Anticoherence measures for spin states

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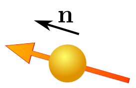
Coherent vs anticoherent spin-j states

Coherent state



$\langle \mathbf{J} \rangle = j\hbar\mathbf{n}$

$|\psi_j\rangle$ is a spin-j coherent state $|\mathbf{n}\rangle$ if it is eigenstate of $\mathbf{J}\cdot\mathbf{n}$ for some $\mathbf{n}$ with the highest eigenvalue $j\hbar$.

Anticoherent state

$\langle \mathbf{J} \rangle = 0$

$|\psi_j\rangle$ is a spin-j t-anticoherent state if [1]

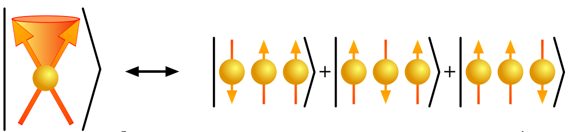
$$\langle\psi_j|\,(\mathbf{J}\cdot\mathbf{n})^k\,|\psi_j\rangle$$

does not depend on $\mathbf{n}$ for $k = 1,\ldots,t$.

One-to-one mapping and geometrical representation

One-to-one mapping

spin-j state $\leftrightarrow$ symmetric state of $2j$ spin-$\frac{1}{2}$
$\leftrightarrow$ symmetric state of $N = 2j$ qubits



state of a spin-$\frac{3}{2}$ — symmetric state of 3 spin-$\frac{1}{2}$

Any N-qubit symmetric state $|\psi_S\rangle$ can be expressed in the Dicke basis $\{|D_N^{(k)}\rangle\}$ with $|D_N^{(k)}\rangle = \mathcal{N}\sum_{\text{perm.}}|\mathbf{0}_{N-k}\mathbf{1}_k\rangle$, where $\mathbf{a}_\ell$ is a string of ℓ characters a, N is the number of qubits and k the excitation number :

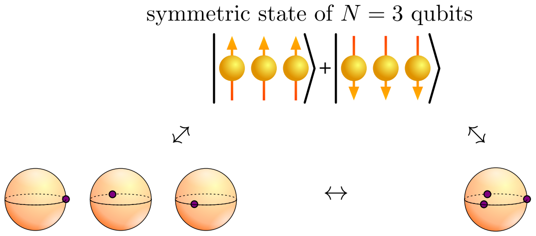
$$|\psi_S\rangle = \sum_{k=0}^{N} d_k |D_N^{(k)}\rangle, \qquad \sum_{k=0}^{N}|d_k|^2 = 1$$

There is a formal correspondence between the standard $|j,m\rangle$ basis (common eigenstates of J_z and $\mathbf{J}^2$) and the Dicke basis :

$$|j,m\rangle \;\leftrightarrow\; |D_{2j}^{(j-m)}\rangle \qquad |\psi_j\rangle \;\leftrightarrow\; |\psi_S\rangle \tag{1}$$

Majorana representation

symmetric state of $N = 3$ qubits



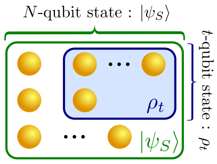
$N = 3$ single-qubit states $\leftrightarrow$ $N = 3$ points on the Bloch sphere

Majorana points can be located at the same position on the Bloch sphere, and the largest number of degenerated points is denoted by $d_{\max}$ ($\leqslant N$).

- Spin-j coherent states are characterised by $d_{\max} = 2j$.
- Anticoherent states *can* only exist if $d_{\max} \leqslant j$ [2].

Conditions for anticoherence

A multiqubit symmetric state $|\psi_S\rangle \leftrightarrow |\psi_j\rangle$ is t-**anticoherent** iff all t-qubit states are maximally mixed in the symmetric subspace [2,3], or if the expectation value of spin operators fulfill the following conditions [4] :



$$\rho_t = \frac{\mathbb{1}_{t+1}}{t+1} \overset{(1)}{\Longleftrightarrow} \langle\psi_j|J_+^r J_z^q|\psi_j\rangle = \frac{\mathrm{Tr}(J_z^q)}{2j+1}\delta_{r0}$$

for $r = 0,\ldots,t$, $q = 0,\ldots,t-r$, where N is the number of qubits and δ_{r0} the Kronecker symbol.

Measures of t-anticoherence

Anticoherence conditions only allow to determine whether a state anticoherent is or not. In order to quantify the amount of anticoherence of a state, we introduce measures of anticoherence on the set of pure spin-j states $|\psi_j\rangle$. We require a measure of t-anticoherence, $\mathcal{A}_t$, to fulfill the following conditions:

i. $\mathcal{A}_t(|\psi_j\rangle) = 0 \Leftrightarrow |\psi_j\rangle$ is coherent
ii. $\mathcal{A}_t(|\psi_j\rangle) = 1 \Leftrightarrow |\psi_j\rangle$ is t-anticoherent
iii. $\mathcal{A}_t(|\psi_j\rangle) \in [0,1]$ for any state $|\psi_j\rangle$
iv. $\mathcal{A}_t(|\psi_j\rangle)$ is invariant under global phase changes and arbitrary spin rotations

Measures of anticoherence based on purity

A measure satisfying all conditions above, based on the mapping (1) and the purity of the t-qubit reduced density matrix of $|\psi_S\rangle\langle\psi_S|$, $R_t \equiv \mathrm{tr}(\rho_t^2)$, reads

$$\boxed{\mathcal{A}_t(|\psi_j\rangle) = (t+1)(1-R_t)/t} \tag{2}$$

- $t = 1$

In this case, (2) is a linear function of the total variance [5], $\mathbb{V} = \sum_{i=x,y,z}\langle J_i^2\rangle - \langle J_i\rangle^2$, and reads

$$\mathcal{A}_1(|\psi_j\rangle) = (\mathbb{V} - j)/j^2$$

- $t = 2$

In this case, (2) involves two-point correlators of spin operators, and reads

$$\mathcal{A}_2(|\psi_j\rangle) = (\mathbb{W} + \alpha)/\beta$$

where $\alpha = \frac{j(j^2-2j+3)}{2(j-1)}$, $\beta = \frac{(2j-1)^2 j}{3(j-1)}$, and

$$\mathbb{W} = \mathbb{V} - \sum_{i,k=x,y,z}\langle J_i J_k\rangle\langle J_k J_i\rangle/(2j(j-1))$$

Exemples: GHZ and W states

$\mathcal{A}_1((\|j,j\rangle + \|j,-j\rangle)/\sqrt{2}) = 1$	$\mathcal{A}_1(\|j,1-j\rangle) = (2j-1)/j^2$
$\mathcal{A}_2((\|j,j\rangle + \|j,-j\rangle)/\sqrt{2}) = 3/4$	$\mathcal{A}_2(\|j,1-j\rangle) = 3(j-1)/j^2$

Anticoherence and degeneracy of Majorana points

Anticoherence measures can be numericaly optimized to find anticoherent states of given j and $d_{\max}$.

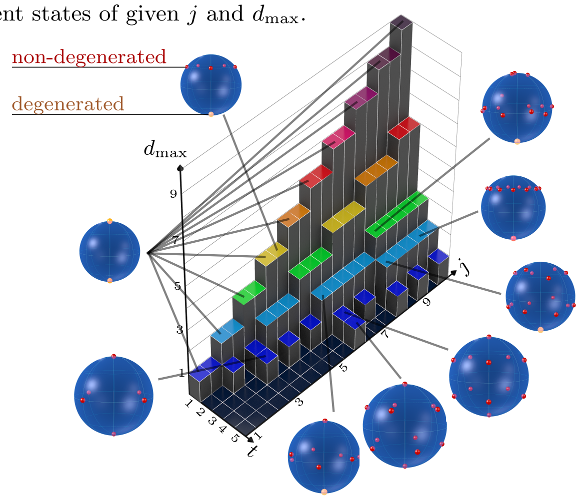


Fig.: Maximum degeneracy allowed for the existence of t-anticoherent states with a spin quantum number j.

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