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LOW ENERGY PIONS AND DENSITY EVOLUTION IN
RELATIVISTIC NUCLEAR COLLISIONS

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The recent discovery of differences in the π⁺ and π⁻ yield at 3°, 11° as well as the observation of different low energy π⁺ and π⁻ spectra in relativistic nucleus-nucleus collisions raise the possibility that such data are sensitive to the matter density distribution during the collision process. There is no way at present to extract such information from proton inclusive cross sections. In a symmetric N = Z system, π⁺/π⁻ differences arise solely from electromagnetic forces and reflect, in principle, the properties of the charge distribution after the pion is created. We report here on some aspects of a classical calculation of these phenomena.

1. Model for the collision process. The nuclear collision is pictured as a succession of relativistic, on-shell, binary, baryon-baryon collisions. The evolution of the system is calculated by means of a Monte-Carlo method, which embodies the following important features: (i) relativistic kinematics, (ii) empirical elementary cross sections, (iii) pionic degrees of freedom are accounted for by allowing Δ-production, (iv) 3's are considered stable against pion emission until the end of the collision process; they may, however, be destroyed in collisions with nucleons. This last point is a reasonable approximation given the present knowledge of the behavior of 3 resonances in nuclear matter [5].

The present model is a very successful parameter-free description of inclusive cross sections (see fig. 1) and two proton correlations at 3MeV/A [4].

2. Matter distribution. During the collision process, our calculation reveals that the matter can be compressed substantially \( n \approx \frac{N}{A} \) and then expands rapidly. However, pions should not be sensitive to these early stages of the process. At the end of the collision process and at later times, the calculated matter (and charge) distribution of a symmetric system can be

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fairly well parametrized as

$$
\rho(r_j, t) = \frac{Q_j}{(r_j \gamma \pi)^{3/2}} \exp \left( -\frac{r_j^2}{\gamma} \right) \cdot \frac{q - Q_j}{2} \cdot \frac{\gamma}{(\sqrt{\pi})^3} \exp \left[ -\frac{(x - x_j)^2 + y^2 + z^2 (z + z_j + vt)^2}{a^2} \right]
$$

Here, $q$ is the total charge of the system, $v$ is the initial velocity in the c.m. frame, and $\gamma$ is the corresponding Lorentz factor. The beam direction is $\hat{x}$ and the impact parameter is along $\hat{x}$. By fitting this form to the Monte Carlo calculations at each time for a given impact parameter, we find that the participant charge, $Q_j$, is almost independent of time, while $r_j$, $x_j$, and $a$ are linear functions of time. These results support qualitatively the participant-spectator model, except that they predict an expansion of the three parts of the system. Typical expansion velocities are $v_{r_j} \approx 0.4$ and $v_a \approx 0.2$. $Q$ is

Fig. 1. Invariant proton cross section for $Ar + Ar$ at 8.5 MeV. The dots are the experimental values and the histograms are the result of the calculation.
close to that predicted by a "clean cut" geometry, except at small impact parameters, where it is somewhat smaller.

3. Electromagnetic effects on the pion spectrum. Let \( S(\vec{r}, t, b, \vec{p}_f) \) be the source function for creating pions of momentum \( \vec{p}_f \) at space-time point \((\vec{r}, t)\) in a collision at impact parameter \(b\). The inclusive cross section will be \([5]\)

\[
E_f \frac{d^3 \sigma}{dp_f} = \sum \int d^3 \vec{r} d\tau \int d^2 b \frac{E_f}{E_i} \frac{d^3 p_1}{d^3 p_f} S(\vec{r}, t, b, \vec{p}_1),
\]

where \(\vec{p}_f\) is the final asymptotic momentum after experiencing the electromagnetic field. We have calculated this integral by a Monte Carlo method, sampling the initial value of \(\vec{p}_1, \vec{z}, t, b\) and solving classical equations of motion in order to find \(\vec{p}_f\). The pion source has been factored into functions depending upon each of the arguments. The spatial and impact parameter dependences of the source are described by gaussians, whose parameters are consistent with those of the participant charge distribution at the instant of \(\Delta\)-decay. The time dependence of the source is taken to be a delta function in time, assuming that the pions are released at the end of the strong interaction process. This is in keeping with the above-mentioned picture of long-lived delta resonances which decay at the end of the decompression stage. The momentum dependence of the pion source is assumed to be thermal (with temperature \(T\)), or direct (i.e., related to the \(N+N\rightarrow \bar{n}+N+N\) data) or a combination of both.

Fig. 2 shows the results for \(Ar + Ca \rightarrow \pi^+ + \pi^\pm + \pi\) at 1.25 GeV. The lines are contours of constant invariant cross sections. The upper left corner shows an undistorted thermal distribution with \(T = 70\) MeV. The distorted cross sections corresponding to different initial spectra as shown in the rest of the figure: Upper Right, the thermal spectrum; Lower Right, Lower Left, two mixtures of thermal and direct spectra, respectively \(6.8\) and \(3.8\) thermal. The electromagnetic forces produce a depression in the low energy region and therefore a maximum at \(90^\circ\) near \(p_\perp \approx m_c\). The position of the peak is sensitive to the initial spectrum. Comparison with experiment \([6]\) would favor a highly thermal initial spectrum, as far as the location of the maximum is concerned. However, the shape of the contour lines near this maximum is not well reproduced. Beyond the region experimentally investigated for this system, we predict a depletion of the pion yield at \(J^0\) and \(18,\bar{\nu}\) at rapidities
Fig. 1. Calculated invariant $\pi^+$ cross sections for Ar + Ca at 1.05 GeV. Normalization is arbitrary. Triangles indicate a maximum and arrows indicate the initial c.m. rapidity.

slightly beyond the incident c.m. rapidity. This is consistent with the measured ratio of $\pi^-$ to $\pi^+$ yields at $\gamma^0$ for $^{23}$NaF at 400 MeV [1], as Fig. 2 strikingly shows. We find, remarkably, that this ratio is fairly independent of the initial pion spectrum. Negative pions are "captured" by the spectator parts of the system travelling with the incident velocity. Thus, it is very likely that $\pi$-mesic atoms are formed in relativistic nucleus-nucleus collisions.

Although there is some uncertainty in the pion source and a classical treatment may be suspect in some regions of phase space, our calculations have revealed that the low energy pion cross sections are sensitive to the evolution of the charge distribution after the strong interaction process. In particular:

(1) the height and the shape of the peak in the $\pi^-/\pi^+$ ratio at $\gamma^0$ are
sensitive to the spatial extent of the spectator part of the system,
(2) the high-momentum $\pi^-/\pi^+$ ratio at $0^\circ$ depends upon the spatial charge
distribution at the end of the strong interaction process,
(3) the $\pi^-/\pi^+$ ratio at small momentum depends mainly upon the charge and
expansion of the participant part.

We have not yet quantified these dependences and they deserve further
attention. The location and the shape of the maximum in the $\pi^+$-spectrum at
$90^\circ$ c.m. also depends upon the properties of the charge distribution, but is
quite sensitive to the source parameters. The fact that we cannot reproduce
the detailed shape of the maximum may indicate some unusual properties of
the pion source.

References
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