# Effect of Ground State Correlations on the Imaginary Part of the Optical-Model Potential

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The effect of the ground state correlations on the imaginary part of the nucleon-nucleus optical-model potential is qualitatively discussed within the random phase approximation (RPA). We also investigate the effect of terms of order higher than two in the microscopic nucleon-nucleus interaction. A rough estimate of both effects is given for the case  $K^{39} + p$ .

## 1. Introduction

A large effort has been devoted to the microscopic calculation of the imaginary part of the nucleon-nucleus optical-model potential [1-5]. All those calculations are, however, limited to second order in the microscopic nucleon-nucleus interaction. Moreover, the description of the nucleus excitations is often restricted to particle-hole excitations (the so-called Tamm-Dancoff approximation (TDA)). Sometimes, more complicated descriptions are used for the target excited states, but the ground state is generally taken as a shell-model state. In a previous paper [6], we have already indicated how the formula giving the opticalmodel potential (OMP) should be changed if correlations are introduced in the ground state of the compound system. These correlations were described within a RPA model previously investigated by several authors [7-12]. In the present paper, we use this model in the frame of a simple assumption for the nuclear interactions in order to discuss qualitatively, and to estimate roughly, the effect of correlations on the imaginary part of the OMP. We separate this correction from the high (larger than the second one) order (in nucleon-nucleus interaction), corrections which are present even when correlations are neglected. The imaginary part is a sum of two contributions, one from the direct inelastic excitations of the target, and the other from the average resonant scattering. We show that the corrections can be very different for these two contributions.

In Section 2, we briefly recall the RPA model. In Section 3, we introduce a simple assumption for the nuclear interactions. In Section 4,

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we discuss qualitatively the effect of the correlations and of the high orders on the imaginary part and we give a rough estimate of those effects in the case of  $K^{39} + p$ . Section 5 contains our conclusions.

## 2. The OMP and the RPA in the Continuum

We briefly recall the main formulae obtained in Ref. [6], from which we borrow the definitions and notation. Let  $\psi_0$  be the true ground state wave function of the nucleon-nucleus system and let  $\psi_E^{c(+)}$  be the scattering wave function. In addition, let  $A_c^+(E)$  be the creation operator of a particle-hole pair, (see the exact definition in Ref. [6]), which when applied to  $\psi_0$  is assumed to describe the elastic channel state. The OMP in a channel c can be written as [6]:

$$\mathscr{V}_{E}^{\text{opt}(c)}(E', E'') = v_0(E', E'') + \mathscr{V}_{E+iI}^{c}, \tag{2.1}$$

$$\mathscr{V}_{E}^{c} = (1,0)\mathbf{Z}(E) \begin{pmatrix} 1\\0 \end{pmatrix}, \tag{2.2}$$

where the  $2 \times 2$  matrix Z(E) is given by [6]

$$Z(E) = V^{e} + V^{e} D^{c-1} V^{e} + Z'.$$
 (2.3)

The matrix Z' arises from the fact that the wave function  $\psi_E^{c(+)}$  contains a component, both in the channel state  $A_c^+(E) | \psi_0 \rangle$  and in the conjugate boson state  $A_c(E) | \psi_0 \rangle$ . It has a very complicated form. We will neglect it in the following, since it is presumably smaller than the other corrections. The matrices  $V^e$  and  $D^c$  are given by (Ref. [6]):

$$V^e = V + V E_C^{-1} V^e, (2.4)$$

$$D^c = E_S - V^e. (2.5)$$

Finally, let us recall the perturbative series for  $V^e$ :

$$V^{e} = \begin{pmatrix} V_{cc}^{(1)}(E', E'') & V_{cc}^{(2)}(E', E'') \\ -V_{cc}^{(2)}(E', E'') - V_{cc}^{(1)}(E', E'') \end{pmatrix}$$

$$+ \sum_{c' \neq c} \int_{\varepsilon_{c'}}^{\infty} dE''' \begin{pmatrix} V_{cc'}^{(1)}(E', E''') & V_{cc'}^{(2)}(E', E''') \\ -V_{cc'}^{(2)}(E', E''') & -V_{cc'}^{(1)}(E', E''') \end{pmatrix}$$

$$\cdot \begin{pmatrix} \frac{1}{E^{+} - E'''} & 0 \\ 0 & \frac{1}{E + E'''} \end{pmatrix} \begin{pmatrix} V_{c'c}^{(1)}(E''', E'') & V_{c'c}^{(2)}(E''', E'') \\ -V_{c'c}^{(2)}(E''', E'') & -V_{c'c}^{(1)}(E''', E'') \end{pmatrix} + \cdots.$$
(2.6)

The effect of the correlations brought in by the RPA is contained in the interaction  $V^{(2)}$ .

# 3. The Interaction $V^{e}$ in a Simple Case

We study a simple approximation which allows us to calculate  $V^e$  exactly. We assume that the matrix elements  $V^{(1)}$  and  $V^{(2)}$  can be factorized:

$$V_{cc'}^{(1)}(E', E'') = -\chi f_{E'}^c f_{E''}^{c'}, \tag{3.1}$$

$$V_{cc'}^{(2)}(E', E'') = -\chi f_{E'}^c f_{E''}^{c'}.$$
 (3.2)

This is achieved with a two-body interaction like:

$$v(i,j) = v(\mathbf{r}_i, \mathbf{r}_j) = \sum_{L} \frac{2L+1}{4\pi} f_L(\mathbf{r}_i, \mathbf{r}_j) P_L(\cos \Theta), \qquad (3.3a)$$

with

$$f_L(r, r') = -2\pi \chi_L D_L(r) D_L(r'),$$
 (3.3b)

provided the antisymmetrization of the matrix element is neglected. Then, only one multipole is effective, for which L is equal to the total spin of the system. Interactions of this type are often used [13, 14]. For L=2 it reduces to the quadrupole-quadrupole interaction. With the help of (3.1) and (3.2), we can solve the integral equation (2.4) by rewriting it as:

$$(1|V^e|1) = (1|V|1) + (1|V|1)(1|E_c^{-1}|1)(1|V^e|1) + (1|V|2)(2|E_c^{-1}|2)(2|V^e|1),$$
(3.4a)

$$(2|V^{e}|1) = (2|V|1) + (2|V|1)(1|E_{c}^{-1}|1)(1|V^{e}|1) + (2|V|2)(2|E_{c}^{-1}|2)(2|V^{e}|1),$$
(3.4b)

with

$$(1|V|1) = -(2|V|2) = V^{(1)}, -(2|V|1) = (1|V|2) = V^{(2)}, (3.4c)$$

$$(1|E_C^{-1}|1) = (E^+ - E')^{-1}, \quad (2|E_C^{-1}|2) = (E + E')^{-1}.$$
 (3.4d)

In the Eqs. (3.4a) and (3.4b), and in the following, the integration over the variable E', and the summation over  $c' \neq c$  have not been

written explicitly. It is easy to see that these operations must be done whenever  $E_C$  appears. The quantity  $(1 \mid V^e \mid 1)$  is equal to:

$$(1|V^e|1) = (1|V^a|1)(1|E_c^{-1}|1)(1|V^e|1)$$
 (3.5a)

with

$$(1|V^{a}|1) = (1|V|1) + (1|V|2)(2|E_{c}^{-1}|2)[1 + (2|V|2)(2|E_{c}^{-1}|2) + (2|V|2)(2|E_{c}^{-1}|2)(2|V|2)(2|E_{c}^{-1}|2) + \cdots](2|V|1).$$
(3.5b)

With the help of Eqs. (3.4a) and (3.4b), we have:

$$(1|V^a|1) = -\chi^a f_{E'}^c f_{E''}^c, \tag{3.6a}$$

with

$$\chi^{a} = \chi \left[ 1 - \chi \sum_{c' \neq c} \int_{E_{c'}}^{\infty} dE'' \frac{(f_{E''}^{c'})^{2}}{E + E''} \right]^{-1}.$$
 (3.6b)

Then, Eq. (3.5a) becomes

$$(1|V^e|1) = -\chi^e f_{E'}^c f_{E''}^c, \tag{3.7a}$$

with

$$\chi^{e} = \chi^{a} \left[ 1 + \chi^{a} \sum_{c' \neq c} \int_{g_{c'}}^{\infty} dE'' \frac{(f_{E''}^{c'})^{2}}{E^{+} - E''} \right]^{-1}.$$
 (3.7b)

Eqs. (3.7b) and (3.4b) yield successively

$$(2|V^e|1) = \chi^e f_{E'}^c f_{E''}^c, \tag{3.8}$$

$$(k|V^e|k') = (-)^k \chi^e f_{E'}^c f_{E''}^c.$$
 (3.9)

We note that  $\chi^a$  is identical to  $\chi$  if the correlations introduced by the RPA are removed, and this happens when the off-diagonal terms of V are put equal to zero, or when the "propagator"  $(E+E''')^{-1}$  is multiplied by zero. Thus,  $\chi^a$  contains all the effects of the correlations.

# 4. Effect of Correlations on the Imaginary Part of the OMP

a) General Discussion

From Eqs. (2.1) to (2.3), we find:

$$\mathscr{V}_{E}^{\text{opt}(c)}(E', E'') = v_{0} - \chi^{e} f_{E'}^{c} f_{E''}^{c} + \sum_{ss'} \sum_{kk'=1}^{2} (-) \chi^{e} f_{E'}^{c} f_{s} 
\cdot (D^{c-1})_{ss'kk'} (-)^{k'} \chi^{e} f_{s'}^{c} f_{E''}^{c},$$
(4.1)

where

$$(D^{c})_{ss'kk'} = (E + iI + (-)^{k}E_{s})\delta_{ss'} + (-)^{k}\chi^{e}f_{s}f_{s'}.$$
(4.2)

We assume in the following that the quantities  $f_s$  have random signs. Then, we can neglect the second term in Eq. (4.2) and obtain:

$$\mathcal{V}_{E}^{\text{opt}(c)}(E', E'') = v_{0} - \chi^{e} f_{E'}^{c} f_{E''}^{c} + \sum_{s} (\chi^{e})^{2} \frac{f_{s}^{2}}{E + iI - E_{s}} f_{E'}^{c} f_{E''}^{c} - \sum_{s} (\chi^{e})^{2} \frac{f_{s}^{2}}{E + E_{s}} f_{E'}^{c} f_{E''}^{c}.$$
(4.3)

In the last term and in  $\chi^e$ , the quantity iI should be added to the argument E. We neglect it since I is much smaller than  $E+E_s$ , and since  $\chi^e$  is slowly varying with the energy. The imaginary part of the OMP is given by:

$$\operatorname{Im} \mathscr{V}_{E}^{\operatorname{opt}(c)}(E', E'') = f_{E'}^{c} f_{E''}^{c} \left\{ -\operatorname{Im} \chi^{e} + \sum_{s} f_{s}^{2} \operatorname{Im} \left( \frac{\chi^{e^{2}}}{E + iI - E_{s}} \right) + \sum_{s} \frac{f_{s}^{2}}{E + E_{s}} (-\operatorname{Im} \chi^{e^{2}}) \right\}.$$

$$(4.4)$$

In the absence of correlations due to the RPA, we would have

$$\operatorname{Im} \mathscr{V}_{E}^{\operatorname{opt}(c)} = f_{E'}^{c} f_{E''}^{c} \left\{ -\operatorname{Im} \widetilde{\chi}^{e} + \sum_{s} f_{s}^{2} \operatorname{Im} \left( \frac{\widetilde{\chi}^{e^{2}}}{E + iI - E_{s}} \right) \right\}, \tag{4.5}$$

where the quantity

$$\tilde{\chi}^{e} = \chi \left[ 1 + \chi \sum_{c' + c} \int_{\varepsilon_{c'}}^{\infty} dE'' \frac{(f_{E''}^{c'})^{2}}{E^{+} - E''} \right]^{-1}$$
(4.6)

has the same value  $\chi^e$  when  $\chi^a = \chi$ . Finally, we write the formula (4.5) when the calculation is limited to second order in V (or in  $\chi$ ):

Im 
$$\mathcal{V}_{E}^{\text{opt}(c)} = f_{E'}^{c} f_{E''}^{c} \chi^{2} \left\{ \text{Im } X - \sum_{s} f_{s}^{2} \frac{I}{(E - E_{s})^{2} + I^{2}} \right\},$$
 (4.7)

with

$$X = \sum_{c' \neq c} \int_{E_{c'}}^{\infty} dE' \frac{(f_{E'}^{c'})^2}{E^+ - E'}.$$
 (4.8)

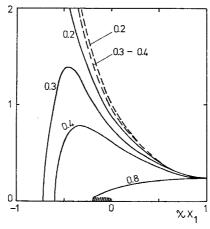


Fig. 1. Typical variation of  $\beta$  (dashed curves) and  $\alpha$  (full curves) versus  $\chi X_2$  and  $\chi X_1$  (see Section 4). The value of  $\chi X_2$  is given for each of the curves

The difference between formulae (4.4) and (4.5) is due to the RPA correlations. The difference between (4.5) and (4.7) reflects the role of multiple collisions, since each collision corresponds to a matrix element of V.

The effect of this "multiple scattering" can be summarized in two parameters  $\alpha$  and  $\beta$ . The first one is the ratio between the compound nucleus contribution to the imaginary part with "multiple scattering" (the term which contains a summation over s in (4.5)) and the compound nucleus contribution without multiple scattering (summation over s in (4.7)). The parameter  $\beta$  is the same ratio for the direct contribution

$$\beta = -\frac{\operatorname{Im} \widetilde{\chi}^e}{\gamma^2 \operatorname{Im} X}.$$
 (4.9)

With the help of Eqs. (4.6) and (4.8), and writing  $X = X_1 - iX_2$ , we have:

$$\alpha = \frac{(1 + \chi X_1)^2 - \chi^2 X_2^2}{\left[ (1 + \chi X_1)^2 + \chi^2 X_2^2 \right]^2},\tag{4.10}$$

$$\beta = \frac{1}{(1 + \chi X_1)^2 + \chi^2 X_2^2}. (4.11)$$

The characteristic curves of  $\alpha$  and  $\beta$  versus  $\chi X_1$  and  $\chi X_2$  are given in Fig. 1. The variation of the parameter  $\beta$  is rather simple. Roughly, it

is larger than unity for  $\chi X_1$  negative, and is smaller than unity, when  $\chi X_1$  is positive. The situation for  $\alpha$  is more complicated:  $\alpha$  is smaller than unity for  $\chi X_1$  positive. It can be larger than unity if  $\chi X_2$  is small and if  $\chi X_1 > -0.3$ . However, not all values of  $\chi X_2$  and  $\chi X_1$  are physical. Firstly, they must be such that  $|\chi X|^2$  is smaller than one, in order to ensure the convergence of the series of  $\chi^e$ . This limits the values of  $|\chi X_2|^2$  and  $|\chi X_1|^2$ . Secondly, we have:

$$X_1 = \sum_{c' \neq c} P \int_{c_{c'}}^{\infty} \frac{(f_{E'}^{c'})^2}{E - E'} dE' = \sum_{c' \neq c} \mathscr{I}_{c'}.$$

For a closed channel, the quantity  $\mathscr{I}_{c'}$  is negative. For an open channel  $(\varepsilon_{c'} < E)$  the integral is sizable only if  $(f_E^{c'})^2$  varies rapidly for  $E' \approx E$ , i.e. if E is just above threshold; then, it is probably negative. In this case, the sign of  $\chi X_1$  is given by the sign of  $\chi$ . If  $\chi$  is positive (attractive interaction),  $\chi X_1$  is negative and  $\beta > 1$ ,  $\alpha < 1$  for  $|\chi X_1| \gtrsim 0.2$ . Then, the multiple scattering increases the direct contribution to the absorption, while it decreases the compound states contribution. If  $\chi$  is negative (repulsive interaction), the multiple scattering decreases both the compound states and the direct contributions to the absorption.

The corrections due to the correlations are manifold. Firstly, a new kind of term appears, namely the last one in Eq. (4.4). This term is expected to be small because of the large denominator. The other corrections can be expressed in terms of two parameters  $\xi$  and  $\zeta$ . The first one is the ratio between the direct contribution to the imaginary part with correlations and the same quantity without them. The parameter  $\zeta$  is the same ratio for the compound nuclear contribution:

$$\xi = \gamma^2 \frac{(1 + \chi X_1)^2 + \chi^2 X_2^2}{(1 + \chi \chi X_1)^2 + \gamma^2 \chi^2 X_2^2},\tag{4.12}$$

$$\zeta = \gamma^2 \frac{\left[ (1 + \gamma \chi X_1)^2 - (\gamma \chi X_2)^2 \right] \left[ (1 + \chi X_1)^2 + \chi^2 X_2^2 \right]^2}{\left[ (1 + \gamma \chi X_1)^2 + (\gamma \chi X_2)^2 \right]^2 \left[ (1 + \chi X_1)^2 - \chi^2 X_2^2 \right]}, \tag{4.13}$$

with

$$\gamma = \frac{\chi^a}{\chi} = \left[1 - \chi \sum_{c' + c} \int_{e_{c'}}^{\infty} dE'' \frac{(f_{E''}^{c'})^2}{E + E''}\right]^{-1}.$$
 (4.14)

The coefficients  $\xi$  and  $\zeta$  reduce to unity if  $\gamma \to 1$ . The effect of the correlations is entirely contained in  $\gamma$ . In order to display the variation of  $\xi$  and  $\zeta$  with  $\gamma$ , we set  $\chi X_1 = 0$ , which is an average acceptable value of

 $\chi X_1$ . We have:

$$\xi = \frac{\gamma^2}{1 + \gamma^2 \chi^2 X_2^2} (1 + \chi^2 X_2^2), \tag{4.15}$$

$$\zeta = \gamma^2 \frac{1 - \gamma^2 \chi^2 X_2^2}{\left[1 + \gamma^2 \chi^2 X_2^2\right]^2} \cdot \frac{(1 + \chi^2 X_2^2)^2}{1 - \chi^2 X_2^2}.$$
 (4.16)

The quantity  $\xi$  is always increasing with  $\gamma(>0)$ . Consequently,  $\xi$  is always larger than 1 for  $\gamma>1$  and smaller than unity for  $\gamma<1$ . The function  $\zeta$  is maximum at  $\gamma^2=(3\chi^2X_2^2)^{-1}$  and is then equal to

$$\frac{1}{8\chi^2 X_2^2} \frac{(1+\chi^2 X_2^2)^2}{(1-\chi^2 X_2^2)}.$$

This quantity is always larger than or equal to unity, since  $0 < \chi^2 X_2^2 < 1$ . If  $3\chi^2 X_2^2 > 1$ , the function  $\zeta$  has no maxima and is always less than 1 in the domain  $\gamma > 1$ . It is larger than 1 for

$$(1-\chi^2 X_2^2)/(\chi^2 X_2^2 (3\chi^2 X_2^2+1)) < \gamma < 1.$$

For the case  $3\chi^2 X_2^2 < 1$ , the function  $\zeta$  is larger than unity for

$$1 < \gamma < (1 - \chi^2 X_2^2) / (\chi^2 X_2^2 (3\chi^2 X_2^2 + 1))$$

and is smaller than unity everywhere below  $\gamma=1$ . Let us show that we can roughly relate the values of  $\gamma$  to the sign of  $\chi$ . We neglect the closed channels in (4.14). Since the denominator E+E'' is always increasing with E'', let us restrict the interval of integration to the domain  $\varepsilon_{c'} < E'' < -\varepsilon_c + 2E$ . We get:

$$\gamma \approx \left[1 - \chi \sum_{\substack{c' \neq c \\ \varepsilon_{c'} > E}} \int_{\varepsilon_{c'}}^{2E - \varepsilon_{c'}} dE' \frac{(f_{E'}^{c'})^2}{E + E''}\right]^{-1}.$$
 (4.17)

Let us give to the function  $(f_{E'}^{c'})^2$  its values at the center of the interval. We have:

$$\gamma \approx \left[1 - \chi \sum_{\substack{c' \neq c \\ c' \neq c}} (f_E^{c'})^2 \frac{2(E - \varepsilon_{c'})}{2E}\right]^{-1}.$$
 (4.18)

Since  $\varepsilon_{c'}$  has values between 0 and E, we have roughly

$$\gamma \approx \left[1 - \frac{\chi X_2}{2\pi}\right]^{-1},\tag{4.19}$$

by using the definition given below (4.9). Then,  $\gamma$  is larger or smaller than unity when  $\chi$  is positive or negative, respectively. In conclusion, the direct contribution is enhanced for  $\chi > 0$ , while the compound states contribution is decreased except if  $3\chi^2 X_2^2 > 1$ . Opposite conclusions must be drawn if  $\chi$  is negative.

The schematic model defined by the Eqs. (3.1) and (3.2) has been used to describe the collective vibrational states in nuclei [13]. The collective states are pushed upwards or downwards, for  $\chi$  negative or positive, respectively. It appears that  $\chi$  should in general be positive, except for the dipole states. However, the non collective 1<sup>-</sup> states have been described with a positive  $\chi$  by Dover and Dietrich [14], who studied the case of Pb<sup>208</sup>. We did a TDA calculation [5] for the low-lying 1<sup>-</sup> and 3<sup>-</sup> states of Ca<sup>40</sup> and found  $\chi$ =1.50 MeV. In this present calculation, we make a commonly adopted choice [14]:

$$D_L(r) = \left(\frac{r}{R_0}\right)^L,\tag{4.20}$$

where  $R_0$  is the nuclear radius. We also calculate the quantities  $X_1$  and  $X_2$  (see Eq. (4.8) and the definitions below (4.9)), in order to obtain a rough estimate of the effects of the correlations and of the "multiple scattering" on the imaginary part of the OMP. The quantity  $X_1$  may be written:

$$X_1^{(J)} = \sum_{c' \neq c} (\mathfrak{f}_{c'})^2 P \int_{\mathfrak{g}_{c'}}^{\infty} \frac{dE'}{E - E'} \left[ \int_0^{\infty} dr \, w_{h_{c'}}(r) \left( \frac{r}{R_0} \right)^J u_{p_{c'}}(r, k_c') \right]^2, \quad (4.21)$$

where we have explicitly indicated the dependence upon J. The functions  $w_{h_c}$ , and  $u_{p_c}$ ,  $(r, k_c')$  are respectively the hole and particle wave functions associated with the boson operator  $A_c^+(E)$  (in addition,  $E' = \varepsilon_c + \frac{h^2 k_c'^2}{2M}$ ). The coefficient  $\mathfrak{f}_c$  is a geometrical factor that is defined in Ref. [14].

J	$X = X_1 - iX_2$	β	α
0	0	1	1
1	-0.114-i0.317	1.09	0.36
2	-0.423 - i 0.476	1.55	-0.17
3	-0.693 - i 0.410	2.63	0.058

Table 1. Values of  $X_1$ ,  $X_2$ ,  $\alpha$  and  $\beta$  for the special case of Section 4b

Because of the properties of the Green functions, we have:

$$X_{1}^{(J)} = \operatorname{Re} \sum_{c' \neq c} \int_{0}^{2} \int_{0}^{\infty} dr \int_{0}^{\infty} dr' w_{h_{c'}}(r) \left(\frac{r}{R_{0}}\right)^{J} G_{E-\varepsilon_{c}, lj}^{+}(r, r') \left(\frac{r'}{R_{0}}\right)^{J} w_{h_{c'}}(r')$$

$$- \sum_{n, c' \neq c} \int_{c'}^{2} \left[ \int_{0}^{\infty} dr w_{h_{c'}}(r) \left(\frac{r}{R_{0}}\right)^{J} w_{lj}^{n}(r, k_{lj}^{n}) \right]^{2} (E - \varepsilon_{c} - \varepsilon_{n})^{-1},$$

$$(4.22)$$

with

$$G_{\varepsilon, lj}^{+}(r, r') = -\left(\frac{2\pi M}{\hbar^2 k}\right)^{\frac{1}{2}} e^{i\delta_{lj}(\varepsilon)} u_{lj}(r_{>}, k) G_{lj}(r_{<}, k),$$
 (4.23)

$$\varepsilon = \frac{\hbar^2 k^2}{2M}.\tag{4.24}$$

The quantum numbers l and j are the same as those of  $u_{pc'}(r, k'_c)$ . The wave functions  $w_{h_c}$ ,  $(r,k_n)$  are those of single-particle bound states with the same l and j and the quantities  $\varepsilon_n$  are the (negative) energies of these bound states. It is easy to see that  $X_2^{(J)}$  is given by minus the imaginary part of the first term in (4.22). We have computed  $X_1$  and  $X_2$ along these lines for the case  $^{39}K + p$ , since an evaluation exists for the imaginary part in the second order in V [5]. We have considered all the inelastic channels c' open at 24 MeV above the Ca40 ground state, which is the energy considered in Ref. [5]. The results are given in Table 1. We also quote the values of  $\alpha$  and  $\beta$  (see Eqs. (4.10) and (4.11)) for a value of  $\gamma = 1.50$  MeV for all J. For J = 0, X is equal to zero because of the orthogonality between the orbitals  $w_{h_{c'}}$  and  $u_{p_{c'}}$  (see Eq. (4.21)). The quantities  $X_1$  are negative as we noticed in Section 4a. We can also see that the condition of convergence for the series of  $V_{\rm eff}$  ( $|\chi X|$  < 1) becomes less well fulfilled when J increases. This may be due to the choice of  $D_L(r)$ . It seems that  $\alpha$  is negative for J=2 and 3, because of the same choice.

We conclude that, in general, multiple scattering increases the direct contribution to the absorption. The values of  $\alpha$  are, however, not

accurate enough to draw definite conclusions concerning the compound nucleus contribution. Another remark is of interest. We also multiplied the direct contribution evaluated in Ref. [5] by an average value of  $\beta$  (1.55), keeping the compound nucleus contribution unchanged. We then calculated the absorption cross section and found  $\sigma_A = 662$  mb, while the previous value was 600 mb.

From Table 1 and Eq. (4.19), we find that  $\gamma$  ranges between 1 and 1.13, while  $3\chi^2 X_2^2 < 1$ . For  $\gamma = 1.13$  and  $\chi^2 X_2^2 = 1/3$  (average value for J=1, 2, 3), we have  $\xi = 1.14$  and  $\zeta = 0.98$ . One can thus expect that ground state correlations slightly enhance the direct contribution while they leave the compound nuclear contribution practically unchanged.

#### 5. Conclusions

We have estimated the corrections which must be made to the calculations of the imaginary part of the optical-model potential, when these calculations are limited to the second order in the microscopic nucleon-nucleus interaction and to a simple shell-model structure of the nucleus. These corrections are essentially due to the multiple scattering and to the ground state correlations. The first effect increases the direct contribution and decreases the average resonant (or compound nuclear) contribution. The ground state correlations are expected to increase slightly the direct contribution and to leave the compound states contribution practically unchanged.

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