The exclusion of competing one-way essential complements: implications for net neutrality☆

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Abstract

We analyze the incentives of internet service providers (ISPs) to break net neutrality by excluding competing one-way essential complements, i.e. internet applications competing with their own products. A typical example is the exclusion of VoIP applications by telecom companies offering internet and voice services. A monopoly ISP may want to exclude a competing internet app if it is of inferior quality and the ISP cannot ask for a surcharge for its use. Competition between ISPs never leads to full app exclusion but it may lead to a fragmented internet where only one ISP offers the application. We show that, both in monopoly and duopoly, prohibiting the exclusion of the app and surcharges for its use does not always improve welfare.

JEL classification: L12 L13 L51 L96

Keywords: Net Neutrality, Foreclosure, One-Way Essential Complements

1. Introduction

In 2005, Madison River, a US internet service provider (ISP), excluded Vonage, a Voice over IP (VoIP) application, from its network, which resulted in a conflict between stakeholders over the control of the bundle of services offered on the internet. Most ISPs offer multiple services – internet, phone, television, video, etc.– and applications such as Vonage are competing with these services. These apps are "competing one-way essential complements" (Chen and Nalebuff, 2006): competing because Vonage is a substitute to the phone, and one-way essential complement because the internet is "essential" for the app to work but the opposite is not true. On the one hand, they create a business stealing effect and excluding them is a way for the ISP to limit unwanted competition. On the other hand, they create value for internet users who are willing to use and to

☆We thank participants at CISS, Connected Life 2014, the 18th Centre for Competition and Regulatory Policy Workshop, the 29th Jornadas de Economia Industrial, the EARIE meeting and the Ninth IDEI-TSE-IAST conference on the Economics of Intellectual Property, Software and the Internet. We also thank Juan José Ganuza, Jacques Crémer, Martin Peitz and Nicolas Petit for helpful discussions and, more particularly, Florian Schuett who discussed the paper twice. We are also grateful to the Co-editor (Martin Peitz) and two reviewers for their helpful comments and suggestions. We thank Isabelle Peere for proofreading the manuscript. This research was funded through the ARC grant for Concerted Research Actions, financed by the French-speaking Community of Belgium.

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pay for these new services. That value can possibly be extracted by the ISP through higher internet prices and, therefore, exclusion might not necessarily be optimal. The interplay of these two types of incentives is the main object of this article.

The concept of exclusion rings multiple bells. In this paper, we link the literature on vertical foreclosure and one-way essential complements with the literature on net neutrality. Indeed, the exclusion of competing applications is part of the larger debate on "net neutrality". Because it is still a very lively dispute, net neutrality does not have a unified definition. Still, Schuett (2010) summarizes it as "the principle that all data packets on an information network are treated equally". Accordingly, content exclusion is a breach of the net neutrality principle. The literature (Choi and Kim (2010); Economides and Hermalin (2012); Reggiani and Valletti (2016) for instance) has generally focused on two implications of net neutrality: the **non-discrimination rule** and the **zero-price rule**.

The first interpretation simply means that a bit is a bit and that contents should be treated similarly, regardless of their nature, origin and destination. For example, there should be no prioritization: the bits sent by Youtube should not be transferred faster than those sent by Vimeo. Similarly, traffic management should be limited to isolated cases and the exclusion of particular applications –the most extreme form of discrimination– should be forbidden. Furthermore, the non-discrimination rule also implies that internet users can use applications without paying an extra fee to the ISP. Stated differently, the ISP cannot condition the use of an application to the payment of a surcharge. The non-discrimination rule prohibits the exclusion of competing apps (content-based discrimination, which we henceforth refer to as condition NN_1) and price surcharges for using such apps (financial discrimination, condition NN_2). We say that an ISP fully complies with net neutrality if there is no exclusion of the app and no surcharge to use it. An ISP partially complies with net neutrality if there is no exclusion but a surcharge to use the app.[3]

The zero-price rule prohibits financial transfers between residential ISPs and content producers (CPs). On the internet, CPs pay a backbone provider to be connected to the network and residential consumers pay to be connected to an ISP.[4] According to the zero-price rule, ISPs do not have the right to make CPs pay a termination fee for the access to internet consumers. The zero-price rule implies that there is a "missing price"[5] prohibiting financial transfers between CPs and ISPs. The zero-price rule and the non-discrimination rule have been criticized for prohibiting the emergence of value-added services on the internet. In our model, the zero-price rule is always enforced and therefore our focus is exclusively on the no-discrimination rule.

The literature has generally focused on the implications of net neutrality on congestion (Choi and Kim (2010); Choi et al. (2015a); Peitz and Schuett (2016) and Economides and Hermalin (2012)) and innovation and investment (Reggiani and Valletti (2016); Bourreau et al. (2015) and Choi et al.

[3]This distinction echoes the distinction between weak and strong net neutrality of Gans (2015) and Gans and Katz (2016) who state that net neutrality is *strong* if content-based price discrimination is outlawed both with regard to CPs and consumers, and that it is *weak* if discrimination is outlawed with regard to one group only.

[4]See Faratin et al. (2008) and Economides and Hermalin (2012) for more on the structure of the internet and net neutrality.

[5]For an analysis of net neutrality as a case of missing prices (websites cannot use prices to regulate the usage of consumers because most websites are free), see Jullien and Sand-Zantman (2016).

(2015b)). By contrast, we will concentrate our analysis on the exclusion of competing applications. Indeed, as highlighted in a BEREC report (BEREC, 2012), most of the alleged net neutrality breaches are concentrated in two areas: data-intensive services and applications competing with ISPs' own services. This was also highlighted in Krämer et al. (2013): "[...] there exist several examples of ISPs that have blocked voice over IP (VoIP) traffic which is in competition to their regular telephone service." Our focus is on this second category.

Let us consider, for instance, the first famous net neutrality breach, committed by Madison River, which we highlighted at the beginning of this introduction. After the blocking of Vonage, the FCC intervened, fined and made Madison River sign a consent decree to stop the throttling (FCC, 2005). Exclusion was also the starting point of the net neutrality law in the Netherlands (International Telecommunications Union, 2012). In 2010, KPN, a Dutch ISP, started to develop a new strategy towards competing applications: users either had to pay to use Skype and WhatsApp or face blocking. The Dutch parliament reacted by enacting one of the first net neutrality laws in the world, effectively putting a halt to KPN's strategy. The reaction has not been so prompt in Spain where ISP Yoigo is still making mobile users pay for access to VoIP applications: users have to pay a fee for mobile data and an additional fee if they want to use VoIP. Yoigo's Swedish counterpart, Teliasonera, also tried to set the same pricing scheme but had to withdraw it after a public uproar (Grundberg, 2012). Hence, the intertwining of applications and of ISPs' own services is a major issue. When Yoigo applies a surcharge for VoIP applications, it respects NN_1 but not NN_2. When Madison River excludes Vonage, it respects neither NN_1 nor NN_2.

To better understand the issue, we build a model that focuses on the interaction of two markets: that for a communication-based service[6] ("the phone") and that for internet-based services ("the internet"). The ISP has an installed network and offers internet and phone services to consumers. An alternative firm competes on the communication market by offering some VoIP software to internet users. Consumers are thus offered three products: the internet, the phone and the VoIP application (hereafter "the app" or "the application").

Our model has four specific features. First, the app and the phone are both horizontally and vertically differentiated substitutes. Second, the app needs the internet to work but the phone does not. The internet and the app are therefore one-way essential complements (Chen and Nalebuff, 2006) and the incentives of the ISP are complex because the app is complementary to one of its products but is a substitute to another. This separates our set-up from more traditional vertical foreclosure models Rey and Tirole (2007). Third, the price of the app is exogenous. Finally, consumers' valuations for the internet are heterogeneous. Network congestion is not explicitly incorporated in the model but the possibility for the ISP to degrade the quality of the competing app may be interpreted as network congestion, e.g. through a lower bandwidth or a higher jitter/delay.

This paper is organized around three questions. First, does an ISP have incentives to exclude a competing application? Second, should it charge a premium to consumers to use the app? Last, is net neutrality welfare improving? Each question is considered in a monopolistic and a duopolistic

[6]Note that we have picked this service for illustrative purposes but we could also have chosen Netflix and TV, Spotify and music services, WhatsApp and SMS, etc.

setting, i.e. competing ISPs. These questions are studied in a framework where the zero-price rule is always enforced, which is presumably the most appropriate set-up to analyze incentives to exclude the app. If the app is available in our setting, it will a fortiori also be available if the zero-price rule is relaxed and ISPs can extract revenue from CPs.

We show that a monopoly ISP will not exclude the app if it is a superior alternative to its own product. In this case, the value added by the app can be extracted through a higher internet price and it more than compensates the competition on the communication market. If the app is an inferior alternative, exclusion is a concern although not a systematic one. And, if the ISP can apply a surcharge for enabling the app, exclusion will never occur. Finally, we show that prohibiting such surcharges is not beneficial to the firm and, more surprisingly, can also hurt consumers. Therefore in a monopoly setting, implementing net neutrality rules can hurt welfare.

When several ISPs compete, we first show that it is not possible to have, in equilibrium, exclusion of the app by both ISPs. Complete exclusion of the app therefore is not an issue in duopoly. With competition between ISPs, offering the app is a way for firms to differentiate their products and, should one firm exclude the app, the other has no incentives to do so. Indeed, this other firm can escape fierce competition from the rival ISP by offering an improved product –the internet with the app. This product is a source of profit if the firm can sell it at a premium, i.e. if the firm can apply a surcharge for the use of the app.

We then characterize the equilibrium under competition, considering both symmetric and asymmetric ISPs. ISPs are symmetric when they both offer the phone; they are asymmetric when only one offers the product competing with the app. We first show that both firms offering the app without surcharge is a Nash equilibrium only in the symmetric case but this equilibrium is not unique. There also exist equilibria with a fragmented internet where the app is only made available at one ISP. In the asymmetric case, only fragmented internet equilibria exist. Consumer surplus is always highest when both ISPs comply with NN_1 and NN_2. On the contrary, firms have a higher profit in a fragmented internet. Consumers and firms have –in contrast to the monopoly case– different interests with regard to net neutrality obligations. Regarding total welfare, we show that in the symmetric case, welfare is highest when net neutrality is enforced but this may not be true in the asymmetric case.

Net neutrality, NN_1 and NN_2, can thus be seen as a "competition intensifier" which sometimes works well –as in the symmetric duopoly case– but sometimes hastens the pace too much –as in the monopoly and the asymmetric duopoly case. We therefore conclude that net neutrality should not be seen as a one-size-fits-all rule and that having a fragmented internet where apps are only available at some ISP does not necessarily hurt welfare. Regarding exclusion, an ex-post regulation assessing breaches case by case may thus be preferable to imposing a strong ex-ante rule on all market participants.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the basic model. In Section 4, we analyze the case of a monopolistic ISP. Section 5 extends the model to include competition between ISPs. Section 6 concludes. All omitted proofs are in the Appendix.

2. Literature review

The theoretical foundation of our approach is the literature on one-way essential complements initiated by Chen and Nalebuff (2006). They study the competition (à la Bertrand) between one-way essential complements, i.e. two goods that are complements but where one is essential for the other to be useful. We reach two conclusions that are reminiscent of theirs. First, they show that if the firm producing the essential good (A) can enter the other firm's (B) market, and product B's value is not too high, A will give away a substitute to product B and raise the price of A. In most cases, this resembles the equilibrium in our monopoly set-up: the monopolist has an interest in raising the price of the internet rather than that of the phone, the major difference being that, because of differentiation, prices will still be positive. Second, they find that if it has the choice, firm A will always set one price for a version of A compatible with B and another (lower) price for an incompatible product. We find a similar result: the price for a version of the internet which is compatible with the app is always higher than the price for a version that is not compatible. The reason is intuitive: the higher price is a way for the owner of the essential good to extract surplus from the presence of the competing product.

We adjust our model to better reflect the realities of the market we examine. We assume that the second good owned by the ISP does not need the internet to work, and therefore does not suffer when the price of the internet is raised. Also, we model competition between non-essential goods in a Hotelling framework. Finally, we extend their set-up to include competing ISPs, or in their terminology, competing essential goods.

By relating our model to the net neutrality literature,[7] we are able to compare our results to articles that examine the issue of net neutrality and vertical integration (Dewenter and Rosch (2016), Guo et al. (2010), Brito et al. (2014), Fudickar (2015)). Compared to these articles, our major contribution is to highlight the link between the one-way essential complements literature and net neutrality. Instead, the main driver of most of that literature seems to be the competition for advertising revenues.

Dewenter and Rosch (2016) consider the incentives of a monopoly ISP, integrated with a CP, to exclude competing CPs from its network in a two-sided model where CPs compete for advertisers and their content is free. They show that a monopolistic ISP may find it profitable to exclude the rival's content if there is (i) little product differentiation on the content market and consumers only value differentiated products (ii) limited network externalities from consumers to advertisers and (iii) strong network externalities from advertisers to consumers.[8] In that case, the competitor steals a large fraction of the ISP's business on the advertising market because contents are close substitutes. This effect cannot be compensated by higher access fees since consumers do not obtain much value from the additional content because contents are very homogeneous, i.e. the competition effect on the advertising market is less than compensated by the complementarity effect. Thus, the ISP finds it profitable to limit competition by excluding contents. The major divergence with us,

[7] See Schuett (2010), Krämer et al. (2013) and Greenstein et al. (2016) for literature reviews.

[8] There are other cases where exclusion can happen in their model but we do not comment on them since they only arise under extreme assumptions, e.g. if content is perfectly homogeneous and the ISP's users do not use the services of the competing app.

besides the fact that we also consider a duopoly of ISPs, is that in our model when the competitive pressure exerted by the rival app is intense, its value to users is also high and therefore there exist possibilities to monetize the app. Still, exclusion is a concern both in the monopoly case, when price discrimination is limited by net neutrality rules, and in the duopoly case.

Guo et al. (2010) and Brito et al. (2014) consider, respectively, the case of a vertically integrated monopolist competing with another CP, and a two ISP-two CP situation where one of the ISPs is integrated with one of the CPs. Each ISP can offer a fast lane, against payment, to CPs. The leading factor in both papers is the heterogeneity in ad revenues that CPs can generate. Hence, the identity of the integrated CP matters. Both articles show that net neutrality is not always respected by ISPs but that it is not true that they always have incentives to discriminate competing applications. ISPs could even discriminate *against* their own CP if the advertising revenue difference is sufficiently strong, in order to extract more from the competing CP. Both articles also show that vertical integration is not *per se* bad for welfare.

Taking a slightly different point of view, Fudickar (2015) looks at the effect of vertical integration on prioritisation in the absence of advertising rents. Even if the integrated ISP favours its CP to reduce the congestion it faces, consumers are always unambiguously better off with the prioritisation regime. However, welfare may decrease because of the loss in profit of the non-integrated CP.

That one-way essential complements are an intrinsic part of the internet is indisputable. The major difference with the previous studies (Guo et al. (2010), Brito et al. (2014), Fudickar (2015), Dewenter and Rosch (2016)) is that we consider competition with a good (the phone) that does not require the essential good (the internet) to work while they consider competition between goods that all require the essential good. This leads to two important differences which impact exclusion incentives. First, except for Fudickar (2015), in these articles the integrated ISP or ISPs compete with other CPs for advertising revenues. Because the phone is not financed through advertising, competition is mostly concentrated on the consumers' side. Second, because the app is a one-way complement while the phone is not, if the price of the internet increases, it directly affects the application but not the phone.

This also clearly differentiates our contribution from the standard vertical foreclosure literature (Rey and Tirole, 2007). Usually, this literature considers that all upstream goods have to go through a downstream firm before reaching consumers. This is clearly not the case for the phone in our set-up.

Kourandi et al. (2015) study the problem of internet fragmentation whereby some applications are only available through a particular ISP because of bilateral exclusivity contracts, and not on the internet as a whole. They build a two ISP-two CP model where there are two forces at play. On the one hand, CPs want to be available at both ISPs to maximize exposure and to increase advertising revenues. On the other hand, if both ISPs are available at an ISP, they compete for advertisers and prices for ads go down. They show that the zero-price rule cannot always prevent fragmentation, for instance if competition leads to very low ad prices, and that having no fragmentation is always beneficial to consumers but not always to total welfare. These two conclusions are in line with ours but for different reasons. In our case, fragmentation is not driven by advertising revenues but by the will to differentiate product lines to reduce competition. The consumer surplus result, in their case, arises because (i) consumers enjoy the joint consumption of CPs' contents more than they enjoy the consumption of the content of a single CP and (ii) competition between ISPs does not allow them to increase prices too much in case of no fragmentation. In our model, consumer surplus is always highest under no fragmentation, although consumers do not consume the phone and the app at the same time, because competition between ISPs leads to lower prices. Finally, their welfare result depends on advertising competition: if it is strong, ad prices –and hence ad revenues– may decrease so much that welfare decreases. In our setting, welfare may decrease because no fragmentation may lead to a situation where consumers are not matched with the right good, i.e. that which is closest to their location.

D'Annunzio and Russo (2015) also study fragmentation under the prism of competition for ads. Their crucial insight is that the decreasing marginal value of advertising may lead ISPs to fragment the internet to protect "their" CP from ad competition and be able to extract more revenue from it. They also obtain, as we and Kourandi et al. (2015) do, that the zero-price rule is not always sufficient to prevent fragmentation.

Finally, this work is related to studies on exclusivity in two-sided markets (Hagiu and Lee, 2011) and to the literature on access provision (e.g. Lewis and Sappington (1999)) in the telecommunications sector (e.g. de Bijl and Peitz (2004, 2009, 2010)). In particular, de Bijl and Peitz (2010) consider the case of an integrated ISP selling both the phone and VoIP, and a VoIP competitor. Their conclusion is that the incumbent might choose to underinvest in VoIP quality –even though it also affects its own VoIP business– so that the competitor cannot enter because it will not be able to set a high enough price to recover its entry cost. Note that a major difference with our model is that they do not examine peer-to-peer VoIP, and hence for each call the VoIP application has to pay a termination fee to the ISP.

3. Model

There are three products –the internet, the phone and the app– which cater for the demand of consumers for two services: internet-based services (the internet) and communication-based services (the phone and the app). The application and the internet are one-way essential complements: the app cannot be used without the internet but the internet and the phone can be used on their own. The internet and the phone are offered by a monopolistic residential ISP: we relax this assumption in Section 5 where we introduce competing ISPs. The app is made available on the internet. It is financed through exogeneous means such as advertising or a given price paid by consumers. In the latter case, the gross utility of the app, u^a (see *infra*), should be understood as a net utility. The only assumption needed is that the owner of the application does not pass-through internet price variations to consumers. In line with this assumption, despite very different prices for the internet, there is a single price for WhatsApp worldwide. We note that since we consider a single app and there are no network externalities, our model is not two-sided. Production costs are all normalized to zero.

The app and the phone are both horizontally and vertically differentiated substitutes. With respect to vertical differentiation, all consumers obtain gross utility $u^a \in [0, 1]$ if they consume the app and $u^t \in [0, 1]$ if they consume the phone. Consumers are therefore homogeneous with regard to the utility provided by the app and the phone. They also single-home: they use either the app, the phone or nothing. While multi-homing may be pervasive in the VoIP/phone industry, it is less so for television/VOD[9] or SMS/WhatsApp.[10] Single-homing is the set-up where the competition exerted by the app is the strongest and, as for the zero-price rule, the worst case for not excluding the app.

The utility difference $\Delta u = u^a - u^t$ is our measure of vertical product differentiation. With respect to horizontal differentiation, the app and the phone are located at the extremes of a Hotelling line of size 1. A consumer located at x incurs a disutility τx when he consumes the app and a disutility $\tau(1-x)$ when he consumes the phone. Let τ be our measure of horizontal product differentiation.

Consumers have heterogeneous valuations $\theta \in [0,1]$ for the internet. Consumers are uniformly distributed on the unit square with the vertical axis measuring the consumers' heterogeneous valuations for the internet θ and the horizontal axis being the Hotelling line. The population size is normalized to 1.

To limit the number of cases to consider, we restrict the parameter set by assuming the following.

Assumption 1.
$u^a, u^t \geq 2\tau$

Assumption 1 guarantees that the market will be fully covered in the monopoly case without the app. It also implies that –when the app is enabled– a consumer switching to the app was a previous phone user. There is therefore no demand expansion when the app is available. Hence, the business stealing effect of the app is maximized, which is presumably the most appropriate case to study incentives to exclude. Note that this assumption implies $\tau \leq 1/2$ since we assumed $u^a, u^t \in [0,1]$.

Let us denote the internet by i, the phone by t and the app by a. The consumer can choose between four combinations of goods, as consumers will not use the app and the phone together: (i,t), (i,a), (i), (t); they could also consume nothing $(\emptyset)$. The ISP offers the internet and the phone at prices $p_i \geq 0$ and $p_t \geq 0$, and consumers choose whether to subscribe to the services. The associated net utilities for a consumer located at (x,θ) are:

$$U(x,\theta) = \begin{cases} U(i,t) = \theta - p_i + u^t - \tau(1-x) - p_t, \\ U(i,a) = \theta - p_i + u^a - \tau x, \\ U(i) = \theta - p_i, \\ U(t) = u^t - \tau(1-x) - p_t, \\ U(\emptyset) = 0. \end{cases}$$

We ignore the possibility for the ISP to bundle the phone and the internet, but we will consider the possibility for the ISP to offer a version of the internet where the app is disabled. In that case,

[9] Wall Street Journal, 2015, Streaming Services Hammer Cable-TV Ratings, http://www.wsj.com/articles/streaming-services-hammer-cable-tv-ratings-1426042713

[10] Bloomberg, 2015, WhatsApp Shows How Phone Carriers Lost Out on $33 Billion. http://www.bloomberg.com/news/articles/2014-02-21/whatsapp-shows-how-phone-carriers-lost-out-on-33-billion

there are two prices for the internet: p_i without the app and $\tilde{p}_i$ with it. With a price surcharge, the utility $U(i,a)$ is:

$$U(i,a) = \theta - \tilde{p}_i + u^a - \tau x. \quad (1)$$

4. Monopoly ISP

In this section, we consider a monopolistic ISP. If the ISP offers the internet and the phone at prices $p_i \geq 0$ and $p_t \geq 0$, its profit is $\Pi = d_i p_i + d_t p_t$, where d_i and d_t are respectively the demand for the internet and for the phone.

4.1. Exclusion

Let us start with the case where the app is excluded by the ISP. The demand for the internet at price p_i is $d_i = 1 - p_i$ and the ISP's profit is maximized for $p_i^e = 1/2$ and $d_i = 1/2$. The demand for the phone at price p_t is $d_t = \min[(u^t - p_t)/\tau, 1]$. Under Assumption 1, the profit maximizing price is $p_t^e = u^t - \tau$ and the market is fully covered ($d_t = 1$). The total profit of the ISP is

$$\Pi^e = 1/4 + u^t - \tau. \quad (2)$$

Figure 1 represents consumers' product choices. Consumers with a high valuation of the internet buy both goods while those with a low valuation only buy the phone. As the transportation cost is low enough ($\tau \leq u^t/2$), all consumers buy (at least) one product.

4.2. No Exclusion

The impact of the app's entry on the ISP's profit is difficult to assess *a priori* because of two competing effects. On the one hand, there is a **complementarity** effect. Some users obviously benefit from the availability of the free app. This higher utility, or higher willingness to pay, can be extracted through a rise in the price of the internet, which will increase profit. On the other hand, the app's presence leads to a **competition** effect, whereby some consumers switch from the phone to the app. The impact of these two effects is *a priori* unclear as their relative importances are both linked to Δu. A higher Δu means that the app has relatively more value compared to the phone, amplifying both the competition effect (more consumers switch to the free app) and the complementarity effect (consumers are ready to pay more to use the internet). To assess the relative importance of these two effects, we first consider the case where the market is still covered when the app is available. This implies that the price of the phone satisfies $p_t \leq u^t - \tau$.

Lemma 1. *In a fully covered market situation,*

1. *If $\Delta u \geq -\tau$ or $u^a > -\Delta u(2 + \Delta u)/4$, the monopoly ISP charges prices $p_i^{ne} = 1/2 + u^t - \tau + \Delta u/2$ and $p_t^{ne} = u^t - \tau$ and realizes profit $\Pi^{ne} = u^t - \tau + (1 + \Delta u)^2/4$.*
2. *If $\Delta u < -\tau$ and $u^a \leq -\Delta u(2 + \Delta u)/4$, the monopoly ISP charges prices $p_i = 1/2$ and $p_t = -\Delta u - \tau > 0$ and realizes profit $\Pi^{ne2} = \frac{1}{4} - (\tau + \Delta u)$.*

The two situations characterized by Lemma 1 differ substantially. In the first, the app, as compared to the phone, is either a superior or, at least, not a too inferior product. In the second situation covered by Lemma 1, the value of the app, relative to that of the phone, is so low that the

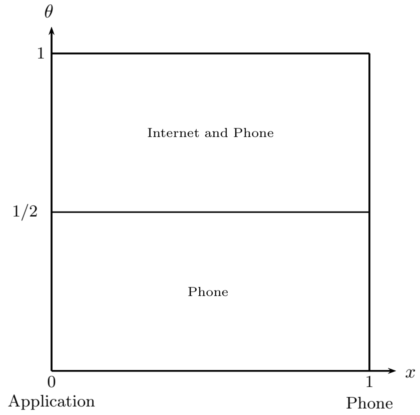


Figure 1: Monopoly ISP, app exclusion.

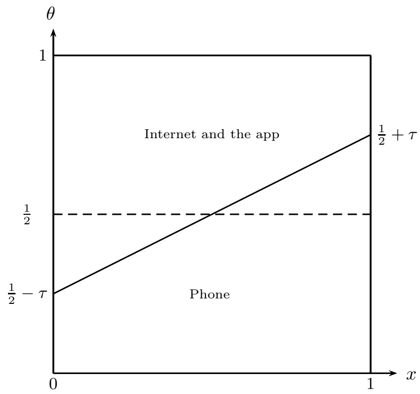


Figure 2: Rebalancing prices to exactly offset the app's entry for $\Delta u = 0$

ISP chooses prices such that no consumer purchases the application. The market configuration is similar to the exclusion case. As this scenario adds nothing to the discussion, we henceforth ignore it and we focus on the situations where the app is sufficiently valuable for the consumers.

Assumption 2. $\Delta u \geq -\tau$ *or* $u^a > -\Delta u(2+\Delta u)/4$.

4.3. Comparisons

In the covered market case, the price of the phone is left unchanged compared to the situation where the app is not available, implying that $u(i,a) > u(i,t)$ for all x. Thus, all internet users choose the free app rather than the phone, which reduces the ISP's profit (competition effect). Yet, the internet has more value as it enables the free app. The ISP is therefore able to compensate its losses on the phone market by rebalancing its prices to extract the extra surplus due to the app's entry. If the app and the phone are not vertically differentiated ($\Delta u = 0$), the ISP increases the price of the internet by $u^t - \tau$, which is exactly the price of the phone. In that case, the losses due to the competition effect are perfectly compensated by the extra surplus extracted via the complementarity effect.[11] Figure 2 illustrates this case. If the app is more valuable than the phone ($\Delta u > 0$), the internet price rises even more and the complementarity effect more than compensates the competition effect.

Proposition 1. *If $\Delta u \geq 0$, app exclusion is not a profitable strategy for the ISP.*

Proof. Suppose that the ISP sets the prices according to part 1 of Lemma 1 ($p_i = p_i^{ne} = 1/2 + u^t - \tau + \Delta u/2$, $p_t = p_t^{ne} = u^t - \tau$). Then, its profit is $u^t - \tau + (1 + \Delta u)^2/4$ which is higher than $\Pi^e = 1/4 + u^t - \tau$ because $\Delta u > 0$. ■

When the app has less value than the phone ($\Delta u < 0$), incentives to exclude depend on the market being covered or not. In a covered market, it is clear that the profit is smaller than Π^e but this is not necessarily the case if the market is not covered. We therefore proceed in two steps. First, we derive the condition for a covered market at equilibrium. In particular, it can be shown (see the Appendix) that if $\Delta u \leq \Delta \bar{u} = \frac{(1-2\tau)u^t - \tau(3-2\tau)}{u^t - \tau}$ the market will be covered. Second, we use a numerical example to illustrate that, if the market is not covered and $\Delta u < 0$, app exclusion is not always the preferred option.

Proposition 2. *If $\Delta u \leq \Delta \bar{u}$, the market is fully covered at equilibrium and the equilibrium prices are (p_i^{ne}, p_t^{ne}), giving a profit of Π^{ne}. If this condition is not satisfied, the market is not fully covered at equilibrium, $p_t > u^t - \tau$ and $\Pi > \Pi^{ne}$.*

Propositions 1 and 2 are summarized in Figure 3. To sum up, if $\Delta u \geq 0$, exclusion never arises. If $\Delta u < 0$ and $\Delta u < \Delta \bar{u}$, the market is covered at equilibrium and the ISP's profit is higher when it excludes the app. Last, for $\Delta \bar{u} < \Delta u < 0$, the market is not covered at equilibrium and exclusion is not systematic. The equilibrium analysis of this case ($\Delta \bar{u} < \Delta u < 0$) is rather involved and we use a numerical example to illustrate the incentives to exclude the app. Suppose that $\tau = 1/4$ and $u^t = 3/4$. Without the app, the profit is equal to $\Pi^e = 3/4$. Let us consider two values for $u^a < u^t$: 0.74 and 0.70 with, in both cases, $0 > \Delta u > \Delta \bar{u} = -1/2$. Equilibrium prices, demands and the corresponding profits are reported in Table 1. From the example, it is clear that exclusion is not profitable for $u^a = 0.74$ as the ISP manages to compensate the competition effect with higher prices even if the app has (slightly) less value than the phone. For the lower value of u^a, the ISP's profit is higher with exclusion. The competition effect is too strong and the ISP cannot compensate through price rebalancing.

	$u^a = 0.74$	$u^a = 0.70$
p_i	0.98	0.96
p_t	0.56	0.55
d_i	0.56	0.53
d_t	0.37	0.39
Π	$0.76 > \Pi^e$	$0.73 < \Pi^e$

Table 1: Numerical simulations for $\Delta u < 0$: $u^t = 0.75$ and $\tau = 0.25$.

[11] Note also that by increasing the price of the internet, the ISP reduces competition on the voice market. Indeed, because the app and the internet are one-way essential complements, a rise in the price of the essential good is similar to one in the price of the non-essential product.

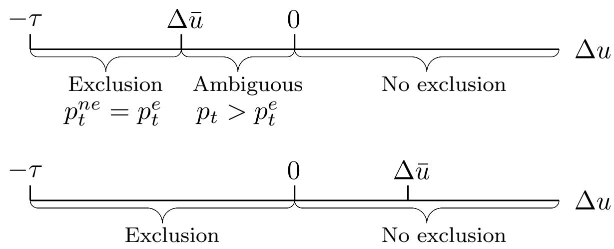


Figure 3: Graphical Summary of Propositions 1 and 2

4.4. Price surcharge

An alternative for the ISP to monetize the app is to apply a surcharge to internet subscribers using the app. We show that selling two versions of the internet, one where the app is enabled at price $\tilde{p}_i$ and one where it is disabled at price p_i, increases consumer segmentation and hence profits. Thus, a monopoly ISP has no incentive to exclude a competing app, regardless of the quality differential, i.e. with a price surcharge, exclusion is not an issue for all values of Δu.

Proposition 3. *With a price surcharge, the ISP always realizes a profit weakly higher than the exclusion profit* Π^e.

Proof. Suppose that the monopolist applies the following prices: $p_t = u^t - \tau$, $p_i = 1/2$ and $\tilde{p}_i = 1/2 + u^t - \tau$, i.e. the phone price and the internet prices are equal to p_t^e and p_i^e, the surcharge is equal to the phone price: $\tilde{p}_i = p_i^e + p_t^e$.

At these prices, the market is fully covered with all consumers using either the phone (with or without internet) or the app (with internet at price $\tilde{p}_i$). To prove Proposition 3, it suffices to show that the number of consumers buying one version of the internet is at least as high as in the exclusion case (where $d_i = 1/2$). In the Appendix, we show that it is the case for the whole parameter space. Therefore, with a price surcharge for the internet, the profit is always higher or (in the worst case) equal to Π^e and app exclusion is never a profitable strategy. ∎

Proposition 3 shows that a price surcharge for the internet with the app is preferred to app exclusion as the surcharge reduces competition and allows the ISP to extract part of the surplus created by the app. In particular, the ability to set two different prices for the internet enables the ISP to better extract surplus from the different types of consumers, i.e. to price discriminate.

4.5. App exclusion and the net neutrality debate

Does net neutrality improve welfare and consumer surplus? To investigate this question, we focus on the covered market situation for which the welfare analysis remains tractable. In that case, $p_t = u^t - \tau$. Let us start by examining the optimal behaviour of the firm if it is able to set three prices. We know from Proposition 3 that a price surcharge is always more profitable than exclusion. The next Proposition compares profits in the no-exclusion case if the monopolist can set two or three prices.

Proposition 4. *Suppose that the market is covered ($p_t = u^t - \tau$). With a price surcharge, the ISP always realizes a profit weakly higher than* Π^{ne}. *If $-\tau < \Delta u < \tau$, profit is strictly higher.*

Proof. The proof is similar to that of Proposition 3. First, we know that the possibility to use three prices never lowers profit because in that case, the monopolist can simply set $\tilde{p}_i = +\infty$ and revert to the two price situation. Second, suppose that $\tilde{p}_i = p_i^{ne}$, $p_i = p_i^{ne} - p_t^{ne}$ and $p_t = p_t^{ne}$. At these prices, the market is covered ($p_t^{ne} = u^t - \tau$) and the prices of the bundles (i, a) and (i, t) are identical. At these prices, the consumer indifferent between (i, t) and (i, a) is located at $\tilde{x}_1 = \frac{\Delta u + \tau}{2\tau}$. For a price surcharge, with these prices, to be (strictly) profitable, we need $0 < \tilde{x}_1 < 1$, which implies $-\tau < \Delta u < \tau$.

With a price surcharge, consumers in $[\tilde{x}_1, 1]$ buy the internet and the phone if $\theta \geq 1/2 + \Delta u/2$ while with two prices, they buy the internet and the app if $\theta(x) \geq 1/2 + \Delta u/2 - \tau(1-2x)$. Consumers located in in $[\tilde{x}_1, 1]$ have a higher demand for internet when the ISP uses a price surcharge, while consumers in $[0, \tilde{x}_1]$ have the same demand.

Formally, denoting demand for the internet in the two price case, for the internet with the app and the internet with the phone in the three price case as, respectively, d_i, d_{ia} and d_{it}, we have that, if $\tilde{p_i} = p_i^{ne}$, $p_i = p_i^{ne} - p_t^{ne}$ and $p_t = p_t^{ne}$ and $-\tau < \Delta u < \tau$:

$$d_i < d_{it} + d_{ia}. \tag{3}$$

■

Therefore, condition NN_2 –no price surcharge to use the app– will not be voluntarily enforced by a monopoly ISP under the assumption that $-\tau < \Delta u < \tau$, i.e. so long as the app and the phone are not very strongly differentiated. Otherwise, the monopolist is indifferent to the price surcharge. If $-\tau < \Delta u < \tau$, the ISP has incentives to perform financial-based discrimination between contents. On the other hand, if it can impose a surcharge, then content-based discrimination (condition NN_1) is not a concern. If the ISP cannot apply a price surcharge, incentives to exclude depend on the relative value of the app and the phone to consumers. If $\Delta u \geq 0$, the ISP will not exclude the app (Proposition 2). If $\Delta u < 0$, app exclusion might be a concern depending on parameter values (see Table 1 for an example). In a nutshell, if $-\tau < \Delta u < \tau$, net neutrality requirements *always* constrain the behaviour of a monopolistic ISP.

By changing the price structure but not the price level, the monopolist can better segment the consumers and manages to increase demand for the internet without having to decrease its prices and consequently, its profit increases. With prices $\tilde{p_i} = p_i^{ne}$, $p_i = p_i^{ne} - p_t^{ne}$ and $p_t = p_t^{ne}$, consumer surplus also increases because some consumers start buying the internet (demand expansion effect) and consumers are better segmented, which therefore decreases transportation costs. However, the following Lemma establishes that the impact of a price surcharge on consumer surplus is difficult to assess because prices $\tilde{p_i} = p_i^{ne}$, $p_i = p_i^{ne} - p_t^{ne}$ are not optimal.

Lemma 2. *In the covered market situation ($p_t = u^t - \tau$), compared to the case where $\tilde{p_i} = p_i^{ne}$, $p_i = p_i^{ne} - p_t^{ne}$, the profit of the monopolist decreases if it sets (both):*

1. $\tilde{p_i} < p_i^{ne}$,
2. $p_i < p_i^{ne} - p_t^{ne}$.

Proof. See the Appendix. ■

This Lemma adds another difficulty to the assessment of the effect of a price surcharge on consumer surplus: at least some, perhaps all, internet users will face higher prices. This, however, has to be balanced with the improved matching. Some consumers who used to buy the phone or the internet with the app can now switch to the internet with the phone. Depending on the values of the parameters, any of the effects can prevail. As the example in Table 2 shows, even in the case where all consumers face higher prices, consumer surplus (and profit) can still increase because of the better matching. Firms' and consumers' interests may thus sometimes be aligned *against* net neutrality.

	Three Prices	Two Prices
p_t	0.26	0.26
p_i	0.507	0.765
$\tilde{p}_i$	0.768	/
Π	0.518	0.515
CS	0.152	0.146
W	0.67	0.661

Table 2: Numerical simulations for $u^a = 0.31$, $u^t = 0.3$ and $\tau = 0.04$.

From this example, it appears that net neutrality may intensify competition (prices decrease) but leads to an inefficient repartition of consumers (a transport costs increase) which, in this case, reduces consumer surplus. We will reach a similar conclusion in Section 5.2.

4.6. Sabotage vs. app exclusion

We have so far considered that, to reduce competition on the app/phone market, an ISP has no alternative but to exclude the app or to ask for a price surcharge. Another strategy to reduce competition is to sabotage the rival app by diminishing its quality. With the development of high bandwidth applications and content, the internet is becoming more and more congested and traffic management by ISPs is a growing concern, especially because net neutrality rules aim at prohibiting discrimination between contents. In this context, an ISP can reduce a rival product's quality either by slowing down traffic delivery or by increasing jitter/delay. This is particularly a concern when (1) the app provides time-sensitive or bandwidth-intensive content such as real-time audio/video streaming or VoIP services and (2) when this content competes with the ISP's own services.

Sabotage in the context of our model can be modeled as lowering the utility of the app u^a. Let us consider the covered market case ($\Delta u < \Delta \bar{u}$). Without exclusion, the ISP's profit is Π^{ne} and this profit unambiguously increases with u^a:

$$\frac{\partial \Pi^{ne}}{\partial u^a} = \frac{1 + \Delta u}{2} > 0. \tag{4}$$

This means that downgrading the rival app is not a profitable strategy. On the contrary, a higher app quality always benefits the ISP when the app is enabled. The recent deal between Comcast and Netflix whereby Netflix agreed to pay to be *directly* connected to Comcast's network illustrates our point. The agreement may be beneficial to Comcast even in the absence of any payment because it increases the value of Netflix through the improved connection, which is equivalent to an increase in u^a.[12]

[12] See for instance The Wall Street Journal (2014), Netflix to Pay Comcast for Smoother Streaming, available at http://on.wsj.com/1ZdoCTx or Dan Rayburn (2014), Here's How The Comcast and Netflix Deal Is Structured, With Data and Numbers, available at http://blog.streamingmedia.com/2014/02/heres-comcast-netflix-deal-structured-numbers.html

Finally, if we are in the parameter space where, without surcharge, the ISP would exclude the app with certainty ($\Delta u < \Delta \bar{u} < 0$) but exclusion is not allowed, the ISP will not have incentives to degrade the quality of the rival app.

5. Duopoly

We now consider that there are two ISPs, ISP_1 and ISP_2, which compete à la Bertrand. We look at two different cases. In the first, the symmetric case, ISPs are both offering the internet and the phone. In the second, the asymmetric case, ISP_1 offers the internet and the phone while ISP_2 only offers the internet. Although the first case is more likely if we think of examples such as the phone and a VoIP app, the second aims at representing interactions of goods such as Netflix and ISPs' VOD products that are not offered by all ISPs.[13]

The internet offered by the ISPs is similar up to the availability of the app. Therefore, if both exclude or both admit it, the internet is considered by consumers as a homogeneous product. It is only when the application is available at one ISP and not at the other that the internet has two different versions. The same holds for the phone: it is a differentiated product with regard to the app but consumers see no difference between the phone offered by ISP_1 and that offered by ISP_2.

The reason why the contrast between the two cases is interesting is related to the complementarity and competition effects of the previous section. In the symmetric case, competition on the phone and the internet markets already exists, and admitting the app does not create additional competition. Only the complementarity effect remains, but for that, only one ISP should offer the app, otherwise all benefits would be dissipated by competition, fully in the case of Bertrand competition and partially in the case of less extreme competition, e.g. if ISPs have different bandwidth capacities. In contrast, in the asymmetric case, only one ISP offers the phone and admitting the app still creates competition. Both the complementarity *and* the competition effect remain present and incentives to admit the app are more ambiguous.

We use the concept of *fragmented internet* (Kourandi et al., 2015) to refer to a situation where the application is available at one ISP but not at the other. Therefore, there is *no fragmentation* if the application is admitted by both ISPs. Our NN conditions are closely linked to fragmentation: fragmentation implies that one ISP does not respect NN_1 or in other words, that one ISP has some exclusive content, the application.

Finally, note that in this section, we only use Assumption 1.

5.1. Symmetric ISPs

We first consider two symmetric ISPs –each ISP offers the internet and the phone– competing à la Bertrand. We assume that consumers are one-stop shopping: they cannot buy the internet at one ISP and the phone at the other. The game is played in the following way:

1. ISPs decide to exclude or admit the application,

[13] The ISPs may decide in a previous stage on the bundles of services they want to offer. Our two structures can thus be endogenized.

		ISP 2	
		Admitting	Excluding
ISP 1	Admitting	0 ; 0	$\frac{(\tau+\Delta u)^2}{8\tau}$; 0
	Excluding	0 ; $\frac{(\tau+\Delta u)^2}{8\tau}$	0 ; 0

Table 3: Pay-off Matrix in the Symmetric Case

2. ISPs set the prices for the internet (p_i), the internet with the app ($\tilde{p}_i$) and the phone (p_t).

Let us start with the analysis of the second stage of the game. If both ISPs adopt the same policy towards the app –exclusion or no exclusion– they are perfectly symmetric, Bertrand competition leads to marginal cost pricing and profits are zero. All consumers are buying the internet and, if the app is available, consumers choose their preferred voice solution, the app for the consumers located at $x \in [0, 1/2 + \Delta u/2\tau]$ and the phone for consumers located at $x \in [1/2 + \Delta u/2\tau, 1]$. If the app is not available, all consumers buy the phone.

But homogeneity is not a definitive curse: one ISP could exclude the app from its network. In this case, the internet with the app is only offered by one firm. Still, the internet without the app and the phone are offered by the two firms leading to $p_i = p_t = 0$. The firm offering the app chooses a surcharge equal to $\tilde{p}_i = \tau/2 + \Delta u/2$, consumers located at $x \in [0, 1/4 + \Delta u/4\tau]$ buy the app and the ISP realizes a profit equal to $(\tau + \Delta u)^2/8\tau$.

Through this exclusion, the ISP creates differentiated internet products catering to different consumers. One could see these products as a "high-quality" internet with the app and a "low-quality" internet without the app. Admitting the app enables the ISP to sell the internet with the app at a positive price, $\tilde{p}_i = \tau/2 + \Delta u/2$, yielding a positive profit. Notice that because $p_i \neq \tilde{p}_i$, the ISP offering the app only complies with the NN_1 condition while the other ISP complies with none.

The above results are summarized in the pay-off matrix in Table 3. The details of the computations are relegated to the Appendix.

Let us turn to the first stage of the game. Except if $\Delta u = -\tau$,[14] it is clear that there are three possible Nash equilibria: one where both firms admit the app and offer it for free, and two where only one firm admits the app and offers it at a premium price while the other excludes it. Thus, as in the monopoly case, the application will not be completely excluded. It is however possible that the internet becomes fragmented with only one ISP offering the app. Intuitively, if one ISP excludes the app, the other can offer a differentiated product, thereby making positive profits by charging a positive price for this good. Compared to the monopoly case, there is no longer a competition effect created by the app, as competition already exists on the phone market. Only the complementarity effect and the possibility to monetize the value created by the app remain. This possibility only exists if there is reduced competition, i.e. if only one firm offers the app. Therefore, total exclusion

[14] We ignore this particular case from here on.

		ISP 2	
		Admitting	Excluding
ISP 1	Admitting	$\frac{(\tau-\Delta u)^2}{8\tau}$; 0	$\frac{u^a+u^t}{2} - \frac{\tau}{2}$; 0
	Excluding	$\frac{(3\tau-\Delta u)^2}{18\tau}$; $\frac{(3\tau+\Delta u)^2}{18\tau}$	$u^t - \tau$; 0

Table 4: Pay-off Matrix in the Asymmetric Case

is not an equilibrium.

Moreover, net neutrality may be enforced in equilibrium even without regulation or government intervention because "admit" is a weakly dominant strategy for both ISPs.

Proposition 5. *If two symmetric ISPs compete à la Bertrand, there are two classes of equilibria: one where the internet is not fragmented and both firms respect NN_1 and NN_2, and another where the internet is fragmented and one firm respects NN_1 while the other excludes the app and therefore respects neither NN_1 nor NN_2.*

Computing the consumer surplus and social welfare effects of each class of equilibrium, we show that:

Proposition 6. *Consumer surplus and welfare are highest under the equilibrium where both firms admit the application, i.e. when both NN_1 and NN_2 are respected.*

Imposing the net neutrality conditions is pro-competitive in the sense that this drives all prices down to marginal cost. Consumers choose their preferred voice solution. Because of the low prices and the better matching, consumer surplus is highest in that situation. Therefore, net neutrality is always pro-competitive and welfare enhancing in this case. We will see that this result may not hold if ISPs are asymmetric.

5.2. Asymmetric ISPs

Let us now suppose that ISP_1 sells the phone and the internet while ISP_2 only sells the internet. First, because of the asymmetric situation, ISP_1 has the upper hand: whatever ISP_2's choice, ISP_1 will always get a positive pay-off because it has one differentiated good, the phone. Second, the only way for ISP_2 to have a positive pay-off is to offer the app when ISP_1 excludes it. The equilibrium in the pricing game is represented by the pay-off matrix in Table 4.

Proposition 7. *If two asymmetric ISPs compete à la Bertrand, the only pure Nash equilibria are those with fragmentation.*

The first important result is that the application is never *completely* foreclosed from the market, i.e. at least one firm will always admit it. The reason is simple: the only chance for ISP_2 to obtain a positive profit is to admit the app. In other words, NN_1 will always be respected by at least one ISP.

Second, the situation where both firms admit the app is not a Nash equilibrium and therefore, NN_1 and NN_2 are never respected by both ISPs. ISP_1 does not have incentives to admit the app if ISP_2 also does. The reason is that, on the one hand, admitting will not lead to a complementarity effect because Bertrand competition will drive the price of the internet with the app to zero. On the other hand, because the price of the internet with the app will be zero, competition on the voice market will be extremely strong and the price of the phone will have to decrease. Both effects have a negative impact on the profit of ISP_1 and it therefore has no incentive to admit the app if ISP_2 also does.

Regarding consumer surplus and social welfare, the main question is whether imposing NN_1 and NN_2, i.e. whether imposing the admit-admit situation, increases consumer surplus and welfare.

Proposition 8.

1. *Consumer surplus is always highest when both firms admit the application, i.e. when NN_1 and NN_2 are respected.*
2. *Consumer surplus and welfare are not necessarily highest in the same situation.*
3. *Imposing NN_1 and NN_2 can increase or decrease welfare.*

The first part Proposition 8 is intuitive. When both firms admit the app, $\tilde{p}_i = 0$ due to Bertrand competition. Therefore the only good with a positive price is the phone but that price cannot be too high because of the very strong competition coming from the app. Consumers benefit from these low prices. However, as in Section 4.5, they lose some surplus due to higher transportation costs (they consume disproportionately the free app). Overall, the former effect is always stronger than the latter.

While net neutrality is pro-competitive and beneficial to consumers, it does not always enhance total welfare. Indeed, the gain in consumer surplus due to the lower prices can be outweighed by the lower profit and the higher transportation costs.

5.3. Exclusivity

It can be shown that both in the symmetric and the asymmetric cases, the profit of an ISP is highest if it is the sole provider of the app. Therefore, ISPs may compete to obtain the exclusivity to offer the app. Exclusivity can be obtained either by paying the rival in exchange for a commitment to block the app or by signing an exclusivity contract with the app developer (as in Kourandi et al. (2015)). Since they are always better off under the net neutrality/no-fragmentation equilibrium, this type of exclusivity agreement is always detrimental to consumers but may be good for total welfare if it increases sufficiently profits and does not reduce consumer surplus too much (Proposition 8). Kourandi et al. (2015) find similar conclusions (see pp.6-7 for more details).

Though in principle the zero-price rule prohibits financial transfers between ISPs and CPs, and hence also exclusivity contracts, telecom operators in Belgium and in France have signed contracts with Netflix to include its VOD catalogue on their internet/TV box, thereby admitting subscribers to watch Netflix on their TV. For these operators, it can therefore be expected that the complementarity effect exerted by Netflix will more than compensate the losses created by an intensified competition.

6. Conclusion

Applications and content are one-way essential complements to the internet. This paper analyzes a classic issue: the incentives of a vertically integrated essential good provider to exclude competing content. While most of the literature focuses on competing complements –competition between content providers–, we analyze the incentives to exclude an essential complement when it competes with a substitute that does not need the internet to be consumed. Examples of such situations abound: video on demand and Netflix, SMS and messaging apps, telephone and VoIP, etc. This paper studies the important question of the welfare effect of applying net neutrality regulation in those situations. We reach two main conclusions. First, exclusion of the app is not an issue. In monopoly, the ISP will simply adapt its price structure to take advantage of the presence of the app. In duopoly, at least one ISP will always be willing to offer the app to differentiate itself from its rival. Second, net neutrality is a competition intensifier in that it reduces prices. Unsurprisingly, its application hurts ISPs. More surprisingly however, net neutrality can also hurt consumers. Indeed, despite lower prices, the matching between consumers and products may be less efficient when net neutrality is enforced. Therefore, the interests of firms and consumers may sometimes be aligned *against* net neutrality. It is interesting to note that this arises in the monopoly case and that competition leads to a divergence of the interests of ISPs and consumers.

For simplicity's sake, we have overlooked a number of important issues. First, our model is static and does not encompass investment issues. Imposing net neutrality through regulation may decrease profits so much that investment could plummet. This argument against net neutrality has been put forward in the literature by, for instance, Choi and Kim (2010). In the monopoly setting, so long as the investment needed to accommodate the app is not too costly, our results should not change. This is especially the case if the cost of accommodating the app is linked to its value, in which case the ISP could choose between investing or degrading. In the duopoly setting, investment could change the selection of equilibrium but should not affect our main conclusions. Indeed, so long as investment is not too costly, the only difference would be to make firms which are indifferent between admitting and excluding, because they earn zero anyway, lean towards exclusion. That would not change the outcomes in the asymmetric case but would make the net neutrality equilibrium less likely in the symmetric scenario.

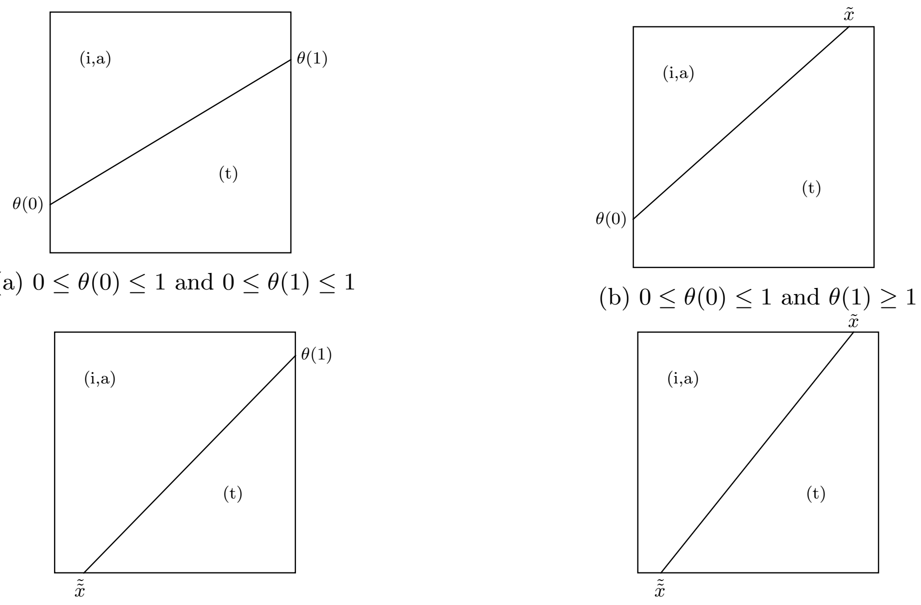
Second, we have maximized the possibility of exclusion through multiple assumptions: the zero-price rule, the covered market and the single-homing. To derive more policy recommendations, it may be a good idea to relax these assumptions.

Finally, it might also be interesting to generalize the model to include network effects. Indeed, while a phone user can be reached via Skype and vice versa, a WhatsApp user cannot send a message to someone who does not own the application. Thus a fragmented internet might lower users' willingness to pay and the profits of ISPs, thereby modifying incentives to exclude.

7. Appendix

7.1. Proof of Lemma 1

To prove the Lemma, we derive candidate equilibrium prices for the covered market situation. To be covered, the following condition must be satisfied for all x: $u(t) \geq u(\emptyset)$, implying that $p^t \leq u^t - \tau$. We prove the proposition in two steps. First, we identify a candidate equilibrium where the market is fully covered and consumers buy the internet (with the app) *or* the phone.



(a) $0 \leq \theta(0) \leq 1$ and $0 \leq \theta(1) \leq 1$

(b) $0 \leq \theta(0) \leq 1$ and $\theta(1) \geq 1$

(c) $\theta(0) \leq 0$ and $0 \leq \theta(1) \leq 1$

(d) $\theta(0) \leq 0$ and $\theta(1) \geq 1$

Figure 4: Configurations when $-\Delta u + \tau \leq p_t \leq u^t - \tau$

Second, we derive a candidate equilibrium where consumers can also purchase the phone with the internet.

Step 1. If $p_t \geq -\Delta u + \tau$, then by Assumpation 1 $u(i,t) \leq u(i,a)$ for all $x \in [0,1]$. We will start our analysis by searching for candidate equilibrium prices satisfying $-\Delta u + \tau \leq p_t \leq u^t - \tau$. Under these conditions, the market is fully covered and consumers buy either the internet and the app or the phone. The firm's profit is given by:

$$\Pi = p_i d_i + p_t d_t.$$

Solving $u(i,a) = u(t)$, we identify those consumers who are indifferent between the two options.

$$\theta(x) = (p_i - p_t) - \Delta u - \tau(1 - 2x). \tag{5}$$

From this equation, we can identify two boundary values corresponding to the extremes of the Hotelling line: $\theta(0) = (p_i - p_t) - \Delta u - \tau$ and $\theta(1) = (p_i - p_t) - \Delta u + \tau$ that will be used to derive the demand functions. Four configurations (see Figure 4) for the demands need to be considered:

(a) $0 \leq \theta(0) \leq 1$ and $0 \leq \theta(1) \leq 1$,

(b) $0 \leq \theta(0) \leq 1$ and $\theta(1) \geq 1$,

(c) $\theta(0) \leq 0$ and $0 \leq \theta(1) \leq 1$,

(d) $\theta(0) \leq 0$ and $\theta(1) \geq 1$.

Note first that case (d) can be ruled out because the conditions for $\theta(0) \leq 0$ and $\theta(1) \geq 1$ to be simultaneously respected are respectively $(p_i - p_t) \leq \Delta u + \tau$ and $(p_i - p_t) \geq \Delta u + 1 - \tau$, which is impossible given $\tau < 1/2$. If $\tau = 1/2$, case (d) is just a boundary case of the others.

In case (a), the demands are given by:

$$\begin{aligned} d_i &= \frac{(1-\theta(0)) + (1-\theta(1))}{2}, &(6)\\ d_t &= 1 - d_i. &(7)\end{aligned}$$

Profit maximizing prices are $(p_i, p_t) = (\frac{1}{2} + u^t - \tau + \frac{\Delta u}{2}, u^t - \tau)$ and Assumption 1 guarantees that at these prices $\theta(0)$, $\theta(1) \in [0, 1]$. The corresponding demands are given by $(d_i, d_t) = (\frac{1}{2} + \frac{\Delta u}{2}, \frac{1}{2} - \frac{\Delta u}{2})$ and the firm's profit is equal to $\Pi^{ne} = u^t - \tau + \frac{(1+\Delta u)^2}{4}$.

In case (b), $\theta(1) > 1$ and the demands are given by:

$$\begin{aligned} d_i &= \frac{(1-\theta(0))}{2}\tilde{x}, &(8)\\ d_t &= 1 - d_i. &(9)\end{aligned}$$

where $\tilde{x}$ is the consumer with the highest internet valuation $\theta = 1$ indifferent between (i, a) and (t):

$$\tilde{x} = \frac{1 + p_t - p_i + \Delta u + \tau}{2\tau}. \tag{10}$$

Profit-maximizing prices for this demand configuration are given by $(p_i, p_t) = (1 + u^a, u^t - \tau)$ resulting in a profit equal to $u^t - \tau$ which is lower than the profit in case (a).

In case (c), $\theta(0) < 0$ and the demands are given by:

$$\begin{aligned} d_i &= 1 - d_t, &(11)\\ d_t &= \frac{\theta(1)}{2}(1 - \tilde{\tilde{x}}). &(12)\end{aligned}$$

where $\tilde{\tilde{x}}$ is the consumer with the lowest internet valuation $\theta = 0$ indifferent between (i, a) and (t):

$$\tilde{\tilde{x}} = \frac{p_t - p_i + \Delta u + \tau}{2\tau}. \tag{13}$$

Profit-maximizing prices for this demand configuration are given by $(p_i, p_t) = (u^a - 2\tau, u^t - \tau)$ resulting in a profit equal to $u^a - 2\tau$ which is lower than the profit in case (a).

There is thus a unique equilibrium candidate for this covered market situation: $(p_i, p_t) = (\frac{1}{2} + u - \tau + \frac{\Delta u}{2}, u - \tau)$.

Step 2.

At price $p_t = u^t - \tau$, we have $u(i, a) > u(i, t)$, for all x. To have $u(i, a) < u(i, t)$ for some x, p^t must decrease by at least $u^a - 2\tau$. This means that there is a discontinuity in the phone demand at $p_t = u^t - \tau$. For $p_t < -\Delta u + \tau$, the consumer indifferent between (i, a) and (i, t) is located at

$$x^* = \frac{1}{2} + \frac{\Delta u + p_t}{2\tau}. \tag{14}$$

From this, we can derive the demand functions. Two cases must be considered depending on whether $\theta(0)$ is positive or negative. Suppose first that $\theta(0) \geq 0$. Then, we have:

$$
\begin{aligned}
d_i &= \frac{(1-\theta(0)) + (1-\theta(x^*))}{2} x^* + (1-x^*)(1-p_i), && (15)\\
d_t &= 1 - \frac{(1-\theta(0)) + (1-\theta(x^*))}{2} x^*. && (16)
\end{aligned}
$$

In Equation (15), the first term is the internet demand of consumers using the app; the second term is the internet demand of those buying the phone. Let us notice that in this case, $d_i + d_t \geq 1$.

The profit is equal to $p_t d_t + p_i d_i$. Taking the first order conditions of the profit-maximization problem, we can show numerically[15] that for all admissible parameter values satisfying $u^a, u^t \in [0,1]$, $\tau \leq \frac{1}{2}$ and Assumption 1, there is no interior maximum giving a higher profit than Π^{ne}. Then, the only possible solution is a corner solution where $u(i,a) \leq u(i,t)$ for all x. If no consumers are using the app, then the internet price is $p_i = \frac{1}{2}$ as there is no longer a complementarity effect. The phone price solves the equation $u(i,a) = u(i,t)$ for $x = 0$ giving $p_t = -(\Delta u + \tau)$. This solution is admissible if p_t is non-negative, requiring $-(\Delta u + \tau) \geq 0$. For this to be true, it is necessary for the phone to have more value than the app ($\Delta u < 0$). At these prices, $d_t = 1$ and $d_i = \frac{1}{2}$, giving a profit equal to $\Pi = \frac{1}{4} - (\tau + \Delta u)$. This profit is higher than the profit derived above if:

$$
\frac{1}{4} - (\tau + \Delta u) \geq \Pi^{ne} \Rightarrow u^a \leq -\Delta u \frac{(2 + \Delta u)}{4}.
$$

If this condition holds true, then the equilibrium prices in a covered market are $(p_t, p_i) = (-\Delta u - \tau, \frac{1}{2})$.

The corresponding profit is obviously lower than the profit with exclusion as the partition of consumers is the same but the price of the phone is lower.

Suppose now that $\theta(0) < 0$. In that case, demands and profit are:

$$
\begin{aligned}
d_i &= (1-p_i) + \left(\frac{x^* + \tilde{\tilde{x}}}{2}\right) p_i, && (17)\\
d_t &= (1-p_i)(1-x^*) + \frac{(1-\tilde{\tilde{x}}) + (1-x^*)}{2} p_i, && (18)\\
\Pi &= p_i d_i + p_t d_t. && (19)
\end{aligned}
$$

It is then easy to check using any standard mathematical software that the first-order conditions are never satisfied. Therefore, we have corner solutions. There are two possibilities. Either $x^* = 1$ but then no one consumes the internet and the phone together, and we are back to Step 1, or $\tilde{\tilde{x}} = 0$ and we are in the previous case where $\theta(0) \geq 0$.

[15] We proceed in the following way. We compute analytically the first-order conditions. Because the profit function is a cubic, the first-order conditions are quadratic in prices. This makes the analytical computation of the optimal prices particularly complex. Therefore, we proceed numerically. The software is fed numerical values and delivers optimal prices. We then check that, at these optimal prices, the relevant indifferent consumers are within their bounds (0 and 1) and therefore, that demands are positive and weakly smaller than 1. For many parameter values, this is not the case and the optimal prices are not admissible. Then, we have corner solutions. For other parameter values, we obtain admissible prices but they always lead to a lower profit than with $p_t = u^t - \tau$.

7.2. Proof of Proposition 2

If the price of the phone is above $u^t - \tau$, then the market is no longer fully covered and consumers have three options: (i, a), (t) and $(\emptyset)$. Solving the equations $u(i, a) = 0$ and $u(t) = 0$, we have the indifferent consumers defined as:

$$\hat{\theta}(x) = p_i - u^a + \tau x,$$

and

$$\hat{x} = 1 - \frac{u^t - p_t}{\tau}.$$

Under the conditions $\hat{\theta}(0) \in [0, 1]$, $\theta(1) \in [0, 1]$ and $\hat{x} \in [0, 1]$, the demands are given by:

$$d_i = \frac{(1 - \hat{\theta}(0)) + (1 - \theta(\hat{x}))}{2}\hat{x} + \frac{(1 - \theta(\hat{x})) + (1 - \theta(1))}{2}(1 - \hat{x}), \tag{20}$$

$$d_t = \frac{\theta(\hat{x}) + \theta(1)}{2}(1 - \hat{x}). \tag{21}$$

In this case, $d_i + d_t \leq 1$.

It is important to note that the candidate equilibrium with covered market described in Part 1 of Lemma 1: $(p_i^{ne} p_t^{ne}) = (1/2 + u^t - \tau + \Delta u/2, u^t - \tau)$ corresponds to the limit case where $\hat{x} \to 0$. This means that if we use the demand functions defined in (20) and (21) to compute the profit and if $\partial \Pi / \partial p_t |_{p_i = p_i^{ne}, p_t = p_t^{ne}} < 0$, setting p_t above $u^t - \tau$ does not increase the profit. Consequently, $(p_i^{ne}, p_t^{ne}) = (1/2 + u^t - \tau + \Delta u/2, u^t - \tau)$ is the unique equilibrium.

Conversely, if $\partial \Pi / \partial p_t |_{p_i = p_i^{ne}, p_t = p_t^{ne}} > 0$, then increasing p_t above $u^t - \tau$ gives a strictly higher profit. Therefore in that case (p_i^{ne}, p_t^{ne}) is not an equilibrium. Performing the adequate computations we obtain that:

$$\left.\frac{\partial \Pi}{\partial p_t}\right|_{p_i = p_i^{ne}, p_t = p_t^{ne}} < 0 \text{ if } \Delta u \leq \Delta \bar{u} = \frac{(1 - 2\tau)u^t - \tau(3 - 2\tau)}{u^t - \tau}. \tag{22}$$

The candidate equilibrium when the market is not covered can be derived numerically but a complete analytical characterization is particularly complex as many cases should be considered.

7.3. Complement to the proof of Proposition 3

With prices $(p_i, p_t, \tilde{p}_i) = (1/2, u^t - \tau, 1/2 + u^t - \tau)$, the consumer indifferent between (i, a) and (i, t) is located at

$$\tilde{x}_1 = \frac{\Delta u + \tau}{2\tau}. \tag{23}$$

Depending on the location of $\tilde{x}_1$, three cases should be considered (see Figure 5).

(1) $\tilde{x}_1 < 0$ if $\Delta u < -\tau$. In this case, the app has too little value and no consumer uses it. The outcome is similar to the exclusion case.

(2) $\tilde{x}_1 > 1$ if $\Delta u > \tau$. In this case, the phone has too little value and all internet users choose the app. The consumer indifferent between (i, a) and (t) is characterized by $\theta(x)$ given in Equation (5). Evaluated at the prices considered, we have that $\theta(1) = 1/2 - (\Delta u - \tau) < 1/2$. The fact that $\theta(1) < 1/2$ implies that the demand for internet at price $\tilde{p}_i$ is larger than $1/2$ leading to a larger profit than Π^e.

(3) $\tilde{x}_1 \in [0,1]$ if $-\tau \leq \Delta u \leq \tau$. For these parameter values, the demand configuration is represented on Figure 5. Referring to Figure 5, it is immediate that the demand for internet services is larger than in the exclusion case as $\theta(\tilde{x}_1) = 1/2$. Therefore, profit is larger than Π^e.

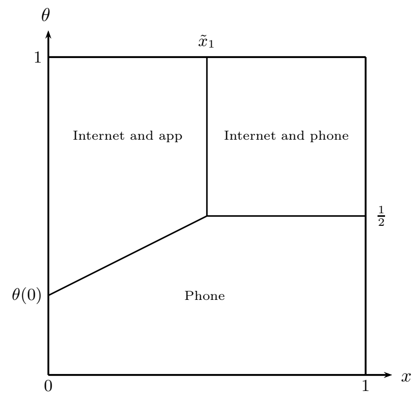


Figure 5: Demand with three prices and $\tilde{x}_1 \in [0,1]$

7.4. Proof of Lemma 2

To prove Lemma 2, we show that in a covered market situation, i.e. a situation where $p_t = u^t - \tau$, increasing prices above $\tilde{p}_i = p_i^{ne}$ and $p_i = \bar{p}_i = p_i^{ne} - p_t^{ne}$ is profitable.

Importantly, one must recall that in the case with only two prices, p_i^{ne} and p_t^{ne} are optimal. In that case, the profit and the first-order conditions with regard to the price of the internet are the following:

$$\Pi = p_i d_i + p_t d_t, \tag{24}$$

$$\frac{\partial \Pi}{\partial p_i} = p_i \frac{\partial d_i}{\partial p_i} + d_i + p_t \frac{\partial d_t}{\partial p_i} = 0. \tag{25}$$

We know that the market is covered and therefore that $\frac{\partial d_i}{\partial p_i} = -\frac{\partial d_t}{\partial p_i}$. Hence, optimal prices (p_i^{ne}, p_t^{ne}) are characterized by:

$$(p_i^{ne} - p_t^{ne}) \left(\frac{\partial d_i}{\partial p_i} \right) = -d_i. \tag{26}$$

With three prices, d_{ia} is demand for the internet with the app, d_{it} demand for the internet with the app disabled and d_t demand for the phone. The demand configuration is represented on Figure 5 and profit takes the following form:

$$\Pi = \tilde{p}_i d_{ia} + p_i d_{it} + p_t d_t. \tag{27}$$

We evaluate the impact of raising the internet prices p_i and $\tilde{p}_i$ by an equal amount. Taking the total differential of the profit, the impact of increasing both prices by $\mathrm{d}p_i$ on the profit is:

$$\left(\frac{\mathrm{d}\Pi}{\mathrm{d}\tilde{p_i}} + \frac{\mathrm{d}\Pi}{\mathrm{d}p_i}\right)\mathrm{d}p_i \tag{28}$$

This expression should be evaluated at $\tilde{p_i} = p_i^{ne}$, $p_i = \bar{p_i} = p_i^{ne} - p_t^{ne}$ and $p_t = p_t^{ne}$. Therefore, we know that:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\tilde{p_i}} + \frac{\mathrm{d}\Pi}{\mathrm{d}p_i} \quad = \quad d_{ia} + p_i^{ne}\frac{\partial d_{ia}}{\partial \tilde{p_i}} + \bar{p_i}\frac{\partial d_{it}}{\partial \tilde{p_i}} + p_t^{ne}\frac{\partial d_t}{\partial \tilde{p_i}} + d_{it} + \bar{p_i}\frac{\partial d_{it}}{\partial p_i} + p_i^{ne}\frac{\partial d_{ia}}{\partial p_i} + p_t^{ne}\frac{\partial d_t}{\partial p_i}. \tag{29}$$

We want to show that Equation (29) is always positive. First, because the market is covered, we have that:

$$\frac{\partial d_t}{\partial \tilde{p_i}} \quad = \quad -\frac{\partial d_{ia}}{\partial \tilde{p_i}}, \tag{30}$$

$$\frac{\partial d_{ia}}{\partial p_i} \quad = \quad -\frac{\partial d_t}{\partial p_i}. \tag{31}$$

Using this and the fact that $\bar{p_i} = p_i^{ne} - p_t^{ne}$, we can rewrite Equation (29) as

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\tilde{p_i}} + \frac{\mathrm{d}\Pi}{\mathrm{d}p_i} = d_{ia} + d_{it} + (p_i^{ne} - p_t^{ne})\left(\frac{\partial d_{ia}}{\partial \tilde{p_i}} + \frac{\partial d_{it}}{\partial \tilde{p_i}} + \frac{\partial d_{ia}}{\partial p_i} + \frac{\partial d_{it}}{\partial p_i}\right). \tag{32}$$

Given that $\tilde{p_i}$ and p_i increase by the same amount, the consumer indifferent between (i, a) and (i, t), $\tilde{x}_1$, is unchanged. Consequently, the impact of increasing both internet prices on the total demand for internet is the same as the impact of increasing the price p_i on the demand for internet in the two price case. Equation (32) then simplifies to:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\tilde{p_i}} + \frac{\mathrm{d}\Pi}{\mathrm{d}p_i} = d_{ia} + d_{it} + (p_i^{ne} - p_t^{ne})\frac{\partial d_i}{\partial p_i}. \tag{33}$$

Using the first order condition (Equation 26), we have:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\tilde{p_i}} + \frac{\mathrm{d}\Pi}{\mathrm{d}p_i} = d_{ia} + d_{it} - d_i. \tag{34}$$

Because we know from Proposition 4 that $d_{ia} + d_{it} > d_i$, the Lemma is proven.[16]

7.5. Proof of Proposition 5

In the case where both exclude or neither does, prices and profits are zero and all consumers buy the internet. Let us consider now that ISP_1 admits the app and ISP_2 does not. The price of the internet alone is zero, $p_i = 0$ and therefore, demand will be divided among internet with the app and internet with the phone. Comparing utilities in each case yields an indifferent consumer located at

$$x = \frac{1}{2} - \frac{\tilde{p_i}}{2\tau} + \frac{\Delta u}{2\tau}. \tag{35}$$

[16] To check that the market is covered with three prices, we can proceed as for the proof of Proposition 2 but in this case, it is complicated to derive the condition analytically.

The profit of ISP_2 is zero and ISP_1's profit is:

$$\Pi_1 = x\tilde{p}_i. \tag{36}$$

This yields a unique solution, $\tilde{p}_i = \tau/2 + \Delta u/2$ and $\Pi_1 = (\tau + \Delta u)^2/8\tau$. ISP_2 makes no profit since $p_i = p_t = 0$ because of Bertrand competition. The indifferent consumer is located at $1/4 + \Delta u/4\tau$.[17] When ISP_2 admits the app, the proof is *mutatis mutandis* similar.

7.6. Proof of Proposition 6

When there is no fragmentation (both ISPs admit the app), which we denote by an upper-script "nf", the indifferent consumer is located at $x = 1/2 + \Delta u/2\tau$. Then we have that:

$$\begin{aligned} W^{\mathrm{nf}}_{sym} &= CS^{\mathrm{nf}}_{sym} = \int_0^{\frac{1}{2}+\frac{\Delta u}{2\tau}} \int_0^1 \theta + u^a - x\tau \, d\theta \, dx + \int_{\frac{1}{2}+\frac{\Delta u}{2\tau}}^1 \int_0^1 \theta + u^t - \tau + x\tau \, d\theta \, dx & (37) \\ &= \frac{\Delta u^2 - \tau^2 + 2\tau(1 + u^a + u^t)}{4\tau}. & (38) \end{aligned}$$

Throughout the Appendix, the superscripts "a" and "e" respectively stand for admit and exclude. The first letter refers to ISP_1 and the second letter to ISP_2.

In the exclusion/no-exclusion or no-exclusion/exclusion cases, welfare is equal to:

$$\begin{aligned} W^{a/e}_{sym} &= \int_0^{\frac{\tau+\Delta u}{4\tau}} \int_0^1 \theta + u^a - x\tau - \frac{\tau + \Delta u}{2} \, d\theta \, dx + \int_{\frac{\tau+\Delta u}{4\tau}}^1 \int_0^1 \theta + u^t - \tau + x\tau \, d\theta \, dx + \Pi_1 & (39) \\ &= \frac{-5\tau^2 + 3\Delta u^2 + 2\tau(4 + 3u^a + 5u^t)}{16\tau}. & (40) \end{aligned}$$

Comparing the two, we have: $W^{\mathrm{nf}}_{sym} - W^{a/e}_{sym} = \frac{(\Delta u+\tau)^2}{16\tau} > 0$.

7.7. Proof of Proposition 7

Both exclude. Let us first consider that both exclude. Then, $p_i = 0$ because both ISPs supply the internet and homogeneous Bertrand competition takes place. Therefore, all consumers buy the internet and some also buy the phone. Comparing utilities in each case, we find the indifferent consumer between (i, t) and (i) to be located at:

$$\dot{x} = 1 - \frac{u^t - p_t}{\tau}. \tag{41}$$

Thus, the profit of ISP_1 can take two forms:

$$\Pi_1 = \begin{cases} p_t = u^t - \tau & \text{if}\, p_t \leq u^t - \tau, \\ p_t\,(1 - \dot{x}) & \text{if}\, p_t > u^t - \tau. \end{cases}$$

[17] We ignore the corner solutions where all consumers buy exclusively the app or the phone.

The first order condition of the second form of profit yields $p_t = u^t/2$ which cannot be since $u^t/2 < u^t - \tau$. Therefore, the corner solution, $p_t = \Pi_1 = u^t - \tau$ is the only solution.

Both admit. Suppose we have an interior solution. The only product offered for a price other than zero is the phone, for the usual Bertrand reasons. Everyone consumes the internet and demand is separated between those who consume it with the phone and those who consume it with the app. The indifferent consumer is located at:

$$\ddot{x} = \frac{1}{2} + \frac{p_t}{2\tau} + \frac{\Delta u}{2\tau}. \tag{42}$$

The profit of ISP_1 is:

$$\Pi = p_t(1 - \ddot{x}). \tag{43}$$

The first-order condition yields $p_t = \tau/2 - \Delta u/2$, $\Pi_1 = (\tau - \Delta u)^2/8\tau$ and $\ddot{x} = 3/4 + \Delta u/4\tau$.

If $\Delta u > \tau$, we have a corner solution, all prices are equal to 0 and consumers do not buy the phone.

ISP_1 excludes, ISP_2 admits. The price of the internet is zero again. Consumers are divided between those who consume it with the phone and those who consume it with the application. The indifferent consumer is located at:

$$\ddddot{x} = \frac{1}{2} + \frac{p_t - \tilde{p}_i}{2\tau} + \frac{\Delta u}{2\tau}. \tag{44}$$

Firms' profits are:

$$\begin{aligned} \Pi_1 &= p_t(1 - \ddddot{x}), & (45) \\ \Pi_2 &= \tilde{p}_i \ddddot{x}. & (46) \end{aligned}$$

Computing firms' best responses yields the equilibrium $\tilde{p}_i = \tau + \Delta u/3$, $p_t = \tau - \Delta u/3$, $\ddddot{x} = 1/2 + \Delta u/6\tau$, $\Pi_1 = (3\tau - \Delta u)^2/18\tau$ and $\Pi_2 = (3\tau + \Delta u)^2/18\tau$. Another possible candidate equilibrium would be higher prices with some consumers buying the internet only. In that case, the indifferent consumer (between the internet and the app and the internet alone) and the profit of ISP_2 is:

$$\begin{aligned} \overset{:}{x} &= \frac{u^a - \tilde{p}_i}{\tau}, & (47) \\ \Pi_2 &= \tilde{p}_i \overset{:}{x}. & (48) \end{aligned}$$

This yields an optimal price of $\tilde{p}_i = \frac{u^a}{2}$, implying that ISP_2 will want to cover the whole market with the app. The same holds true *mutatis mutandis* for ISP_1 and therefore this is not an equilibrium.

Finally, there are two corner solutions that we must examine: the app or the phone covering the whole market. These situations do not interest us but we provide the bounds for completeness. These cases happen respectively if $u^a \geq 3\tau + u^t$ and $u^t \geq 3\tau + u^a$. If we want to restrict the solution to be interior we must therefore impose $-3\tau \leq \Delta u \leq 3\tau$.

ISP$_1$ admits, ISP$_2$ excludes. Because it only supplies the internet, the profit of ISP$_2$ is zero. ISP$_1$ thus sets the prices of the internet with the app and of the phone freely. Two cases are possible: consumers are divided among app-users and phone users or some consumers choose to buy the internet alone. This second option can immediately be discarded for the same reason as in the exclusion/no-exclusion case. Let us consider therefore that no one consumes just the internet. One can think of the problem as finding the highest prices so that the consumers who indifferent between the internet and the internet with the app –or between the internet and the internet with the phone– are located at $x = 1/2$. At $x = 1/2$, we know that $U(i, a) = U(i, t) = 0$. Therefore, we have that $\tilde{p}_i = u^a - \tau/2$, $p_t = u^t - \tau/2$ and $\Pi_1 = (u^a + u^t)/2 - \tau/2$.

Admit-Admit is not a Nash equilibrium. Suppose that ISP 2 admits the application, if ISP 1 also admits, its profit is $(\tau - \Delta u)^2/8\tau$. If ISP 1 excludes, its profit is $(3\tau - \Delta u)^2/18\tau$. Computations show that the former profit is higher than the latter if either $\Delta u < 0$ and $\tau < -\Delta u/3$ or $\Delta u > 0$ and $\tau < 5\Delta u/9$. To have an interior solution in the Admit-Admit (or no-fragmentation) case,[18] we need the consumer who is indifferent between the phone and the app to be located between 0 and 1: $0 < 3/4 + \Delta u/4\tau < 1$. This implies $-3\tau < \Delta u < \tau$.

Take the first set of conditions. Clearly, $\tau < -\Delta u/3$ is in contradiction with $-3\tau < \Delta u$. For the second set of conditions, $\tau < 5\Delta u/9$ is in contradiction with $\Delta u < \tau$ because $\Delta u > 0$. Therefore, we cannot have an interior equilibrium and higher profits in the Admit-Admit case. It is therefore not a Nash equilibrium.

7.8. Proof of Proposition 8

Under no fragmentation (both ISPs admit the app), and assuming that we have an interior solution, consumer surplus and welfare are:

$$CS^{nf}_{asym} = \int_0^{\frac{3}{4}+\frac{\Delta u}{4\tau}} \int_0^1 \theta + u^a - x\tau \, d\theta \, dx + \int_{\frac{3}{4}+\frac{\Delta u}{4\tau}}^1 \int_0^1 \theta + u^t - \tau + x\tau - \frac{\tau}{2} + \frac{\Delta u}{2} \, d\theta \, dx \quad (49)$$

$$= \frac{1}{2} + \frac{7u^a + u^t}{8} + \frac{\Delta u^2}{16\tau} - \frac{7\tau}{16}. \quad (50)$$

$$W^{nf}_{asym} = \frac{1}{2} + \frac{5u^a + 3u^t}{8} + \frac{3\Delta u^2}{16\tau} - \frac{5\tau}{16}. \quad (51)$$

Under the exclusion/no-exclusion case, they are:

[18] Because the price of the internet with the app is 0, the only corner solution is $p_t = 0$ which would lead to a profit of 0 for ISP$_1$ and is therefore clearly not a Nash equilibrium.

$$
\begin{aligned}
CS^{e/a}_{asym} &= \int_0^{\frac{3\tau+\Delta u}{6\tau}} \int_0^1 \theta + u^a - x\tau - \tilde{p}_i \, d\theta \, dx + \int_{\frac{3\tau+\Delta u}{6\tau}}^1 \int_0^1 \theta + u^t - (1-x)\tau - p_t \, d\theta \, dx & (52)\\
&= \frac{1}{2} + \frac{u^a + u^t}{2} - \frac{5\tau}{4} + \frac{\Delta u^2}{36\tau}, & (53)\\
W^{a/e}_{asym} &= \frac{1}{2} + \frac{u^a + u^t}{2} - \frac{\tau}{4} + \frac{5\Delta u^2}{36\tau}. & (54)
\end{aligned}
$$

Under the no-exclusion/exclusion case, they are:

$$
\begin{aligned}
CS^{a/e}_{asym} &= \int_0^{\frac{1}{2}} \int_0^1 \theta + u^a - x\tau - u^t + \frac{\tau}{2} \, d\theta \, dx & (55)\\
&+ \int_{\frac{1}{2}}^1 \int_0^1 \theta + u^t - (1-x)\tau - u^t + \frac{\tau}{2} \, d\theta \, dx & (56)\\
&= \frac{1}{2} + \frac{\tau}{4}. & (57)\\
W^{a/e}_{asym} &= \frac{1}{2} - \frac{\tau}{4} + \frac{u^a + u^t}{2}. & (58)
\end{aligned}
$$

1. Consumer surplus is always highest when both firms admit the application. In the no-fragmentation case, if the profit maximization problem leads to a corner solution, we have that all prices are equal to 0 and consumers are unambiguously better off in that situation compared to any other.

Let us now consider the case of an interior solution. We have that $CS^{nf} < CS^{a/e}$ if:

$$
\frac{7u^a + u^t}{8} + \frac{\Delta u^2}{16\tau} - \frac{7\tau}{16} < \frac{\tau}{4}. \tag{59}
$$

It can be shown that this in contradiction with the conditions that $u^a > 2\tau$ and $u^t > 2\tau$.

We have that $CS^{nf} < CS^{e/a}$ if:

$$
\frac{7u^a + u^t}{8} + \frac{\Delta u^2}{16\tau} - \frac{7\tau}{16} < \frac{u^a + u^t}{2} - \frac{5\tau}{4} + \frac{\Delta u^2}{36}. \tag{60}
$$

This can be shown to be in contradiction with the conditions that 1) $u^a > 2\tau$ and $u^t > 2\tau$ and 2) $-3\tau \leq \Delta u \leq 3\tau$. Therefore, the proposition is proven.

2. Consumer surplus and welfare are not necessarily highest in the same situation. We know that consumer surplus is highest in the Admit/Admit situation. Let us simply provide an example where welfare is highest in the Exclusion/Admit case. If $\tau = 1/8$, $u^a = 1/2$ and $u^t = 3/4$ then $W^{e/a} = 335/288 \simeq 1.16319$, $W^{nf} = 147/128 \simeq 1.14844$ and $W^{a/e} = 35/32 \simeq 1.09375$.

3. Imposing NN_1 and NN_2 can increase or decrease welfare. This is a direct result from the two previous parts of the proposition.

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