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GLOBAL VARIABLES AND THE DYNAMICS OF RELATIVISTIC NUCLEUS–NUCLEUS COLLISIONS

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Abstract: Various global variables providing a simple description of high multiplicity events are reviewed. Many of them are calculated in the framework of an intra-nuclear cascade model, which describes the collision process as a series of binary on-shell relativistic baryon–baryon collisions and which includes inelasticity through the production of Δ-resonances. The calculations are first made for the Ar+KCl system at 0.8 GeV/A, with global variables including either all the nucleons or only the participant nucleons. The shape and the orientation of the ellipsoid of sphericity are particularly investigated. For both cases, on the average, the large axis of the ellipsoid is found to point in the beam direction. This result is discussed in comparison with hydrodynamics predictions and in relation with the mean free path. A kind of small "bounce-off effect" is detected for intermediate impact parameters. The possibility of extracting the value of the impact parameter b from the value of a global variable is shown to depend upon the variation of this variable with b and upon the fluctuation of the global variable for a given impact parameter. A quality factor is defined to quantify this possibility. No current global variable seems to be more appropriate than the number of participant nucleons for the impact parameter selection. The physical origin of the fluctuations inside the intranuclear cascade model is discussed and the possibility of extracting useful information on the dynamics of the system from the fluctuations is pointed out. The energy dependence of our results is discussed. Some results of the calculations at 250 and 400 MeV/A are also presented for the same system Ar+KCl.

1. Introduction

The second generation of high-energy nucleus–nucleus collision experiments is marked by the advent of 4π detectors: plastic ball, streamer chamber, Diogène. This has stimulated the interest of the nuclear physicists for the so-called global variables. The latter have been invented by the particle physicists in order to characterize with one or two numbers a high multiplicity event (whose total description requires the knowledge of the momentum, the mass, the charge, etc. of all the ejectiles) in a way which is intuitively comprehensive. In particular, the global variables are very useful to single out events corresponding to a special dynamics, like the two-jet and three-jet events in e^+–e^- collisions [1]).

In relativistic nucleus–nucleus collisions, the study of the global variables may give answers to several questions. The first one is the long-standing problem of

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discriminating between hydrodynamics and cascade dynamics (often called gas dynamics). A partial answer to this question is provided by the recent works of refs. [2-5]) and the preliminary and fragmentary analyses of the streamer chamber experiments [6]). These results *seem* to favour the cascade dynamics, although further measurements are needed to give a definite and *precise* answer. After all, the physical reality may not be purely of one type, but may incorporate aspects of both.

Another interesting question connected with the analysis of experimental data, is the possible selectivity of the global variable with respect to the impact parameter. The knowledge of the latter is highly desirable, since dense matter effects are believed to occur for small impact parameters.

A more general problem is the analysis of the data event by event. So far, experimentalists have been measuring average (over events) quantities, like the inclusive cross sections. They are now going to face the problem of selecting the useful quantities from the overabundant information they will be able to collect. It is highly desirable to have a preliminary theoretical guide before starting the analysis.

In the present work, we want to study these three questions from the point of view of the intranuclear cascade (INC) model. More precisely, we address to ourselves the following questions:

(i) What are the differences between the INC predictions and the hydrodynamical calculations for the average (over the runs of the INC calculations) global variables? How different are the impact parameter dependences in the two models? Do not the fluctuations inherent to the INC model and *to the physical world*, mask the possible differences?

(ii) Is there a global variable which may serve as a better impact parameter discriminator than the usual fragment multiplicity?

(iii) Is there any physical information contained in the fluctuations of the global variables?

The paper is organized as follows. In sect. 2, we present the most widely used global variables. Sect. 3 is the core of the paper. After a short description of the INC model, we present the results, i.e. the average values of several global variables and their fluctuations for different impact parameters. We study the Ar + KCl system at 0.8 GeV per nucleon. We differentiate between two cases: (a) all the particles are included; (b) only the participants are retained. Case (a) is useful in the comparison with the hydrodynamics, since the latter hardly distinguishes between participants and spectators. Case (b) is of more practical interest since spectators are sometimes hardly detectable. Moreover, it can give more information on the dynamics. In sect. 4, we compare our results with the hydrodynamical approach. We discuss the possibility of deducing an impact parameter from an observed value of a global variable. To quantify this possibility, we define a quality factor. We finally analyze the fluctuations of the global variables and try to show that they carry important and interesting information about the dynamics of the collision.

In sect. 5 we discuss the energy dependence of our results for the same system Ar+KCl. Sect. 6 contains our conclusion.

2. The global variables; definitions

Many different global variables have been proposed [7]). Some are well suited to detect collision events of a particular type. We give here the definition as well as the main properties of the most widely used global variables. The latter may be classified in three main categories:

(i) The ones associated with the *momentum tensor*,

$$Q_{ij} = \sum_{\nu=1}^{N} \gamma(p^{(\nu)}) p_i^{(\nu)} p_j^{(\nu)} . \tag{2.1}$$

In this relation, the sum runs over the ejectiles, and $p_i^{(\nu)}$ is the ith Cartesian coordinate of the momentum $\boldsymbol{p}^{(\nu)}$ of fragment (ν). The quantity $\gamma(p)$ is a scalar weighting factor which may be used for emphasizing some parts of the momentum space. Most of the time, γ is chosen between

$$\gamma(p^{(\nu)}) = 1 , \quad \frac{1}{m_\nu} , \quad \frac{1}{p^{(\nu)}} , \quad \frac{1}{p^{(\nu)2}} , \tag{2.2}$$

where m_ν is the mass of the fragment. The symmetric tensor Q_{ij} is totally defined by six numbers. This already large number and the non-evident physical interpretation of each number is unsatisfactory for our intuition. Interestingly, the tensor Q_{ij} can be represented by an ellipsoid. Hence, the six variables may be split into those associated with the shape of the ellipsoid and those associated with its orientation in the three-dimensional space. This is usually done by referring to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$ of tensor (2.1) and to three angles like the Euler angles defining the eigenvectors $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$. The overall size of the tensor is of little interest, since it is mainly given by conservation laws. Two parameters are left out for the shape. Similarly, out of three angles, only one is retained usually, namely the angle θ_1 between the largest axis and the beam axis (see however our discussion in sect. 3).

The two variables associated with the shape of the ellipsoid may be conveniently taken as the two aspects:

$$q_1 = \frac{\lambda_1}{\lambda_3} , \quad q_2 = \frac{\lambda_2}{\lambda_3} , \quad q_1 \geq q_2 . \tag{2.3}$$

The larger q_1 is, the more elongated the ellipsoid is. Very frequently, one would like to have only one shape parameter. If one is interested in a limited description, one shape parameter and one angle may be sufficient. Preferably, the shape parameter then involves the two aspects. It can alternately be the sphericity

$$S = \frac{3}{2} \frac{\lambda_2 + \lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} , \tag{2.4}$$

or the flatness

$$F = \tfrac{1}{2}\sqrt{3}\,\frac{\lambda_2-\lambda_3}{\lambda_1+\lambda_2+\lambda_3}, \tag{2.5}$$

or the "jettiness"

$$j = \frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2+\lambda_3}, \tag{2.6}$$

or the "prolateness"

$$\phi = \frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}, \tag{2.7}$$

or the eccentricity

$$\varepsilon = \frac{\lambda_s-\frac{1}{2}(\lambda_2+\lambda_r)}{\lambda_1+\lambda_2+\lambda_3}. \tag{2.8}$$

In the last relation, λ_s is the eigenvalue corresponding to the axis of quasi-symmetry. It is λ_1 when $\lambda_1-\lambda_2>\lambda_2-\lambda_3$ (prolate event, $\lambda_r=\lambda_3$) and it is λ_3 when $\lambda_1-\lambda_2<\lambda_2-\lambda_3$ (oblate event, $\lambda_r=\lambda_1$). We show in table 1, where we have used the eigenvalues $\lambda_1', \lambda_2', \lambda_3'$ normalized to $\lambda_1'+\lambda_2'+\lambda_3'=1$, the values of the global variables for the three limiting cases of a spherical (isotropic emission), a pencil-like (two opposite momenta) or a disk-like event (isotropic emission in a plane). This table reveals that the aspect q_1 is well suited for the detection of spherical events. The sphericity, the jettiness and the thrust are suited to isolate pencil-like events and the flatness is suited to disk-like events. The eccentricity ε has the appealing property of having well separated values for the three limits, but shows a discontinuity when passing from prolate to oblate shapes (except if it does through the spherical shape). On the contrary, the "prolateness" is continuous for this transition and is equal to $\frac{1}{2}$ at the transition, no matter what q_1 is.

The shape of an ellipsoid may be represented by a point in a (q_1, q_2) plane as is shown in fig. 1b. A given value of the global variables cited above corresponds to a straight line in this graph. Hence, it shows that any of the variables (2.3)–(2.8) alone yields a limited information which sometimes turns out to be sufficient. An alternative representation is given by a kind of Dalitz triangle, as in fig. 1a.

(ii) The *thrust* (or alike) which uses only some component of the momenta. The thrust is defined as

$$T = \max_{\hat{n}} \frac{\sum_{\nu=1}^{N} |\boldsymbol{p}^{(\nu)}\cdot\hat{\boldsymbol{n}}|}{\sum_{\nu=1}^{N} |\boldsymbol{p}^{(\nu)}|}, \tag{2.9}$$

where $\hat{\boldsymbol{n}}$ is a unit vector. The global variable here includes a scalar quantity, T, and the direction $\hat{\boldsymbol{n}}_T$ which maximizes expression (2.9), i.e. 3 quantities (instead

TABLE 1

Values of the global variables for limiting events

Global variable	Sphere	Pencil	Disk
$q_1 = \frac{\lambda_1}{\lambda_3}$	1	∞	∞
$q_2 = \frac{\lambda_2}{\lambda_3}$	1	undefined	∞
$S = \frac{3}{2}(\lambda'_2 + \lambda'_3)$	1	0	$\frac{3}{4}$
$F = \frac{1}{2}\sqrt{3}(\lambda'_2 - \lambda'_3)$	0	0	$\frac{\sqrt{3}}{4}$
$j = \lambda'_1 - \lambda'_2$	0	1	0
$\phi = \frac{\lambda_2 - \lambda_3}{\lambda_1 - \lambda_3}$	undefined	0	1
$\varepsilon = \lambda'_s - \frac{1}{2}(\lambda'_2 + \lambda'_r)$	0	1	$-\frac{1}{2}$
T (thrust)	$\frac{1}{2}$	1	$2/\pi$

of the 6 variables we had above). The values of the thrust for the limiting events is also shown in table 1.

(iii) The simplest of the global variables is the *number of ejectiles* (or alike) and is given by one number only. It is useful in the case of the nuclear collisions, to consider also the *number of participants* N_p, which is the number of nucleons whose momentum is significantly changed during the process. The very definition is not obvious and is a matter of convention.

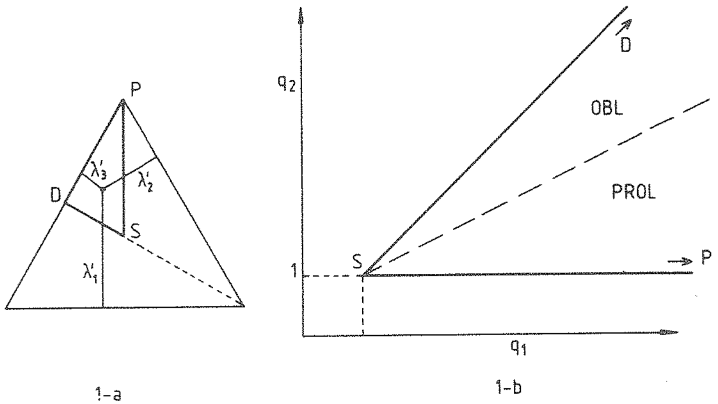

Fig. 1. Two ways of characterizing the shape of an ellipsoid. See text for more detail. The heavy lines delineate the accessible area. P = pencil, S = sphere, D = disk.

We will also consider the longitudinal kinetic energy fraction [4])

$$E_e = \frac{\sum_\nu E^{(\nu)}_{\mathrm{kin}\|}}{\sum_\nu E^{(\nu)}_{\mathrm{kin}}}, \qquad (2.10)$$

where $E^{(\nu)}_{\mathrm{kin}}$ is the kinetic energy of the nucleon (ν) and $E^{(\nu)}_{\mathrm{kin}\|}$ the kinetic energy of its motion parallel to the beam direction. In the non-relativistic approximation we made, we have

$$E_e = \frac{1}{\mathrm{Tr}\,(\underset{\sim}{Q})} \boldsymbol{n}_\| \cdot \underset{\sim}{Q} \cdot \boldsymbol{n}_\| = \lambda'_1(\boldsymbol{e}_1 \cdot \boldsymbol{n}_\|) + \lambda'_2(\boldsymbol{e}_2 \cdot \boldsymbol{n}_\|) + \lambda'_3(\boldsymbol{e}_3 \cdot \boldsymbol{n}_\|),$$

where the $\boldsymbol{e}_i$ ($i = 1, 3$) are the eigenvectors of tensor Q with weight $\gamma(p^{(\nu)}) = 1$, and $\boldsymbol{n}_\|$ is the unit vector along beam axis.

The physical meaning of the global variables is more or less transparent. The sphericity tensor Q gives an idea of the distribution of the particle ($\gamma = 1/p^2$), the momentum ($\gamma = 1/p$) and the energy ($\gamma = 1/2m$) flows. More precisely, for weight $\gamma = 1$, the tensor Q is just N times the covariance matrix of the N nucleons momentum distribution. The mean of this distribution is zero since we are in the center of mass (c.m.) frame. We have a similar result for the weights $1/p$ and $1/p^2$. For instance, Q/N is the covariance matrix of the $\boldsymbol{p}^{(\nu)}/p^{(\nu)}$ vectors for $\gamma = 1/p^2$. In this case the mean value of these vectors, $(1/N)\sum_{(\nu)} \boldsymbol{p}^{(\nu)}/p^{(\nu)}$ is close to zero because target and projectile are the same. The thrust angle gives the direction of the main momentum flow and the thrust itself gives a rough idea of the distribution of the flow around this direction. It is rather obvious that the global variables lose their meaning when they are calculated in a frame which moves quickly with respect to the c.m. frame.

In relativistic nuclear collision physics it is interesting to consider global variables which are invariant under clusterisation. By this one means that such a quantity does not change, when two nuclear fragments of masses A_1 and A_2 (in units of the nucleon mass) and of same velocity are replaced by a particle of mass $(A_1 + A_2)$ with the same velocity. This is particularly useful when considering the formation of clusters. Sometimes, one may consider that the cluster formation results from a coalescence of individual nucleons. If this is true, the predictions of models which do not *explicitly* include the cluster formation, like the INC, will more naturally be compared with experiment through quantities invariant under clusterisation. From the weights given in (2.2), $1/p^{(\nu)}$ and $1/m_\nu$ renders the sphericity tensor invariant under clusterisation. The thrust and the longitudinal kinetic energy fraction are also invariant under clusterisation. However, if the experimental results contain not only the momenta of the compound ejectiles, but also their number of nucleons, it will be possible to use global variables which are *not* invariant under clusterisation. But, in this case, a deuteron for instance will be naturally considered as two separated nucleons of same velocity in order to make comparisons with theoretical models that do not treat the composite formation.

3. Numerical results at 800 MeV/*A*

3.1. THE INC MODEL

In order to calculate the global variables, we have used the version (CR) of the INC model described in refs. [8,9]), where a detailed account can be found. In just a few words, this model describes the collision process by a succession of binary on-shell relativistic baryon–baryon collisions. It gives a good description of the observables in the GeV range [8]). The inelastic processes are introduced through the creation of Δ-resonances which can scatter and disappear through collisions with nucleons. The Δ-resonances are given a lifetime larger than the collision time. This simplification gives rise to a too small low-energy pion yield, but is not important for the properties of the nucleon spectra which are rather insensitive to the lifetime of the Δ's. This also applies to the momentum–energy flow.

We consider here collisions between $A=40$ nuclei at 800 MeV (250 and 400 in sect. 5) incident kinetic energy per nucleon. We refer to this system as the neighbouring Ar+KCl system, since the latter has been extensively studied. The results shown below are based on 1280 collisions spread over 8 values of the impact parameter.

In all the calculations, the values of b are: 0, 1.02, 2.04, 3.06, 4.08, 5.11, 6.13 and 7.15 fm. For each value of b, we have 160 collisions. For each collision, we calculate the global variable and we present the means and rms of those global variables over the 160 collisions. In the whole paper, the "P+S case" means that we used participants and spectators in our calculations, and the "P-case" that we used participants only. The global variables were calculated in the two nuclei center of mass, for nucleons only. We verified at 800 MeV/A that the results for participants (subsect. 3.3) are not significantly modified if the calculation is made in the participant center of mass. We verified also at 800 MeV/A that the addition of pions to participants did not modify significantly the results for tensor-like (weights $\gamma=1$, $1/p$ or $1/p^2$) and thrust global variables.

3.2. GLOBAL VARIABLES FOR ALL THE NUCLEONS

We have first investigated the properties of the global variables calculated with all the nucleons, no matter they have participated or not to the collision. These results, which might be far from what can be measured, are comparable with the predictions of hydrodynamics, which treats all the nucleons on the same footing.

Fig. 2 shows the results of the INC calculations for the two aspects q_1 and q_2 (eq. (2.3)) as functions of the impact parameter b. While on the average the aspect q_1 increases very much with b, the aspect q_2 remains remarkably constant. The ellipsoid associated with the sphericity tensor is close to a sphere at zero impact parameter. It is strongly elongated, but close to axial symmetry (q_2 close to 1), for large impact parameters. This is largely due to the relative weight of the spectators

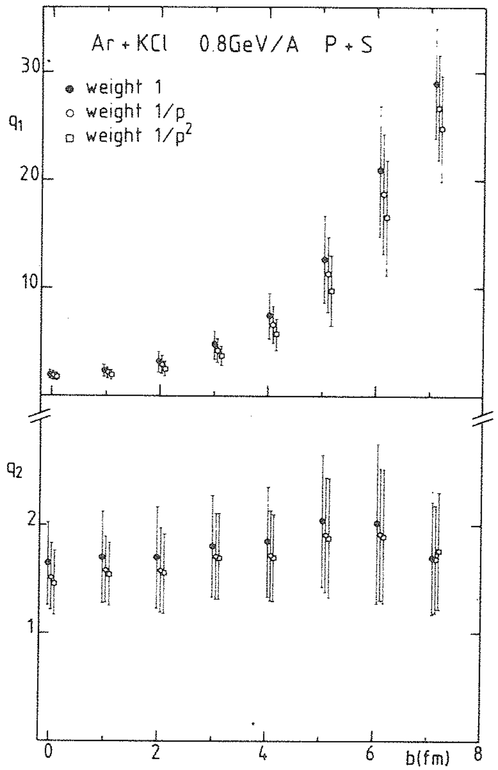

Fig. 2. Ar+KCl at 800 MeV per nucleon. Average values of the two aspects q_1 and q_2 (eq. (2.3)) as a function of the impact parameter for three weights $\gamma = 1, 1/p, 1/p^2$ (eqs. (2.1), (2.2)). The bars indicate two times the calculated standard deviations. The variables q_1 and q_2 are defined for all the nucleons (P+S means participants plus spectators).

(see below). As seen from fig. 2, there is no striking difference between the three weights $\gamma = 1$, $1/p$, $1/p^2$ for the sphericity tensor. This is a general observation that we will not repeat further. It shows that the particle, momentum and energy flow are grossly similar.

We display in fig. 3, the calculated values of the sphericity, one of the global variables involving the three eigenvalues λ_i. It shows more clearly the definite departure from purely spherical events at zero impact parameter.

The main information about the orientation of the ellipsoid is carried by the angle θ_1 between the eigenvector e_1 for the largest eigenvalue λ_1 and the incident direction. The other angles may be taken as the azimuthal angle of e_1 and one of the angles defining e_2. The latter is somewhat irrelevant since the ellipsoid is nearly symmetric. We have checked that the distribution of the angle between the reaction plane and the (e_1, e_2) plane is roughly flat. The average angle $\bar{\theta}_1$ (where the average is taken over the runs) as well as its fluctuations are given in fig. 4. It is maximum

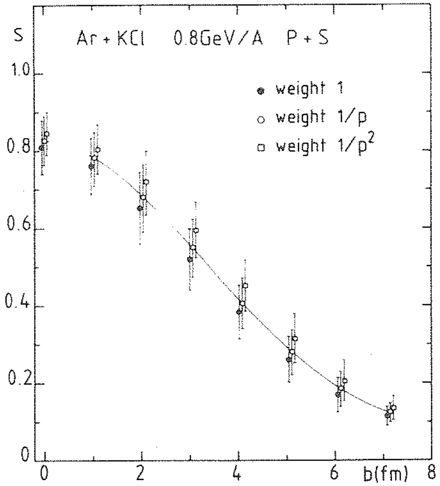

Fig. 3. Ar+KCl, 800 MeV per nucleon. Average value of the sphericity (eq. (2.4)) for all the nucleons. Same conventions as in fig. 2.

for $b=0$, where it amounts to 18° and gradually decreases as b is increasing. We want to stress, however, that this result does not imply a general tilt of the ellipsoid axis $\boldsymbol{e}_1$ away from the beam axis. On the contrary, as fig. 5 illustrates, the direction $\boldsymbol{e}_1$ preferentially points along the beam axis. We discuss later the physical implications of such a pattern. There is however a slight deviation of the most probable direction of $\boldsymbol{e}_1$ from the beam axis. For intermediate values of b (~3–4 fm), the vector $\boldsymbol{e}_1$ lies preferentially *in the reaction plane* on the impact parameter side. This situation is clearly depicted by fig. 6, which shows the projection of $\boldsymbol{e}_1$ on the impact parameter axis (upper part) and on the vector perpendicular to the reaction

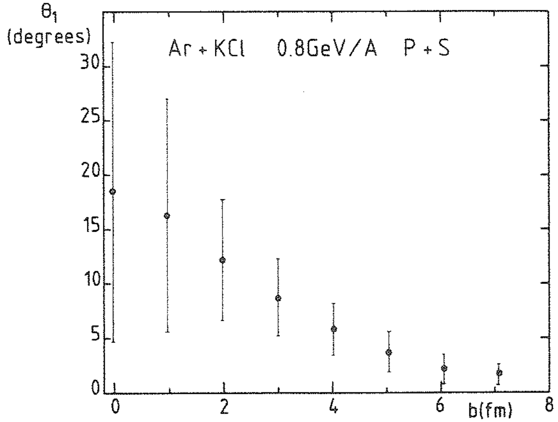

Fig. 4. Average value of the angle θ_1 between the largest axis of the ellipsoid (vector $\boldsymbol{e}_1$) and the beam axis. Same conventions as in fig. 2.

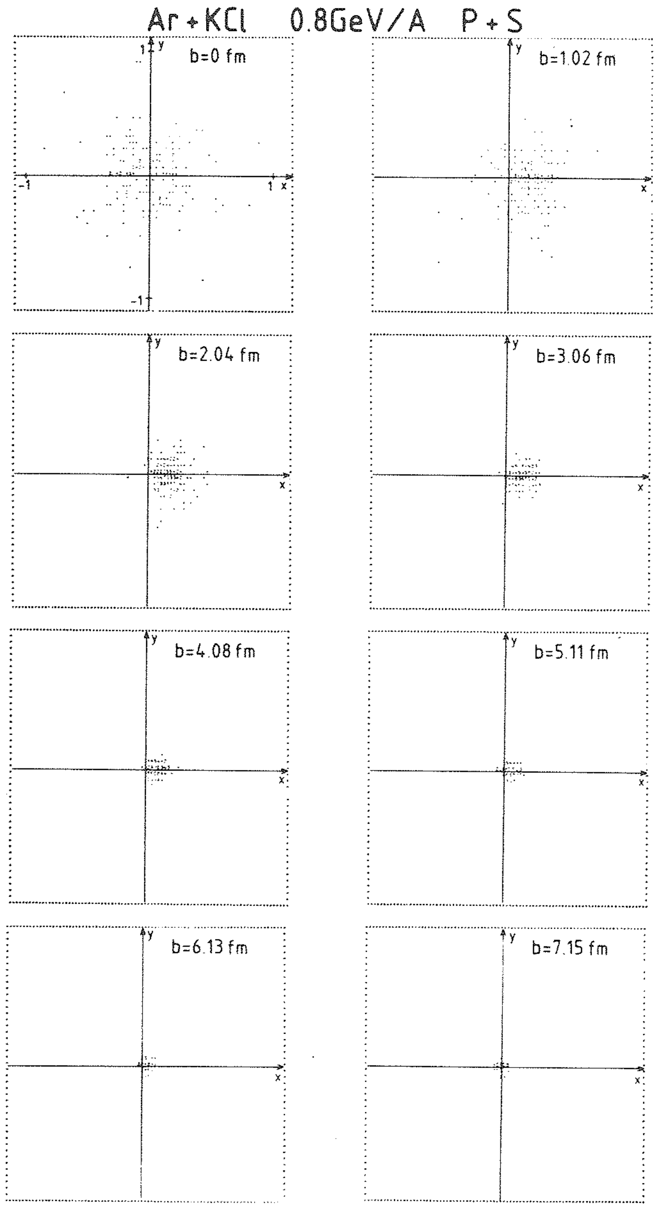

Fig. 5. Ar+KCl, 800 MeV per nucleon. Two-dimensional plot of the projection of the vector e_1, unit vector along the largest axis of the ellipsoid of sphericity (weight 1) on the plane perpendicular to the beam axis. The dots denote the density of the end point of this projection. The center of each square is the projection of the beam axis. The impact parameter lies along the horizontal axis on the right side of the center of the square ($0x$).

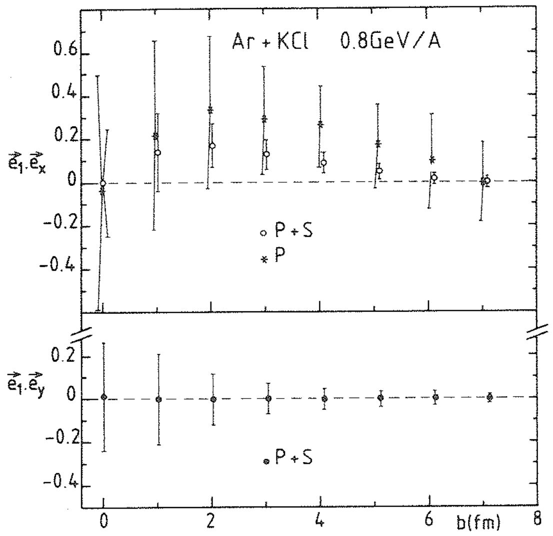

Fig. 6. Average value of the projection of the e_1 axis (see fig. 5 and text) on the two axes e_x (along the impact parameter) and e_y (perpendicular to the impact parameter and the beam axis).

plane (lower part). It is clear from figs. 4 and 6, that for $b \approx 0$, the positive and relatively large value of $\bar{\theta}_1$ is spurious, and just comes from the fluctuating orientation of e_1 around the beam axis, while, for $b \approx 3$–4 fm, the value of $\bar{\theta}_1$ really corresponds to an overall shift of e_1 from the beam axis. Finally, for very large

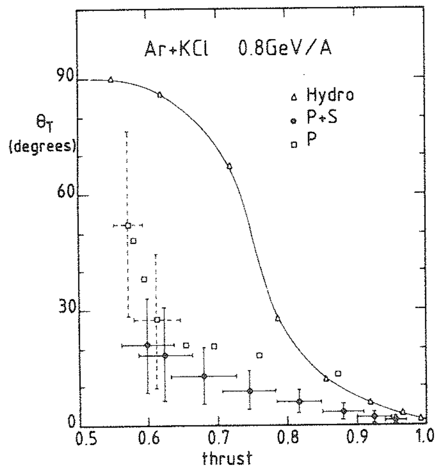

Fig. 7. Value of the thrust T and of the thrust angle θ_T for the Ar + KCl system at 800 MeV per nucleon and for the impact parameters given in the preceding figures. The zero impact parameter points are on the left and the points for the largest one are on the right. The bars indicate two times the standard deviation. P means participants and S spectators. The full line gives the results of ref. [5]). The triangles correspond to the same eight impact parameters than those of our calculations.

impact parameters, $\bar{\theta}_1$ is still spuriously different from zero because of the fluctuations of the direction of e_1.

In fig. 7, we give the values of the thrust T and of the thrust angle θ_T (angle between the direction of the thrust and the beam axis) and make a comparison between our calculation and the predictions of hydrodynamics [5]) for the same system. As already observed in ref. [3]), the thrust T itself is not very different in the two calculations. In contrast, the thrust angle is very different for small ($b \leq 2$ fm) impact parameters. In hydrodynamics, the thrust angle points at 90° because of pressure which forces the nucleons to escape in the direction perpendicular to the beam. As for the angle of e_1, the average thrust angle for zero impact parameter is relatively large, but arises from the large fluctuation of the direction of the thrust around the beam axis, which however is the most probable direction. In fact, the direction of the thrust is always very close to the direction of e_1. We found that the average angle between e_1 and the thrust direction is maximum for central collisions and does not exceed 12°. The same result is valid for the angle between two e_1 vectors corresponding to two different weights γ.

Finally, the number of participants is displayed in fig. 8. We have studied two possibilities for the definition of participants. The simplest is to consider as a participant a nucleon which has collided at least once. But this may be a too strong definition. For instance, if a nucleon of the projectile collides only once and receives a very small momentum transfer, it is counted as a participant with the above definition although, in reality, such a nucleon will travel with a rapidity close to the incident rapidity and will experimentally appear in the so-called fragmentation region. Even more, it is likely that a small momentum transfer can be absorbed coherently by the whole projectile or by a part of it. In this case, the nucleon would

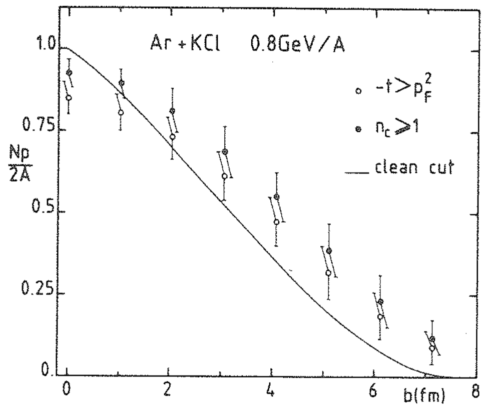

Fig. 8. Average value of the number of participants for the two definitions of the participants (see text: $n_c \geq 1$ means one collision at least; $-t > p_F^2$ means that the invariant momentum transfer squared $-t$ between "before" and "after" the collision is larger than p_F^2). The bars indicate two times the standard deviation. The full curve is the number of participants as given by the clean-cut geometry.

not appear as a free nucleon. Hence, we have considered a second definition of a participant: a nucleon is said to be a participant if its final and initial momenta differ by an amount larger than the Fermi momentum p_F, which is the only parameter characterizing the coherent initial states of the projectile and target nuclei. More precisely, we took: $-t > p_F^2$ where $-t$ is the invariant momentum transfer squared between "before" and "after" the collision. The results are given in fig. 8 compared to the clean-cut geometry prediction. The departure from the latter may be interpreted as follows. Nucleons from the central geometrical region defined by the clean-cut have a non-zero probability of flying undisturbed because of a relatively long mean free path. On the other hand, the finite width of the angular distribution in nucleon–nucleon collision gives rise to a transverse propagation of the momentum which affects nucleons sitting in the "cold" parts of the system as defined by the clean-cut picture. For central collision, this feature is inoperant and the neat result is a higher transparency. At intermediate impact parameters the transverse communication dominates the "longitudinal" transparency.

The number of participants is thus not precisely defined. Let us remind here that other definitions, based on the separation in r-space at the end of the collision [10]), or based on criteria on all the collisions undergone by the nucleon [11]), are even stronger than the second definition adopted above and consequently involve a slightly higher transparency.

Whatever the definition is, it is clear that even for central collisions, a few nucleons are practically not scattered, due to a large effective mean free path. This transparency is responsible for the constant quasi-alignment of the large axis of the sphericity ellipsoid with the incident direction. For intermediate impact parameter, however, this transparency cannot completely prevent the momentum to flow sidewards, despite of the fluctuations. This is very reminiscent of the bounce-off effect [12]). The possible emergence of a collective phenomenon, though of a limited amplitude, out of a cascade calculation was already pointed out in ref. [10]).

3.3. GLOBAL VARIABLES FOR PARTICIPANTS

It is of interest to try to remove the effect of the spectators, which overwhelmingly dominates the global variables at large impact parameter, though we have seen in subsect. 3.2 that looking at the contribution of all the nucleons may reveal some aspects of the dynamics. Here, we present results for the Ar+KCl case at 0.8 GeV/A, using the second definition for the participants. This gives rise to a technical difficulty in the INC calculations. For instance, when two nucleons i and j collide elastically at a small angle, this may be viewed as a small momentum transfer collision if the nucleons i and j emerge at small angles with their initial directions or as a large momentum transfer collision if the labelling of the nucleons in the exit channel is interchanged. As long as the transport of momentum is concerned, the latter situation is irrelevant and we systematically avoided it.

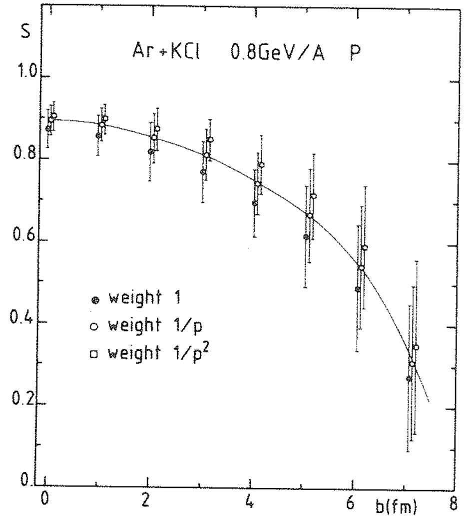

Fig. 9. Sphericity (eq. (2.4)) for the participants only (second definition, see text) as a function of the impact parameter. Same conventions as in fig. 2.

The shape of the sphericity tensor for the participants is described by fig. 9, which displays the average values and the fluctuations of the sphericity. As compared to the participant+spectator (P+S) case, the sphericity is roughly the same at $b=0$, but remains closer to the spherical ($S=1$) limit as the impact parameter increases. This is in keeping with a similar observation made in ref [3]) on the eccentricity. Finally, the sphericity drops off quite abruptly at large impact parameters. For central collisions, the ellipsoid is close to a sphere. The departure of S from unity mainly comes from the fluctuations. This is demonstrated by table 2, which gives the percentage of oblate events (weight $\gamma=1$) for different impact parameters.

TABLE 2

Percentage of oblate events for different impact parameters

b (fm)	0	1.02	2.04	3.06	4.08	5.10	6.12	7.15
percentage	41	42	27	23	5	14	12	5

There is however, even for very central collisions, an overall tendency for having prolate events. The average values and fluctuations of q_2 are the same than in the P+S case for $b<2$–3 fm. Beyond 3 fm, they increase slowly: for $b=5$ fm, $\bar{q}_2$ is 1.8. For every b, the distribution of the angle between the reaction plane and the (e_1, e_2) plane is roughly flat.

The orientation of the ellipsoid is best described by the projection of the vector e_1 on a plane perpendicular to the beam axis. The two-dimensional plot is given

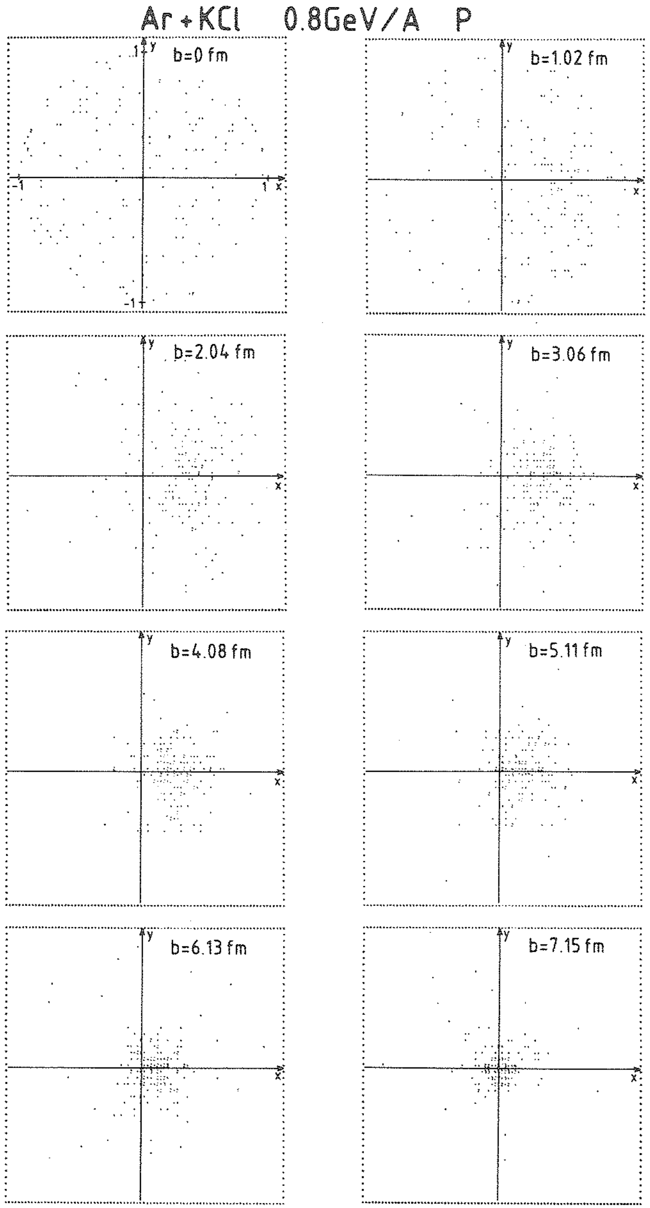

Fig. 10. Ar+KCl, 800 MeV per nucleon. Two-dimensional plot of the projection of vector e_1 (weight $\gamma = 1$) for the participants only. Same conventions as in fig. 5.

by fig. 10. The removal of the spectators has practically left the vector to point freely in any direction for central ($b = 0$) collisions. We have to be careful however when interpreting the plots of fig. 10 because the projection of a unit vector on a plane makes the density of the events appear thicker at the boundary of the accessible circle than around the centre. Actually, the average angle $\bar{\theta}_1$ is 50° (for weight $\gamma = 1$). This angle would be 57° for an isotropic distribution.

As the impact parameter increases, the vector e_1 tends to point at some angle in the reaction plane and *on the impact parameter* side. Because of large fluctuations, the pattern is not as clear as it is for the P+S case. On the other hand, the amplitude of this effect is larger than in the P+S case. While the maximum value of the projection of the unit vector e_1 on the impact parameter axis is 0.17 for the P+S case, it raises up to 0.32 for the participants alone (see fig. 6). Finally, at large impact parameter, the ellipsoid points towards the beam axis with a marked preference.

The results for the thrust T and for the thrust angle θ_T are of similar appearance. For $b = 0$, the thrust is closer to 0.5 than it is for the P+S case and the fluctuations are larger, indicating a tendency to isotropy with a small reminiscence of the initial beam direction. For intermediate impact parameters the memory of the beam direction is more evident and for large impact parameters, the thrust reaches 0.9. We stress once more that the average value of the thrust angle is directly related to its fluctuation and that the full description of the thrust direction reveals that the latter preferentially points in the beam direction for all impact parameters, except for $b = 0$, where it is close to isotropy.

3.4. THE PHYSICAL INTERPRETATION

We have already pointed out the relatively long mean free path prevailing in INC calculations. It forbids total randomisation and allows the system to keep a memory of the incident beam direction: the global variables for P+S never reach the sphere limit for central collisions and the axis of the thrust and of the sphericity tensor points to the beam direction with a relatively small fluctuation.

When the influence of the spectators is removed, the global variables are close to the spherical limit for a wide range of impact parameter b and only for large b do they come back to the pencil limit. The latter behaviour is easy to understand. Just a few nucleons are colliding, once or twice at the most. And as the differential cross section is strongly forward peaked, the global variables are rather close to the pencil limit, the value it would take for a single nucleon–nucleon collision. As the impact parameter decreases, the number of collisions increases and the global variables get away from the pencil-like limit. For $b \lesssim 4$ fm, the values of the global variables indicate that the system is pretty randomized, with always however a reminiscence of the incident direction. For $b \approx 4$ fm, the participant nucleons have made ~3–4 collisions. Hence, one is led to consider that a really small amount of collisions are sufficient to randomize the participants. Not completely however, because a non-negligible fraction has made only one collision and bears the memory of the initial state. Let us finally stress that the study of the sphericity tensor alone brings poor information about a possible thermalization. The latter involves a good knowledge of the momentum distribution, while the sphericity tensor deals with the second moments of the distribution only.

4. The fluctuations

4.1. THE COMPARISON BETWEEN INC AND HYDRODYNAMICS

It appears from previous works and from the present study that the shape global variables are not very different in hydrodynamics and in INC calculations for (P+S), with perhaps an exception for the eccentricity. Before proceeding further, we want to emphasize that when comparing with hydrodynamics, we have to consider participants *and spectators*.

The most important difference between *average* quantities as predicted by the INC and as given by hydrodynamics occurs for the *angles*, like the thrust angle (fig. 7) or the projection of e_1 (figs. 5, 10). The question is then to know whether the fluctuations blur out the differences. The error bars of fig. 7, which represent twice the standard deviation are sufficiently small for $b \leq 4$ fm to guarantee a neat difference in the theoretical predictions. In a plot like fig. 10, the difference is also very clear as hydrodynamics predicts, for small b, events concentrated on a corona boundary of the accessible area.

However, the situation may be experimentally less favourable. Without entering into the detail, we can consider that the experimental uncertainties will increase the observed fluctuations on the global quantities. If the latter are twice the fluctuations for the participants case, the discrimination between INC and hydrodynamics will appear hard, but still feasible. The question is then to know whether for events with thrust ~0.6, one is able to locate the thrust angle with an uncertainty of 20° or less. Only a detailed study of the experimental biases can give a valuable answer.

4.2. THE DETERMINATION OF THE IMPACT PARAMETER

The impact parameter selection is usually done with the help of the fragment multiplicity. Here we want to see whether there is a global variable which is more powerful in this respect. Obviously, the discriminative power of a global variable is larger when its variation with b is stronger and when the fluctuations of this variable are smaller. We have tried to quantify these considerations by defining a quality factor f. Let $\bar{V}(b)$ be the average value of a given global variable for a given impact parameter b and σ its rms deviation. The dimensionless quantity

$$f = \frac{R_1 + R_2}{2\sigma} \frac{d\bar{V}}{db}, \tag{4.1}$$

where R_1 and R_2 are the nuclear radii, gives an idea of the uncertainty in the possibility of attaching a value of b to a given value $\bar{V}$ of the global variable. In first order of the variation of $\bar{V}$ versus b,

$$f=\frac{R_1+R_2}{2\tilde{\sigma}}, \tag{4.2}$$

where $\tilde{\sigma}$ is half of the horizontal width (in b) of the strip determined by the curves $\bar{V}(b)+\sigma(b)$ and $\bar{V}(b)-\sigma(b)$. For a gaussian stochastic variable, the $2\tilde{\sigma}$ width gives already 68.3% of the total probability. Hence, the quality factor can be interpreted as the ratio between b_{max} and the uncertainty with which the impact parameter may be identified with a ~70% confidence level.

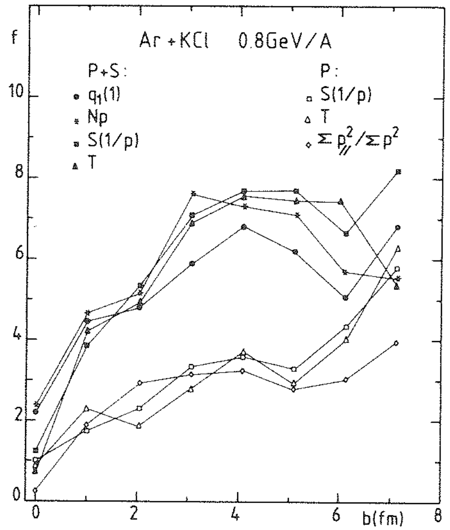

Fig. 11. Quality factor (eq. (4.1)) for several global variables. The system is Ar+KCl at 800 MeV per nucleon.

We show in fig. 11 the quality factor f for various global variables. For the (P+S) case, f is small (2–3) at small impact parameter and increases up to ~6–7 for larger impact parameters. We have calculated f for all the global variables mentioned in sect. 2 (and even for other variables) and for the three weights 1, $1/p$, $1/p^2$. Except for the variables like q_2 or the angles, which have a small b-dependence, all the other (essentially shape) variables have roughly the same quality factor. This may be understood by the following considerations. A shape is essentially given by two variables, for instance q_1 and q_2. If they are uncorrelated, the impact parameter is best determined by q_1, which has the largest quality factor ($f \approx 0$ for q_2). If on the other hand they were correlated, it would be possible to construct another global variable, a function of q_1 and q_2, which would have a smaller dispersion and a better quality factor. We have found that the correlation coefficient between q_1 and q_2,

$$C(q_1, q_2) = \frac{\overline{q_1 q_2} - \bar{q}_1 \times \bar{q}_2}{\sigma(q_1)\sigma(q_2)}, \tag{4.3}$$

is around 0.4 (for a given b). A linear function of q_1 and q_2 can have a quality factor larger than the one of q_1 by a factor $(1-C^2)^{-1/2}$ at the most, which amounts to an increase of 10% only. The quasi absence of correlation and the weak b-dependence of q_2 has an important consequence: all the global variables (2.3)–(2.8) carry essentially the same information as q_1, and it is then not surprising that they all have the same quality factor.

What is more surprising is the fact that they have the same quality factor as the number of participants. We have no simple explanation of this observation.

When participants only are considered, the fluctuations increase as shown from figs. 7, 9, 10. Also, the b-dependence is weaker for a broad range of impact parameter. Therefore, the quality factor substantially worsens and it seems to us that there is very few chance for having a global variable with a better quality factor than the number of participants. None of the ones we have investigated here has this property.

4.3. THE NATURE OF THE FLUCTUATIONS

We have so far referred very often to the fluctuations, without discussing their nature. Technically, in INC calculations, the fluctuations arise because of the random description of the initial state and because of the random scattering of particles. By random scattering, we mean that two baryons are scattered in a way which is not precisely defined by the conditions before the collision. The first source of randomness may be understood as a practical way of describing the initial state of the system, where the nucleons cannot be located precisely because of the quantum fluctuations. But the second source of randomness may at first sight be criticizable once a classical picture has been adopted. Indeed if this picture is pushed forward, there should not be any indetermination on the variables (momentum and position) of the particles once their initial values have been fixed (and provided the hamiltonian is known). This is the point of view of the so-called equation-of-motion approach. The second source of randomness may however be justified for two reasons. First quantum fluctuations bring uncertainty upon the path of a particle. But also even in a completely classical picture, fluctuations arise because of the neglect of N (>2)-body correlations, like in the Boltzmann approach. In the INC calculations one may consider to treat explicitly the effect of the influence of the two-body distribution function (including some of the two-body correlations) on the evolution of the one, two, . . . A-body distribution function. Of course, we do not know whether these sources of fluctuations are quantitatively well treated in the INC model. Nevertheless, the fluctuations are related to the dynamics of the system. We here want to draw the attention on the fact that interesting information about this dynamics *could* be contained in the experimental fluctuations.

Let us make a simple model to illustrate these considerations. We assume, for simplicity, that for a given impact parameter, the participants make a given number of collisions $n(b)$. For large impact parameter, $n=1$ and the distribution of the momentum is directly related to the NN cross section. For the second, third... collisions, we may consider that the nucleons lose energy with a totally randomized system. This model has been proposed by Pirner and Schürmann [14]) and studied extensively in the limit of the heat bath for the randomized system. Hence, we have borrowed results from their study and applied them to the study of longitudinal kinetic energy. We consider $X_\parallel$ which is defined as follows:

$$X_\parallel = \sum_{(\nu)} p_\parallel^{(\nu)2} (=\lambda_1 \boldsymbol{e}_1 \cdot \boldsymbol{n}_\parallel + \lambda_2 \boldsymbol{e}_2 \cdot \boldsymbol{n}_\parallel + \lambda_3 \boldsymbol{e}_3 \cdot \boldsymbol{n}_\parallel) , \tag{4.4}$$

where $p_\parallel^{(\nu)}$ is the momentum component of the νth nucleon along the beam axis. λ_i and $\boldsymbol{e}_i$ $(i=1,3)$ correspond to the sphericity tensor with weight $\gamma=1$, and $\boldsymbol{n}_\parallel$ is a unit vector along the beam direction.

From ref. [15]), we find that the average value after n collisions is

$$\bar{X}_\parallel = \bar{N}_\mathrm{p}\{m\tau[1-\mathrm{e}^{-2\alpha(n-1)}] + \langle p_\parallel^2\rangle_1 \,\mathrm{e}^{-2\alpha(n-1)}\} , \tag{4.5}$$

where τ is the temperature of the heat bath, α is some drift coefficient which is related to the NN cross section and $\langle p_\parallel^2\rangle_1$ is the average value of $p_\parallel^2$ after one collision. The rms deviation is given (for large n) by

$$\sigma_{X_\parallel} = \frac{\sqrt{2}}{\sqrt{\bar{N}_\mathrm{p}}} \bar{X}_\parallel , \tag{4.6}$$

where, of course, $\bar{N}_\mathrm{p}$ is the average participant number. Hence, the ratio $\sigma_{X_\parallel}\sqrt{\bar{N}_\mathrm{p}}/\bar{X}_\parallel$ should be independent of n in the large-n limit. In our case, this ratio has been calculated only in the 250 and 400 MeV/A cases. The results are given in tables 3a for 250 MeV/A and 3b for 400 MeV/A. In each case, $\sigma_{X_\parallel}\sqrt{\bar{N}_\mathrm{p}}/\bar{X}_\parallel$ has been calculated for two definitions of the participants. In the first one ($n_\mathrm{c} \geq 1$), the nucleon has collided at least once. In the second one ($p > p_\mathrm{F}$) the nucleon is out of the two original Fermi spheres.

We do not pretend that the INC dynamics reduces to the model of ref. [15]) for small impact parameters, nor that we have found a model analysis of the INC calculations. Our purpose is simply to stress the *possible importance* of the fluctuations as a source of information on the dynamics. How this can be done is not a simple matter, but is worth being investigated.

Let us finally notice that the fluctuations shown in this calculation are not simply an artifact due to the finite number of runs. We have checked that our statistics

TABLE 3

Value of $\sigma_{X_\parallel}\sqrt{N_p}/\bar{X}_\parallel$ for different impact parameters and for two definitions of the participants

b (fm)		0	1.02	2.04	3.06	4.08	5.10	6.12	7.15
(a) At 250 MeV/A									
$\frac{\sigma_{X_\parallel}\sqrt{N_p}}{\bar{X}_\parallel}$	$n_c \geq 1$	1.38	1.34	1.43	1.43	1.55	1.73	1.71	1.55
	$p > p_F$	1.61	1.59	1.75	1.72	1.74	1.86	1.74	1.29
(b) At 400 MeV/A									
$\frac{\sigma_{X_\parallel}\sqrt{N_p}}{\bar{X}_\parallel}$	$n_c \geq 1$	1.32	1.22	1.33	1.35	1.61	1.67	1.77	1.38
	$p > p_F$	1.49	1.49	1.57	1.47	1.63	1.77	1.79	1.10

guarantees that the rms bars shown throughout this paper are an accurate estimate of the standard deviation of the probability distribution for the global variables under consideration.

5. Energy dependence

We have studied a case in the GeV/A range, because it seems to us that this is the best range for the validity of our INC model. At higher energy, say larger than 2 GeV/A, resonances heavier than the Δ's must play an important role. At low energy, the problem of the Pauli principle and quantum motion in the mean field may become delicate, since it is not reducible to a purely classical picture. The low-energy range is often presented as the range where hydrodynamics could be valid because of smaller transparency. The question is worth being discussed. When going from the energy we have examined (800 MeV/A) to higher energy, the NN elastic scattering is more and more forward peaked. However the inelastic process like Δ-production are becoming more and more important (in the secondary collisions, if not in the first ones). Hence, the transparency is expected to increase, but not dramatically.

When going to lower energy the NN elastic cross section opens significantly, but this is to a large extent compensated by a decrease of the total cross section. At 250 MeV/A, the total NN cross section is half the magnitude it has at energies $\geq$800 MeV/A. Hence, the transparency is not expected to decrease dramatically at 250 MeV/A. Fig. 12 shows the number of participants. Here we have displayed the results for two definitions of the participants: those which make one collision or more ($n_c \geq 1$), and those who have escaped from the original Fermi spheres ($p > p_F$). We see that for the central collisions, the transparency is larger than at 800 MeV/A. This confirms the result obtained in ref. [10]) where another (stronger) definition of the participants is used. At 400 MeV/A, the fraction of nucleons that have made at least one collision is intermediate between the 250 and 800 MeV/A values.

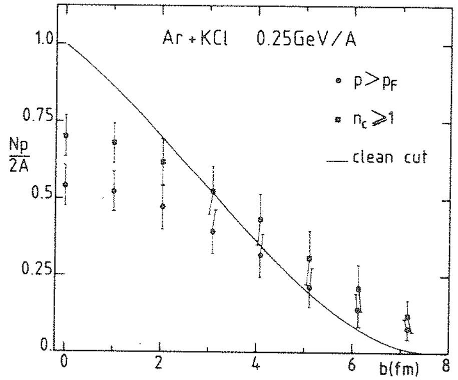

Fig. 12. Ar+KCl, 250 MeV per nucleon. Average number of participants for two different definitions (see text) as a function of impact parameter. The bars indicate two times the standard deviation.

We have calculated the global variables for the 250 and 400 MeV/A cases for two $A=40$ nuclei. The results are *similar* to those prevailing at 800 MeV/A. There is still no striking difference between the three weights $\gamma=1$, $1/p$ and $1/p^2$ for the sphericity tensor. Fig. 13 displays the results at 250 MeV/A for the aspect q_1 and the angle θ_1 between the largest ellipsoid axis and the beam.

In the P+S case, the aspect q_1 and the thrust T have the same mean values and fluctuations at the three energies up to $b=2$–3 fm. For $b>3$ fm, q_1 and T decrease when the incident energy decreases. At $b=6$ fm, $\bar{q}_1$ (weight $\gamma=1$) is respectively

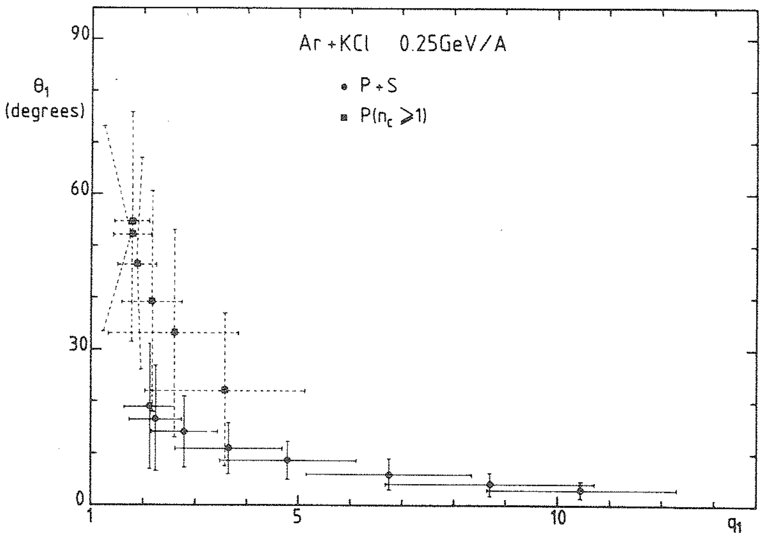

Fig. 13. Ar+KCl, 250 MeV per nucleon. Values of the first aspect q_1 and the angle θ_1 (between the large axis of the ellipsoid of sphericity and the beam axis) for the different impact parameters. The zero impact parameter point is on the left and the point for the largest one is on the right. Same conventions as in fig. 7.

8.7, 12.0, 29.0 for 250, 400 and 800 MeV/A. The fluctuations of q_1 also decrease with decreasing incident energy. For the largest impact parameters, these results are simply related to the ratio between incident momentum (in the c.m. frame) and Fermi momentum. The aspect q_2 has the same mean values and fluctuations than at 800 MeV/A for all impact parameters. The angle θ_1 also has the same mean values (within 2°) and fluctuations. The general orientation of the ellipsoid, as depicted on fig. 5 stays the same at 250 and 400 MeV/A.

In the P-case, the comparison with the 800 MeV/A results is a little biased since the definition of participants we used at 250 and 400 MeV/A is to take those which have made one collision at least (at 800 MeV/A, we used: $-t > p_F^2$, see subsect. 3.2). The shape variables (thrust, sphericity) have the same mean values and fluctuations for $b < 2$–3 fm. For $b > 3$ fm, they stay close to the 800 MeV/A case with a slight tendency to a less peaked shape at 250 MeV/A; at $b = 6$ fm, the average sphericity is respectively 0.65, 0.57 and 0.55 for 250, 400 and 800 MeV/A. The angle θ_1 has roughly the same mean values (within 12°) and fluctuations. Our result displayed on fig. 13 for the P-case at 250 MeV/A is consistent with the one of ref. [2]), if we consider the dispersion and especially, as we have checked, the rather sensitive dependence of the angle θ_1 with the very definition of the participants.

As far as the impact parameter selection is concerned, the general results at 800 MeV/A (see subsect. 4.2) stay valid. The global variables in the P+S case have the same quality factor f as the number of participants. This quality factor increases with b up to ~3 fm and reaches a roughly constant value: 4–5 at 250 MeV/A, 5–6 at 400 MeV/A and 6–7 at 800 MeV/A. f is lower in the P-case than in the P+S case: its values are similar at 400 and 800 MeV/A and a little lower (~2 for $b = 4$ fm) at 250 MeV/A. We also calculated the quality factor for the first eigenvalue λ_1 (weight $\gamma = 1$) at 400 MeV/A. All the previously presented shape variables were normalized; on the contrary, λ_1 is not normalized and for a given shape it is proportional to the number of nucleons. In the both P+S and P-cases, f is not larger for λ_1 than for the other global variables. We did not examine the possibility of using both a shape and an angular global variables for obtaining b. But we determined that for a given impact parameter, the aspect q_1 and the angle θ_1 are poorly correlated: the absolute value of their correlation coefficient (same definition as in subsect. 4.2) at 250 and 400 MeV/A is lower than 0.4 and 0.3 respectively for the P+S and P-cases.

6. Conclusion

We have made a systematic study of the global variables for the relativistic nucleus–nucleus collisions in the frame of an INC model. We have concentrated our attention on a symmetric system in the 0.25–1.00 GeV incident kinetic energy per nucleon. The main results are the following:

(i) When all the nucleons (P+S case) are considered, the ellipsoid of sphericity is close to a sphere for $b = 0$, and gets more and more elongated as b increases. The direction of the large axis is on the average aligned with the incident momentum except for the intermediate impact parameter, where a kind of "bounce-off" effect appears.

(ii) When only participants are considered, the ellipsoid of sphericity remains close to a sphere for a wide range of impact parameters and gets elongated for peripheral collisions only. *On the average*, the orientation of the ellipsoid remains qualitatively the same.

(iii) The difference with hydrodynamics predictions lies essentially in the orientation of the ellipsoid (or of the thrust) for small impact parameter.

(iv) The global variables fluctuate around their average value, more in the participant case than in the (P+S) case. The size of the fluctuations are not expected to mask completely the difference between hydrodynamics and the INC model. The sensitivity of the detectors are called, however, to play an important role in their ability to discriminate between the two models.

(v) The quality factor, i.e. the capacity of a global variable to give a selection on the impact parameter, is the highest for the number of participants and for the shape variables when all nucleons are considered.

(vi) The fluctuations are potentially carrying information about the dynamics of the collisions.

Our study of the global variables is far from being complete. Many points have still to be investigated in detail, before a useful comparison with the present and future experimental data. We especially think to the uncertainty brought by the detectors. Other interesting points are the definition of global variables adapted to asymmetric systems, the search for global variables characterizing the clusterisation of the matter. Finally, further investigation of the physical content of the fluctuations are highly desirable.

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