European Journal of Psychology of Education
2005, Vol. XX, n° 4, 355-367

The use of mathematical symbolism in problem solving: An empirical study carried out in grade one in the French community of Belgium

Annick Fagnant
Université de Liège, Belgium

This article relates to an empirical study based on the use of mathematical symbolism in problem solving. Twenty-five pupils were interviewed individually at the end of grade one; each of them was asked to solve and symbolize 14 different problems. In their classical curriculum, these pupils have received a traditional education based on a "top-down" approach (an approach that is still applied within the French Community of Belgium): conventional symbols are presented to the pupils immediately with an explanation of what they represent and how they should be used. Teaching then focuses on calculation techniques (considered as a pre-requisite for solving problems). The results presented here show the abilities (and difficulties) demonstrated by the children in making connections between the conventional symbolism taught in class and the informal approaches they develop when faced with the problems that are put to them. The limits of the "top-down" approach are then discussed as opposed to the more innovative "bottom-up" type approaches, such as those developed by supporters of Realistic Mathematics Educations in particular.

Introduction

In mathematics, in contrast to other sciences, objects do not have a tangible existence and are not directly accessible to perception. That being the case, the only access route is via symbolic representation. Mathematics belongs to what Sfard (2000, p. 39) calls "Virtual Reality", which she opposes to tangible reality called "Actual Reality": "*actual reality communication may be perceptually mediated by the objects that are being discussed, whereas in the virtual reality discourse perceptual mediation is scare and is only possible with the help of what is understood as symbolic substitutes of objects under consideration*". Symbols are therefore an integral part of mathematical reasoning. At the present time, a number of authors (see the work of Cobb, Yackle, & Mc Clain, 2000) advocate the idea according to which "*the*

This paper is based an a research project which was financed by the National Found for Scientific Research.

ways that symbols are used and the meanings they come to have are mutually constitutive and emerge together" (Cobb, 2000, p. 18). In this work that sets out the dominant positions currently held with regard to symbolism, several authors advocate the idea according to which symbols represent nothing in and by themselves. Attention should not therefore be focussed on the symbols and their meaning but rather on the activity of symbolizing and meaning making (Yackel, 2000).

This position differs strongly from the traditional vision of teaching where conventional symbols are presented to the pupils immediately with an explanation of what they represent and how they should be used. Some authors (Gravemeijer, 1997, 2002; Gravemeijer, Cobb, Bowers, & Whitenack, 2000) describe this type of approach as a "top-down" one in contrast with the more innovative "bottom-up" approaches to which we will return at the end of this paper. In the traditional approach, pupils are presented with tangible models which are intended to help them to acquire abstract mathematics: "*manipulative materials and visual models are used to make the abstract mathematics to be taught more concrete and accessible for the students*" (Gravemeijer, 2002, p. 8). In this "top-down" approach, "*mathematical symbols are treated as referring unambiguously to fixed, given referents. The teacher's role... is typically cast as that of exploring what symbols mean and how they are able to be used by linking them to referents*" (Gravemeijer et al., 2000, p. 226). In the domain which concerns us, this can be translated by the use of materials that are more or less tangible such as blocks or counters to illustrate the formal adding and subtracting operations. Addition is then presented as "putting together" two sets of objects (or by adding one set to another), symbolized by 3+4=7 (3 and 4 representing the two initial sets and 7 their union). In the case of subtraction, conventional symbolism is introduced in a similar way, after first illustrating the action of "taking away" using tangible objects. Today, these approaches are widely criticized in favour of approaches leaving pupils with much wider scope for initiative in constructing the symbols (the "bottom-up" approach to which we shall return later).

The study developed in this article is set in the flourishing field of research linked with solving arithmetical problems (see Barrouillet & Camos, 2002; Fayol, 1990; Fuson, 1992; Verschaffel & De Corte, 1997; Verschaffel, Greer, & De Corte, 2000). Several research studies focus on exploring informal strategies for problem solving (essentially based on counting) developed by young children (Carpenter & Moser, 1982, 1984; De Corte & Verschaffel, 1985a; Dellarosa-Cummins, 1991; Riley, Greeno, & Heller, 1983). The results of these studies have shown that pupils had significant informal skills in this field: they develop a variety of strategies which, in general, model the actions or relationships described in the problems. Other researches (Bebout, 1990; Carey, 1991; Carpenter, Moser, & Bebout, 1988; De Corte & Verschaffel, 1985b; Fagnant, 2002a,b, 2005) take an interest in the abilities that pupils demonstrate in using mathematical symbolism. They evidence specific difficulties at the symbolization stage: children are not always capable of producing a correct number sentence when they are confronted with a problem, even if they have solved it correctly. In other words, these studies show that pupils experience difficulties in making connections between their informal approaches to problem solving and their use of mathematical symbolism.

On the basis of the difficulties demonstrated by children, several authors (Bebout, 1990; Carpenter, Moser, & Bebout, 1988; Verschaffel & De Corte, 1997) have advocated early teaching of problems in order to give a variety of meanings to mathematical symbolism. These recommendations have not always been followed: why? For Resnick (quoted by De Corte, Greer, & Verschaffel, 1996), a persistent idea in educational thinking is that knowledge should be first acquired, and that applications for reasoning and problem solving should be delayed. As far as early experiences with learning mathematics are concerned, it can be noted that currently, the traditional "top-down" approach described at the beginning of this paper is still being applied in many classes in the French Community of Belgium: conventional symbols are introduced as representing the actions of "adding" and "taking away" and are illustrated by acting on manipulatives. Calculation techniques are thus widely developed and, too often, this is done outside any context of problem solving. Indeed, even if this approach to teaching is not officially dominant internationally, a significant number of teachers still think that mastery of calculation techniques is a prerequisite for solving problems. In this context, how do children succeed in using the conventional symbolism that they have learned in this way?

Problem solving at the beginning of primary school may be considered as an appropriate ground for a study of "the activity of symbolizing": do pupils succeed in making connections between problem situations put to them (or the informal counting approaches they develop to solve them) and the mathematical symbolism used in class to teach calculation techniques?

The creation of links between problems and mathematical symbols is a complex process which cannot be reduced to a simple translation (Julo, 1995). This complexity has been demonstrated in a number of fields that we are not going to enumerate here (by way of very contrasting examples, see the work of Nunes, 1997, who took an interest in distinguishing the difficulties of transition between "oral and written arithmetic"; see also the work done by Julo, 1995, and by Clément, 1982, quoted by Booth, 1984, in the field of expressing problems in terms of algebraic equations). Even if the difficulties associated with mathematical symbolism cannot be reduced to the use of conventional symbols to solve problems, it is interesting to analyze pupils' skills in this field. Indeed, even if the traditional approach they have received is immediately open to criticism, it should be noted that they have been widely confronted with conventional symbolism and it is consequently legitimate to ask questions about the use they may make of it. In this case, the ability to make links between problems and/or counting strategies developed to resolve them, provides an indication shedding a great deal of light on the issue.

While recognizing the research results emphasized by various studies carried out previously in the field, we have opted for an original analysis in terms of "pupil profiles". Faced with a variety of problems, the aim is to see if pupils are capable of making adequate use of the conventional symbols they have learned in class in order to represent adding and subtracting operations.

Method

The study presented here is an empirical study focussing on 25 first graders, selected randomly from 6 classes from 4 schools in the French Community of Belgium. In their classical curriculum, these pupils have been subject to a traditional approach to teaching (a "top-down" type approach) as described at the beginning of this paper. They received no specific teaching in problem solving in grade one; problems are not therefore used to promote understanding of the basic operations (Verschaffel & De Corte, 1997).

The methodology used consists of individual interviews based around the 14 problems as classified by Riley, Greeno, and Heller (1983): 2 combine type problems, 6 change type problems and 6 compare type problems (see appendix 1 for the list of problems). The data that interests us here was collected at the end of the school year; it enables us to take stock of what pupils had learned after a year in school. The approach was fairly similar to that developed by De Corte and Verschaffel (1985a,b): the interviewer reads the problem aloud, the child repeats the problem and tries to solve it using tangible materials (dolls and blocks). The child is then invited to write a number sentence corresponding to the problem and/or to the strategy developed to solve it (*"can-you write a number sentence corresponding to the story or to what you counted – with the blocks, out loud or in your head"*). The interviewer then repeats the question to the child and the child must encapsulate the answer within the number sentence. Finally, the last stage of the interview aims to analyze to what extent the child is capable of demonstrating flexibility (Carey, 1991); he/she is then invited to produce one or more further number sentence(s) in line with the problem and/or the strategy developed. An immediate drawback of the interview technique does need to be pointed out: the task of producing a number sentence did not constitute a real challenge for the pupils and as a consequence, some of them were not able to fully comprehend the purpose of this stage of the approach. It should be noted however that the interviewer insisted that the children should write an initial number sentence after having solved the problem and that they subsequently should suggest other comparable number sentences (flexibility).

The solution was considered to be correct as soon as it gave the expected result. If the child made a counting error, the interviewer invited them to recount so as to avoid "technical errors".

For number sentences, several criteria were taken into account: the number sentence should evidence a correct structure and be built on the three expected numbers (the two data items in the problem and the expected answer). In addition, the answer should be correctly identified within the number sentence. This means that some number sentences were considered as being correct even if they were in no way linked with the problem or the strategy for solving it (i.e., even if it was neither a question of a relational number sentence linked to the structure of the problems, nor a standard number sentence giving an answer following an equal sign). For example, the number sentence "5+7=12" was considered as a correct solution to change 2 problem (*Peter had 12 cherries. Peter gave 7 cherries to Ann. How many cherries does Peter have now?)*, even if it had been solved by a clearly subtracting strategy (e.g., take 12 blocks, take away 7 blocks and count the ones left).

Results

This article focuses on the task of producing a number sentence: are children capable of using mathematical symbolism in problem solving situations? In another words, can they succeed in making connections between the conventional symbols they have learned in class and the problems presented to them (and/or the informal approaches developed to solve them)?

Overall results

Before taking a close look at the analysis by pupil profile that constitutes the heart of this paper, it is important to mention several general results that shed light on the rest of our paper (see Fagnant, 2002b, 2005, for more details).

First of all, it is important to note the significant abilities shown by the young pupils concerning the development of informal approaches to finding solutions, while recalling that they have not yet been subject to explicit and intensive teaching in the area of problem solving. Most strategies are material or verbal and consist in modelling actions or relationships described in the problems. These results confirm the inventiveness of young children in this field, already extensively described in the 80s (see principally the works of Carpenter & Moser, 1982, 1984 and De Corte & Verschaffel, 1985a).

Concerning the links between these informal strategies and the production of number sentences, the first finding to be noted is that, for all the problems, the number of correct number sentences is always less important than the number of correct answers. This finding does not just translate into an overall trend: it is a reality that is met for each problem and for each child. Concretely, this means that no pupil succeeded in producing a correct number sentence if they had not first solved the problem correctly. The difference between the number of correct number sentences and correct answers indicates difficulties of symbolization which are distinguished here by an inability to make connections between informal problem solving approaches (or the problems thus solved) and the mathematical symbolism used in class for teaching calculation techniques.

The links between problems, strategies and number sentences were analyzed in a much more precise manner than in terms of the numbers of correct answers and number sentences. Given the type of strategies developed by pupils (direct modelling of actions or relationships described in the problems), most of the number sentences produced enabled problems to be "matched" to strategies (it was a question of relational number sentences). Faced with certain problems, the children nevertheless suggested number sentences that "matched" neither the problem nor the strategy (it was a question of "additive reconstructions"). Let's take three contrasting cases to illustrate our thesis:

- In the problems corresponding to direct addition (e.g., Change 1 – *Peter had 4 apples. Ann gave 9 apples to Peter. How many apples does Peter have now?*), pupils in general develop addition strategies consisting of "counting all" or of "counting on" starting with one of the two numbers mentioned [e.g., 9=>10(1), 11(2), 12(3), 13(4=stop)=>*R*=13]. The correct number sentences produced are 4+9=(13) or 9+4=(13); relational number sentences that correspond both to the problem and the strategy for solving it (possibly by commutation).
- In problems corresponding to indirect addition (e.g., Change 3 – *Peter had 5 sweets. Ann gave several sweets to Peter. Now Peter has 11 sweets. How many sweets did Ann gave to Peter?*), pupils almost exclusively develop indirect addition strategies consisting in "counting up" from 5 to 11 [e.g., 5=>6(1), 7(2), 8(3), 9(4), 10(5), 11=stop(6)=>*R*=6]. The correct number sentences produced as a result are 5+(6)=11 or possibly (6)+5=11 by commutation; relational number sentences which, as in the previous example, correspond both to the problem and the strategy. No subtraction strategy (consisting in taking 5 away from 11) was observed and, in the same way, no pupil produced a correct number sentence of the standard type:11-5=(6).
- In problems corresponding to direct subtraction (e.g., Change 2 – *Peter had 12 cherries. Peter gave 7 cherries to Ann. How many cherries does Peter have now?*), pupils almost exclusively develop subtractive strategies consisting in taking 7 away from 12 and in counting what is left. Two types of number sentence are encountered in these circumstances: the number sentence 12-7=(5) (a relational number sentence that "matches" the problem and the strategy) and the number sentences 7+(5)=12 or (5)+7=12 (number sentences that don't "match" either the problem or the strategy – these are number sentences we have called "additive reconstructions"). Unlike to the relational number sentences suggested in the previous example (faced with a change 3 problem), these number sentences pose a question insofar as present they don't a connection with either the problem or the strategy. The same type of analysis may be carried out for problems suggesting indirect subtraction: the subtracting number sentences a-c=(b) and a-(b)=c generally "match" the strategy (and also the problem in the second case) while "additive reconstructions" of the type c+(b)=a or (b)+c=a do not "match" either of them.

This way of considering number sentences goes beyond the traditional dichotomy between relational and numerical – or standard – number sentences (De Corte & Verschaffel, 1985b; Vergnaud, 1982); it permits clarification of the analysis by pupil profile which is presented here after.

Analysis by pupil profile

To carry out this analysis, interest was focused on the type of number sentence produced by each child when confronted with the various problems. Three types of situation were distinguished:

- additive situations corresponding to relational adding number sentences (direct or indirect addition) or standard adding number sentences (indirect subtraction);
- subtractive situations corresponding to relational and standard subtracting number sentences (direct or indirect subtraction);
- indeterminate situations in the face of which relational number sentences is difficult to determine.

The following table presents the various types of situation, the problems associated with them, as well as the various types of number sentences produced by the pupils.

Table 1
The various types of situations and problems connected with them

Types of situations situations		Problems involved	Number sentences produced by the pupils
Additive situations	Direct addition	Combine 1 Change 1 Compare 3	a+b=? or b+a=?
	Indirect addition	Combine 2 Change 3 Compare 1 Change 5	a+?=b or ?+a=b
	Indirect subtraction	Change 6	a+b=? or b+a=? ?-a=b
Subtractive situations	Direct subtraction	Change 2 Compare 4	a-b=? b+?=a or ?+b=a
	Indirect subtraction	Change 4	a-?=b, a-b=? b+?=a or ?+b=a
Indeterminate situations		Compare 2	a+?=b, ?+a=b
		Compare 5 Compare 6	b-a=? a+b=? or b+a=?

An analysis of the types of number sentence produced in these different situations has permitted the determination of six clearly hierarchized pupil profiles, evidencing an ever widening use of symbolism: no correct number sentence (profile 0)=>an addition in only one additive situation (profile I)=>some additions in a variety of additive situations (profile II)=>some additions in additive and subtractive situations (profile III)=>some additions and subtractions in a variety of situations (profiles IV and V).

Profile 0 – No problem is correctly symbolized (3 pupils)

Pupils in this profile did not suggest any correct number sentence; they demonstrated no adequate use of mathematical symbolism in problem solving situations. Two children were only able to solve 1 or 2 problems, leading to questions concerning their understanding, even uniformal, of adding and subtracting operations. The third child really constitutes an atypical case: he was able of solving 10 of the 14 problems correctly. The strategies developed are generally material strategies which consist in acting on the actions or relationships described in the problem (e.g., "counting everything" for change 1, "taking away" for change 2, "counting up from a to b" for change 3, etc.). One may really wonder why a pupil who shows such informal skills was not able of suggesting any correct number sentence, despite repeated requests from the interviewer. Should this be seen as a criticism of the insufficiently "challenging" nature of the task or is it a sign of fundamental difficulties, indicating a huge gap between the conventional symbolism used in class and the more "tangible" activities put forward here?

Profile I – Addition is only used in a classical additive situation (direct addition) (2 pupils)

Both pupils in this profile were able to solve several problems (combine 1, change 1, change 2, change 5, compare 1 for one of them and combine 1, change 1, 2, 3 and 4 and compare 2 for the other one) by developing counting strategies consisting in acting on the actions or relationships described in the problem. However, they just suggested a single correct number sentence when faced with a situation involving direct addition: combine 1 for one pupil and change 1 for the other one. The capacities for symbolization demonstrated by these pupils make one think that for them, addition corresponds only to a single type of situation and covers a single meaning: "*combine meaning*" for one, and "*change add to meaning*" for the other (Fuson, 1992).

Profile II – Addition is used in a variety of additive situations (1 pupil)

The pupil in this profile solved and symbolized five additive situations correctly: three involving direct addition (a+b=? faced with combine 1, change 1 and compare 3, linked with the various adding strategies), one involving indirect addition (a+?=b faced with compare 1, linked with a strategy of counting up from a to b) and one of the indirect subtraction type (a+b=? faced with change 6, linked with an adding strategy representing a degree of abstraction in relation to the problem – inversion of the action involved?). Addition is therefore used correctly in a variety of situations, and linked with the problem solving strategies. Alongside this relatively significant mastery of addition, the pupil shows much lower skills when faced with subtractive situations: he is not able to symbolize change 2 and 4 problems while he solved them correctly by material strategies of the take away type (direct or indirect), corresponding to modelling the problem.

Profile III – Addition is used in a variety of addive situations and in subtractive situations ("additive reconstructions") (6 pupils)

As in profile II, the six pupils in this profile displayed good mastery of addition, which they used adequately in a variety of additive situations: they all solved and symbolized correctly the two "classical" additive situations involving direct addition (combine 1 and change 1, linked with an adding strategy) and several situations involving indirect addition (combine 2 and change 3 for five pupils and only combine 2 for the sixth – linked with strategies of counting up from a to b). Several pupils also solved and symbolized change 5 and/or 6 correctly, as well as compare 1, 3 and/or 5 (using direct or indirect adding strategies – counting up from a to b – and using adding number sentences linked to these strategies: a+b=? or a+?=b, depending on the situation).

The "new feature" of profile III in comparison with II is that a degree of "adaptability" of the adding (which we have called "additive reconstruction") allowed the children to suggest correct number sentences in subtractive situations (change 2 and 4 for four pupils and change 4 alone for the two others). The strategies used for solving these problems are clearly subtracting: mainly material strategies of the direct take away type for change 2 (take away 7 blocks from a pile of 12, then count what is left) and indirect taking away for change 4 (take some blocks away from the pile of 11 so that 7 of them remain, then count the ones that have been taken away). "Additive reconstructions" of the type b+?=a or ?+b=a "match" the configuration of the blocks that the pupils have in front of them after having solved the problem, but they do not correspond either to the strategies for solving the problems, or to the problems themselves.

Profile IV – Addition is used in a variety of situations and subtraction is used correctly at least once (4 pupils)

The particular nature of this profile in comparison with the preceding ones is that there is at least one correct number sentence using the minus sign; there is, therefore, the correct use of two conventional symbols which constitutes an advance for this profile in comparison with the preceding one. As with profile III, a suitable use of addition can be noted in additive situations, linked with the problem solving strategies developed: a+b=? faced with problems involving direct addition (combine 1 and change 1, as well as compare 3 for some); a+?=b for problems involving indirect addition (combine 2, change 3 and 5, as well, possibly, as compare 1); a+b=? faced with change 6 for one pupil. The new feature in comparison with the preceding profile is that one or more subtractive situations are symbolized by subtraction linked to the problems and to the strategies for solving them: a-b=? for change 2 (and possibly compare 4 – linked with a direct take away type strategy) and a-?=b or a-b=? for change 4 (linked with a direct or indirect take away type strategy). Several difficulties remain however with pupils in this profile because some problems were solved correctly and were not symbolized or badly symbolized (it is a specific question of combine or change type problems because errors concerning the more complex compare type problems still occur in profile V): change 4 for one pupil, change 6 for two pupils and combine 1 and change 5 for the last one. It is not easy to explain these difficulties: why were pupils not able to link a number sentence with their problem solving strategy while they were able to do so when faced with other problems that were sometimes more complex? These difficulties indicate a certain form of instability in the use of symbols; this is what distinguishes pupils in this profile from those in the next profile.

Profile V – Addition and subtraction are used adequately – All the problems correctly solved are correctly symbolized (5 pupils)

The particular feature of profile V is that all the combine and change type problems are symbolized correctly. Some problems of the compare type may be also correctly symbolized but not necessarily all of them (some difficulties occur here when pupils are faced with these particularly complex problems – Lewis & Mayer, 1987; Stern, 1993). Four pupils in this profile always suggest number sentences linked with the problem solving strategies and with the problems: a+b=? for combine 1 and change 1 (+ possibly compare 3, linked with adding strategies); a+?=b or ?+b=c for combine 2, change 3 and 5 (+ possibly compare 1, 2 and 5, linked with indirect adding strategies involving counting up from a to b); a-b=? for change 2 (and possibly compare 4, linked with direct take away strategies) and a-?=b for change 4 (linked with direct or indirect take away strategies). The change 6 problem itself produces two types of correct number sentence: a+b=? linked with adding strategies and ?-a=b linked with the story and strategies of the hypothetical test type.

The fifth pupil displays similar skills concerning adding, but he demonstrates an "ambiguous" use of subtraction which, curiously, he only uses when faced with the compare 2 problem, preferring "additive reconstructions" when faced with change 2 and 4, and compare 4. All the problems are, nevertheless, symbolized correctly, which means that he is allocated to this profile.

The pupils in this profile can be considered as proving good use of mathematical symbolism in problem solving. They succeeded in creating a number of connections between conventional symbols and the problems and therefore, in using mathematical symbolism in situations where various meanings are encountered (Fuson, 1992).

Particular cases

Four pupils constitute particular cases and do not come within any of the profiles: the simplest additive situations (direct addition – change 1 and combine 1) are not symbolized correctly, while one or several more complex situations are (compare 1 for one pupil, change 4 for two pupils and combine 2 and change 5 for the last). These results are difficult to explain: on the one hand, instability in the use of symbols is doubtless the sign of very imperfect mastery of their use; on the other hand, the lack of "challenge" in the task perhaps also contributed to destabilizing some pupils.

A difficult use of subtraction...

The 12 pupils in profiles 0, I, II and III (as well as the 4 that are unclassified, that is 16 pupils out of a total of 25) did not suggest any single correct number sentence using the "-" sign. The difficulties relating to the use of this sign are clearly demonstrated thanks to the last stage of the interviews having to do with pupils' capacity for flexibility (Carey, 1991). When the child suggested an "additive reconstruction" when faced with a subtractive situation [for example 7+(5)=12 when faced with a change 2 problem], we asked him to produce another number sentence, closer to the problem and/or the strategy developed to solve it (almost always a subtracting strategy: take a pile of 12 blocks, take away 7 blocks from this pile and count the blocks left). Some children were able to develop the commutative property of the addition [here, (5)+7=12], but none of them who had produced an "additive reconstruction" was able to demonstrate sufficient flexibility to produce the corresponding subtracting number sentence [here, 12-7=(5)]. These pupils seemed, therefore, to experience real difficulties in using the minus sign in problem solving situations (put another way, to make connections between this symbol and a problem or problem solving strategy).

Conclusion and discussion

The empirical study described in this article was carried out within the French Community of Belgium, in grade 1 classes subject to a traditional teaching approach of the "top-down" style as described at the beginning of this paper. The aim was to see how pupils succeeded in using conventional symbolism learned in this way. As we have shown at the beginning of this paper, the problems associated with mathematical symbolism cannot be reduced to the use of conventional symbols when faced with problems. Linking a number sentence to a problem is not simply an act of translation; the ability to make connections between symbols learned in class and problem solving strategies developed when faced with a variety of problems constitutes an indication that enables light to be shed on pupils' understanding of conventional symbols and on the use they can make of them. We have tried to shed this light by focusing on an analysis "by pupil profile". If this analysis provides evidence of hierarchization regarding the abilities demonstrated by the children at the level of the use of symbols, it should be pointed out that the low number of children interviewed (as a reminder: 25) does not allow the results to be generalized. Additional studies would be necessary to refine these profiles and attest to their relevance. The aim was not to create a model, but to provide some "original" analysis of the abilities and difficulties demonstrated by the children.

The results observed may be briefly summarized as follows: the 9 pupils in profiles IV and V demonstrate relatively accurate use of conventional symbols (mainly children in profile V); as far as the other 16 pupils are concerned, making connections with problems and/or strategies proves to be clearly more problematic. For 9 pupils (the 5 in profiles 0 and I, but also including the 4 that are unclassifiable), even the use of addition proves problematic; for the other 7 (profiles II and III), adding symbols are used adequately but subtraction is never suggested.

The overall results are therefore somewhat mitigated and show the limits of the traditional approach to which the children questioned had been subject.

In the past, a number of studies (for an overall view, see Barrouillet & Camos, 2002; Fayol, 1990; Fuson, 1992; Verschaffel & De Corte, 1997) had already shown the informal skills of young pupils in problem solving before receiving any formal teaching. Our results confirm this finding. As some have already advocated (see in particular Bebout, 1990; Carpenter, Moser, & Bebout, 1988; Verschaffel & De Corte, 1997), it would be appropriate to rely on the informal skills thus demonstrated in order to build learning and therefore, to develop an earlier approach to problem solving with a view giving a variety of meanings to the symbols.

But would developing problem solving to help children make connections be enough? Wouldn't there be a risk of continuing to develop a "top-down" approach in which pupils are told how to link the conventional symbols (the meaning of which has been predefined) to the informal strategies they develop? Would it not be better to leave them more room to build the symbols as well? From this perspective, several authors currently advocate the so-called "bottom-up" approach, as opposed to the traditional "top-down" type approach (Gravemeijer et al., 2000). In the "bottom-up" approaches, Gravemeijer (1997) states that learners' knowledge and informal strategies should constitute the starting point for more formal mathematical learning; a significantly greater share in the initiative is thus granted to the pupils. Problem solving in which pupils are able to develop personal and meaningful approaches constitutes the starting point favoured by these approaches. From this perspective, "*the challenge is to support individual students' transition to forms of mathematical activity in which the use of conventional symbols carries the significance of acting on experientially real mathematical entities*" (Gravemeijer et al., 2000, p. 226). It's in this "bottom-up" perspective that you can put the "inventive" approaches such as those developed by Bednarz, Dufour-Janvier, Poirier, and Bacon (1993): "*Instead of presenting symbolizations as external representation of preexisting mathematical relationships, they argued that symbolizations should come to the fore as tools constructed by the students themselves... The starting point should be communicative situations in which students symbolize their mathematical understanding*" (Gravemeijer et al., 2000, p. 232). The study by Bednarz et al. (1993) was carried out among first grade pupils, based on a situation involving passengers getting on and off a bus. The children had to invent symbolization allowing them to keep track of the movements in the bus, in such a way as to be able to communicate them to the other students. The results of the study showed that the pupils succeeded in inventing a variety of symbolizations, combining conventional symbols with which they were familiar with different stylized representations of the bus. This type of approach seems promising but some questions remain unanswered, in particular *the question of how conventional ways of symbolizing might eventually emerge* (Gravemeijer et al., 2000, p. 232-233).

It is also in the "bottom-up" perspective that the Realistic Mathematics Education theory is situated (see the various works by the Gravemeijer team) for which one of the key concepts relating to the problematic nature of symbolization that is our concern, is the emergence of meanings shared by the learner's community (*taken-as-shared meanings*) that emerge when learners and teacher negotiate the different interpretations and solutions possible when faced with a problem. Here, the transition towards conventional symbols is envisaged through a *learning trajectory* which is thought through in advance; the teacher may play a proactive role *that might involve introducing ways of symbolizing that fit with students' activity* (Gravemeijer et al., 2000, p. 248).

These various innovative approaches give a completely different status to symbolization: "*the metaphors of 'transmission' of knowledge with help of symbols that functions as 'carriers of meaning' are replaced by the image of students constructing their own way of symbolizing as part of their mathematical activity*" (Gravemeijer, Lehrer, van Oers, & Verschaffel, 2002, p. 1). If these approaches seem promising, the jury is still out on the question of knowing how to bring such change into classroom practice...

Appendix 1

The 14 types of problem suggested

Combine 1	Peter has 4 apples. Ann has 9 apples. How many apples do Peter and Ann have together?
Combine 2	Peter has 5 sweets. Ann also has some sweets. Together Peter and Ann have 11 sweets. How many sweets does Ann have?
Change 1	Peter had 4 apples. Ann gave 9 apples to Peter. How many apples does Peter have now?
Change 2	Peter had 12 cherries. Peter gave 7 cherries to Ann. How many cherries does Peter have now?
Change 3	Peter had 5 sweets. Ann gave several sweets to Peter. Now Peter has 11 sweets. How many sweets did Ann give to Peter?
Change 4	Peter had 11 lollipops. Peter gave several lollipops to Ann. Now Peter has 7 lollipops. How many lollipops did Peter give to Ann?
Change 5	Peter had several books. Ann gave 6 books to Peter. Now Peter has 11 books. How many books did Peter have to start with?
Change 6	Peter had several pencils. Peter gave 5 pencils to Ann. Now Peter has 9 pencils. How many pencils did Peter have to start with?
Compare 1	Peter has 5 sweets. Ann has 11 sweets. How many more sweets does Ann have than Peter?
Compare 2	Peter has 7 lollipops. Ann has 11 sweets. How many less lollipops does Peter have than Ann?
Compare 3	Peter has 4 apples. Ann has 9 apples more than Peter. How many apples does Ann have?
Compare 4	Peter has 12 cherries. Ann has 7 cherries less than Peter. How many cherries does Ann have?
Compare 5	Ann has 11 books. Ann has 6 more books than Peter. How many books does Peter have?
Compare 6	Ann has 9 pencils. Ann has 5 pencils less than Peter. How many pencils does Peter have?

References

Barrouillet, P., & Camos, V. (2002). *Savoirs, savoir-faire arithmétiques et leurs déficiences*. Paris: Rapport pour le Ministère de la Recherche. Programme cognitique, école et sciences cognitives, document non publié.

Bebout, H.C. (1990). Children's symbolic representation of addition and subtraction word problems. *Journal for Research in Mathematics Education, 2*(2), 123-131.

Bednarz, N., Dufour-Janvier, B., Poirier, L., & Bacon, L. (1993). Socioconstructivist viewpoint on the use of symbolism in mathematics education. *Alberta Journal of Educational Research, 39,* 41-58.

Booth, L.R. (1984). *Algebra: Children's strategies and errors*. Oxford, UK: Nfer-Nelson Publishing Company.

Carey, D.A. (1991). Number sentences: Linking addition and subtraction word problems and symbols. *Journal for Research in Mathematics Education, 22*(4), 266-280.

Carpenter, T.P., & Moser, J.M. (1982). The development of addition and subtraction problem solving skills. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), *Addition and Subtraction. A cognitive perspective* (pp. 9-24). Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Carpenter, T.P., & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grade one through three. *Journal for Research in Mathematics Education, 15*(3), 179-202.

Carpenter, T.P., Moser, J.M., & Bebout, H.C. (1988). Representation of addition and substraction word problems. *Journal for Research in Mathematics Education, 4*, 345-357.

Cobb, P. (2000). From representation to symbolizing: Introductory comments on semiotics and mathematical learning. In P. Cobbs, E. Yackel, & K. Mc Clain (Eds.), *Symbolising and communicating in mathematics classrooms. Perspectives on discourse, tools and instructional design* (pp. 17-36). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Cobb, P., Yackel, E., & Mc Clain, K. (2000, Eds.). *Symbolizing and communicating in mathematics classrooms. Perspectives on discourse, tools and instructional design*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

De Corte, E., & Verschaffel, L. (1985a). Beginning first grader's initial representation of arithmetic word problems. *Journal of Mathematical Behavior, 4*, 3-21.

De Corte, E., & Verschaffel, L. (1985b). Writing number sentences to represent addition and substraction problems. In J.K. Damarin & M. Shelton (Eds.), *Proceeding of the Seventh Annual Meeting of North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 50-56). Columbus, USA: University of Colombus.

De Corte, E., Greer, B., & Verschaffel, L. (1996). Mathematics teaching and learning. In D.C. Berliner & R.C. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 491-549). New York: MacMillan.

Dellarosa-Cummins, D. (1991). Children interpretation of arithmetic word-problems. *Cognition and Instruction, 8*(3), 261-289.

Fagnant, A. (2002a). *Quelle compréhension du symbolisme mathématique au travers de la résolution de problèmes arithmétiques?* Thèse de doctorat non publiée. Université de Liège. Belgique.

Fagnant, A. (2002b). Mathematical symbolism: A feature responsible for superficial approaches? In A.D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 345-352). Norwich, UK: University of East Anglia.

Fagnant, A. (2005). Résoudre et symboliser des problèmes additifs et soustractifs en début d'enseignement primaire. In M. Crahay, E. De Corte, J. Grégoire, & L. Verschaffel (Eds.), *Enseignement et apprentissage des mathématiques: Que disent les recherches?* (pp. 131-150). Bruxelles: De Boeck.

Fayol, M. (1990). *L'enfant et le nombre*. Paris: Delachaux et Niestlé.

Fuson, K.C. (1992). Research on whole number addition and subtraction. In D.A. Grows (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243-275). New York: MacMillan.

Gravemeijer, K. (1997). Mediating between concrete and abstract. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics. An international perspective* (pp. 315-345). Hove, East Sussex: Psychology Press Ltd.

Gravemeijer, K. (2002). Preamble: From models to modelling. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 7-22), Dordrecht, The Netherlands: Kluwer Academic Publishers.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). *Symbolizing, modelling, and instructional design.* In P. Cobbs, E. Yackel, & K. Mc Clain (Eds.), *Symbolising and communicating in mathematics classrooms. Perspectives on discourse, tools and instructional design* (pp. 225-273). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Gravemeijer, K., Lehrer, R., van Oers, B., & Verschaffel, L. (2002). Introduction and overview. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 1-5), Dordrecht, The Netherlands: Kluwer Academic Publishers.

Julo, J. (1995). *Représentation des problèmes et réussite en mathématiques. Un apport de la psychologie cognitive à l'enseignement.* PU de Rennes: Psyhcologies.

Lewis, A.B., & Mayer, R.F. (1987). Student's miscomprehension of relational statements in arithmetic word problems. *Journal of Educational psychology, 75*(4), 363-371.

Nunes, T. (1997). Systems of signs and mathematical reasoning. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 29-44). Hove, East Sussex: Psychology Press Ltd.

Riley, M.S., Greeno, J.G., & Heller, J.I. (1983). Development of children's problem-solving ability in arithmetic. In H.P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.

Sfard, A. (2000). Symbolizing mathematical reality into being. Or how mathematical discourse and mathematical objects create each other. In P. Cobbs, E. Yackel, & K. Mc Clain (Eds.), *Symbolising and communicating in mathematics classrooms. Perspectives on discourse, tools and instructional design* (pp. 37-98). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Stern, E. (1993). What makes certain arithmetic word problems involving the comparison of sets so difficult for children? *Journal of Educational Psychology, 85*(1), 7-23.

Vergnaud, G. (1982). A classification of cognitive tasks and operation of thought involved in addition and subtraction problems. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), *Addition and Subtraction. A cognitive perspective* (pp. 39-59). Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Verschaffel, L., & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 69-97). Hove, East Sussex: Psychology Press Ltd.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, The Netherlands: Swets & Zeitlinger.

Yackel, E. (2000). Introduction: Perspectives on semiotics and instructional design. In P. Cobbs, E. Yackel, & K. Mc Clain (Eds.), *Symbolising and communicating in mathematics classrooms. Perspectives on discourse, tools and instructional design* (pp. 1-15). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Cet article relate une étude empirique centrée sur l'utilisation du symbolisme mathématique en résolution de problèmes. Vingt-cinq élèves ont été interviewés individuellement en fin de première année primaire; ils ont chacun été amenés à résoudre et à symboliser 14 problèmes différents. Dans leur curriculum classique, ces élèves ont reçu un enseignement traditionnel basé sur une approche de type "top-down" (approche encore couramment développée en Communauté française de Belgique): les symboles conventionnels sont proposés d'emblée aux élèves à qui on explique ce qu'ils représentent et comment ils doivent les utiliser. L'enseignement se focalise alors sur les techniques de calculs (considérées comme un pré requis à la résolution de problèmes). Les résultats présentés ici montrent les capacités (et difficultés) démontrées par les élèves pour créer des connexions entre le symbolisme conventionnel enseigné en classe et les démarches informelles qu'ils développent face aux problèmes qui leur *sont proposés. Les limites de l'approche "top-down" sont alors discutées en opposition avec des approches plus novatrices de type "bottom-up", telles que celles développées par les tenants de la Realistic Mathematics Education notamment.*

Key words: Arithmetical problems, Empirical study, Pupil profile, Top-down vs. bottom-up type approaches, Use of symbolism.

Received: April 2004

Revision received: December 2004

Annick Fagnant. Service de Pédagogie Expérimentale, Université de Liège, Bd du Rectorat, 5, B32, B4000 Liège, Belgique. E-mail: afagnant@ulg.ac.be, Web site: www.ulg.ac.be/pedaexpe/

Current theme of research:

Development of problem solving teaching/learning activities in primary school. Analysis of PISA results. Construction of external evaluation about mathematics and sciences.

Most relevant publications in the field of Psychology of Education:

Demonty, I., Fagnant, A., & Lejong, M. (2004). *Résoudre des problèmes: Pas de problème!* Guide méthodologique et documents reproductibles – 8-10 ans. Bruxelles: De Boeck.

Fagnant, A. (2002). Mathematical symbolism: A feature responsible for superficial approaches? In A.D. Cockburn & E. Nordi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education, 2,* 345-352. Norwich, UK: University of East Anglia.

Fagnant, A. (2005). Résoudre et symboliser des problèmes additifs et soustractifs en début d'enseignement primaire. In M. Crahay, L. Verschaffel, E. De Corte, & J. Grégoire (Eds.), *Enseignement et apprentissage des mathématiques: Que disent les recherches?* (pp. 131-150). Bruxelles: De Boeck

Fagnant, A., & Vlassis, J. (2000). Utiliser les connaissances antérieures des élèves pour construire les mathématiques: Quelques réflexions sur les calculs relationnels au primaire et sur les équations au secondaire. *Actes du premier congrès des chercheurs en éducation* (pp. 227-232). Bruxelles, du 24 au 25 mai 2000: www.agers/cfwb/be.

Fagnant, A., Vlassis, J., & Crahay, M. (2005). Using mathematical symbols at the beginning of arithmetical and algebraic learning. In L. Verschaffel, E. De Corte, G. Kanselaar, & M. Valcke (Eds.), *Developing powerful learning environments to promote deep conceptual and strategic learning in major curricular domains* (pp. 81-95). Leuven University Press: Studia Paedagogica.