THE PARTICLE–VIBRATION COUPLING FORM FACTOR

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In the framework of a simple model, it is shown that the particle–vibration coupling form factor is intermediate between the so-called surface and volume forms. A simple parametrization of our results is also given.

The coupling of a valence (or incoming) nucleon with the vibrations of the core (or of the whole nucleus) has been studied extensively. This coupling plays an important role in many nuclear phenomena, like the contraction of the valence orbits, which has recently received attention [1], and the radiative capture of 10–20 MeV nucleons, which is also of current interest. The theoretical approach to this coupling has nowadays reached a high level of sophistication, where both the valence nucleon motion and the vibration are treated microscopically [2,3]. However, for many purposes, the vibration can be treated macroscopically as a phonon field. This is especially true for the investigation of the radiative capture, and also of the deep inelastic collisions within the model elaborated by Broglia et al. [4]. In this simplified picture of the particle–vibration interaction, one has to define a form factor, which in a sense describes how vibration is excited when the distance between the nucleon and the core is varied. For E1, \( T = 1 \) vibration, this form factor is in general taken as the derivative of the core density. One then speaks of a surface peaked form factor. This choice relies on a simple hydrodynamical or macroscopic model [5,6]. However, it has recently been argued [7,8], in relation with the semi-direct model of the fast neutron capture, that a volume form factor would be preferable. The experimental test of the two forms is obscured by the introduction of an imaginary particle–vibration coupling, whose theoretical foundation is not yet established. It is our purpose to investigate this form factor in the frame of a simple model which had been introduced in ref. [5]. The idea is to write the interaction in a microscopic and in a macroscopic form and to identify the matrix elements of the two forms between the ground state and the giant resonance state, the latter being interpreted as a one-phonon state.

To be more precise, the particle–vibration interaction has the following microscopic form:

\[
H_{\text{int}} = \sum_i v(x, x_i),
\]

where \( x \) represents the coordinates, including spin and isospin, of the valence (or incoming nucleon), and \( x_i \) those of the core nucleon \( i \). For simplicity, we assume a static, zero-range interaction:

\[
v(x, x_i) = \delta(r - r_i)
\]

\[
\times (P_0 + P_5 \sigma \cdot \sigma_i + P_T \tau \cdot \tau_i + P_{\tau 5} \sigma \cdot \sigma_i \tau \cdot \tau_i).
\]
The interaction Hamiltonian $H_{\text{int}}$ is a scalar product of irreducible tensors relative to the valence nucleon, $\mathcal{M}_\mu$, and of irreducible tensors relative to the core, $A_\mu(r)$:

$$H_{\text{int}} = \sum_{\mu=LMTM_TSM_S} P^\mu A_\mu(r) \mathcal{M}_\mu,$$

where

$$\mathcal{M}_\mu = Y_{LM}^\mu(\Omega) \otimes \xi_{SM_S} \otimes \eta_{MT_T},$$

$$\xi_{SM_S} = 1,$$

$$\xi_{SM_S} = \sigma_{MS},$$

$$\eta_{MT_T} = \tau_{MT_T}$$

for $S = 1$, $T = 1$. The quantity $P^\mu$ depends only on the indices $S, M_S, T, M_T$ of the set $\mu$ and $A_\mu(r)$ is given by:

$$A_\mu(r) = \sum_i \frac{\delta(r - r_i)}{r^2} \xi_{SM_S}(i) \eta_{MT_T}(i) Y_{LM}(\Omega_i).$$

We assume that for a given multipole $\mu$, we can relate the giant resonance state $|G_\mu\rangle$ and the ground state by

$$|G_\mu\rangle = Q_\mu |0\rangle / \langle 0|Q_\mu^\dagger Q_\mu |0\rangle^{1/2},$$

where $Q_\mu$ is a one-body operator of multipolarity $\mu$:

$$Q_\mu = \sum_i f(r_i) \xi_{SM_S}(i) \eta_{MT_T}(i) Y_{LM}(\Omega_i).$$

Approximation (5) amounts to assuming that the state $|G_\mu\rangle$ exhausts the non-energy-weighted sum rule for the operator $Q_\mu$.

The macroscopic form of $H_{\text{int}}$ is

$$H'_{\text{int}} = \sum_\mu k_\mu h_\mu(r) \mathcal{M}_\mu \alpha_\mu,$$

where $\alpha_\mu$ is the collective coordinate associated with the vibration of the core, and where $k_\mu$ is a coupling constant. We make the assumption that the zero-phonon state is the ground state and the one-phonon state $|1\mu\rangle$ can be identified with the giant resonance state $|G_\mu\rangle$. Then, we require that

$$\langle G_\mu|H_{\text{int}}|0\rangle = \langle 1\mu|H'_{\text{int}}|0\rangle,$$

or for the multipole $\mu$:

$$P^\mu \langle G_\mu|A_\mu(r)|0\rangle = k_\mu h_\mu(r) \langle 1\mu|\alpha_\mu|0\rangle.$$

The next step is to assume that $\alpha_\mu$ and $Q_\mu$ have the same matrix elements. If we choose $k_\mu$ equal to $P^\mu$,

we then get the following expression for the form factor:

$$h_\mu(r) = \langle G_\mu|A_\mu(r)|0\rangle / \langle G_\mu|Q_\mu|0\rangle,$$

or, using eq. (5):

$$h_\mu(r) = \langle 0|Q_\mu^\dagger A_\mu(r)|0\rangle / \langle 0|Q_\mu^\dagger Q_\mu|0\rangle.$$

Noting that, according to eq. (4)

$$\int r^2 f(r) A_\mu(r) \, dr = Q_\mu,$$

we have the following normalization:

$$\int f(r) h_\mu(r) r^2 \, dr = 1.$$
Fig. 1. Form factor for the $L = 1$, $T = 1$ multipoles. The full curve corresponds to our calculation. The long dashed curve is the volume form factor of ref. [6], while the short dashed curve is the surface form factor.

$$H_L = \sum_{[a]} \sum_{[b]} \delta_{\tau_a \tau_b} \langle (rL)_{ab} \rangle^2 \frac{1}{4\pi} \times (2l_a + 1)(2l_b + 1)(2j_a + 1)(2j_b + 1) \times \left[ \begin{array}{cc} j_a & L \\ 1/2 & j_b \end{array} \right]^2 \left[ \begin{array}{cc} l_a & L \\ 0 & 0 \end{array} \right]^2.$$  

(17)

This formula is valid for $L = 1$ ($T = 1$), $L = 2$ ($T = 0$, 1), $L$ odd $\geq 3$ ($T = 0$, 1) multipoles; for all of these, $s = 0$. It can be easily checked that (14) is normalized as (13). We note that the volume form of ref. [6] is obtained by dropping the terms $F_L$ and $H_L$.

In figs. 1 and 2 we show numerical results for the dipole and quadrupole operators, respectively; in the case of several magic nuclei. The single particle wave functions are calculated in a Woods–Saxon potential well whose parameters are taken from ref. [9]. We have neglected the energy dependence of the well depth for simplicity. We compare our calculated $h(r)$ with the volume form $h_v$ of Longo and Saporetti [7] as well as with a surface form $h_s$ proportional to $r^{L-1}$ $\times$ dp/dr. All the forms are normalized in the same way. One can see that our model yields curves which are in between the volume and the surface form factors. For the lightest nuclei ($^{40}$Ca, $^{48}$Ca, $^{56}$Ni) our result lies closer to the surface form factor. This is even more true for $^{16}$O (not shown). We observed that when harmonic oscillator wave functions are used for nuclei from $^{16}$O to $^{56}$Ni, one obtains $h(r) = h_s(r)$ for E1

Fig. 2. Same as fig. 1 for the $L = 2$ ($T = 0$ or 1) multipole.
tipolarity. This property follows from the fact that in this case \( E_1, f(r) = r \), the sum rule is exhausted by a single oscillator state. In other words, the state \( |G_\mu\rangle \) (eq. (5)) is essentially a 0p state in the collective variable.

For the quadrupole operators, the situation is the same, the quantity \( h(r) \) lying a little bit closer to \( h_s \). This trend is even more pronounced for the \( L = 3 \) multipoles (results not shown).

It is interesting to parametrize our results in a manner simple enough to be introduced in the existing codes for calculating semi-direct capture cross sections. Since \( h(r) \) enters in a matrix element weighted by \( r^2 \), we have sought an overall fit covering all the nuclei, the \( L = 1 \) and \( L = 2 \) operators and the domain where \( r \) is larger than one half of the half-density radius. We found that a good fit is provided by the arithmetic mean

\[
h(r) = 0.5 h_s + 0.5 h_v. \quad (18)
\]

For the \( L = 3 \) multipole, the surface form factor should be weighted by a factor \( 0.5 \).

We have examined the question of knowing whether our results are tied to the zero range forces. In fig. 3, we show the result of a calculation based on a Yukawa-type of interaction. We see that even a small range parameter \( \mu = 2 \text{ fm}^{-1} \) (corresponding to a mass of \( \approx 200 \text{ MeV} \)) does not change the shape of \( h(r) \) drastically (the normalization (13) is not valid for a Yukawa force). The appropriate range parameter is probably much larger. Indeed, for the nucleon–nucleon force, the pions do not contribute to the \( \tau \cdot \tau_l \) on purely scalar parts. For the \( \tau \cdot \tau_l \) part, the light boson that contributes is the \( \rho \) meson with a range parameter of \( \mu = 3.83 \text{ fm}^{-1} \). This range is likely to remain roughly the same for the effective nucleon–nucleon interactions in a nucleus. Fig. 3 shows that for such a value of \( \mu \), the delta interaction gives a sufficiently good approximation of \( h(r) \).

In some cases, especially for \( L = 2 \) transitions, a substantial part of the non-energy-weighted sum rule lies outside the giant resonance state and hence, other states can be excited in the semi-direct capture. To take this into account, we propose to include a coupling of the form (7) for all the states and multiply the coupling constant by a factor \( \sqrt{x} \), where \( x \) is the fraction of non-energy-weighted sum rule contained in the corresponding state. This approximation is reasonable but has a classical nature and some phases are neglected.

References