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DOORWAY PARAMETERS OF THE RESONANCES IN SUBTHRESHOLD NEUTRON-INDUCED FISSION

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Abstract: A two-channel model is used to extract doorway parameters from resonating subthreshold neutron-induced fission. The decaying width is found to be large compared to the spreading width. Information on the fission strength function is also obtained from this analysis and is compared with the results of an optical-model-type calculation.

1. Introduction

The groups of narrow resonances observed in subthreshold neutron-induced fission [refs. [1-5]] are suggestive evidence for a double-humped fission barrier [6,7]. The existence of two humps in the fission barrier of some nuclei is accounted for in Strutinsky's calculations on the shell corrections to the liquid-drop mass formula [8]. The discovery of fission isomers with short half-lives [9-12] and of broad sub-barrier fission resonances [13-14] had previously provided arguments for the existence of such a double-peaked barrier.

In the case of slow-neutron fission displaying narrow resonances below threshold, it has been proposed [6,7] that compound states of two kinds are formed: the first is concentrated in the internal well and the second is in the external well. The evolution of the system from compound states of the first kind to compound states of the second kind, and finally to fission is a good picture of the doorway phenomena, with the states in the external well playing the role of doorway states for the fission channel. There exist many theories of the doorway resonances in nucleon channels, but none is strictly valid for doorway resonances in fission channels. However, one may expect that the doorway state equations (intermediate structure, fine structure, ...) are independent of the dynamical assumptions of the theory. Indeed, the formulae obtained in the case of the shell-model approach [15] are identical in form to those obtained from Feshbach's theory [16], although in the latter theory, degrees of freedom other than independent particles motion are allowed. One attempt [17] has been made to apply doorway formulae to resonating neutron-induced fission. This attempt failed, since the author could not determine whether one has $\Gamma^{\uparrow} > \Gamma^{\downarrow}$ or $\Gamma^{\uparrow} < \Gamma^{\downarrow}$,

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where $\Gamma^{\uparrow}$ and $\Gamma^{\downarrow}$ are the escape width and the spreading width of the doorway state respectively. In this paper, we propose a more detailed model than in ref. [17]). It leads to a reaction cross section which contains many parameters (e.g. the positions of individual resonances and the coupling matrix elements). Since many of these parameters are not important from the physical point of view, we reduce the complexity of the problem by looking at the average reaction cross sections. Within our model, if we make use of reasonable assumptions, this quantity depends only upon four parameters which include $\Gamma^{\uparrow}$ and $\Gamma^{\downarrow}$. We apply this model to the case of ^{240}Pu. The decaying width is found to be very large and the spreading width $\Gamma^{\downarrow}$ small. We also make a comparison with another model which provides fission strength functions.

In sect. 2, we describe the model and analyse the structure of the S-matrix. In sect. 3, the average reaction cross section is investigated. Sect. 4 is devoted to a refinement of the model. We indicate in sect. 5 how to extract the doorway parameters from the data and we apply our model to the case of ^{240}Pu. In sect. 6, we discuss the fission strength function for a double-humped barrier. Finally, sect. 7 contains our conclusions.

2. The model

2.1. THE NOTATION

The two-channel model described here has been constructed by Lejeune and Mahaux [18]). We summarize those features which are relevant here. The Hamiltonian H is divided into two parts:

$$H = H_0 + V, \tag{2.1}$$

where H_0 is a model Hamiltonian and V is the residual interaction. The Hamiltonian H_0 possesses bound states ϕ_i and scattering states $\chi_E^{\rm d}$. We have the following definitions and expressions:

$$\begin{gathered} H_0\,\chi_E^{\rm d} = E\chi_E^{\rm d}, \qquad H_0\,\Phi_i = E_i\,\Phi_i, \\ \langle\Phi_i|\Phi_j\rangle = \delta_{ij}, \qquad \langle\Phi_i|\chi_E^{\rm d}\rangle = 0, \qquad \langle\chi_E^{\rm d}|\chi_{E'}^{\rm d'}\rangle = \delta_{\rm dd'}\,\delta(E-E'), \\ \langle\Phi_i|V|\Phi_j\rangle = V_{ij}, \qquad \langle\Phi_i|V|\chi_E^{\rm d}\rangle = V_i^{\rm d}(E), \qquad \langle\chi_E^{\rm d}|V|\chi_{E'}^{\rm d'}\rangle = 0. \end{gathered} \tag{2.2}$$

The last equation in (2.2) is a reasonable assumption since it is unlikely that direct processes are important in neutron-induced fission. The scattering matrix is given by:

$$S_{\rm dd'} = \exp{(i\delta_{\rm d}+i\delta_{\rm d'})}[\delta_{\rm dd'} - i2\pi \sum_{j,m=1}^{M} V_j^{\rm d}(E)[D^{-1}(E)]_{jm}\, V_m^{\rm d'}(E)], \tag{2.3a}$$

with

$$[D(E)]_{jm} = (E-E_j)\delta_{jm} - V_{jm} - \sum_{\rm d} F_{jm}^{\rm d}, \tag{2.3b}$$

$$\begin{aligned} F_{jm}^{\rm d} &= \int_{\varepsilon_{\rm d}}^{\infty} {\rm d}E'\, \frac{V_j^{\rm d}(E')V_m^{\rm d}(E')}{E^+ - E'} = {\rm P}\int_{\varepsilon_{\rm d}}^{\infty} {\rm d}E'\, \frac{V_j^{\rm d}(E')V_m^{\rm d}(E')}{E-E'} - i\pi V_j^{\rm d}(E)V_m^{\rm d}(E) \\ &= R_{jm}^{\rm d} - i\pi V_j^{\rm d}(E)V_m^{\rm d}(E). \end{aligned} \tag{2.3c}$$

The quantities ε_d and δ_d are the threshold energy of channel d and the background phase shift in channel d respectively.

2.2. SPECIALIZATION OF THE MODEL

We admit only two channels: d = c, c′ and a doorway state Φ_0 in channel c coupled to complicated states $\Phi_j, j = 1, \ldots, M$ which in turn are coupled to channel c′ only. So we get the following matrix elements:

$$\langle\Phi_0|V|\chi_E^c\rangle = V_0^c \neq 0, \qquad \langle\Phi_0|V|\chi_E^{c'}\rangle = 0, \qquad \langle\Phi_j|V|\chi_E^c\rangle = 0,$$
$$\langle\Phi_j|V|\chi_E^{c'}\rangle = V_j^{c'}, \qquad \langle\Phi_0|V|\Phi_j\rangle = V_{j0}. \tag{2.4}$$

The first and fourth matrix elements are assumed to be energy-independent. Moreover it is assumed that $(V_j^{c'})^2 \ll (V_0^c)^2$, so that Φ_0 has the character of a strong doorway state.

We have, because of eq. (2.4):

$$D(E) = \begin{pmatrix} E-R_{00}-E_0+i\pi(V_0^c)^2 & V_{l0} \\ V_{j0} & (E-E_j)\delta_{jl}-V_{jl}-R_{jl}^{c'}+i\pi V_j^{c'}V_l^{c'} \end{pmatrix} = \begin{pmatrix} A & V_{l0} \\ V_{j0} & (\Delta^{c'})_{jl} \end{pmatrix}. \tag{2.5}$$

The matrix $\Delta^{c'}$ may be diagonalized by a complex orthogonal matrix $\mathcal{O}$:

$$\mathcal{O}\Delta^{c'}\tilde{\mathcal{O}} = (E-e_l)\delta_{jl}, \qquad e_l = \varepsilon_l - \tfrac{1}{2}i\gamma_l. \tag{2.6}$$

The matrix $U = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{O} \end{pmatrix}$ is complex orthogonal and one has

$$\mathscr{D} = UD\tilde{U} = \begin{pmatrix} E-\varepsilon_0+\frac{1}{2}i\Gamma^{\uparrow} & V_l \\ V_j & (E-e_j)\delta_{jl} \end{pmatrix}, \tag{2.7}$$

with

$$\varepsilon_0 = E_0 + R_{00}, \qquad \Gamma^{\uparrow} = 2\pi(V_0^c)^2, \qquad V_l = \sum_j \mathcal{O}_{lj} V_{j0}. \tag{2.8}$$

The scattering matrix takes the form:

$$S_{dd'} = \exp(i\delta_d + i\delta_{d'})[\delta_{dd'} - i2\pi \sum_{j,l=0}^{M} v_j^d (\mathscr{D}^{-1}(E))_{jl} v_l^{d'}], \tag{2.9a}$$

with

$$v_j^d = \sum_{l=0}^{M} U_{jl} V_l^d. \tag{2.9b}$$

Performing the calculations, one finds [18]:

$$S_{cc}(E) = \exp(i2\delta_c) \frac{E-\varepsilon_0-\frac{1}{2}i\Gamma^{\uparrow} - \sum_{l=1}^{M} \frac{V_l^2}{E-e_l}}{E-\varepsilon_0+\frac{1}{2}i\Gamma^{\uparrow} - \sum_{l=1}^{M} \frac{V_l^2}{E-e_l}}, \tag{2.10a}$$

$$S_{cc'}(E) = -i2\pi \exp(i\delta_c + i\delta_{c'}) \frac{V_0^c \sum_{l=1}^{M} \frac{V_l v_l^{c'}}{E-e_l}}{E-\varepsilon_0+\frac{1}{2}i\Gamma^{\uparrow} - \sum_{l=1}^{M} \frac{V_l^2}{E-e_l}}. \tag{2.10b}$$

All the S-matrix elements have the same poles $\mathscr{E}_j$ which are the solutions of

$$d(\mathscr{E}_j) = \mathscr{E}_j - \varepsilon_0 - \tfrac{1}{2}i\Gamma^{\uparrow} - \sum_{l=1}^{M} \frac{V_l^2}{\mathscr{E}_j - e_l} = 0. \tag{2.11}$$

2.3. POLES OF THE S-MATRIX AND SUM RULES

We say a few words here about the fine structure of the doorway resonance. As the general model described above will be applied to neutron-induced fission, we shall write n and f for the channel indices instead of c′ and c (where n stands for neutron and f for fission). We suppose here that all the fission channels can be treated as a single one. We shall discuss this assumption later.

The matrix elements S_{nf} may be written

$$S_{nf} = -i \exp(i\delta_n + i\delta_f) \sum_{l=0}^{M} \frac{\Gamma_{ln}^{\frac{1}{2}} \Gamma_{lf}^{\frac{1}{2}}}{E-\mathscr{E}_l}. \tag{2.12}$$

From eq. (2.10b) and (2.12) we find the sum rule:

$$\sum_{l=1}^{M} \gamma_l + \Gamma^{\uparrow} = \sum_{l=0}^{M} \Gamma_l. \tag{2.13}$$

Provided γ/d is small compared to unity, this relation can be decomposed into two other ones [17,18]):

$$\sum_{l=0}^{M} \Gamma_{lf} = \Gamma^{\uparrow}, \qquad \sum_{l=0}^{M} \Gamma_{ln} = \sum_{l=1}^{M} \gamma_l. \tag{2.14}$$

Let us assume now, that the real [18]) decaying width $\Gamma^{\uparrow}$ is very large compared to the spreading width:

$$\Gamma^{\downarrow} = 2\pi \frac{\overline{V_l^2}^{\,l}}{d}. \tag{2.15}$$

In the last equation the bar denotes an average over the index l and the quantity d is the mean distance between the complicated states. The S-matrix has a pole located approximately at the energy $\varepsilon_0 - \frac{1}{2}i\Gamma^{\uparrow}$. The residue of this pole is very small compared to the residues of the other poles [see eq. (2.10b)] in the non-diagonal S-matrix element and practically zero in S_{nn}. Moreover the widths Γ_l for $l \neq 0$ are smaller than d [ref. [15])]. By looking at eq. (2.12), we see that in the case $\Gamma^{\uparrow} \gg \Gamma^{\downarrow}$ no background is observable in the reaction cross section and the observed resonances are well isolated (in contrast with the one-channel case). It can be checked that the background is also not observable, if the quantity $2\pi\langle \Phi_0|V|\chi_E^n\rangle^2$, which has been put somewhat arbitrarily equal to zero in eq. (2.4), is allowed to be of the order of γ.

For $\Gamma^\uparrow \ll \Gamma^\downarrow$, the resonances are also well isolated. The difference is that the sum of the observable fission partial widths is equal to $\Gamma^\downarrow$ in the case $\Gamma^\uparrow \gg \Gamma^\downarrow$ and equal to $\Gamma^\uparrow$ if $\Gamma^\uparrow \ll \Gamma^\downarrow$. Since the experimental reaction cross section exhibits well-isolated resonances, one does not know which case one is facing and the sum rules (2.14) are not useful.

3. Average reaction cross sections

We start from the value of $|S_{nf}|^2$:

$$|S_{nf}|^2 = 2\pi\Gamma^\uparrow \frac{\left|\sum_{l=1}^{M} \frac{V_l v_l^{c'}}{E-e_l}\right|^2}{\left|E-\varepsilon_0+\frac{1}{2}i\Gamma^\uparrow - \sum_{l=1}^{M}\frac{V_l^2}{E-e_l}\right|^2}. \tag{3.1}$$

Assuming that the quantities $V_l v_l^{c'}$ have random signs, we can write

$$\langle|S_{nf}|^2\rangle = 2\pi\Gamma^\uparrow \left\langle \frac{\sum_{l=1}^{M} \frac{V_l^2 (v_l^{c'})^2}{|E-e_l|^2}}{\left|E-\varepsilon_0+\frac{1}{2}i\Gamma^\uparrow - \sum_{l=1}^{M}\frac{V_l^2}{E-e_l}\right|^2}\right\rangle. \tag{3.2}$$

Making the assumption

$$2\pi(v_l^{c'})^2 \approx \gamma_l \approx \bar{\gamma}_l = \gamma, \tag{3.3}$$

and taking the first-order expression of $\langle|S_{nf}|^2\rangle$ in the variable γ, we obtain

$$\langle|S_{nf}|^2\rangle = \Gamma^\uparrow\gamma \left\langle \frac{\sum_{l=1}^{M} \frac{V_l^2}{(E-\varepsilon_l)^2}}{\left(E-\varepsilon_0 - \sum_{l=1}^{M}\frac{V_l^2}{E-\varepsilon_l}\right)^2 + \frac{1}{4}\Gamma^{\uparrow 2}}\right\rangle. \tag{3.4}$$

Utilizing a picket-fence model for the complicated states, we get:

$$\langle|S_{nf}|^2\rangle = \frac{\pi\gamma}{d}\frac{\Gamma^\uparrow\Gamma^\downarrow}{2}$$
$$\times \left\langle \frac{1}{\left[(E-\varepsilon_0)\sin\frac{\pi(E-\varepsilon_0)}{d} - \frac{1}{2}\Gamma^\downarrow \cos\frac{\pi(E-\varepsilon_0)}{d}\right]^2 + \frac{1}{4}\Gamma^{\uparrow 2}\sin^2\frac{\pi(E-\varepsilon_0)}{d}} \right\rangle. \tag{3.5}$$

The average cross section is now:

$$\langle\sigma_{\mathrm{nf}}\rangle = K\frac{\pi\gamma\Gamma^{\uparrow}\Gamma^{\downarrow}}{2d}\,\frac{I}{2\arctan 2}\int_{E-2I}^{E+2I}\frac{\mathrm{d}E'}{(E'-E)^2+I^2}$$

$$\times\frac{1}{\left[(E'-\varepsilon_0)\sin\frac{\pi(E'-\varepsilon_0)}{d}-\frac{1}{2}\Gamma^{\downarrow}\cos\frac{\pi(E'-\varepsilon_0)}{d}\right]^2+\frac{1}{4}\Gamma^{\uparrow 2}\sin^2\frac{\pi(E'-\varepsilon_0)}{d}}, \qquad (3.6)$$

where K is a trivial kinematical factor. Eqs. (3.5) and (3.6) can be considered as first-order expressions in γ/d rather than γ. We shall see in an example that γ/d is small, which provides confidence in the validity of the expressions.

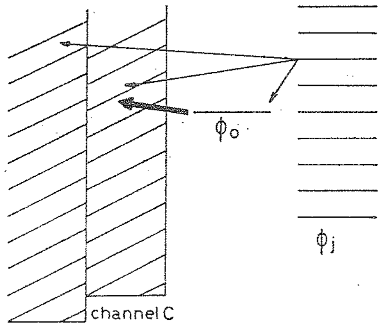

Fig. 1. Strength of the different couplings. The state Φ_0 is the doorway state. The Φ_j are the complicated states. The strength of the coupling is represented by the width of the arrows.

4. Refinement of the model

We modify the model described in sect. 2 by assuming that the complicated states are weakly coupled to channel c:

$$V_j^{\mathrm{c}}(E) = \langle\Phi_j|V|\chi_E^{\mathrm{c}}\rangle \neq 0,$$

$$(V_j^{\mathrm{c}}(E))^2 \ll (V_0^{\mathrm{c}})^2, \qquad (V_j^{\mathrm{c}})^2 \ll V_{l0}\,. \qquad (4.1)$$

We represent in fig. 1 the strength of the different couplings. Because of this modification we now obtain, with the help of definition (2.9):

$$S_{\mathrm{cc}}(E) = \exp(i2\delta_{\mathrm{c}})\left[1-i\frac{\Gamma^{\uparrow}}{d(E)}-i2\pi\sum_{l=1}^{M}\frac{(v_l^{\mathrm{c}})^2}{E-e_l}\right.$$

$$\left.-\frac{i2\pi}{d(E)}\sum_{l=1}^{M}\frac{2V_0^{\mathrm{c}}v_l^{\mathrm{c}}V_l}{E-e_l}-\frac{i2\pi}{d(E)}\sum_{l,j=1}^{M}\frac{v_l^{\mathrm{c}}V_lv_j^{\mathrm{c}}V_j}{(E-e_l)(E-e_j)}\right]. \qquad (4.2)$$

The meaning of the different terms is clear: the first term corresponds to the processes $\chi^{\mathrm{c}}\to\Phi_0\to\chi^{\mathrm{c}}$; the second term to the processes $\chi^{\mathrm{c}}\to\Phi_j\to\chi^{\mathrm{c}}$, which are now allowed. The other ones correspond to more the complicated sequences $\chi^c \to \Phi_0 \to \Phi_j \to \chi^c_E$ and $\chi^c \to \Phi_j \to \Phi_0 \to \Phi_l \to \chi^c$. The non-diagonal matrix element of S is given by:

$$S_{cc'}(E) = -i2\pi \exp[i\delta_c + i\delta_{c'}] \left\{ \sum_{l=1}^{M} \frac{v_l^c v_l^{c'}}{E-e_l} + \frac{V_0^c}{d(E)} \sum_{l=1}^{M} \frac{v_l^{c'} V_l}{E-e_l} + \frac{1}{d(E)} \sum_{\substack{l=1\\ j=1}}^{M} \frac{v_l^c v_j^{c'} V_l V_j}{(E-e_l)(E-e_j)} \right\}. \tag{4.3}$$

The first term corresponds to the processes $\chi^c \to \Phi_j \to \chi^{c'}$, the second one to the processes $\chi^c \to \Phi_0 \to \Phi_j \to \chi^{c'}$ and the last one to the processes $\chi^c \to \Phi_l \to \Phi_0 \to \Phi_j \to \chi^c$. The first process is present everywhere, while the other two occur only in the vicinity of the doorway state. Elsewhere, the average reaction cross section can be put in the form:

$$\langle \sigma_{cc'} \rangle = K2\pi \frac{S_c S_{c'}}{S}, \tag{4.4}$$

with

$$S_c = \frac{2\pi\overline{(v_l^c)^2}^{l}}{d}, \qquad S_{c'} = 2\pi \frac{\overline{(v_l^{c'})^2}^{l}}{d}, \qquad S = S_c + S_{c'}. \tag{4.5}$$

We add the term (4.4) to expression (3.6) in order to improve the fit of the average cross section in the tails of the intermediate resonance. This amounts to keeping in eq. (4.3) only the first two terms in the brackets and to neglecting the interference between them. We justify this approximation by the usual statistical assumptions ($\overline{v_l^c V_l}^{l} = \overline{v_l^{c'} V_l}^{l} = 0$) and by the fact that the contribution of the term (4.4) appears to be very small in practice compared to the contribution of the term (3.6). In the examples given below, the ratio between them at the resonance energy is of the order of 0.05. Then, we get for the (n, f) cross section:

$$\langle \sigma_{nf} \rangle = K \left\{ 2\pi \frac{S_n S_f}{S} + \frac{\pi\gamma\Gamma^{\uparrow}\Gamma^{\downarrow}}{d} \times \frac{I}{2 \arctan 2} \int_{E-2I}^{E+2I} \frac{dE'}{(E-E')^2 + I^2} \times \frac{1}{\left[(E'-\varepsilon_0)\sin\frac{\pi(E'-\varepsilon_0)}{d} - \frac{1}{2}\Gamma^{\downarrow}\cos\frac{\pi(E'-\varepsilon_0)}{d}\right]^2 + \left[\frac{1}{4}\Gamma^{\uparrow 2}\sin^2\frac{\pi(E'-\varepsilon_0)}{d}\right]} \right\}, \tag{4.6}$$

where the notation is obvious.

5. A numerical example

We apply our model to the case of ^{240}Pu(n, f). The measurements have been made by Migneco and Theobald [1]). Groups of strong resonances have been found at several

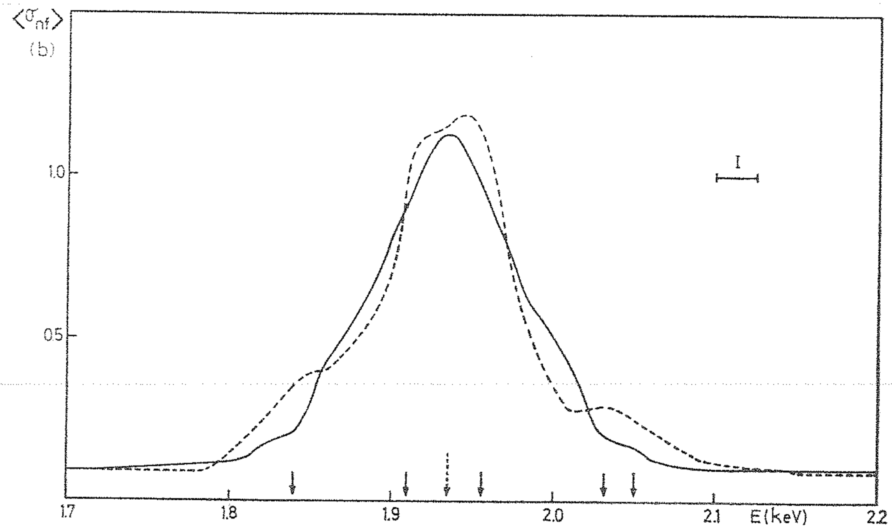

Fig. 2. Dotted curve: average ^{240}Pu(n, f) cross section in the region of 1900 eV. Full curve: best fit with the help of eq. (5.1). The dotted arrow indicates the value of the parameter ε_0. The full arrows indicate the location of the individual resonances.

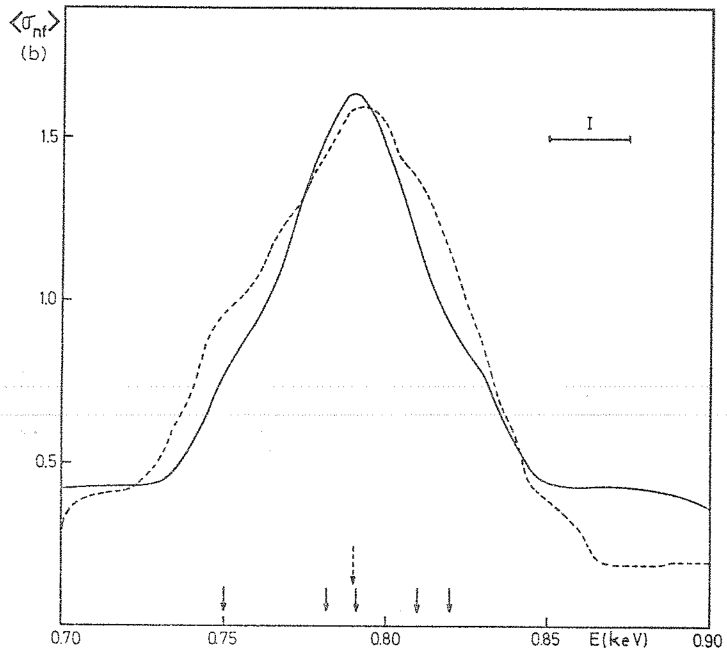

Fig. 3. The same as fig. 2 for the region of 800 eV.

energies of the incident neutron. We have selected two of them: those at about 800 eV and 1900 eV. Since the observed narrow resonances are not mixed with a broad one, we may deduce that we are facing one of the extreme cases: $\Gamma^{\uparrow} \gg \Gamma^{\downarrow}$, $\Gamma^{\uparrow} \ll \Gamma^{\downarrow}$. We fitted the average experimental reaction cross sections with the help of eq. (4.6) that we rewrite as:

$$\langle \sigma_{\mathrm{nf}} \rangle = \sigma_{\mathrm{B}} + \sigma_{\mathrm{res}}, \qquad \sigma_{\mathrm{B}} = 2\pi K \frac{S_{\mathrm{n}} S_{\mathrm{f}}}{S}. \tag{5.1}$$

We first determined σ_{B} from the background between the groups of resonances. The parameter ε_0 is set equal to the position of the experimental peak. We are then left with three free parameters γ, $\Gamma^{\uparrow}$ and $\Gamma^{\downarrow}$. In fact, according to the discussion of sect. 3, we should have fixed one of the two parameters $\Gamma^{\uparrow}$ and $\Gamma^{\downarrow}$. But we preferred to leave them free in order to check the predictions of subsect. 2.3.

We have taken for K:

$$K = \frac{\pi}{k_{\mathrm{n}}^2} g, \tag{5.2}$$

where k_{n} is the wave number of the incident nucleon and g is a degeneracy factor. We have put g equal to 2, which is a reasonable average value. We then obtain the following results:

(i) group at 800 eV

$$\varepsilon_0 = 790 \text{ eV}, \qquad \Gamma^{\uparrow} = 3 \text{ eV}, \qquad \Gamma^{\downarrow} = 400 \text{ meV}, \qquad \gamma = 7 \text{ meV};$$

(ii) group at 1900 eV

$$\varepsilon_0 = 1935 \text{ eV}, \qquad \Gamma^{\uparrow} = 31 \text{ eV}, \qquad \Gamma^{\downarrow} = 400 \text{ meV}, \qquad \gamma = 20 \text{ meV}.$$

The corresponding curves are given in figs. 2 and 3.

In this case $\Gamma^{\uparrow} \gg \Gamma^{\downarrow}$, the sum of the observed fission partial widths is equal to $\Gamma^{\downarrow}$ [refs. [17,18])]. By applying this sum rule we should have found $\Gamma^{\downarrow} = 170$ meV for the group at 800 eV and $\Gamma^{\downarrow} = 86$ meV for the group at 1900 eV. The agreement with the above values can be considered as good, owing to the approximations made in the derivation of eq. (5.1). We emphasize that the quantity γ/d is typically of the order of 10^{-3} which justifies the approximation made in eq. (3.4).

We can also extract two other interesting parameters: the fission strength function for the states in the external well $S_{\mathrm{e}}^{\mathrm{f}}$ and the fission strength function for the states in the internal well $S_{\mathrm{i}}^{\mathrm{f}}$. Indeed, the quantity $S_{\mathrm{e}}^{\mathrm{f}}$ is given by:

$$S_{\mathrm{e}}^{\mathrm{f}} = \frac{\langle \Gamma^{\downarrow} \rangle}{\langle D \rangle}, \tag{5.3a}$$

where $\langle \Gamma^{\uparrow} \rangle$ is the average of the decaying width over a few groups of resonances, and $\langle D \rangle$, their average distance. Roughly, we get

$$S_{\mathrm{e}}^{\mathrm{f}} \approx \frac{16 \text{ eV}}{650 \text{ eV}} \approx 0.025. \tag{5.3b}$$

The quantity S_i^f can be obtained from the analysis of the average background between groups of resonances. If this background is due to (n, f) reaction through the complicated states, we roughly have $S \approx S_n$ [see eq. (4.5)], and:

$$\sigma_B \approx 2\pi K S_i^f. \tag{5.4}$$

Such a formula gives in our case

$$S_i^f \approx 10^{-5}. \tag{5.5}$$

6. Comparison with another model

In order to check the results of our model, we compute the fission strength functions with the help of another model. Similarly to what is done for the nucleon strength function, the fission strength function can be given by:

$$S^f = \int_0^\infty W(\delta)|u(\delta)|^2 d\delta, \tag{6.1}$$

where $u(\delta)$ is the wave function corresponding to the deformation potential (δ = deformation parameter), and $W(\delta)$ is an imaginary part which is added to this potential to describe the absorption by the compound nucleus processes. For a double-humped barrier (see fig. 4), one has two kinds of compound states. Hence, one can define two fission strength functions corresponding to the two wells:

(i) an "internal" fission strength function

$$S_i^f = \int_0^\infty W_i(\delta)|u(\delta)|^2 d\delta, \tag{6.2}$$

(ii) an "external" fission strength function

$$S_e^f = \int_0^\infty W_e(\delta)|u(\delta)|^2 d\delta. \tag{6.3}$$

The quantities W_i and W_e differ from zero only in the internal and the external wells respectively. We have computed S_e^f using eq. (6.3). The deformation potential is taken from the work by Strutinsky [8]) for ^{240}Pu † and is reproduced in fig. 4. The imaginary potential W is equal to 1 MeV as in ref. [19]) in the limits indicated in fig. 4. The results are plotted on the same figure.

Several interesting features arise from this calculation. First the quantity S_e^f shows a remarkable structure. Maxima occur in the vicinity of the normal modes in the second well. This was already emphasized by Gai *et al.* [22]), using a quasiclassical picture, although they predict a simpler shape around the maxima. In our example, the second well is characterized by a vibrational quantum of $\hbar\omega = 0.35$ MeV. The normal modes are then located at 1.17, 1.52, 1.87 and 2.22 MeV. This agrees quite

† In our case, the compound nucleus is ^{241}Pu. Because of the lack of information concerning this nucleus, we take the deformation potential of ^{240}Pu, with the expectation that an extra nucleon does not destroy the shape of the potential very much.

well with bumps at 1.2, 1.47 and 2.05 MeV, if one takes into account the fact that the well does not have an exact parabolic form. This can be explained by saying that the system excites a quasistationary state, which means that the transmission coefficient is enhanced. These quasistationary states have a vibrational nature. In the region of

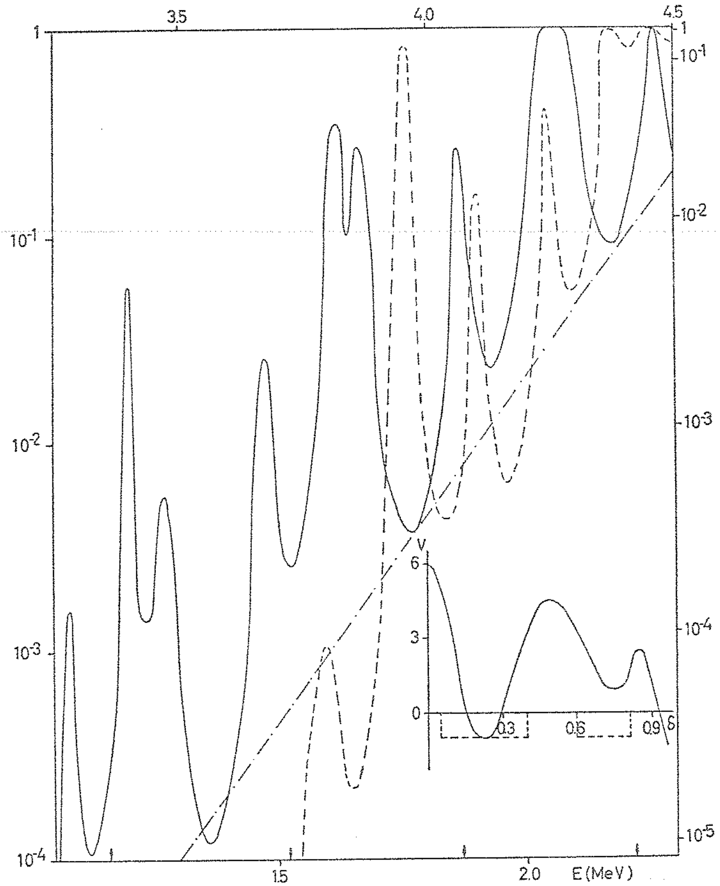

Fig. 4. The full curve represents the external fission strength function S_e^f corresponding to the barrier shown in the corner. The scale is on the right side. The scale on the left side is for $T_e^f = 1 - \exp\{-4\pi S_e^f\}$ which is represented by the same curve. The dotted line represents the internal fission strength function S_i^f for the energy region indicated on the top of the figure. The arrows indicate the energies of the normal modes in the external well.

the vibrational state, the fission widths of the compound nucleus states in the second well are broadened. This situation favours the occurrence of doorway state phenomena. Let us indicate that a doorway group of resonances covers at most 1 keV, which is a small part of fig. 4.

Such a situation also explains the broad subthreshold resonances in neutron-induced fission. Indeed, in the region below the first hump and above the second one, the internal strength function S_i^f is quite sizable and also displays resonating structures, which correspond to vibrational states in the first well (see fig. 4). This generates large fission partial widths for the compound states formed in the first well. Since at this energy they are very close to each other, only a gross structure appears in the shape of the strength function S_i^f. Thus it seems that broad subthreshold resonances occur when the first bump is higher than the second one. This corresponds to the equations of sect. 5 when doorway states are removed. One is left with an average cross section given by

$$\langle \sigma_{nf} \rangle \approx 2\pi K \frac{S_n S_f}{S}, \tag{6.4}$$

where S_f contains a penetration factor and is proportional to S_i^f. Even in the region of the doorway phenomenon, the quantities S_i^f can be extracted from experiment. The background between resonances (provided one is sure it is only due to fission) is given by eq. (6.4). Since $S_n \approx S$, the background amplitude is given by S_f (up to a kinematical factor).

In order to compare the two models, one has to answer the question: what is the compound nucleus excitation energy corresponding to a neutron incident energy of 1 or 2 keV? Let us only indicate that the fission threshold (the top of the external hump) corresponds to about 700 keV for the incident neutron energy [1]). That gives for an incident neutron energy of 1 keV, a compound nucleus energy of about 1.8 MeV in the scale of fig. 4. Eq. (6.3) leads to a value of about 10^{-3} for S_c^f (see fig. 4) which agrees with a value in eq. (5.3b). It is worthwhile to point out that eq. (6.3) predicts a "doorway region" at about 1.8 MeV and also at 2.1 MeV. Calculations indicate that the value of (5.5) for S_i^f is overestimated by two or three orders of magnitude, but this is not surprising, since the background is probably not due to fission only. Let us finally notice that eq. (6.2) predicts broad subthreshold resonances at 3.9, 4.1 and 4.3 MeV. This could be checked experimentally.

7. Discussion

We have justified the application of our model to neutron-induced fission by the fact that the equations of the doorway phenomenon are expected to be independent of the nature of the degrees of freedom involved in the reaction. This assumption is supported by the fact that the average (n, f) cross sections are remarkably well fitted by the formulae obtained in the framework of the shell-model approach to nuclear reactions. This agreement is much better in our case than in the cases of ^{56}Fe(n, n) [ref. [20])] and ^{206}Pb(n, n) [ref. [21])].

We have assumed that all the open fission channels can be treated as a single one. This is safe here [19]) since only one saddle-point channel occurs in the case of ^{240}Pu [ref. [1])] but can be wrong in other cases. We have also neglected the photon channels. This should have the effect of decreasing the (n, f) cross section. As a consequence, the value of γ [see eq. (5.1)] is probably underestimated.

Since $\Gamma^{\uparrow} \gg \Gamma^{\downarrow}$, the (n, f) intermediate resonances have the character of strong doorway resonances. Such a value of $\Gamma^{\uparrow}$ may seem doubtful, since it would not allow the existence of shape isomerism. In fact, there is no contradiction, since isomer states lie much lower in the bottom of the second well. We see from fig. 4 that the fission width of the states in the bottom of the well is very small. We finally conclude that our model enables us to extract the doorway parameters from experiment.

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