INSTABILITY OF POINT NUCLEI

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We discuss the instability of point nuclei with charge $Z > 68$ due to inverse β-decay and spontaneous π^+ production.

The problem of charged particles in supercritical Coulomb fields has been intensively studied in the last few years [1,2]. In particular, it was recently shown that a charge Z of arbitrarily small radius ("point" nucleus) cannot exceed $Z = 137$ [3]. This result is the consequence of the study of the stability of solutions to the Dirac equation for a charged particle in the external Coulomb field of the nucleus.

In this letter we wish to carry out a corresponding study of Klein–Gordon (KG) particles. We are aware that there is so far no evidence in nature for the existence of a "fundamental" charged scalar boson, which could be identified with a point-like KG particle. We shall therefore in the following identify our KG particles with pions, thus neglecting their composite structure. Our study leads to the following conclusions.

(1) The first source of instability of point nuclei with respect to KG particles (say, $\pi^{\pm}$) of mass greater than the electron mass is the "inverse β-decay" [2] instability arising for $Z > Z_{\mathrm{cr}}^{(1)} = 68$:

$$\mathrm{N} \to (\mathrm{N}, \pi^-) + \mathrm{e}^+ + \nu_\mathrm{e} . \tag{1}$$

In reaction (1), N denotes the supercritical ($Z > 68$) point nucleus, (N, π^-) a π^- bound state in the Coulomb field of the nucleus.

(2) This instability sets in just before Z reaches $Z_{\mathrm{cr}}^{(2)}$, the threshold value for spontaneous π^+ production. In general, for arbitrary $Z > Z_{\mathrm{cr}}^{(2)}$, both instabilities coexist.

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In order to see this, let us start with the radial KG equation for the ground-state ($l = 0$) wavefunction of a particle of mass m and energy E in the Coulomb field of a nucleus with charge Z and radius a:

$$u'' + [(E - V)^2 - m^2]u = 0 , \tag{2}$$

where

$$V = -\alpha Z/r \quad r > a ,$$

$$V = -\alpha Z/a , \quad r < a . \tag{3}$$

We use a system of units where $\hbar = c = 1$; α is the fine structure constant.

The solution to eq. (2) with interaction (3) is standard. Bound states are given by the equation [4]:

$$[\mathrm{d}W_{k,\mu}(\rho)/\mathrm{d}r] / W_{k,\mu}(\rho)|_{r=a} = K \cot Ka , \tag{4}$$

where: $W_{k,\mu}(\rho)$ is the Whittaker function, and

$$k = E\alpha Z/(m^2 - E^2)^{1/2} , \tag{5}$$

$$\mu = (\tfrac{1}{4} - \alpha^2)^{1/2} , \tag{6}$$

$$\rho = 2(m^2 - E^2)^{1/2} r , \tag{7}$$

$$K = [(E + \alpha Z/a)^2 - m^2]^{1/2} . \tag{8}$$

Fig. 1 gives the bound states spectrum coming from solving eq. (4) with decreasing values of the nuclear radius starting at $a = 10^{-3}$ fm down to $a = 10^{-2}$ fm. One can see that one reaches $E = -m$ at $\alpha Z_{\mathrm{cr}}^{(2)} \approx 1$ for extremely small values of ma ($ma \approx 10^{-27}$!).

This result is to be compared with the corresponding result from the Dirac equation, for which one

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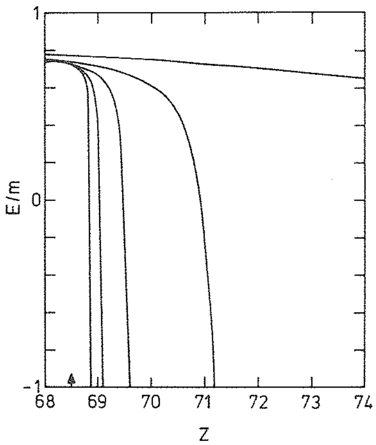

Fig. 1. 1s bound-state energy spectrum for a Klein–Gordon particle with interaction (3) as a function of the external charge Z. The different curves (from upper right to lower left) refer to a radius $a = 10^{-3}$, 10^{-9}, 10^{-15}, 10^{-21} and 10^{-27} fm, respectively, and to m = pion mass. Note that our results hold for any mass, provided that the radius a is rescaled in such a way that ma keeps a constant value.

finds $\alpha Z_{cr} = 1$ [3]. It illustrates how, for given values of m and a the specific form of the KG equation can decrease (by a factor $\leqslant 2$) the critical Z-value for spontaneous pair creation with respect to the critical Z-value for Dirac particles. On the other hand, our curves also illustrate how $Z_{cr}^{(2)}$ increases with am. Note that for $a \lesssim 10^{-25}$ fm, the critical value $Z_{cr}^{(2)}$ is always less than $Z = 69$. This range of values of a can be conveniently taken as defining a point nucleus. It is known [2,4] that for $Z > Z_{cr}^{(2)}$ complex eigenvalues occur. A numerical illustration for a p-wave square-well interaction can be found in ref. [5]. These complex eigenvalues imply that the system (point nucleus) becomes unstable. Specifically, spontaneous emission of positively charged KG particles becomes possible for $Z > Z_{cr}^{(2)}$, as the vacuum of the theory then consists of negatively charged bosons bound to the nucleus. The repulsive force between like bosons ultimately ensures the stability of the system [2]. However, for boson masses $m > m_{electron}$ (a condition so far always satisfied in nature), fig. 1 shows that $E = -m_{electron}$ occurs slightly before $E = -m$, so that (neglecting of course strong interactions) the inverse β-decay process (1) is the first source of instability for point nuclei.

More precisely, the situation is as follows. There exists a critical value of Z ($Z = Z_{cr}^{(1)}$) at which E, the pion energy, becomes negative and equal in magnitude to the electron mass. This value of $Z \equiv Z_{cr}^{(1)}$ is the threshold value for reaction (1), since, for $Z = Z_{cr}^{(1)} + \epsilon$, it does not cost any energy to emit a (e^+, ν_e) pair of total energy $m_e(1 + \epsilon')$, as the corresponding pion energy is then: $E = -m_e(1 + \epsilon')$, ϵ and ϵ' being arbitrarily small quantities. For example, one has, with $a = 10^{-27}$ fm, $E = -0.5$ MeV ($E/m = -0.00358$) for $\alpha Z_{cr}^{(1)} = 0.50244$. For arbitrary values of $Z > Z_{cr}^{(2)} = 68$ and arbitrarily small nuclear radius, both β-decay and π^+ production instabilities will coexist ‡1.

This is illustrated in fig. 1, where we find that for $a = 10^{-15}$ fm $\alpha Z_{cr}^{(2)} = 0.508759$. We have computed for this $Z_{cr}^{(2)}$-value that the 2s-level crosses $E = 0$ for $a = 10^{-27}$ fm so that 0.508759 is also a critical value $\alpha Z_{cr}^{(1)}$ for reaction (1) for the first excited bound state. It is beyond the scope of this letter to discuss the relative importance of these two instabilities. In any case, we can conclude that there is no stable point nucleus with $Z > 68$.

‡1 Strictly speaking, another instability occurs between $Z_{cr}^{(1)}$ and $Z_{cr}^{(2)}$. It simply corresponds to replacing e^+ and ν_e in (1) by μ^+ and ν_μ and need not be discussed further.

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