CENTRAL AND PERIPHERAL COLLISIONS

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ABSTRACT

The experimental as well as theoretical status of the heavy ion physics around one GeV per nucleon incident beam energy is reviewed. Emphasis is put on the possibility of extracting information on dense matter. It is argued that such a possibility calls for a good knowledge of the reaction dynamics.

1 - INTRODUCTION

We want here to discuss some aspects of the heavy ion physics in the GeV range. For convenience, we define the latter as the range of incident beam energy spreading between 250 MeV and 2 GeV per nucleon. At first sight, this part of nuclear physics may appear of limited interest. The success of Glauber theory and other multiple scattering models when applied to proton-nucleus interaction in the same energy range show that, most of the time, nuclei are behaving as a collection of nucleons and their dynamics can be described with a single basic quantity: the nucleon-nucleon scattering amplitude. So the question often raised by some physicists is: if the dynamics is largely dominated by nucleon-nucleon scattering, what is the interest of studying nucleus-nucleus rather than proton-nucleus reactions? The answer was given for the first time in a remarkable paper (1) ten years ago, which already contained many of the concepts extensively used in the study of the relativistic nucleus-nucleus reactions. For the same energy per nucleon, a beam of nuclei carries a much larger flux and deposits more energy, offering the opportunity of creating for a short time a dense and very excited nuclear system. Not always clearly stated, the aim of the experimental effort was to get information about this object. New phases of matter and equation of state of dense matter appeared as kinds of Holy Grail that the experimentalists were trying to quest (the reader may push this comparison further and try to identify King Arthur or Lancelot!). The developments were rather disappointing. It is more and more clear now, that in order to get information about the properties
of dense matter, we need a good picture of the dynamics of the collision. The reason is that the collision process is very explosive and that we measure only the remnants of a (possible) previously formed unusual phase.

In the following, we want to analyze the main developments of the past few years, both experimentally and theoretically, in relation with two important questions: the dynamical evolution of the collision and the properties of dense matter. Of course, we shall be far from being complete because of lack of space and because some points are discussed by other speakers in this meeting. But we shall try to underline the principal aspects of the actual situation.

In section 2, we discuss the participant-spectator picture, which may serve as a useful picture of lowest level of sophistication. Section 3 is a short discussion of the inclusive cross sections for various particle production. Emphasis is put on what they may contain as indications on the formation of a hot dense system built with participant nucleons. In section 4, we discuss some general properties of the equation of state and mention a few attempts to determine the equation of state from existing data. In section 5, we compare two important theoretical models: hydrodynamics and intranuclear cascade (INC) model. We show that they are rather contradictory on the dynamics at the microscopic level and that global analysis of multifragment events is potentially able to discriminate between the two models. In section 6, we say a few words about the physical problems attached to the spectators. Finally, section 7 contains our conclusion.

2 - GENERAL FEATURES

As an introduction, we may look at the characteristic parameters of a nucleus-nucleus system. Some of them are given in fig. 1 as functions of the beam energy per nucleon (in the lab frame). One can see that around 1 GeV per nucleon, the de Broglie wave length is about 0.3 fm, shorter than the range of NN force, ~1 fm (at least for the part of the force which is responsible for large momentum transfer), which in turn is shorter than the internucleon distance (d ~ 1.8 fm). This fact indicates, but only indicates, that the dynamics of the collision is dominated by the classical collision regime: a collision event is a succession of binary on-shell classical (relativistic) nucleon-nucleon (or more generally baryon-baryon) collisions.

What can we expect from such a picture? Due to their high longitudinal
velocity, the nucleons which are sitting in the non-overlapping regions between the projectile and the target will just pass through, keeping their initial velocity. These nucleons are called (projectile or target) spectators. On the other hand, the nucleons in the overlapping region are expected to interact violently with each other, to acquire large momentum transfer and to be scattered over a wide range of angles and momenta. In other words, as outlined by fig. 2, these nucleons come to populate the central region in momentum.

Fig. 1: Value of the de Broglie wave length $\lambda$ and the mean free path $\lambda$ for a nucleon in a projectile with the indicated incident energy. The dotted line represents qualitatively the influence of the Pauli blocking. The quantities $r_d$, $\lambda$, $R$ are the range of the nucleon-nucleon force, the average nucleon-nucleon interdistance and the Pb nuclear radius.

Fig. 2: Schematic representation of the PS model: (a) separation in $r$-space; (c) separation in $p$-space. The expansion and the eventual desintegration of the participant system are sketched in (b).

Do we have evidence of such a picture? The early observation of fragments close to 0° with a velocity very close to the projectile velocity (3) and the copious production of protons and light fragments at any angle in the c.m.
system (for a review, see ref. (4)) support the basic idea of the PS model. But, how many nucleons are assigned to be participant? The simplest idea is the clean-cut geometry assumption. A nucleon of the projectile, which on its longitudinal motion encounters the target, is a participant. The number of participants is then an impact-parameter dependent quantity. Therefore, only the average number of participants is accessible experimentally. According to ref. (4), the average number of participant protons \( \overline{Z_p} \) from the projectile is equal to the proton number of the projectile \( Z_p \) multiplied by the ratio of the target cross section \( \pi r_0^2 A_T^{2/3} \) to the geometrical cross section

\[
\overline{Z_p} = Z_p \frac{\pi r_0^2 A_T^{2/3}}{r_0^2 (A_T^{1/3} + A_p^{1/3})^2}.
\]  

(2.1)

The yield of participant charge is then the total reaction cross section (which may be taken to the geometrical cross section) multiplied by the average projectile and target charge participant number:

\[
\sigma_{ch}^p = \pi r_0^2 (A_T^{1/3} + A_p^{1/3}) \left( \overline{Z_p} + \overline{Z_T} \right) = \pi r_0^2 (Z_p A_T^{2/3} + Z_T A_p^{2/3}).
\]  

(2.2)

This formula is in remarkable agreement with the measurements of the cumulated charge particle yield at large angles, as shown by fig. 3, for light systems.

Fig. 3: Sum of charges of \( p, d, t \) and \( ^3\text{He} \) emitted at large angles, compared to formula (2.2). Taken from ref. (5).

However, a formula similar to (2.2) can be obtained easily for the charge spectator yield. The predicted \( A \) dependence is also in very good agreement with experimental data, even for large systems, but the fit requires a lower value for \( r_0 \) (= 0.95 fm). Despite of the good predictability of these formulae, they do not constitute a rigorous proof of the clean-cut geometry picture, because they deal with inte-
grated quantities. We shall come back to this point in section 6. One can take, however, the clean-cut picture as a good starting approximation.

We close this introduction to general features with a sketchy model for the evolution of the participant system, the main points of which are accepted by most of the people working in the field. The system suffers a large momentum transfer: it is first compressed and highly excited, then expands and finally reaches a density, called the freeze-out density, at which it supposedly fragments or disassembles, emitting nucleons, light fragments and pions, and occasionally other particles. The details of this evolution are however badly known and are a matter of controversy. However, they are very important, since they directly condition the possibility of getting information about the properties of the compressed excited matter. In the next section, we review the main features of the light particle emission cross sections in relation with the properties of nuclear matter in the compressed stage, the so-called fireball, and with the dynamics of the collisions itself.

3 - LIGHT-PARTICLE EMISSION

3.1. Protons

We first study the proton spectra. In fig. 4, we present the 90° c.m. spectrum of the protons emitted in three different symmetric systems. The spectra are very similar to each other, which implies that the important quantity is the beam energy per particle and not the total energy. They are essentially exponential, though significant departures exist at low energy. One can observe a copious production beyond the kinematical limit (∼180 MeV in this case). Even if a (generous) Fermi motion is introduced, the single NN collision cannot reasonably generate the observed high energy tail (dash-and-dot curve in fig. 4). This situation suggests that the high energy protons are generated by multiple collisions. In an extreme picture, numerous repeated collisions lead to the idea of thermal equilibrium. This seems to be in agreement with
the exponential law \( e^{-E/E_0} \) which fits the high energy tail. The extracted parameter \( E_0 \) can tentatively be interpreted as the temperature of the system, and is in fact consistent with what would be expected from the available energy on the basis of an equilibrium situation. However, the situation is not so simple, as emphasized by Nagamiya. The 90° c.m. cross section has been parametrized in the following way

\[
E \frac{d^3\sigma}{dp^3} = C(E) A^a(E),
\]

in order to study the dependence of the mass number of the (here symmetric) system. The value of the exponent \( a \) is given in fig. 5. In the pure thermal equilibrium hypothesis, the parameter \( E_0 \) should be the same and the cross sections are proportional to the total cross section, which gives a factor \( A^{2/3} \) and to the average number of participants, which gives an additional factor \( A \). Totally, a \( A^{5/3} \) dependence is expected, which in fig. 5 is given by the dotted line.

![Graph showing A-dependence of the exponent α]

**Fig. 5**: A-dependence of the exponent \( α \) (see text and fig. 4). Taken from ref. (5).

### 3.2. Pions

In fig. 6, the negative pion spectra at 90° c.m. in Ne+NaF collisions are displayed for various incident energies. They are rather exponential (\( \sim \exp(-E/E_0) \)). The observed values of \( E_0 \) are plotted in fig. 7 versus the c.m. beam energy along with the values extracted from the proton spectra. Also is shown (in dotted line) the value of the temperature predicted by a pure thermal model with the nucleon translational degrees of freedom \( (T \approx 2/3 E_{\text{cm}}/A) \). The observed value of \( E_0 \) both for protons and pions diverges from this line as energy is increasing, a result of the cooling of the system.
Fig. 6: Negative pion spectra at 90° c.m. in the Ne + NaF system at different beam energies. Taken from ref. (5).

due to pion production itself. Fig. 6 also reveals that the pion parameter $E_0$ is systematically smaller than the proton parameter. This has been explained by Siemens and Rasmussen (6) who proposed the picture of an expanding fireball. To the same internal energy an additional radial expansion energy is to be added. Since the velocity is the same for both kinds of particles, less energy is added to the pions. Alternative explanations involve the delta isotopes as the pion source (7), the phase space of individual collisions (8) or the mean free path of the produced particles (4).

Something unusual is however happening in the pion production process. As fig. 6 shows, pions may be produced at low energy ($E_{lab} = 180$ MeV/A) with a sizeable kinetic energy (up to 250 MeV !). This can only happen in NN collisions if the Fermi momentum amounts to 600 MeV/c. Considerations based on the phase space model show that at least 8 nucleons are required to share their energy in a very correlated way in order to produce such pions. Another embarassing aspect is illustrated by fig. 8, which shows the beam energy dependence of the $\pi^-$ multiplicity in central Ar + KCl collisions. Currently available models overestimate the $\pi^-$ yield, unless unrealistic assumptions
Fig. 8: Negative pion multiplicity for central collisions. Experiment (full dots) is from ref. [9]. Open circles and open triangles show the results of an intranuclear cascade calculation [10] for two different values of the lifetime of the $\Delta$-resonance.

are used (11,12). The figure shows the example of an intranuclear cascade calculation. The discrepancy between experiment and theory is probably interesting. At least, one explanation has been advanced in relation with the equation of state (see section 4). But, in our opinion, the question is more complex. Possibility of many-body effects on the pion source and on the pion propagation should be studied in more detail.

3.3. Composite particles

In general, $d$, $t$, $^3\text{He}$ and $\alpha$ particles are emitted with a substantial yield from the fireball. The experimental yields have been found to follow the so-called coalescence law, which relates the cross section for production of a mass $A$ composite to the proton cross section (5):

$$E_A \frac{d^3\sigma_A}{dp_A^3} = C_A (E_p \frac{d^3\sigma_p}{dp_p^3})^A. \quad (3.2)$$

In this relation, $p_A = A p_p$ and $C_A$ is a quantity which does not depend upon the angle and the energy of the emitted particles. Relation (3.2) is particularly well fulfilled for the deuterons, where it may be written as

$$E_d \frac{d^3\sigma_d}{dp_d^3} = C_d (E_p \frac{d^3\sigma_p}{dp_p^3}) (E_n \frac{d^3\sigma_n}{dp_n^3}), \quad (3.3)$$

since the neutron and proton emission pattern are expected to be very similar. Relation (3.3) strongly suggests that the deuteron formation happens very late during the interaction process close to the freeze-out. If, by chance, at this moment, a neutron and a proton have momenta not very different from each other, they are very likely to form a deuteron. This leads to the concept of the coalescence radius, which is the maximum relative momentum $p_0$ beyond which a
proton-neutron pair cannot fuse. This quantity can be extracted from the experimental value of \( C_d \) and lies around \( p_0 \approx 200 \text{ MeV}/c \), which is roughly equal to what can be considered as the deuteron radius in momentum space.

One has to note that a relation such as (3.3) is also consistent with the mass action law ruling chemical and thermal equilibrium of a mixture of different species reacting with each other. In this case, the quantity \( C_d \) has the appealing property of being related to the volume of the system at the freeze-out. However, this approach is questionable because the proton spectra when studied at all angles, show a definite (and important) departure from thermal equilibrium. The corresponding departure can be traced back in the deuteron emission pattern.

The coalescence idea which assumes that composites appear at the end of the interaction process is not without difficulty either. First, the proper idea of coalescence requires a proximity of the neutron and the proton in phase space and not in momentum space only. This question is studied in refs. (13,14). Furthermore, relation (3.2) holds between observed cross sections, with the observed proton yield. A strict application of the coalescence idea would imply the proton yield prior to coalescence, i.e. the so-called primordial proton yield.

Another intriguing aspect is the following. If the idea of coalescence is right, why do they not exist before the last stage of the interaction process? Perhaps, the answer lies in the Pauli blocking effect. The interaction between two nucleons should be renormalized due to the presence of the surrounding nucleons. The Pauli blocking does not seem however to play an important role in the evolution of the proton distribution if one believes in the intranuclear cascade calculations. But, in the hot phase, two nucleons have most of the time a high relative kinetic energy, of the order of the temperature \( \sim 80-100 \text{ MeV} \). However, for those pairs with a small relative velocity, a small renormalization is important. The deuteron is a loosely bound object and a small renormalization will make it unbound. Therefore, the system is likely to contain correlated pairs, instead of deuterons. When the density decreases, the Pauli blocking is less important and the correlated pairs may form deuterons. This effect is similar to the Mott transition in metals and has been invoked by several people (15,16), in a slightly different context that we shall allude to in the next section.
4 - THE NUCLEAR EQUATION OF STATE

The likelihood of the existence of a hot and dense phase during the nucleus-nucleus collision has for long been regarded as providing an unique opportunity to determine experimentally the equation of state of nuclear matter. In particular, as suggested by fig. 9 (which has been adapted from (17)), the perspective of exploring new phases of nuclear matter, aside the usual liquid and gas phases, has stimulated the activity of the experimentalists.

We shall review very briefly what is known about the equation of state. The predicted boundaries of the pion condensed phase, characterized by a spin-isospin order, has varied wildly over past years. Now, the critical density for cold matter seems to be very high ($\rho_{cr} \approx 6-8 \rho_0$). Quite sophisticated quantum chromodynamics calculations on a lattice (18-20) indicate that the lower full curve of fig. 9 corresponds to a transition from a normal phase of nuclear matter constituted of quarks confined in hadrons to a phase with deconfined quarks and gluons, the quark-gluon plasma. The critical temperature at zero density is around 200 MeV and the critical density at zero temperature is about 10 times normal matter density. Still above this transition, another phase transition could exist, corresponding to a restoration of chiral symmetry, with quarks being massless. The energy density in the quark-gluon plasma seems to be quite large (~ a few GeV fm$^{-3}$). Such a point does not seem to be reachable in the GeV range (21) in contrast with early theoretical estimates (22).

The equation of state in the normal phase with confined hadrons is not very much known either. What is known is the position of the minimum for zero temperature and the value of the second derivative

$$\frac{U(\rho, T=0)}{A} = U_0 + \frac{K}{13} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 . \quad (4.1)$$

The parameters $U_0$ and $\rho_0$ are -16 MeV and 0.17 fm$^{-3}$ respectively and
the compression modulus $K$ is around 200 MeV (23). Nothing very precise is known for $\rho = a$ few times $\rho_0$ and for $T = 0$. Do we have to introduce additional terms in different powers of $\rho$? Is the thermal component similar to the usual Fermi gas component? Do we have a $p = \rho^\gamma$ law for isoentropic transformations, and what is the value of the parameter $\gamma$? (This is an important question for astrophysicists). What is the chemical composition of the matter at large $T$? Are there many $\Delta$-resonances and pions? What is the isospin dependence? Hopefully these questions can be answered by heavy ion experiments in the GeV range.

In order to study a phase, one has first to build it. In keeping with this simple observation, a favourable situation would happen if the system quickly equilibrates and evolves quasi-statically up to the freeze-out point. Paradoxically, a perfect equilibrium is not advantageous. Indeed, in this case, the only signal one could detect is carried by the particles emitted at the freeze-out. By the very definition of a thermal plus chemical equilibrium, the properties of the system at the freeze-out are totally determined by the instantaneous density and temperature and are not influenced by the past history of the system. Therefore, the ideal situation would be an imperfect equilibrium, the system being equilibrated except for one degree of freedom. Such a possibility happens when the thermodynamic variables $(\rho, T)$ are such that the creation of a special kind of particles is just allowed and when these particles have a long mean free path compared to the linear dimension of the system. They can escape without disturbing too much the rest of the system. Based on this principle, the following signals of the formation of a quark-gluon plasma have been proposed: lepton pairs, $\gamma$ rays and possibly strange quarks have probably a sufficiently long mean free path.

A similar possibility may occur for the confined hadron phase. Measurements of the spectrum of $K^+$ mesons in $\text{Ne}+\text{NaF}$ reactions at 2.1 GeV per nucleon (24) reveal an exponential shape (in the c.m.) with a temperature parameter $E_0 \sim 142$ MeV. The ordering

$$E_0(K^+) > E_0(p) > E_0(\pi)$$

(4.2)

between the slope parameters along with the ordering of the (average effective) mean free path
\[ \lambda(K^+) > \lambda(p) > \lambda(\pi) \] (4.3)

is, according to Nagamiya (4), due to successive emissions of the three kinds of particles from the expanding fireball. Kaons would be emitted when the system is hot and compressed, protons would appear at a latest stage when the fireball has already cooled down. The pions are emitted the latest because of the large interaction cross section. This argument is however critizable because (i) pions are interacting strongly with nucleons and would then be "emitted" at the same time; (ii) the kaons produced by the reactions \( \text{NN} \rightarrow \Lambda \pi K \) and \( \text{NN} \rightarrow N \Sigma K \) create pions at the beginning of the process when the nucleons are very energetic, but the reaction \( \pi N \rightarrow \Lambda K \), which happens later when pions are created, could be an important source of kaons, due to a relatively large cross section. The slope parameter of \( K^+ \) spectra is not reproduced by conventional models based on hadron-hadron cross sections. Similarly, the average transverse kinetic energy of the emitted \( \Lambda \)-particle is also underestimated by the theoretical previsions (25). The production mechanism is certainly unusual and perhaps involves quark degrees of freedom. In any case, we need further measurements of strange particle yields.

An interesting attempt to deduce the equation of state from experiment has been done by Stock's group (26). They interpret the discrepancy between the observed \( \pi^- \) yield and the predictions of the intranuclear cascade calculations, see fig. 6, as indicative of a compressional effect. In such calculations (see sect. 5), the interaction energy is essentially contained in the translational nucleon degrees of freedom. If part of the available energy is used to compress the system, because of repulsive forces, the average kinetic energy is diminished. The pion production yield is therefore reduced, as a thermal model calculation would indicate. The discrepancy of fig. 6 can accordingly be translated in compression energy as a function of the density. The final result is the determination of \( U(\rho, T=0)/A \) up to \( \rho \approx 3 \rho_0 \). Surprisingly enough, the corresponding compression modulus is close to the value mentioned above, coming from the analysis of giant resonances in nuclei.

Instead of considering temperature and density, one may try to study the entropy generated during the collision, as first suggested by Siemens and Kapusta (27). The advantageous feature is that the entropy may vary slowly during the expansion stage of the collision (13, 27), in contrast with \( T \)
They derive an expression relating the entropy per nucleon to the deuteron to proton ratio

$$S/A = 3.95 - \ln \frac{N_d}{N_p}.$$  \hspace{1cm} (4.4)

The physical content of this relation is intuitively clear. The entropy varying inversely with the occupation probability in phase space, the higher the entropy is, the lower the occupation in phase space is, the more distant from each other the nucleons are, the less probable the deuteron formation yield is. Application of eq. (4.4) to experimental data reveals an "experimental" entropy larger than what is expected from ordinary equations of state $U = U(\rho, S)$, on the basis of the available energy and the presumed density ($\rho \approx 3-4 \rho_0$) at the beginning of the expansion stage. The discrepancy was tentatively interpreted as due to the opening of new degrees of freedom. However, eq. (4.4) is derived from general arguments with the help of threee assumptions: (i) bulk equilibrium, which amounts to assume that the mean free path is small compared to the linear dimension of the system; (ii) isoentropic expansion; (iii) no dynamical effect in deuteron formation (or in other words, the deuteron is formed once the neutron and the proton have a relative motion similar to the one they have inside an actual deuteron). In ref. (28), Bertsch arrived at the conclusion that assumption (i) is questionable. The large effective mean free path makes the system to have a thick surface, where the deuteron formation is much less probable. This is also the conclusion of ref. (29).

5 - GAS DYNAMICS OR HYDRODYNAMICS?

As already mentioned at the end of the preceding section, the global equilibrium is probably not realized in the participant systems. This is supported by the observation of anisotropy of the proton angular distribution in the c.m. system for symmetrical systems (5) and to a lesser extent by the p-p correlation measurements in the quasi-elastic domain. If the global equilibrium is not reached, the system may perhaps still be characterized by a local equilibrium. Each small part of the system (whose size is to be defined) is thermally equilibrated, and the properties vary smoothly over the different parts. Even more, the system may be composed of subsystems away from thermal equilibrium or of subsystems in equilibrium but without any interaction with each other. In any case, what we need for sure before being able to get information about
the properties of the hot dense matter is a good description of the collision mechanism, which ultimately will relate the relevant observables to the relevant part of the system in equilibrium. Two detailed theoretical models have emerged from the host of the models which have flourished during the past six years or so. One is hydrodynamics and the second is the intranuclear cascade model (INC), which roughly corresponds to the two off-equilibrium situations described above.

The INC model is a simulation of the collisions, which assumes that the process is a succession of binary on-shell classical relativistic hadron-hadron collisions. In this model, nucleons move freely between NN collisions whose final state is decided randomly according to a law taken from experimental cross sections. Observables and other quantities are calculated by ensemble averaging. The whole procedure is usually believed to be a way of solving the Boltzmann equation (see discussion in ref. (29)) and the underlying dynamics is generally denoted as gas dynamics. Roughly speaking, it corresponds to the situation where the range of forces \( r_0 \), the internucleon distance \( d \) and the mean free path \( \lambda \) can be ordered as

\[
r_0 \ll d \ll \lambda. \tag{5.1}
\]

Numerical estimates agree with this ordering, except that \( r_0 \) is smaller, but not much smaller than \( d \).

The hydrodynamical approach is based on the well-known Navier-Stokes equation. The basic dynamical ingredient is, in this case, the equation of state itself, generally under the form \( p = p(\rho, T) \). Generally speaking, this approach is justified when the mean free path is small compared to the dimension of the system. Numerical estimates of the mean free path \( \lambda \) (see fig. 1), especially of the path for stopping the particle (which is larger than \( \lambda \)), makes an hydrodynamical approach unjustified in the GeV range. However, the possibility of a reduction of the mean free path by dense matter effects (proximity of pion condensation, for instance) should not completely be discarded.

The crucial difference between the two approaches lies in the effective mean free path (we say effective, because conditions are considerably changing during the whole process). If the mean free path is short compared to the dimension of the nuclei, collective flows are expected. The experimental situation in this respect is favourable to an hydrodynamic approach (30), although
this point is controversial (31, 32). If, on the other hand, the mean free path is comparable with the dimension of the nuclei, transparency effects are expected. The case of central collisions between equal size nuclei offers a dramatic case in this respect. If $\lambda \approx R$, the preferred direction of emission for particles is the longitudinal direction, whereas if $\lambda \ll R$, the matter is opaque and the particles will try to escape in the perpendicular direction.

Transparency effects of this kind are hardly detectable in inclusive measurements because they will be blurred out by the copious emission of the spectator fragments in the forward direction. Hence, the suggestion for turning to exclusive measurements in order to isolate the interesting events. Exclusive measurements are, however, difficult to analyze because of the abundant information once the fragment multiplicity is large. The global analysis helps in reducing the large number of quantities necessary to characterize completely such an event to a few well chosen parameters, which can be intuitively comprehensible. A commonly used procedure deals with the sphericity tensor

$$Q_{ij} = \sum \gamma(p^{(v)}) p_{i}^{(v)} p_{j}^{(v)},$$

(5.2)

where $p_{i}^{(v)}$ is the $i$th Cartesian component of the $v$th fragment momentum and where $\gamma$ is a suitably chosen scalar weighting factor (see refs. (33,34)). The symmetric tensor $Q_{ij}$ is defined by 6 numbers and can be represented by an ellipsoid. The 6 numbers may be chosen as the three eigenvalues of the tensor and three angles fixing the orientation of the ellipsoid. The overall size of the ellipsoid being given by conservation laws (for instance, if $\gamma = 1$), the following two ratios

$$q_1 = \frac{\lambda_1}{\lambda_3}, \quad q_2 = \frac{\lambda_2}{\lambda_3},$$

(5.3)

with $\lambda_1 \geq \lambda_2 \geq \lambda_3$, are usually adopted. For a rough description, which is generally sufficient, the ellipsoid is characterized by two numbers, namely $q_1$ and the angle $\theta$ between the large axis and the beam direction. A similar characterization of the emission pattern is based upon the thrust variable. The thrust direction $\hat{n}_T$ is the one which maximises the quantity

$$t = \max_{\hat{n}} \sum_{v} \frac{|p^{(v)}|}{\Sigma_{v} |p^{(v)}|}$$

(5.4)
and the thrust itself is the maximum value. For an isotropic emission \( t = 0.5 \) and \( q_1 = 1 \), and for a back to back emission, \( q_1 = \infty \) and \( t = 1 \). For a more detailed discussion, see refs. (33, 34). Figure 10 shows the predicted

Fig. 10: Representation of the thrust and the thrust angle for the Ar+KCl systems. The predictions of the hydrodynamical calculation of ref. (35) are represented by the triangles. The predictions of the INC calculation of ref. (33) are given by the other symbols : \([P+S]\) means that all nucleons are included in the calculation of the thrust, while \([P]\) means that the participants only are retained. The different points correspond to different impact parameters ranging from 0 to \( \sim 7 \) fm by steps of 1 fm, going from left to right.

pattern of an INC calculation in comparison with the hydrodynamical predictions. What is important to notice is the difference for small impact parameters. In the hydrodynamical picture, the preferential emission direction is perpendicular to the beam direction. In the INC calculations, the effective mean free path is large enough to guarantee a non-negligible transparency, which in turn keeps the preferred emission direction close to the beam direction, even if the spectators are disregarded. The preliminary measurements (36,37) reveal a pattern closer to the one predicted by the INC. This is not the single success of the INC, which is presently the best model for reproducing one-particle and two-particle inclusive cross sections. But this is the first direct manifestation of non-negligible mean free path.

Let us rapidly underline two features of the INC model, in relation with fig. 10. (i) A situation with \( \vec{\theta}_T \) different from 0° may correspond to two different physical realities: either the vector \( \hat{n}_T \) points in directions with \( \theta \approx \vec{\theta}_T \) and random \( \varphi \), or the vector \( \hat{n}_T \) points towards the beam direction, with some fluctuations. In the latter case, by looking at \( \theta \) only, one gets a value of \( \vec{\theta}_T \) which is directly related to the fluctuations of the vector \( \hat{n}_T \) rather than to its average direction. Actually, the analysis of INC results discloses a situation of the second kind. Unfortunately, on the expe-
rimental side, the azimuthal angle of the direction \( \hat{n}_T \) is not accessible because the reaction plane cannot be determined. Efforts to construct global variables which are better indicators of the true average direction are in progress (38). (ii) As suggested by fig. 10, the INC model predicts average global variables but also the fluctuations, in contradistinction with ordinary hydrodynamics. This may be important because fluctuations are related to the dynamics of the collision and possibly to the size (in mass number) of the system (33).

If the dynamics of the system is close to the one of the INC model involving off-equilibrium effects, we may address to ourselves the following question: is it possible to determine a part of the participant system which can be considered as in equilibrium? A possible approach is to analyze the INC differential or integrated cross sections in terms of the number of collisions undergone by the nucleons:

\[
\sigma = \sigma_1 + \sigma_2 + \sigma_3 + \ldots
\]

(5.5)

Figure 11 shows the spectra of the nucleons having made 1, 2, 3, ... collisions.

Fig. 11: Invariant cross sections to produce a proton having made collisions as a function of the c.m. perpendicular momentum \( p_\perp \) and the c.m. rapidity \( y \). They are given in units of \( 0.462 \times 10^4 \) mb \( c^3 \text{GeV}^{-2} \). Since they are symmetrical in \( y \), only the positive values of \( y \) are given. To give an idea of the anisotropy, curves of equal energy (dotted lines) have been drawn. The full curves are smooth interpolations of the results of an INC calculation (39).

For \( n = 1 \), the emission pattern is still very dominated by the angular distribution of the NN differential cross section. But as \( n \) increases, the distribution dis-
torts and tends towards an isotropic thermal distribution. For the system under consideration, already \( n \approx 3 \) yields a thermal-like distribution. It is shown in ref. (39) that the large \( n \) component is dominating at large energy around 90° c.m., as expected. The shape of \( \sigma_1 \) (as a function of the detected proton momentum \( p \)) can also explain the departure from Boltzmann law on the low energy side (see fig. 4) (40).

Another discussion based on interacting subsystems ("clusters", in the literature) is also very instructive in the evaluation of the off-equilibrium component (41).

6 - THE SPECTATOR PARTS

The observation of fragments with rapidity of the projectile was made early, with a substantial fraction of the total cross section, which implies that the spectators separate from the rest for a large domain of impact parameters. Two questions arise, which are intimately related. How many nucleons are spectators? What is the cause of this separation? We indicate in section 2 that the separation should roughly follow the clean-cut geometry. This would result from the very forward NN cross section in this energy range. More detailed predictions can be made by multiple collision models. Figure 12 shows the results of an INC calculation, compared with the clean-cut geometrical value. There is some uncertainty in a satisfactory definition of the participant on the spectators, but the calculation definitely gives more spectators at small impact parameter and less spectators for peripheral collisions. According to ref. (33), this results from two facts: (i) the mean free path, which is not small, manages some transparency of the nuclei; (ii) the differential cross section, which is forward peaked, has nevertheless some opening, which allows momentum.

![Figure 12: Number of participant nucleons \( N_p \) as a function of the impact parameter for the Ar+KCl system and for two slightly different definitions of the participants (33).](image-url)
transfer in the perpendicular direction. These considerations also imply that the number of participants, should roughly remain constant when going down in energy. The mean free path increases but at the same time, the elastic nucleon nucleon cross section opens up. Along the same lines, the number of participants should also be sensitively larger than the clean-cut geometry for large systems. There are already some experimental signs in favour of such a situation in the Ne+U system (42).

We shall not go here into the detail of the more interesting problems of the fragmentation of the spectator part. The latter very rarely appears as a single nucleus, or as a collection of nucleons, but rather as a collection of composite particles. This question cannot be investigated with the help of an INC model. Indeed, the spectator part undergoes generally a very small momentum transfer, which is expectedly absorbed coherently. This process cannot be handled by the INC, which is essentially a classical model and which most of the time, neglects binding effects. The systematics of the experimental invariant cross sections, in a very broad range of energy, seem to indicate a distribution consistent with a thermalization with a temperature of \( T \approx 8 \text{ MeV} \) (43), which is tentatively interpreted as the boiling temperature of nuclear matter. Many models which embody more or less this main idea of thermal equilibrium (44-46) can handle such a situation. However, many questions remain which have not been very much investigated, in the past few years and that we just mention here. Why is there a kind of limiting fragmentation? By which mechanism the spectator parts acquire energy without receiving momentum? Is there indications of non-equilibrium fragmentation?

We finally mention the possibility of studying nuclear matter around normal nuclear matter density and at finite temperature. In this region, nuclear matter is expected to behave as a Vanderwaals fluid (47), with a critical point around \( \rho_c \approx \rho_0 \), \( T_c \approx 15-20 \text{ MeV} \), leaving room for a gas-liquid transition. What is left over when going towards finite nuclei is far from being known. Nevertheless, this question is worth being investigated (28).

7 - CONCLUSION

We have reviewed some of the experimental data, which indicate that a dense hot phase is formed during the collision between two heavy ions in the GeV range. We have presented theoretical arguments indicating that the system is not globally equilibrated and that, if a transitory dense phase is formed, it
does not probably involve all the (participant) system. Consequently, a good
knowledge of the collision dynamics is required in order to relate the dense
phase with the observables.

We can try to enumerate (apart from the proposal for detecting signals from
quark-gluon plasma) the points on which the experimental effort should be pla-
ced and Saturne and Diogene could be useful tools in this respect. Extensive
measurements of multifragment events and their analysis in terms of global
variables are certainly needed in order to improve our knowledge of the basic
dynamics. Further experiments, including multiplicity measurements and corre-
lation experiments, on pion and especially kaon production are necessary,
since the spectra we have at our disposal are not satisfactorily analyzed by
current models. Composite fragments should also be studied extensively, in
order to resolve the question of the mechanism of their formation.

On the theoretical side, the following points have according to us, to be
investigated. First, we have to improve our picture of gas dynamics, perhaps
starting from INC calculations (but other approaches can be adopted). Pauli
principle effect should be studied in more detail. We have to look for the
possibility of implementing off-shell effects properly (perhaps this could not
be done in a pure classical scheme). More generally, we have to study how the
interaction effects can be investigated, and where and when they manifest
themselves in the course of the collision process. Also, we have to look whe-
ther unusual pion and kaon sources can explain the corresponding cross sections.

REFERENCES

(1) C.F. Chapline, H.H. Johnson, E. Teller and M.S. Weiss, Phys.Rev. D8,
4302 (1973)
(2) J.D. Bowman, W.J. Swiatecki and C.F. Tsang, Lawrence Berkeley Laboratory
Report LBL-2908 (1973), unpublished
Rev.Lett. 35, 152 (1975)
(4) S. Nagamiya, in "Quark Matter and Heavy Ion Collisions", ed. by M. Jacob,
(5) S. Nagamiya, M.C. Lemaire, E. Moeller, S. Schnetzer, G. Shapiro, H. Stei-
(8) J. Knoll, Phys.Rev. C20, 773 (1979)
(12) H. Stöcker, Lawrence Berkeley Preprint, LBL-12303 (unpublished)
(22) K.K. Gudima, H. Iwe, and V.D. Lonev, J.Phys. C5, 229 (1978)
(28) G. Bertsch, MSU Conference, September 1982
(31) M. Gyulassy, E.A. Remler and K. Frankel, to be published
(32) V.D. Toneev, MSU Conference, September 1982
(36) H.H. Gutbrod, MSU Conference, September 1982
(37) D. Beavis et al., University of Riverside preprint (1982)
(38) P. Danielewicz and M. Gyulassy, LBL-15721 (1983)
(43) D.K. Scott, MSUCP-364 (1981)