Astron. Astrophys. 134, 189–192 (1984)

ASTRONOMY AND ASTROPHYSICS

Research Note

A comment on systematic errors in determinations of microturbulent velocities

P. Magain*

Institut d'Astrophysique, Université de Liège, 5, avenue de Cointe, B-4200 Cointe-Ougrée, Belgium

Received October 24, 1983; accepted January 9, 1984

Summary. It is shown that the usual method of microturbulent velocity determination from the "abundance versus equivalent width" plot leads to a systematic overestimate of the microturbulent velocity when the observed equivalent widths are affected by random errors. This overestimate is due to the correlation between errors in observed equivalent widths and in line abundances. This error rises rapidly with decreasing quality of the observational material and may be avoided by using theoretical equivalent widths instead of observed ones as abscissa of the "abundance versus equivalent width" plot.

Key words: microturbulence – solar-type stars – errors

I. Introduction

It has been known for a long time (see, e.g., Cowley, 1970) that systematic errors in equivalent width (EW) measurements may lead to systematic errors in the microturbulent velocities if the latter are determined via a curve-of-growth (or similar) technique. However, it is not generally appreciated that *random* errors in EW's may lead to *systematic* errors in microturbulent velocities. The purpose of this note is to discuss the origin of such an effect, to estimate its magnitude in the case of solar-type stars and to show how to avoid it.

II. Origin of the systematic error

The method of microturbulence determination analysed here is the standard method used in "fine" analysis from the EW's. For the chosen stellar model and a guessed microturbulent velocity, the line logarithmic abundances a_i (for a given ion) are plotted versus the measured EW's w_i and a straight line is fitted by least squares through the points. The microturbulent velocity is varied until the correlation between abundance and EW vanishes, that is until the slope of the line is zero.

Let us assume that we know the true EW's W_i and the exact stellar model. If we choose the right value of the microturbulent velocity, the points in the (W, A) diagram will lie along a perfect horizontal line. Now, if some EW measurement w_i is affected by a small *positive* error δ_i:

$$w_i = W_i + \delta_i \tag{1}$$

* Aspirant du Fonds National Belge de la Recherche Scientifique

the corresponding computed abundance a_i will be overestimated (that is, affected by a small positive error $c_i\delta_i$):

$$a_i = A_i + c_i\delta_i\,. \tag{2}$$

Similarly, any *negative* error on W_i will lead to a *negative* error on A_i. So, any point which moves to the right in the (W, A) plane also moves upwards, and any point which moves to the left also moves downwards. This leads to a systematic trend: the slope of the line fitted by least squares will generally be positive. This will require an increase of microturbulent velocity to obtain a horizontal line. So, the correlation between the errors in W and A implies that random errors in EW measurements lead to a systematic overestimate of the microturbulent velocity.

III. Magnitude of the systematic error

To obtain a simple expression for the magnitude of the overestimate of microturbulent velocity, we make some simplifying assumptions. First, we assume that the errors in EW measurements are random and independent of the size of the EW's. That this is a fair approximation is illustrated by Fig. 4 of Magain (1983). Second, we assume that the factor c_i $(=\partial A_i/\partial \delta_i = \partial A_i/\partial W_i)$ in Eq. (2) is the same for all lines under consideration. This factor c_i is plotted versus W in Fig. 1 for Fe I lines of 2 and 3 eV excitation potential at 5000 Å in the solar Holweger-Müller model (Holweger and Müller, 1974). It is seen that this factor is approximately constant for EW's between 20 and 100 mÅ, which is the range of values generally used in stellar microturbulence determinations. Third, we assume that the errors δ_i in EW's are small compared to the range covered by the EW's of the lines used in the analysis. Mathematically, we require that:

$$\delta_i^2 \ll \sigma_w^2, \tag{3}$$

where σ_w^2 is the variance of the EW measurements:

$$\sigma_w^2 = N^{-1}\sum (w_i - \bar{w})^2\,, \tag{4}$$

where w_i are the individual EW measurements, $\bar{w}$ is the mean EW used in the analysis and the sum is over the number N of lines considered.

Let

$$A_i = f\,W_i + g \tag{5}$$

be the linear relation between the exact EW's and the corresponding computed abundances (if the microturbulent

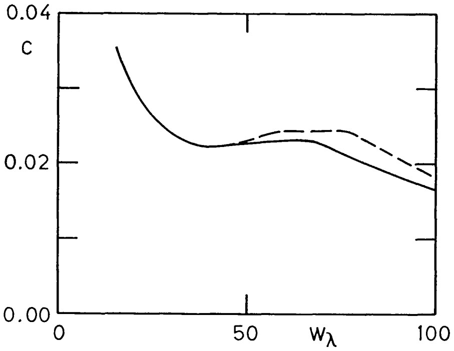

Fig. 1. Plot of $c=\partial A/\partial W$ versus equivalent width for Fe I lines at 5000 Å in the solar Holweger-Müller model. Line excitation potential is 2 eV (dashed line) and 3 eV (full line)

velocity used is the right one, $f=0$). The slope f' of the straight line fitted by least squares through the (w_i, a_i) points is given by:

$$f'=\{\sum w_i a_i - N^{-1}\sum w_i \sum a_i\}/\{\sum w_i^2 - N^{-1}(\sum w_i)^2\}. \quad (6)$$

The denominator of (6) is nothing but N times the variance of the observed EW's. According to the third assumption above, it is approximately the same as N times the variance of the true EW's σ_W^2.

Replacing w_i and a_i by their expressions (1) and (2), with $c_i=c$ according to the second assumption, Eq. (6) thus becomes:

$$f'\simeq\{\sum(W_i+\delta_i)(A_i+c\delta_i)-N^{-1}\sum(W_i+\delta_i)\sum(A_i+c\delta_i)\}/N\sigma_W^2. \quad (7)$$

According to the first and second assumptions above, we have:

$$\langle\sum\delta_i\rangle=0 \quad (8)$$

and

$$\langle\sum W_i\delta_i\rangle=0 \quad (9)$$

(denoting by $\langle X\rangle$ the mean of X).

So, the expectation value of the slope f' is:

$$\langle f'\rangle=f+\langle\delta f\rangle\simeq f+c\sigma_\delta^2/\sigma_W^2, \quad (10)$$

where σ_δ^2 is the variance of the EW errors:

$$\sigma_\delta^2=N^{-1}\langle\sum\delta_i^2-N^{-1}(\sum\delta_i)^2\rangle. \quad (11)$$

So, the expected value of the systematic error on the microturbulent velocity v_t is:

$$\langle\delta v_t\rangle\simeq\frac{\partial v_t}{\partial f}\langle\delta f\rangle\simeq c(\sigma_\delta^2/\sigma_W^2)\frac{\partial v_t}{\partial f}. \quad (12)$$

For a given stellar model and a given set of lines, the quantities c, $\frac{\partial v_t}{\partial f}$ and σ_W can be computed once for all. So, the error on microturbulent velocity goes as the *square* of the dispersion in EW measurements. This implies a rapid rise of the systematic error on microturbulent velocity when the quality of the observational material decreases.

As an illustration of the magnitude of this effect, consider the case of the solar-type subdwarf HD 19445. This star has been analysed by Magain (1983). The microturbulent velocity is determined from a set of 16 Fe I lines with very accurate oscillator strengths from the Oxford group (Blackwell et al., 1982 and references therein). The EW's were measured on the weighted sum of nine photographic spectra, with a reciprocal dispersion of 12 Å mm^{-1}. The stellar model is computed with a version of Gustafsson's programme (Gustafsson et al., 1975). We assume that the right model has the following parameters: $T_{\rm eff}=5750$ K, $\log g=4.0$, $v_t=1.5$ km s^{-1} and that the Fe abundance relative to the sun is [Fe/H] $=-2.35$. With these model parameters, we compute theoretical EW's for the 16 lines in question and compare them to the observed ones. The dispersion between the two sets amounts to 2.4 mÅ, which may be taken as an approximation to σ_δ. The quantities c, $\partial v_t/\partial f$ and σ_W, evaluated from the model and lines discussed above, are: $c\simeq0.022$ (mÅ)$^{-1}$, $\partial v_t/\partial f\simeq180$ mÅ km s^{-1} and $\sigma_W\simeq16.6$ mÅ. So, the mean systematic overestimate of the microturbulent velocity computed from Eq. (12) amounts to $\langle\delta v_t\rangle\simeq0.08$ km s^{-1}, which is a rather small effect for these high quality data. Now, if the star was analysed with one single photographic plate instead of nine, the dispersion in the EW measurements would be roughly $\sqrt{9}=3$ times higher, so $\sigma_\delta\simeq7.2$ mÅ. Then, the expected value of the overestimate of v_t would amount to $\langle\delta v_t\rangle\simeq0.75$ km s^{-1}, which is quite a large effect (50% of the assumed value).

Note the following points.

(1) The systematic error is inversely proportional to the number of spectra used in the analysis (assuming that they are all of similar quality).

(2) Increasing the number of lines used in the determination of v_t does not improve the situation. In fact, $\langle\delta v_t\rangle$ is independent of the number of lines considered, as long as the assumptions above are satisfied.

(3) Due to its origin, this systematic error has the same order of magnitude as the random error on the derived microturbulence. So, the worse procedure in the study of the microturbulence across the HR diagram would be to analyse a large sample of similar stars with low quality material. In such a case, the systematic error would exceed by a large amount the random error on the mean microturbulent velocity for the sample considered.

IV. Unbiased microturbulence determination

As shown above, the systematic error on microturbulent velocity is due to the correlation between the errors in EW and in abundance. The problem comes from the left and right displacement of the points in the (W, A) diagram: the higher points tend to be on the right side and vice-versa. If this horizontal displacement could be avoided, the systematic error would vanish. So, the systematic error can be avoided by choosing as abscissa some quantity related to the observed EW ($W_{\rm obs}$) but free of random errors. We suggest to use the theoretical EW's ($W_{\rm calc}$) computed from the adopted stellar model and some assumed value of the microturbulent velocity.

So, we propose the following scheme for the unbiased determination of the microturbulent velocity:

(1) from the adopted stellar model and observed EW's, compute the line abundances for different values of the microturbulent velocity;

(2) compute the theoretical EW's corresponding to some arbitrary value of v_t;

(3) plot the line abundances (computed from the *observed* EW's) versus the *computed* EW's and choose v_t such that the slope of the line fitted by least squares is zero.

The microturbulent velocity determined in this way is free of the systematic error discussed in this note and should generally be insensitive to the value of v_t adopted in the computation of the theoretical EW's.

This procedure may be illustrated by some numerical examples. In the case of HD 19445 discussed above, the microturbulent velocity determined from the (W_{obs}, A) diagram amounts to 1.55 km s^{-1}, while a value of 1.47 km s^{-1} is deduced from the (W_{calc}, A) diagram. So, the correction amounts to 0.08 km s^{-1}, which is just the mean overestimate computed from the simple model discussed in Sect. III.

A second illustration is given in Fig. 2. Here, 15 fictitious Fe I lines at 5000 Å and of 3 eV excitation potential are used to determine the microturbulent velocity. Their true EW's are given by:

$$W_i = 20 + 5i \quad (i = 0 \text{ to } 14) \tag{13}$$

and the true microturbulent velocity is 1 km s^{-1}. The model used is the solar Holweger-Müller model. To these EW's we add random errors with a rectangular distribution of 6 mÅ standard deviation (typical of many stellar analyses). The (W_{obs}, A) plot is shown in Fig. 2b for a typical case. In this case, we would deduce a microturbulent velocity of 1.41 km s^{-1} for the star, as compared to the exact value of 1 km s^{-1}. In Fig. 2c, the computed abundances are plotted versus the calculated EW's for a microturbulent velocity of 1 km s^{-1}. It is seen that the slope of the line is significantly reduced. This latter diagram would lead to a microturbulent velocity of 1.15 km s^{-1}. So, the correction for the systematic error amounts to 0.26 km s^{-1} in this particular example, in good agreement with the mean value of 0.23 km s^{-1} which is deduced from Eq. (12) for this case. Note that the 0.15 km s^{-1} difference between the exact v_t value and the value deduced from the (W_{calc}, A) diagram is mostly a random error (but see Sect. VI below).

To illustrate the insensitivity of the procedure outlined above to the value of v_t assumed in the computation of the theoretical EW's, we have repeated the analysis with an input value of 1.5 km s^{-1} instead of 1 km s^{-1}. No difference could be detected in the deduced value of the microturbulent velocity.

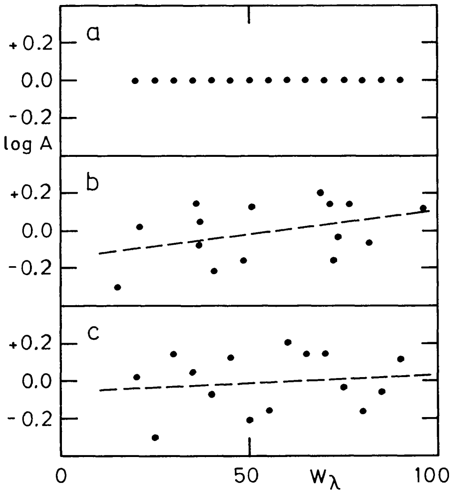

Fig. 2. Plot of abundances versus equivalent widths for a microturbulent velocity of 1 km s^{-1}. Fig. 2a: the true EW's are used to compute the abundances. Figs. 2b and c: random errors are added to the EW's. Fig. 2b: the observed EW's are used as abscissa. Fig. 2c: the systematic error is avoided by using computed EW's as abscissa

V. Some examples

We would like to show how the effect discussed in this note can affect the determination of the microturbulent velocity in some typical examples.

Let us consider first the analysis of Arcturus by Mäckle et al. (1975), as an example of spectroscopic analysis with rather high quality material. To estimate the magnitude of the systematic error on v_t, we select 50 Fe I lines with excitation potentials higher than 3.5 eV and $W < 100$ mÅ. We use the model computed with Gustafsson's programme with the parameters selected by Mäckle et al. (1975). From the (W_{obs}, A) plot, we obtain $v_t \simeq 1.60$ (± 0.14) km s^{-1}, while the (W_{calc}, A) diagram indicates $v_t \simeq 1.43$ (± 0.12) km s^{-1}. So, the systematic error amounts to some 0.17 km s^{-1}. This estimate should not change very much if more lines were used. (Note that part of the difference with the value found by Mäckle et al. may be attributed to the use of a slightly different model atmosphere).

As an example of analysis with much lower quality spectroscopic material, consider the analysis of Groombridge 1830 (=HD 103095) by Peterson (1980). We determine the microturbulent velocity from the Fe I lines with $W < 100$ mÅ and a model with $T_{eff} = 5000$ K, $\log g = 4.5$ and [Fe/H] $= -1$. If we use the Unsöld value of the damping constants γ_6, we obtain $v_t \simeq 2.2$ (± 0.5) km s^{-1} from the (W_{obs}, A) plot and $v_t \simeq 1.3$ (± 0.5) km s^{-1} from the (W_{calc}, A) diagram. Using damping constants γ_6 increased 2 times over the Unsöld formula would lead to $v_t \simeq 1.8$ (± 0.7) km s^{-1} from the (W_{obs}, A) diagram. So, in this case, the systematic error on v_t (~ 0.9 km s^{-1}) is significantly larger than the random error deduced from the scatter of the points in the (W, A) diagram, and it is also larger than the uncertainty coming from the poor knowledge of the damping constants.

Finally, it has been pointed out by the referee, M. Spite, that such an effect could help to explain the discrepancy between the solar microturbulence found from the (W, A) plot ($v_t \simeq 1$ km s^{-1}, Holweger et al., 1978) and the value deduced from a curve-of-growth ($v_t \simeq 0.5$ km s^{-1}, Foy, 1972). According to our best estimate, the systematic error in the determination of Holweger et al. (1978) may amount to some 0.2 km s^{-1}. The remaining discrepancy (~ 0.3 km s^{-1}) may be attributed to systematic errors in the oscillator strengths used by these different authors (see, e.g., Blackwell et al., 1979). We recommend the value of 0.85 km s^{-1} for the microturbulent velocity at the center of the solar disk. This value has been determined by Blackwell et al. (1983) from very high quality data and, so, is essentially free from the systematic error discussed in this note.

VI. Concluding remarks

Some other methods are sometimes used to determine the microturbulent velocity via the EW's. One of these methods consists in determining v_t so as to minimize the dispersion of the computed line abundances (see, e.g., Simmons and Blackwell, 1982). It can be easily seen that this procedure leads to the same kind of systematic overestimate when the observed EW's are affected by random errors. In all the above examples, this method gives the same overestimate of v_t as the (W_{obs}, A) plot. However, we know no simple way to avoid the systematic error in this procedure, so we recommend the use of the (W_{calc}, A) diagram. Note that in the solar case analysed by Simmons and Blackwell, the systematic error is completely negligible due to the very high quality of the observational material.

Another method is to fit a theoretical curve-of-growth through the observed points in the $\left(X, \log\frac{W}{\lambda}\right)$ diagram, where X is the abscissa of the curve-of-growth (see, e.g., Spite and Spite, 1979). The microturbulent velocity is determined so that the flat part of the theoretical curve-of-growth matches the observed one. This procedure is not subject to the kind of systematic error discussed in this note and, so, is a good procedure to determine the microturbulent velocity, even when the observed EW's are affected by random errors. However, it does not take into account the fact that different lines of the same ion may have slightly different curves-of-growth (due, e.g., to their different excitation potentials, wavelengths or damping constants). On the other hand, in the $(W_{\rm calc}, A)$ method, each line is treated individually, which allows to take all these effects into account. So, this latter procedure may be slightly superior to the curve-of-growth method.

Finally, we should point out that another effect may introduce systematic errors in determinations of microturbulent velocities. It is the non-linearity of the flat part of the curve-of-growth. This effect has been studied by Dufton et al. (1981) in the case of B stars. Briefly, on the flat part of the curve-of-growth, positive errors in EW's produce larger abundance errors than the corresponding negative ones. So, the abundances of the strongest lines used in the microturbulent velocity determination are systematically overestimated and so is the deduced microturbulent velocity. In the cases studied here, we estimate this effect to account for roughly one tenth of the systematic error, so it is generally negligible. We therefore conclude that the main cause of the microturbulent velocity overestimate is the correlation between errors in abundances and in EW's, at least in the solar-type stars.

Acknowledgements. We would like to thank C. Arpigny and N. Grevesse for their stimulating advice. We also wish to thank B. Gustafsson for allowing us to use a copy of his model atmosphere programme and D. E. Blackwell for communicating some results in advance of publication.

References

Blackwell, D.E., Petford, A.D., Shallis, M.J.: 1979, *Monthly Notices Roy. Astron. Soc.* **186**, 657
Blackwell, D.E., Petford, A.D., Shallis, M.J., Simmons, G.J.: 1982, *Monthly Notices Roy. Astron. Soc.* **199**, 43
Blackwell, D.E., Booth, A.J., Petford, A.D.: 1983 (submitted to *Astron. Astrophys.*)
Cowley, C.R.: 1970, The Theory of Stellar Spectra, Gordon and Breach, New York
Dufton, P.L., Durrant, A.C., Durrant, C.J.: 1981, *Astron. Astrophys.* **97**, 10
Foy, R.: 1972, *Astron. Astrophys.* **18**, 26
Gustafsson, B., Bell, R.A., Eriksson, K., Nordlund, Å.: 1975, *Astron. Astrophys.* **42**, 407
Holweger, H., Müller, E.A.: 1974, *Solar Phys.* **39**, 19
Holweger, H., Gehlsen, M., Ruland, F.: 1978, *Astron. Astrophys.* **70**, 537
Mäckle, R., Holweger, H., Griffin, R., Griffin, R.: 1975, *Astron. Astrophys.* **38**, 239
Magain, P.: 1983 (submitted to *Astron. Astrophys.*)
Peterson, R.C.: 1980, *Astrophys. J.* **235**, 491
Simmons, G.J., Blackwell, D.E.: 1982, *Astron. Astrophys.* **112**, 209
Spite, M., Spite, F.: 1979, *Astron. Astrophys.* **76**, 150