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EVENT BY EVENT EMISSION-PATTERN ANALYSIS OF THE INTRA-NUCLEAR CASCADE

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An event by event analysis of the emission pattern of participants resulting from an Intra Nuclear Cascade scenario is presented. The procedure, which uses the quadrupole tensor of the momenta of all ejectiles, shows strikingly rigid emission pattern that clearly deviate from the expectations of a collective description like hydrodynamics.

How to quantify macro-dynamical effects, as for instance collective flow effects, in terms of observables is one of the central questions in the field of energetic nuclear reactions. Inclusive measurements are seen to be too insensitive, all the structural differences of quite different models wash out, and even so called "central" spectra [1] show only marginal indications on collective effects [2].

In this note we present some ideas that are capable to quantify the occurrence of collective effects and apply the method to what one might call a genuine non-collective (though cooperative) model: the intra nuclear cascade (INC) [3,4]. The idea is similar to those employed for the jet analysis in particle physics [5]. Clearly, interested in genuine many body effects we have to exploit *many-body* observables. Thus, the technique requires 4π-multiparticle detection. Further, we have to look for bulk properties of the emission pattern, i.e. a few of the lowest moments of the observed momenta. Therefore a kind of quadrupole analysis of *all* emitted momenta seems to be an appropriate tool.

Let us therefore suppose that for each event we know all the momenta P^μ of a certain type of particle (say charged particles). Relative to *their* c.m. frame o can construct the following second rank tensor

$$Q_{ij} = \sum_{\mu=1}^{m} P_i^\mu P_j^\mu \, \gamma(p^\mu) , \quad i,j = \{x, y, z\} , \qquad (1$$

where $\gamma(p)$ is a suitable chosen scalar measure, and m is the multiplicity of the ejectiles in concern. This tensor is precisely of the desired kind (low moments) an it contains some limited information of the emission pattern of that event. Its sensitivity on collective phe mena has been demonstrated by a two thermal source model, the two sources being separated by a collectiv velocity [6].

In fact Q_{ij} represents an ellipsoid and through diagonalisation one can express its content through its three major axis $\{q_1, q_2, q_3\}$ where we use the convention that

$$|q_1 - q_2| \leqslant |q_2 - q_3| \leqslant |q_1 - q_3| . \qquad (2$$

Thus, q_1 and q_2 are the axis with about similar lengt which makes q_3 to the approximate symmetry axis o the ellipsoid.

There are certainly many ways to condense the information of $\{Q_{ij}\}$ down to two variables [5–7]. For this first survey study we chose a representation which shows the amount of deviation from sphericity into the prolate or oblate direction and the spatial direction of the "symmetry axis" relative to the beam.

The quantity

$$\epsilon = [q_3 - \tfrac{1}{2}(q_1 + q_2)]/(q_1 + q_2 + q_3)\ , \qquad (3)$$

serves isotropic events with $\epsilon = 0$, "prolate" shapes [‡1] for positive ϵ with the extreme of pencil like events at $\epsilon = 1$ and "oblate" shapes for $\epsilon < 0$ with flat events ($q_3 = 0$) at $\epsilon = -\frac{1}{2}$. Thus, ϵ plays the role of an eccentricity while its direction gives the direction of the preferred symmetry

$$\boldsymbol{\varepsilon} = |\epsilon|\hat{q}_3\ . \qquad (4)$$

Thus, bringing all events over the eccentricity plane $\{\epsilon_\parallel, \epsilon_\perp\}$ where $\epsilon_\parallel$ denotes the direction of the beam defines a cross section in "eccentricity space"

$$\mathrm{d}^3\sigma/\mathrm{d}^3\epsilon = \mathrm{d}^3\sigma/(\mathrm{d}\rho\epsilon_\perp \mathrm{d}\epsilon_\perp \mathrm{d}\epsilon_\parallel)\ . \qquad (5)$$

Some remarks are in order at this time: Even if a source emits isotropically its particles it does not imply $\boldsymbol{\varepsilon} = 0$. Rather, if only a limited number of particles is emitted, $\boldsymbol{\varepsilon}$ fluctuates in direction (consider the extreme case of two particles only; in their c.m. frame one exactly finds $\epsilon = 1$!) Approximately, the fluctuations due to the finite multiplicity m are of the order

$$\langle(\epsilon - \langle\boldsymbol{\varepsilon}\rangle)^2\rangle = 1/(m-1)\ . \qquad (6)$$

This implies a limited resolution, and one preferentially has to analyze high multiplicity events. This will be seen in the examples shown below. As for the choice of the measure $\gamma(p)$, we studied three cases $(p) = 1$; $1/p$ and $1/p^2$. The last choice is identical to the angular emission analysis suggested by Bertsch and Amsden [7]. The $1/p$-choice has the distinct advantage (as already discussed by Kapusta and Strottman [8] in the context of thrust) to be invariant under coalescence of nucleons into composite particles. Note also that, as far as nucleons are considered, $\gamma = A$, the mass number of the ejectile, also makes eq. (1) invariant under coalescence.

In refs. [7–9] the implications of a hydrodynamic evolution have been studied. However, the above discussed fluctuation has been ignored so far discussing only the ridge lines of the respective observables. The fore it would be desirable to incorporate the finite number effects also into the hydrodynamical scheme, while for our cascade study they are included right away. Besides hydrodynamical flow effects also instabilities toward pion condensation [10] could lead to similar preferred emission patterns for the pions.

We analyze two types of cascade described in refs. [4] and [3], respectively. While the first [4] treats the incident nuclei as a medium, the second [3] starts right away with nucleons in both nuclei, however, froze at positions and momenta relative to the respective nuclear frames. We further make the model-dependent distinction of participants and spectators, and concentrat first on the emission pattern resulting from the participants only, including, however, also the pions produced. The examples selected emphasize the typical features of the cascade scenario.

In contour diagrams over the eccentricity-plane $(\epsilon_\parallel, \epsilon_\perp)$ we show the distribution of events separated into "oblate" and "prolate" shapes in fig. 1 and 2, fo the cascade of ref. [4] at low and high bombarding energies for two different multiplicity cuts and for th cascade of ref. [3] at 800 MeV/nucl. for two differen impact parameter cuts. In fig. 2 (below) also the spectator nucleons are included. The most striking observ tion is that the cascade shows quite rigid emission patterns. Besides the fluctuations discussed above the patterns are almost independent on the selection; be central or non central high or low multiplicities, respe tively. The most frequent shapes are actually pretty close to the shape of the respective inclusive cross section. The emission patterns are fairly isotropic at low bombarding energies (360 MeV/nucl.) and with increasing bombarding energy they show more and mor "collectivity". The latter, however, is sort of trivial in nature; the events recognize the incident beam orie tation due to the forward growing N–N cross section Interesting in particular is the little amount of "oblat shapes. Especially for central collisions of equal mass systems this finding is in strong contradiction with the expectations from a hydrodynamical scenario [2, 9] all the more so as the "symmetry" axis of the "oblate" cascade-events points perpendicular to the bea direction.

[‡1] We use quotation marks since in general all shapes are triaxial and only through (2), (3) we distinguish them to be more on the prolate or on the oblate side.

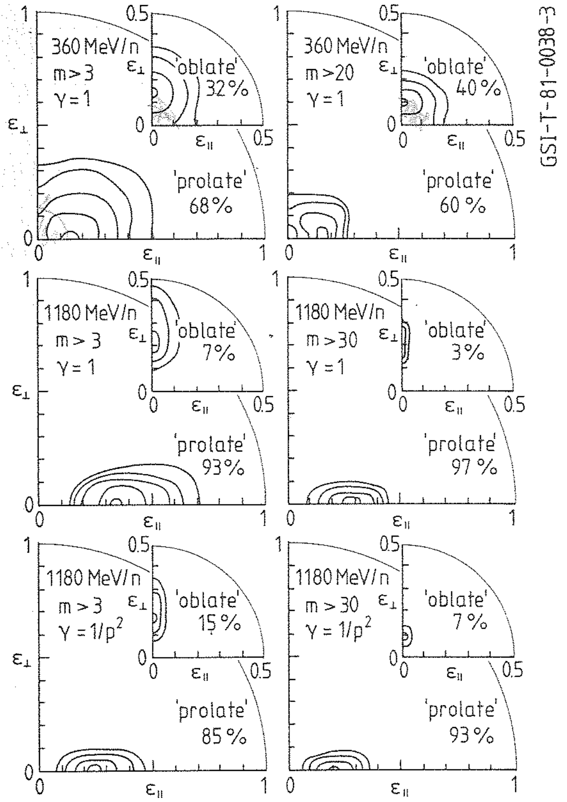

Fig. 1. Distributions of events over the eccentricity plane ($\epsilon_\|$, $\epsilon_\perp$) for prolate and oblate shapes, participants only. A factor of two separates the contour lines, successively. Calculations with cascade [4] for Ar on Ar at 360 and 1180 MeV/nucl. for two multiplicity selections (m) and two choices of the weight γ.

As experimentally one cannot (and may not want to) distinguish participants from spectators we also included the latter in the analysis (fig. 2b) which clearly emphasizes even more the beam direction.

The very same features are reflected in a thrust analysis [8,9]. The thrust t is defined by

$$t = \max_{\hat{n}} \sum_{\mu} |p^{\mu} \cdot \hat{n}| \Big/ \sum_{\mu} |p^{\mu}| \,. \qquad (7)$$

where $\hat{n}$ is an arbitrary unit vector. The thrust angle θ is the angle of the optimal direction $\hat{n}$ and the beam axis. Even in central collisions the thrust angle θ concentrates around 0° with a width at half maximum of about ±10° and no events with $\theta = 90°$, fig. 3.

In summary, we presented some observables that

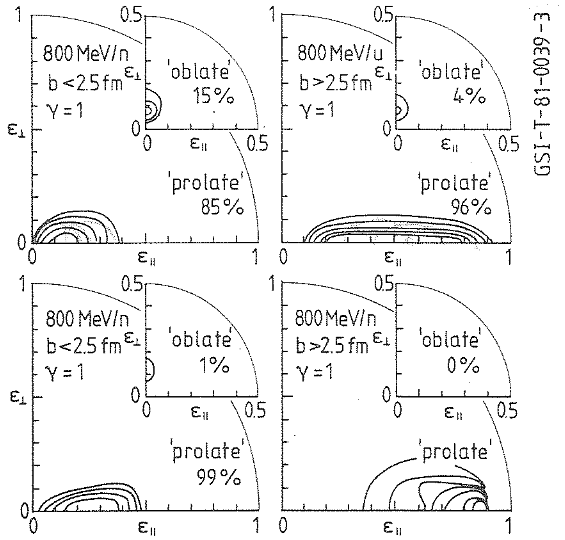

Fig. 2. The same as in fig. 1, however with cascade [3] at 800 MeV/nucl., selected with respect to impact parameter b; above: participants only; below all nucleons.

show striking difference between the expectations fr a cascade and a hydrodynamical scenario. These obse ables be it sphericity, thrust or eccentricity as discus in detail here exploit the eventwise knowledge of the momenta of all particles emitted. With existing (emu sion, streamer chamber) and forthcoming 4π-detecto (plastic ball/wall; Diogen) such an analysis will be feasi and may ultimately isolate collective effects that will

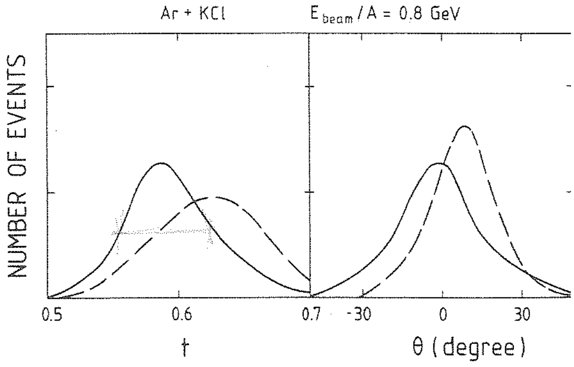

Fig. 3. The events of fig. 2 at b = 0 fm (full lines) and b = 1. fm (dashed lines) subjected to a thrust analysis; left: distribu tion of the thrust t; right: distribution of the thrust angle θ; Note: the thrust distribution is found to lie essentially in th reaction plane, positive angles point towards the impact side

bring us closer to the interesting properties of nuclear matter under extreme conditions.

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