filter and a charge multiplicity $M_p$ in a way which would closely follow the experimental ones. Nevertheless, we have checked in many ways (see fig
that the INC dynamics can produce a sideward collective flow for intermediate impact parameters, which is qualitatively in agreement with the global variable analysis. Refined analyses are required to determine whether the INC dynamics is quantitatively satisfactory or whether equation of state effects are required as well.

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References