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OPTICAL-MODEL POTENTIAL AND SINGLE-PARTICLE RESONANCES

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Abstract: Two methods are described which lead to a definition of the optical-model potential when single-particle resonances are present. They are based on a modification of the single-particle potential. For one method, the properties of the modified potential are extensively discussed.

1. Introduction

Many shell-model theories [1-3]) lead to a microscopic expression for the optical-model potential. But none applies when single-particle resonances (SPR) are present. In that case, it is only possible to define a generalized optical-model potential in channel c [GOMP(c), refs. [1, 2])], which is a one-body potential, the scattering function of which is equal to the diagonal matrix element of the scattering matrix in channel c. When no narrow SPR exists in channel c, it is possible to construct a one-body potential, called the optical-model potential in channel c (OMP(c)), which leads to a scattering function equal to the average diagonal element of the scattering matrix in channel c: OMP(c) is then equal to GOMP(c) evaluated at the complex energy $E+iI$, where I is the averaging interval [4]). When the single-particle potential generates narrow resonances, this is no longer valid because of the presence of rapidly varying matrix elements in the expression for GOMP(c) [ref. [1])]. This is explained in sect. 2.

In this paper, we try to cure this situation. The method described in sect. 3 consists in modifying the single-particle potential in order to remove the SPR. Then an expression for OMP(c) can be derived. In sect. 4, we apply another modification of the single-particle potential recently proposed by Wang and Shakin [5]). Finally, sect. 5 contains our conclusions.

2. The problem

In this section, we establish our notation and recall the reasons which complicate the derivation of OMP(c) when SPR are present.

2.1. DEFINITION OF GOMP(c)

In the shell-model approach to nuclear reactions [1]), the full Hamiltonian is separated into two parts:

$$H = H_0 + V. \tag{2.1}$$

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The "shell-model" Hamiltonian H_0 possesses bound states Φ_i and scattering χ_E^c (with only one unbound nucleon), for which we have the following relations and definitions:

$$\begin{aligned} H_0\Phi_i &= E_i\Phi_i, \qquad H_0\chi_E^c = E\chi_E^c, \\ \langle\Phi_i|\Phi_j\rangle &= \delta_{ij}, \qquad \langle\Phi_i|\chi_E^c\rangle = 0, \qquad \langle\chi_E^c|\chi_{E'}^{c'}\rangle = \delta_{cc'}\delta(E-E'), \\ \langle\Phi_i|V|\Phi_j\rangle &= V_{ij}, \qquad \langle\Phi_i|V|\chi_E^c\rangle = V_i^c(E), \\ \langle\chi_E^c|V|\chi_{E'}^{c'}\rangle &= V(E', E, c', c). \end{aligned} \tag{2.2}$$

The Lippman-Schwinger equation reads:

$$\Psi_E^{c(+)} = \chi_E^{c(+)} + \frac{1}{E^+ - H_0} V\Psi_E^{c(+)}. \tag{2.3}$$

The wave function $\Psi_E^{c(+)}$ contains only outgoing waves in the channels $c' \neq c$. Writing it in the form

$$\Psi_E^{c(+)} = \sum_{i=1}^{M} b_E^c(i)\Phi_i + \sum_{c'}\int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E' a_E^c(E', c')\chi_{E'}^{c'}, \tag{2.4}$$

and taking into account eq. (2.3), one obtains the system:

$$(E-E_i)b_E^c(i) - \sum_{j=1}^{M} V_{ij}\, b_E^c(j) - \sum_{c'}\int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E' V_i^c(E') a_E^c(E', c') = 0, \tag{2.5a}$$

$$(E-E')a_E^c(E', c) - \sum_{m=1}^{M} V_m^{c'}(E') b_E^c(m) - \sum_{c''}\int_{\varepsilon_{c''}}^{\infty} \mathrm{d}E'' a_E^c(E'', c'')V(E'', E', c'', c') = 0. \tag{2.5b}$$

The quantities ε_c are the threshold energies.

The calculation of GOMP(c) amounts to eliminating the coefficients $b_E^c(i)$ and $a_E^c(E', c')$ for $c' \neq c$ in the system (2.5). One gets an integral equation of the form

$$(E-E')a_E^c(E', c) - \int_{\varepsilon_c}^{\infty} \mathrm{d}E'' \mathscr{V}_E^c(E', E'') a_E^c(E'', c) = 0. \tag{2.6}$$

Eq. (2.6) is simply the shell-model representation of the Schrödinger equation obeyed by the generalized optical-model wave function whose definition is [1])

$$\rho_E^c(r) = \int_{\varepsilon_c}^{\infty} \mathrm{d}E' a_E^c(E', c) u_{lj}(r, k_c'), \tag{2.7}$$

where $u_{lj}(r, k_c')$ is the single-particle wave function of the unbound particle in χ_E^c. The manner on which one obtains GOMP(c) starting from the quantity $\mathscr{V}_E^c(E', E'')$ is described in ref. [1]). In the following we restrict ourselves to the latter quantity.

If there is no SPR in channel c, GOMP(c) is given by [1]):

$$\mathscr{V}_E^c(E', E'') = \mathscr{V}_E^{c(\mathrm{dir})}(E', E'') + \sum_{j,m=1}^{M} W_{E,j}^c(E')[{}_s^c D^{-1}(E)]_{jm} W_{E,m}^c(E''), \tag{2.8a}$$

with

$$\mathscr{V}_E^{c(\mathrm{dir})}(E', E'') = \langle \chi_{E'}^c | V + V \sum_{c' \neq c} G_0^{c'}(E) V + V \sum_{c' \neq c} G_0^{c'}(E) V \sum_{c'' \neq c} G_0^{c''}(E) V \ldots | \chi_{E''}^c \rangle, \quad (2.8b)$$

$$W_{E,j}^c(E') = V_j^c(E') + \langle \Phi_j | V \sum_{c' \neq c} G_0^{c'}(E) V + V \sum_{c' \neq c} G_0^{c'}(E) V \sum_{c'' \neq c} G_0^{c''}(E) V \ldots | \chi_{E'}^c \rangle, \quad (2.8c)$$

$$[{}_s^c D(E)]_{jm} = (E - E_j)\delta_{jm} - \langle \Phi_j | V + V \sum_{c' \neq c} G_0^c(E) V + \ldots | \Phi_m \rangle, \quad (2.8d)$$

$$G_0^{c'}(E) = \int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E' \frac{|\chi_{E'}^{c'}\rangle\langle\chi_{E'}^{c'}|}{E^+ - E'}. \quad (2.9)$$

When a SPR is present, the series in eqs. (2.8b, c, d) are divergent. It is still possible [6]) to calculate GOMP(c) using a Weinberg's method to handle the SPR.

2.2. DEFINITION OF OMP(c)

Because of the very definition of OMP(c), the proper optical-model wave function behaves like

$$I_{lj}(r, k_c) - \langle S_{cc} \rangle u_{lj}(r, k_c). \quad (2.10)$$

We assume that it can be written in the form

$$\tilde{\rho}_E^c(r) = \int_{\varepsilon_c}^{\infty} \mathrm{d}E' \tilde{a}_E^c(E', c) u_{lj}(r, k_c'). \quad (2.11)$$

The coefficients $\tilde{a}_E^c(E', c)$ satisfy the following integral equation

$$(E - E')\tilde{a}_E^c(E', c) - \int_{\varepsilon_c}^{\infty} \mathrm{d}E'' \tilde{\mathscr{V}}_E^c(E', E'') \tilde{a}_E^c(E'', c) = 0. \quad (2.12)$$

OMP(c) is then given by [1]):

$$\mathscr{V}_E^{\mathrm{opt}(c)}(E', E'') = v_0(E', E'') + \tilde{\mathscr{V}}_E^c(E', E''), \quad (2.13)$$

$v_0(E', E'')$ being the shell-model representation of the model potential. It is easy to construct $\tilde{\mathscr{V}}_E^c$ if no narrow SPR is present in channel c. In this case, we have [1, 4]):

$$\tilde{\mathscr{V}}_E^c = \mathscr{V}_{E+iI}^c. \quad (2.14)$$

When a SPR is present in channel c, this is no longer valid [1]) because of the rapid variation of the matrix elements $\langle \Phi_i | V | \chi_E^c \rangle$ and $\langle \chi_{E'}^{c'} | V | \chi_E^c \rangle$ and the phase δ_c in the vicinity of the resonance. Let us note that if the SPR is present in channel $c' \neq c$, the relation (2.13) still holds.

The difficulty encountered in the construction of OMP(c) when a SPR is present in channel c is twofold: (i) there is a slow convergence of the Born series, (ii) the phase shift δ_c and the matrix elements $V_i^c(E)$ and $V(E', E, c', c)$ are strongly varying in the vicinity of the SPR energy.

3. Modification of the single-particle potential

3.1. DEFINITION OF THE NEW SINGLE-PARTICLE POTENTIAL

In the present section, we study a method for removing the SPR. This method consists in a modification of the single-particle Hamiltonian. The Hamiltonian H_0 is given by $\sum_i h_0(i)$, where

$$h_0 = t + v_0, \tag{3.1}$$

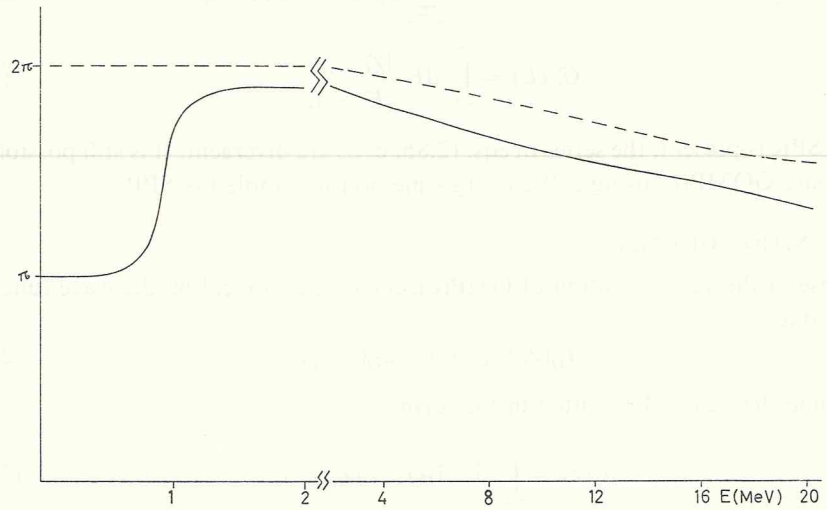

Fig. 1. Full curve: d-phase shift for a square well whose depth = 58 MeV and range = 4.55497 fm. Dotted curve: d-phase shift for the potential $\hat{v}_0$ defined by eq. (3.3), where v_0 is the above square well. To make the comparison easier, the dotted curve has been raised by π.

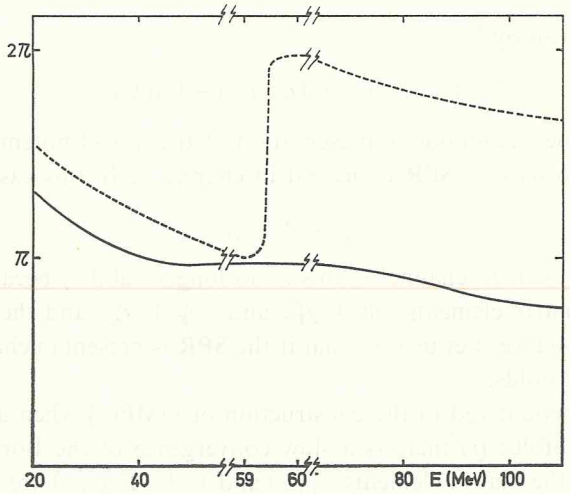

Fig. 2. The same as fig. 1 for higher energies.

the quantities t and v_0 being the kinetic energy and the one-body model potential. We assume that the latter displays a narrow resonance at the complex energy

$$\mathscr{E}_0 = E_0 - \tfrac{1}{2}i\Gamma_0 \tag{3.2}$$

for a given partial wave of orbital and total angular momentum quantum numbers l and j.

Let us define the potential $\hat{v}_0$ by the relation:

$$\hat{v}_0 = v_0 - \frac{v_0|w\rangle\langle w|v_0}{\langle w|v_0|w\rangle}. \tag{3.3}$$

The function w is the Gamow function †. It can easily be seen that operation (3.3) removes a pole of the single-particle Green function, lying nearby the real axis in the complex energy plane.

Let us now discuss the behaviour of the phase shift for the two potentials. In the neighbourhood of E_0, the phase shift $\delta_{lj}(E)$ for potential v_0 is characterized by a rapid increase of the phase shift by $\frac{1}{2}\pi$. What is the energy dependence of the phase shift $\tilde{\delta}$ corresponding to the potential (3.3)? The rapid increase has disappeared as indicated in fig. 1 which exhibits the example of a d-wave narrow resonance in a square well. This arises because of the removal of the pole from the Green function $1/(E^+ - t - v_0)$, which can be explained by the following arguments. It can easily be seen that the phase shift $\tilde{\delta}$ is given by

$$\tilde{\delta} = \delta + \delta_a, \tag{3.4a}$$

$$\delta_a = \arctan \frac{\pi|\langle w|v_0|u_E\rangle|^2}{D + \langle w|v_0(PG_E^+)v_0|w\rangle} \text{ ††}, \tag{3.4b}$$

$$D = \langle w|v_0|w\rangle, \tag{3.4c}$$

where PG_E^+ is the principal part of the Green function corresponding to the unperturbed potential. In the vicinity of the SPR, the numerator in eq. (3.4b) can be parametrized as [1])

$$\pi|\langle w|v_0|u_E\rangle|^2 \approx \frac{1}{2}\frac{\Gamma_0 D^2}{(E-E_0)^2 + \frac{1}{4}\Gamma_0^2}. \tag{3.5}$$

In the same region, the Green function is well approximated by

$$G_E^+ \approx \frac{|w\rangle\langle w|}{E - \mathscr{E}_0} \tag{3.6a}$$

† If v_0 has a long-range part, such as a Coulomb part, only the short-range part must appear in the subtracted term of eq. (3.3).

†† Eq. (3.4b) is obtained by suppressing the bound-state component of the scattering state. It can be seen that to take this component into account, it amounts to renormalize the quantity D, (3.4c).

and its principal part by:

$$PG_E^+ \approx w\rangle\langle w \frac{E-E_0}{(E-E_0)^2+\frac{1}{4}\Gamma_0^2}. \tag{3.6b}$$

Gathering together these results we get for δ_a:

$$\delta_a = \arctan\left\{\frac{1}{2}\frac{\Gamma_0 D^2}{D[(E-E_0)^2+\frac{1}{4}\Gamma_0^2]+D^2(E-E_0)}\right\}. \tag{3.4d}$$

Since Γ_0 is very small compared to D (which is of the order of the well depth) this formula reduces, for $|E-E_0| < \frac{1}{2}\Gamma_0$, to

$$\delta_a \approx \arctan\frac{\Gamma_0}{2(E-E_0)}, \tag{3.4e}$$

except for a very small range of values of E, namely $|E-E_0| \lessapprox \Gamma_0^2/4D$. For instance, for $E = E_0$, eq. (3.4d) gives $\delta_a = \arctan 2D/\Gamma_0$ which deviates very little from eq. (3.4c). We can conclude by saying that the modification (3.3) suppresses the SPR in the potential v_0.

In our numerical example, the phase shift $\tilde{\delta}$ increases rapidly through $\frac{1}{2}\pi$ near 59 MeV (see fig. 2). This can be explained by the following argument. Let us apply the modified Hamiltonian $\tilde{h} = t+\hat{v}_0$ to the Gamow states $|w\rangle$. We get

$$\tilde{h}|w\rangle = \{\mathscr{E}_0 - v_0\}|w\rangle. \tag{3.7}$$

If we cut the Gamow state in the configuration space for $r > a$ (the range of the square well), it becomes an eigenstate of $\tilde{h}$ with the eigenvalue $\tilde{\mathscr{E}}_0 = \mathscr{E}_0 + V_0$, V_0 being the depth of the square well. In our example, Re $\tilde{\mathscr{E}}_0$ is equal to 58.944 MeV. This modified Gamow state does not have the properties of a scattering state, but more or less those of a bound state. This is an example of what is called by Bolsterli [8]) "bound state in the continuum".

Of course, this argument is not rigorous (derivability) but allows to understand simply the effect of the modification (3.3). This bound state in the continuum is not a special property of our model, but it occurs for each Hamiltonian of the type $\tilde{h} = h - g|f\rangle\langle f|$, for which $|f\rangle$ is an eigenstate of h. If we call $E_0 = \langle f|h|f\rangle/\langle f|f\rangle$ and $\tilde{E}_0 = E_0 - g\langle f|f\rangle$, one can show that the denominator of the S-matrix relative to $\tilde{h}$, namely $f^+(E) = 1+g\langle f|G^+(E)|f\rangle$ † vanishes (real and imaginary parts together) at $\tilde{E}_0$ (those conditions ensure, following Bolsterli [8]), the existence of a bound state in the continuum). Indeed we have, if u_E is the scattering state relative to h,

$$\langle u_{\tilde{E}_0}|\tilde{h}|f\rangle = \tilde{E}_0\langle u_{\tilde{E}_0}|f\rangle \tag{3.8a}$$

and by operating with $\tilde{h}$ on the left, we obtain:

$$\langle u_{\tilde{E}_0}|\tilde{h}|f\rangle = \tilde{E}_0\langle u_{\tilde{E}_0}|f\rangle - g\langle f|f\rangle\langle u_{\tilde{E}_0}|f\rangle, \tag{3.8b}$$

† See preceding footnote.

and hence

$$\langle u_{\tilde{E}_0}|f\rangle = 0. \tag{3.8c}$$

It can also be shown that the projection on $|f\rangle$ of the scattering state $|\psi_E\rangle$ relative to the Hamiltonian $\tilde{h}$ is given by

$$\langle f|\psi_E\rangle = \mathrm{e}^{i\delta}\langle f|u_E\rangle(1+g\langle f|G^+(E)|f\rangle)^{-1}. \tag{3.9}$$

Since this projection cannot vanish for $E = \tilde{E}_0$, the real and imaginary parts of the parenthesis must go to zero at this energy.

Let us finally notice that in general bound states are only slightly modified by eq. (3.3). In our numerical example the bound state at -32.7 MeV is shifted to -31.5 MeV (operation (3.3) makes the potential less attractive).

3.2. CALCULATION OF THE OPTICAL-MODEL POTENTIAL

Let us denote by $\hat{H}_0$ and $\hat{V}$ the quantities

$$\hat{H}_0 = H_0 - \sum_{\text{part}} \frac{v_0|w\rangle\langle w|v_0}{\langle w|v_0|w\rangle}, \tag{3.10a}$$

$$\hat{V} = V + \sum_{\text{part}} \frac{v_0|w\rangle\langle w|v_0}{\langle w|v_0|w\rangle}, \tag{3.10b}$$

where $\sum_{\text{part}}$ means that the summation is extended over the coordinates of all the particles. This new splitting of the Hamiltonian defines a new basis $\{\hat{\Phi}_i, \hat{\chi}^{c'}_{E'}\}$ obtained on the same manner as the $\{\Phi_i, \chi^c_E\}$ [ref. [1])]†. We assume that the channel c is the only one which corresponds to a free nucleon with quantum numbers l and j. Now the formulae (2.5) are replaced by

$$(E-\hat{E}_i)\hat{b}^c_E(i) - \sum_{j=1}^{M} \hat{V}_{ij}\hat{b}^c_E(j) - \sum_{c'}\int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E'\hat{a}^c_E(E', c')\hat{V}^{c'}_i(E') = 0, \tag{3.11a}$$

$$(E-E')\hat{a}^c_E(E', c') - \sum_{m=1}^{M} \hat{b}^c_E(m)\hat{V}^c_m(E') - \sum_{c''}\int_{\varepsilon_{c''}}^{\infty} \mathrm{d}E''\hat{a}^c_E(E'', c'')\hat{V}(E'', E', c'', c') = 0, \tag{3.11b}$$

with

$$\Psi^{c(+)}_E = \sum_{i=1}^{M} \hat{b}^c_E(i)\hat{\Phi}_i + \sum_{c'}\int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E'\hat{a}^c_E(E', c')\hat{\chi}^{c'}_{E'}, \tag{3.11c}$$

$$\hat{E}_i = \langle\hat{\Phi}_i|\hat{H}_0|\Phi_i\rangle, \qquad \hat{V}_{ij} = \langle\hat{\Phi}_i|\hat{V}|\hat{\Phi}_j\rangle, \tag{3.11d}$$

$$V^c_m(E') = \langle\hat{\Phi}_m|\hat{V}|\hat{\chi}^c_{E'}\rangle, \qquad \hat{V}(E'', E', c'', c') = \langle\hat{\chi}^{c''}_{E''}|\hat{V}|\hat{\chi}^{c'}_{E'}\rangle. \tag{3.11e}$$

† Since we diagonalize the Hamiltonian within this basis, eqs. (3.10) define more exactly a new problem rather than a new splitting of the Hamiltonian. Nevertheless we hope that the two basis are sufficiently equivalent. This is justified for instance by the fact that the number of resonances is conserved as we show later.

We must now take the continuum-continuum interaction correctly into account. It is clearly not allowed to treat it by perturbation since then eqs. (3.11a, b) will lead to a model with M resonances while the starting model generates $M+1$ resonances. One may exhibit the $(M+1)$th resonance by extracting from $\hat{V}(E'', E', c'', c')$ a part which is separable and thus can be taken exactly into account. For this we write

$$\hat{V}(E'', E', c'', c') = \langle \hat{\chi}^{c''}_{E''} | V | \hat{\chi}^{c'}_{E'} \rangle + \langle \hat{\chi}^{c''}_{E''} | \sum \frac{v_0|w\rangle\langle w|v_0}{\langle w|v_0|w\rangle} | \hat{\chi}^{c'}_{E'} \rangle. \tag{3.12}$$

Since the last operator is a one-body operator, the second term in the right-hand side of the last equation is equal to

$$\frac{\langle \hat{u}_{cE''}|v_0|w\rangle\langle w|v_0|\hat{u}_{cE'}\rangle}{\langle w|v_0|w\rangle} \delta_{cc''}\delta_{cc'}, \tag{3.13}$$

where the functions $\hat{u}_{c'E'}$ are the wave functions of the scattering particle in the states $\hat{\chi}^{c'}_{E'}$. In eq. (3.13), the Kronecker symbols stand because the potential $(v_0|w\rangle\langle w|v_0)/\langle w|v_0|w\rangle$ acts in the space of single-particle states of angular momentum l, j (which correspond to channel c only). Let us now define the following quantities:

$$\tilde{V}(E'', E', c'', c') = \langle \hat{\chi}^{c''}_{E''} | V | \hat{\chi}^{c'}_{E'} \rangle \quad (= \hat{V}(E'', E', c'', c') \quad \text{if} \quad c'' \quad \text{or} \quad c' \neq c), \tag{3.14a}$$

$$\hat{V}^c_{M+1}(E'') = \langle w|v_0|\hat{u}_{cE'}\rangle, \qquad \hat{V}^{c'}_{M+1}(E) = 0 \qquad \text{for} \quad c' \neq c, \tag{3.14b}$$

$$\hat{b}^c_E(M+1) = \int_{\varepsilon_c}^{\infty} \mathrm{d}E'' \hat{a}^c_E(E'', c) \frac{\langle u_{cE''}|v_0|w\rangle}{\langle w|v_0|w\rangle}. \tag{3.14c}$$

We can write eq. (3.11b) in the form

$$(E-E')\hat{a}^c_E(E', c') - \sum_{m=1}^{M+1} \hat{b}^c_E(m)\hat{V}^c_m(E') - \sum_{c''}\int_{\varepsilon_{c''}}^{\infty} \mathrm{d}E'' \hat{a}^c_E(E'', c'')\tilde{V}(E'', E', c'', c') = 0, \tag{3.15a}$$

and eq. (3.14c) in the form

$$\langle w|v_0|w\rangle \hat{b}^c_E(M+1) = \int_{\varepsilon_c}^{\infty} \mathrm{d}E'' \hat{a}^c_E(E'', c)\hat{V}^c_{M+1}(E''). \tag{3.16}$$

The latter equation, together with eq. (3.11a) can be written as follows, with the help of eq. (3.14b):

$$\sum_{j=1}^{M+1} \hat{B}_{ij}\hat{b}^c_E(j) - \sum_{c'}\int_{\varepsilon_{c'}}^{\infty} \mathrm{d}E' \hat{a}^c_E(E', c')\hat{V}^{c'}_i(E') = 0, \tag{3.15b}$$

with

$$\hat{B}_{ij} = \begin{pmatrix} (E-\hat{E}_i)\delta_{ij} - \hat{V}_{ij} & 0 \\ 0 & \langle w|v_0|w\rangle \end{pmatrix}. \tag{3.15c}$$

Eqs. (3.15a, b) define the new model which is shown to display $M+1$ resonances. They are similar in form to eq. (2.5) We can thus take over the procedure of ref. [1])

to calculate the GOMP(c). We get for this quantity an expression similar to the one of eq. (2.8a) provided the quantities Φ, χ, V are replaced by $\hat{\Phi}$, $\hat{\chi}$, $\hat{V}$(or $\tilde{V}$). Now, however, no embarrassing energy variation is present in the matrix elements. Hence, one gets for OMP(c) in shell-model representation, using eqs. (2.8), (2.13) and (2.14):

$$\mathscr{V}_E^{\mathrm{opt}(c)}(E', E'') = \hat{v}_0(E', E'') + \hat{\mathscr{V}}_E^{c(\mathrm{dir})}(E', E'') + \sum_{j,m=1}^{M} \hat{W}_{E,j}^{c}(E')[{}_s^c\hat{D}^{-1}(E+iI)]_{jm} W_{E,m}^{c}(E''), \quad (3.17a)$$

since the quantities $\hat{\mathscr{V}}_E^c$ and $\hat{W}_E^c$ are smoothly energy-dependent. An interesting form of OMP(c) can be obtained by assuming the non-diagonal matrix elements of $\hat{D}_s^c$ to have mutually cancelling effects. One readily gets:

$$\mathscr{V}_E^{\mathrm{opt}(c)}(E', E'') \approx v_0(E', E'') + \hat{\mathscr{V}}_E^{c(\mathrm{dir})}(E', E'') + \sum_{j=1}^{M} \frac{\hat{W}_{E,j}^{c}(E')\hat{W}_{E,m}^{c}(E'')}{E+iI-Ej}. \quad (3.17b)$$

This equation shows that the main effect of the SPR is to replace the old model functions χ_E^c by the new ones $\hat{\chi}_E^c$ in the two last terms of the rhs of eq. (3.17b), without changing the potential term.

4. Orthogonalization method

Recently a method has been proposed by Wang and Shakin [5]) to handle the SPR. In this section, we apply this method to construct OMP(c). Let us first resume the ideas of Wang and Shakin. One defines a resonance wave packet u_R as:

$$u_R = C u_{lj}(r, k_0) \quad \text{for} \quad r < R_c$$
$$= 0 \quad \text{for} \quad r > R_c. \quad (4.1)$$

The quantity C is a normalization constant, $u_{lj}(r, k_0)$ is the wave function at the resonance of the nucleon in the potential v_0 and R_c is some cut-off radius. The wave functions $u_{lj}(r, k)$ can be modified by

$$\hat{u}_{lj}(r, k) = \left[1 - \frac{G_\alpha^+(E)|u_R\rangle\langle u_R|}{\langle u_R|G_\alpha^+(E)|u_R\rangle}\right] u_{lj}(r, k), \quad (4.2a)$$

with

$$G_\alpha^+(E) = \int_0^\infty \mathrm{d}E' \frac{u_{lj}(r, k')u_{lj}(r, k')}{E^+ - E'}. \quad (4.2b)$$

This ensures u_R is orthogonal to $\hat{u}_{lj}(r, k)$. Moreover, u_R is approximately orthogonal to the original bound states $w_{lj}(r, k_n)$ provided R_c is taken large enough †. It is

† This means that the bound states are left approximately unchanged. This is exactly true if one takes for u_R the following definition

$$u_R = C\left\{u_{lj}(r, k_0) - \sum_n w_{lj}(r, k_n)\int_0^{R_c} \mathrm{d}r' w_{lj}(r', k_n)u_{lj}(r', k_0)\right\}$$

shown in ref. [5]) that the modification (4.2) is equivalent to redefining the single-particle Hamiltonian. The modified phase shift $\hat{\delta}$ behaves as in our method (see fig. 1). Because of the quasi-orthogonality of u_R with the original bound states $w_{lj}(r, k_n)$, the modified bound states are approximately the same.

Instead of the model states $\{\Phi_j, \chi_E^c, \chi_{E'}^{c'}\}$ for the full problem, we have now a set $\equiv\{\Phi_j, j = 1, \ldots, M, \Phi_R, \hat{\chi}_E^c, \chi_E^{c'}\}$ where the state Φ_R is obtained by coupling the surface wave function φ_c with the "resonance wave packet" $u_R(r)$. Hence, we may rewrite eq. (2.4) as:

$$|\Psi_E^{c(+)}\rangle = \sum_{j=1}^{M+1} \hat{b}_E^c(j)|\Phi_j\rangle + \sum_{c'} \int_{\varepsilon_{c'}}^{\infty} dE' \hat{a}_E^c(E', c')|\hat{\chi}_{E'}^c\rangle, \tag{4.3a}$$

where

$$|\Phi_{M+1}\rangle = |\Phi_R\rangle. \tag{4.3b}$$

The Hamiltonian is now divided in

$$H = \hat{H}_0 + \hat{V}, \tag{4.4}$$

with

$$\hat{H}_0 = \sum_i \hat{h}_0(i) \tag{4.5}$$

and [5])

$$\hat{V} = \sum_{i<j} V_{ij} + \sum_i (h_0(i) - \hat{h}_0(i)), \tag{4.6}$$

$$\hat{h}_0 = (1 - u_R\rangle\langle u_R) h_0 (1 - u_R\rangle\langle u_R). \tag{4.7}$$

Really, this new splitting is effective only in the space spanned by Φ_R and $\hat{\chi}_E^c$; it coincides with the old one on the rest of the Hilbert space. With the help of eqs. (4.6) and (4.5), one obtains equations similar to eq. (2.5). They write with obvious notation

$$(E - \hat{E}_i)\hat{b}_E^c(i) - \sum_{j=1}^{M+1} \hat{V}_{ij} \hat{b}_E^c(j) - \sum_{c'} \int_{\varepsilon_{c'}}^{\infty} dE' \hat{V}_i^c(E') \hat{a}_E^c(E', c') dE' = 0, \tag{4.8a}$$

$$(E - E')\hat{a}_E^c(E', c') - \sum_{m=1}^{M+1} \hat{V}_m^c(E') \hat{b}_E^c(m) - \sum_{c''} \int_{\varepsilon_{c''}}^{\infty} dE'' \hat{a}_E^c(E'', c'') \hat{V}(E'', E', c'', c') = 0, \tag{4.8b}$$

with, according to eq. (4.7),

$$\hat{E}_{M+1} = 0. \tag{4.9}$$

It is easy to see that $\hat{V} - V$ acts only in the space spanned by Φ_R and $\hat{\chi}_E^c$. One has

$$\begin{aligned} \hat{V}_{M+1, M+1} &= \langle \Phi_R | V | \Phi_R \rangle + \langle \Phi_R | \sum_i (h_0(i) - \hat{h}_0(i)) | \Phi_R \rangle \\ &= \langle \Phi_R | V | \Phi_R \rangle + \langle u_R | h_0 | u_R \rangle. \end{aligned} \tag{4.10}$$

Because of the very definition of u_R, the second term is approximately equal to E_0. Thus we may keep eq. (4.8) with

$$\hat{E}_{M+1} = E_0, \qquad \hat{V}_{M+1, M+1} = \langle \Phi_R | V | \Phi_R \rangle. \tag{4.11}$$

The model described by eqs. (4.8) and (4.11) is the same as the one defined by eq. (2.5). One gets thus for the OMP(c) with obvious notation:

$$\mathscr{V}_E^{\mathrm{opt}(c)}(E', E'') = \hat{v}_0(E', E'') + \sum_{j,m=1}^{M+1} \hat{W}^c_{E,j}(E')[{}^c_s\hat{D}^{-1}(E+iI)]_{jm}\,\hat{W}^c_{E,m}(E'')$$

$$+\langle\hat{\chi}^c_{E'}|\hat{V}|\hat{\chi}^c_{E''}\rangle + \sum_{c'\neq c}\langle\hat{\chi}^c_{E'}|V|\int_{\varepsilon_{c'}}^{\infty} dE'''\,\frac{\chi^{c'}_{E'''}\rangle\langle\chi^{c'}_{E'''}}{E^+ - E'''}\,|V|\hat{\chi}^c_{E''}\rangle + \ldots. \qquad (4.12)$$

Here, we see that the compound nucleus states and the SPR are treated on the same footing.

5. Conclusions

We have examined the question of finding an expression for OMP(c) when a SPR is present in the entrance channel. We have seen that for handling the SPR conveniently, one has to modify the basic states of the unperturbed Hamiltonian. This contrasts with the cases where one is calculating the S-matrix or GOMP(c) and where such a modification is not required [1]).

Our method and that of Wong and Shakin are very similar, since the main idea is to subtract a pole from the single-particle Green function. However our method seems easier in practice, as far as phase shifts are concerned, since our modified potential has a separable form. A method described by Fuller [7]) for handling the SPR is quite similar to the one described in sect. 3. However, it seems to us that the latter is simpler in practice, because it does not require a new definition of the inner product nor a normalized Gamow function.

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