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Revisited mass-loss rates for the nuclei of the planetary nebulae NGC 6210, NGC 6826 and NGC 6543: the first order moment W_1 of subordinate line profiles

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Summary. Using realistic expressions for the velocity and opacity distributions in rapidly expanding atmospheres, we present the results of numerical calculations for the first order moment W_1 of subordinate P Cygni line profiles as a function of the parameter $W_1^0 \alpha \dot{M} \langle n_g \rangle_s$, where $\dot{M}$ represents the mass-loss rate of the central star and $\langle n_g \rangle_s$ the average fractional abundance of the relevant ion across the region of the envelope where the subordinate line profile is formed.

Our calculations clearly show that for unsaturated P Cygni line profiles the relation $W_1 \alpha \dot{M} \langle n_g \rangle_s$ holds irrespective of the type of the opacity and/or velocity distributions. For currently observed P Cygni line profiles, the misuse of this relation can lead to an underestimate of the mass-loss rates by more than one order of magnitude. This is illustrated for the case of 3 selected central stars of planetary nebulae (NGC 6210, NGC 6826 and NGC 6543) for which mass-loss rates have been revisited.

Key words: radiation transfer – mass-loss – moment W_1 – P Cygni profiles – planetary nebulae

1. Introduction

Castor, Lutz and Seaton (1981, referred to below as CLS) have first established that, in the framework of the Sobolev approximation (Sobolev, 1947, 1957, 1958) and for the case of an optically thin line, the first order moment W_1 of a P Cygni line profile provides a good mean of deriving the mass-loss rate $\dot{M}$ of a star, and is particularly well adapted to the interpretation of IUE and other low resolution spectra.

Although CLS have applied this technique for specific distributions of the velocity and line opacity, Surdej (1982, 1983b) has shown that if the line profile is unsaturated, a unique relation does actually exist between W_1 and the quantity $W_1^0 \alpha \dot{M} \bar{n}$ (level) where $\bar{n}$ (level) is the average fractional abundance of an ion in the lower atomic level associated with the given line transition, irrespective of various physical (opacity distribution, collisions, limb darkening, ...) and geometrical (velocity law, rotation, ...) conditions prevailing in the expanding envelope as well as of any Sobolev-type approximations used for the transfer of line radiation.

Adopting realistic expressions for the velocity and opacity distributions, Surdej (1983a, referred to below as Paper I) has constructed "log W_1 – log W_1^0" diagrams for resonance line transitions which allow star observers to derive directly the quantity $\dot{M}\bar{n}$ (level) and to assign an error estimate to the mass-loss rate determinations. However, if the excitation potential of the lower level of a subordinate line transition is large enough, the population of this level is essentially accounted for by photoexcitations from the ground level and therefore shows a strong dependence upon the radial distance such that the quantity $\bar{n}$ (level) is then not defined anymore. Furthermore, subordinate lines such as O IV λ 1341 and O V λ 1371, whenever observed in the ultraviolet spectra obtained with the Copernicus and International Ultraviolet Explorer (IUE) satellites, have intensively been used for the determination of mass-loss rates because they are often unsaturated and because the interesting approximation $\bar{n}$(O IV) + $\bar{n}$(O V) = 1 can reasonably be made, independently of any adopted ionization model.

Therefore, in order to give a direct way of deriving mass-loss rates from the spectra of early-type stars, PN nuclei, quasars, etc., we have computed "log W_1 – log W_1^0" diagrams for the case of subordinate line transitions. On the basis of these diagrams, we have derived the mass-loss rate $\dot{M}$ of three selected central stars of planetary nebulae for which sufficient observational data are available.

2. The first order moment W_1 of subordinate line profiles

The present calculations of the first order moment of a subordinate P Cygni line profile are performed within the framework of the spherical geometry and Sobolev approximations. These assumptions are reasonably correct for the case of the winds surrounding early-type stars and have been recently discussed by various authors (see e.g. Schönberg, 1985 and references therein).

The expression of the first order moment of a line profile due to a transition between a lower level l and an upper level u has been first written by CLS as:

$$W_1 = \left(\frac{c}{\lambda_{1u} v_\infty}\right)^2 \int_{-\infty}^{+\infty} \left(\frac{F_\lambda - F_c}{F_c}\right)(\lambda - \lambda_{1u})\, d\lambda\,, \tag{1}$$

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★★ Also, Chercheur qualifié au Fonds National de la Recherche Scientifique (Belgium) where v_∞ is the terminal velocity of the wind, directly measurable from the extent of the violet wing of saturated resonance line profiles. F_λ and F_c denote the flux in the line and in the continuum, respectively. This equation can be conveniently reduced to the form (cf. Paper I):

$$W_1 = \int_{-X_{\min}}^{1} 4L^2(X')\tau^r_{lu}(X')\gamma_{lu}(X')\left(1 - \frac{\beta^3_{lu}(X')}{\beta^1_{lu}(X')}\right) X'\,dX', \tag{2}$$

taking into account the effects of the finite dimensions of the stellar core. In this relation,

$-X' = -v(r)/v_\infty$ is the dimensionless frequency of a stellar photon scattered in the expanding atmosphere at a radial distance r,

$-X_{\min} = -v_0/v_\infty$, v_0 being the velocity of the wind at the stellar surface,

$-L(X') = r/R^*$ represents the radial distance expressed in stellar radii R^* units,

τ^r_{lu} refers to the radial optical depth of the transition $l - u$ while γ_{lu}, β^3_{lu}, β^1_{lu} are escape probabilities which depend on both the velocity and opacity distributions (these have been defined in Surdej (1982)). This expression of W_1 implicitly assumes the two-level atom model approximation for the line transition $l - u$. CLS have shown this approximation to be essentially good for the most interesting subordinate lines observed in the ultraviolet spectrum of early-type stars, i.e. O IV λ 1341, O V λ 1371 and N IV λ 1718. In terms of stellar and atomic parameters, the expression of the opacity takes the form

$$\tau^r_{lu}(X') = \frac{\pi e^2}{mc} f_{lu}\lambda_{lu} \frac{A(\mathrm{el})\dot{M}}{2\pi\bar{\mu}M_{\mathrm{amu}}v^2_\infty R^*} n_l \frac{d(1/L)}{d(X'^2)}. \tag{3}$$

Within the optically thin approximation ($\tau_{lu} \ll 1$), it is easy to derive the value of the first order moment (cf. Paper I):

$$W^0_1 = \frac{\pi e^2}{mc} f_{lu}\lambda_{lu} \frac{A(\mathrm{el})\dot{M}}{4\pi\bar{\mu}M_{\mathrm{amu}}v^2_\infty R^*} \int_1^\infty n_l \frac{(W(L)-1)}{L^2}\,dL, \tag{4}$$

where $W(L)$ is the geometrical dilution factor, A(el) the abundance of the relevant element and n_l the fractional abundance of the ion in the lower atomic level l; the remaining symbols have their usual meaning.

If the excitation potential of the lower level l of the line transition is high enough such that photoexcitations dominate collisional excitations when populating the level l from the ground level g, we find that the ionization fractions n_l and n_g can be related as follows:

$$\frac{n_l}{n_g} = \frac{g_l}{g_g} \frac{\lambda^3_{gl}}{2hc} I^c_{\nu_{gl}} \frac{\beta^3_{gl}}{\beta^1_{gl}}, \tag{5}$$

where β^1_{gl} (resp. β^3_{gl}) represents the probability that a photon scattered in the line transition $g - l$ will escape locally the expanding atmosphere along any direction (resp. along those directions striking the stellar core); $I^c_{\nu_{gl}}$ is the monochromatic intensity of the stellar continuum at the frequency ν_{gl}; g_l and g_g being the statistical weights of the excited and ground levels. Using Eq. (5), we can rewrite Eq. (4) in the form

$$W^0_1 = \frac{\pi e^2}{mc} f_{lu}\lambda_{lu} \frac{A(\mathrm{el})\dot{M}}{4\pi\bar{\mu}M_{\mathrm{amu}}v^2_\infty R^*} \frac{g_l}{g_g} \frac{\lambda^3_{gl}}{2hc} I^c_{\nu_{gl}} \langle n_g\rangle_s \langle\beta^3_{gl}/\beta^1_{gl}\rangle q^c(\infty) \tag{6}$$

where

$$\langle\beta^3_{gl}/\beta^1_{gl}\rangle = \frac{\int_1^\infty \frac{\beta^3_{gl}}{\beta^1_{gl}} \frac{(W(L)-1)}{L^2}\,dL}{\int_1^\infty \frac{(W(L)-1)}{L^2}\,dL} \tag{7}$$

is an average equivalent dilution factor,

$$\langle n_g\rangle_s = \frac{\int_1^\infty n_g \frac{\beta^3_{gl}}{\beta^1_{gl}} \frac{(W(L)-1)}{L^2}\,dL}{\int_1^\infty \frac{\beta^3_{gl}}{\beta^1_{gl}} \frac{(W(L)-1)}{L^2}\,dL}, \tag{8}$$

and

$$q^c(\infty) = \int_1^\infty \frac{(W(L)-1)}{L^2}\,dL \tag{9}$$

is equal to -0.8927. In Eqs. (6) and (8), $\langle n_g\rangle_s$ represents the fraction of the ion in the ground level averaged over the region where the subordinate line is formed, i.e. in a region which extends from the stellar surface to a few (~ 3) stellar radii. At such a distance, the photoexciting radiation becomes so diluted that it does not affect anymore the formation of the subordinate line profile: for $L > 3$, its contribution to the quantity $\langle n_g\rangle_s$ is found to be of the order of 1%.

3. Determination of the fractional mass-loss rate $\dot{M}\langle n_g\rangle_s$

From Eq. (6) we directly see that a simple linear relation exists between the first order moment W^0_1 of an optically thin subordinate line profile and the quantity $\dot{M}\langle n_g\rangle_s$.

For the case of resonance lines, this relation is irrespective of the velocity and opacity distributions (Surdej, 1982), but for the excited lines, the quantity $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ does depend on both these distributions. In order to estimate the dependence of this relation on $v(r)$ and $\tau^r_{gl}(v)$, we have calculated $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ for 3 typical velocity laws (see Table 1 and next section), assuming both that the photoexciting line is optically thin or optically thick. If the photoexciting line is optically thick, the ratio of the escape probabilities reduces to

$$\frac{\beta^3_{gl}}{\beta^1_{gl}} = W(L) \frac{\alpha + (1-\alpha)(1 + z + z^2)/3}{\alpha + (1-\alpha)/3}, \tag{10}$$

Table 1. Adopted velocity and opacity distributions for calculating the moment W_1 of P Cygni subordinate line profiles

(A) $X' = -X_{\min} + (1 + X_{\min})(1 - 1/\sqrt{L})$
(B) $X' = -X_{\min} + (1 + X_{\min})(1 - 1/L)$
(C) $X' = \sqrt{1 - (1 - X^2_{\min})/L}$

(α) $\tau^r_{gl}(X') \propto 1\bigg/\left(\frac{X'\,dX'}{d(1/L)}\right)$
(β) $\tau^r_{gl}(X') \propto (1 - X')$
(γ) $\tau^r_{gl}(X') \propto 1$
(δ) $\tau^r_{gl}(X') \propto \sqrt{1 - X'}$
(ε) $\tau^r_{gl}(X') \propto (1 - X')^2$
(η) $\tau^r_{gl}(X') \propto 1/X'$

Table 2. Computed values of $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ for the three different velocity laws assuming that the photoexciting line is optically thick or thin

	Velocity law	$\langle\beta^3_{gl}/\beta^1_{gl}\rangle$
Optically thick case	A	0.1209
	B	0.1182
	C	0.1003
Optically thin case	A, B, C	0.0933

where

$$\alpha = \frac{d\ln L}{d\ln X'}, \tag{11}$$

and

$$z = \sqrt{1 - \left(\frac{1}{L}\right)^2}. \tag{12}$$

In the optically thin case, this ratio is independent of the velocity law and is simply given by

$$\frac{\beta^3_{gl}}{\beta^1_{gl}} = W(L). \tag{13}$$

The computed values of $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ are listed in Table 2. We see that $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ is very slightly dependent on the velocity law as well as on the opacity of the photoexciting line. If we choose for $\langle\beta^3_{gl}/\beta^1_{gl}\rangle$ a mean value of 0.103 ± 0.013, and taking $\bar{\mu} = 1.26$ (Allen, 1973), we can easily calculate the quantity $\dot{M}\langle n_g\rangle_s$ from the first order moment W^0_1 of an optically thin subordinate line by means of the relation:

$$\dot{M}\langle n_g\rangle_s = -4.71\,10^{-21}\,\frac{g_g}{g_l}\,\frac{v^2_\infty R^*}{\lambda^3_{gl} I^c_{\nu_{gl}} f_{lu}\lambda_{lu} A(\mathrm{el})}\,W^0_1, \tag{14}$$

which, within a good approximation ($\pm 13\%$), is independent of both the velocity and opacity distributions. In this relation $\dot{M}$ is given in $M_\odot$/yr, v_∞ in km s^{-1}, λ_{gl} and λ_{lu} in 10^3 Å, R^* in $R_\odot$ and $I^c_{\nu_{gl}}$ in erg cm^{-2} s^{-1} Hz^{-1}. Alike the case of resonance line transitions (Surdej, 1985), we immediately see from Eq. (1) that the quantity $\dot{M}\langle n_g\rangle_s$ is also irrespective of v_∞. This is particularly interesting for the analysis of subordinate lines which are formed near the stellar core ($L \lesssim 3$) and which therefore cannot provide a correct determination of v_∞.

4. "log W_1 – log W^0_1" diagrams

4.1. Numerical applications

Combining Eqs. (3) and (6) and with the help of the opacity distribution of the photoexciting line transition (cf. Paper I)

$$\tau^r_{gl}\,\alpha\, n_g \frac{d(1/L)}{X'\,dX'}, \tag{15}$$

we can write the expression of the opacity τ^r_{lu} as

$$\tau^r_{lu} = W^0_1 \frac{n_g}{\langle n_g\rangle_s} \frac{\beta^3_{gl}/\beta^1_{gl}}{\langle\beta^3_{gl}/\beta^1_{gl}\rangle} \frac{d(1/L)}{q^c(\infty)X'\,dX'}, \tag{16}$$

and, finally

$$\tau^r_{lu} = W^0_1 \frac{\tau^r_{gl}(\beta^3_{gl}/\beta^1_{gl})}{\int_{-X_{\min}}^{1} \tau^r_{gl}(\beta^3_{gl}/\beta^1_{gl})(1 - W(L))X'\,dX'}. \tag{17}$$

τ^r_{lu} depends on the opacity of the transition $g - l$ and on the ratio of the escape probabilities β^3_{gl} and β^1_{gl}. In order to avoid the introduction of an additional parameter, we shall consider the two extreme distinct cases: the photoexciting line is optically thick or optically thin such that the ratio $\beta^3_{gl}/\beta^1_{gl}$ reduces to Eqs. (10) or (13), respectively. For such cases, τ^r_{lu} is only dependent on the distribution of the opacity of the photoexciting line as well as on the type of the velocity law. Therefore, for a given set of opacity and velocity laws, the first order moment W_1 (see Eq. (2)) can readily be computed against values of the parameter W^0_1.

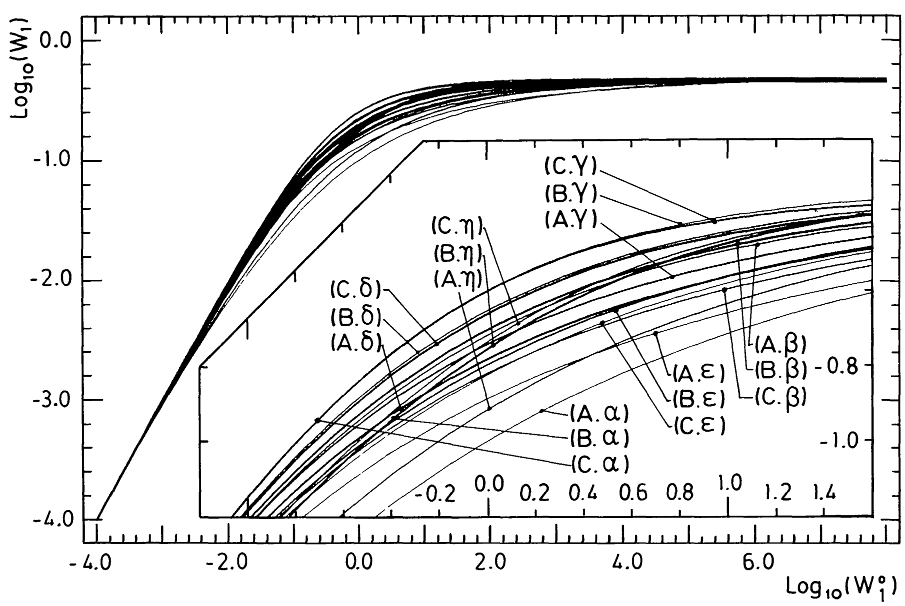

Fig. 1. "$\log_{10} W_1 - \log_{10} W^0_1$" curves computed for the eighteen possible models derived from Table 1 assuming that the $g - l$ photoexciting line transition is optically thin

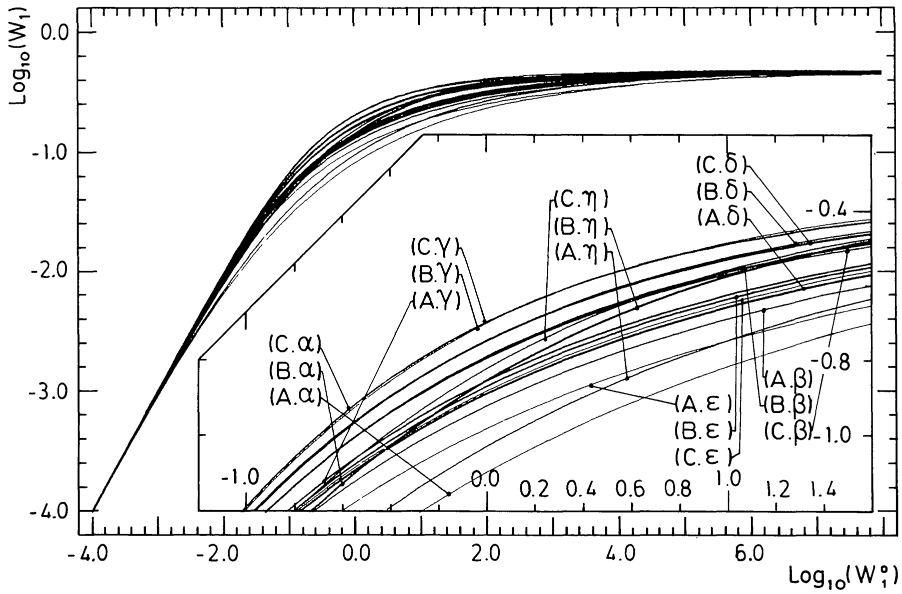

Fig. 2. "$\log_{10} W_1 - \log_{10} W_1^0$" curves computed for the eighteen possible models derived from Table 1 assuming that the $g-l$ photoexciting line transition is optically thick

In Paper I, a series of resonance line opacity and velocity distributions which seem to well represent those characterizing winds of early-type stars have been compiled and are given with suitable notations in Table 1. Let us recall that case (C.α) refers to a radiation-driven wind (Castor et al., 1975) in which there is mass conservation of the relevant species.

The diagrams representing $\log_{10} W_1$ versus $\log_{10} W_1^0$ have been computed for the eighteen possible models assuming alternatively that the photoexciting line is optically thin or thick. These are illustrated in Figs. 1–2. Numerical values are also given in the Appendix. As in Paper I, $X_{\min} = -0.01$ has been adopted, a value currently quoted for the wind of early-type stars.

4.2. Discussion of the results

As expected, Figs. 1 and 2 clearly show that all "$\log W_1 - \log W_1^0$" curves are characterized by a common linear branch – i.e. $W_1 = W_1^0$ – for small values of $\log W_1^0$. As $\log W_1^0 \to \infty$, there results an asymptotic value $\log W_1 \to -0.32$, irrespective of any model and equal to the asymptotic value of $\log W_1$ computed for the case of a resonance line (see Paper I). For intermediate opacities, an observed value of W_1 leads to a precise value of W_1^0 – and consequently of $\dot{M}\langle n_g\rangle_s$ if, and only if, the opacity and velocity distributions are known. Whenever they are not, we can estimate the error affecting the derived value of W_1^0 from the observed dispersion of the curves in Figs. 1 and 2. In doing this, we implicitly assume that the models compiled in Table 1 encompass most of realistic cases.

Comparison between two curves taken from Figs. 1 and 2, computed for the same opacity and velocity distributions, shows that these are not too much dependent on the choice (assumed to be alternatively optically thin or optically thick) of the opacity of the photoexciting line even for intermediate values of W_1^0. For reasonably saturated lines ($W_1 \lesssim 0.20$) the uncertainty affecting $\dot{M}$ never exceeds 50%. We also see that, for each of these eighteen models, the curve computed under the optically thin approximation ($\tau_{gl} \ll 1$) always provides a lower limit for the mass-loss rate.

It is also interesting to note that the curves computed for the subordinate lines (see Figs. 1–2) are more dispersed than those calculated for typical resonance lines (cf. Fig. 7 of Paper I). This is mainly due to the fact that the radial opacity τ_{lu}^r of a subordinate line depends on the velocity distribution (see Eq. (17)) while the opacity τ_{gl}^r of a resonance line does not – except if the opacity distribution α is considered. We also see in Figs. 1–2 that, for a given opacity law, the curves obtained for the velocity laws B and C appear very similar while that calculated for the velocity law A is always shifted towards higher values of $\log W_1^0$ such that the use of this less steep velocity law always provides higher mass-loss rate estimates when determined from the same measured value of W_1.

Therefore, the present "$\log W_1 - \log W_1^0$" curves are model dependent within a larger interval of $\log W_1^0$ values than for the case of a resonance line transition. This implies that a lower value of W_1 must be measured in order to be located on the linear branch. This constitutes a fortunate situation since, whenever a resonance line profile is saturated, weaker features are generally observed for excited lines thus still providing an interesting possibility of deriving an accurate mass-loss rate.

Comparison between the quantity $\langle n_g\rangle_s$ derived from the analysis of subordinate line profiles and $\langle n_g\rangle$ for the case of resonance line profiles of the same ion (cf. Paper I) can give information on the distribution of the ionization fraction across the envelope. The ratio $\langle n_g\rangle/\langle n_g\rangle_s$ is reported in Table 3 for the eighteen models. We see that this ratio is not too much dependent on the opacity approximation ($\tau_{gl} > 1$ or $\tau_{gl} < 1$) made for the photoexciting line. However, larger differences are noticed between the ratios obtained for different model calculations. A ratio $\langle n_g\rangle/\langle n_g\rangle_s$ greater than unity refers to a distribution of the ionization fraction increasing outwards while a ratio smaller than one refers to a decreasing one.

5. Examples of mass-loss rate determinations

5.1. Determination of $\dot{M}$ for three selected central stars of planetary nebulae

In order to apply our method, we have selected in the literature some objects for which measurements of the first order moment

Table 3. The ratio $\langle n_g \rangle / \langle n_g \rangle_s$ computed for the eighteen models listed in Table 1

Model	Optically thin case	Optically thick case
A. α	1.0000	1.0000
B. α	1.0000	1.0000
C. α	1.0000	1.0000
A. β	2.4031	2.6670
B. β	1.1105	1.1443
C. β	0.5615	0.5179
A. γ	4.9557	5.8583
B. γ	2.0870	2.3528
C. γ	1.0000	1.0000
A. δ	3.2754	3.7414
B. δ	1.4297	1.5309
C. δ	0.7046	0.6716
A. ε	1.5695	1.6647
B. ε	0.8020	0.7847
C. ε	0.4240	0.3764
A. η	1.6922	1.7391
B. η	1.0000	1.0000
C. η	0.5707	0.5328

of UV subordinate line profiles were available. The recent paper by Cerruti-Sola and Perinotto (1985, hereafter CP) provides such a list of W_1 values derived from lines observed in low-resolution IUE spectra of central stars of planetary nebulae. NGC 6543 observed by CLS, can also be added to this list.

These UV subordinate lines are probably contaminated by the presence of a nebular emission line and/or an underlying photospheric absorption one. Since the former one arises in a huge nebula located well beyond the expanding atmosphere, its contribution to the first order moment is negligible for reasons of symmetry. The error made by neglecting the presence of a possible strong photospheric absorption line has the effect of underestimating, by less than 20%, the derived mass-loss rate (Surdej, 1982).

The most uncertain parameter in using the UV subordinate lines for mass-loss rate determinations is the intensity $I^c_{v_{g1}}$ which must be evaluated in the unobservable region of the far UV spectrum. CP, as well as CLS, have computed this intensity assuming that the star radiates as a black-body at an effective temperature $T_{\rm eff}$. Unfortunately, this constitutes a very rough approximation whenever extrapolated beyond the Lyman limit. Furthermore, the model calculations, independently of the determination of a reliable $T_{\rm eff}$, remain uncertain because they are quite dependent on an unknown chemical composition (see the discussion by Pottasch, 1984). Therefore, we prefer to use the far UV flux determinations by Natta et al. (1980), based on the analysis of the successive stages of ionization of a given element in the nebula, a method which does not require the knowledge of the effective temperature of the star. As pointed out by these authors, a black-body flux distribution does not match their flux determinations and therefore remains a bad approximation for the prediction of the far UV spectrum.

Finally, we have found in the literature three planetary nebulae which do have a central star analysed by Natta et al. (1980) and for which W_1 measurements have been published. These are NGC 6210, NGC 6826 and NGC 6543.

It should be noticed that, except for C IV in NGC 6210, the resonance lines of C IV and N V are completely saturated in the spectra of these central stars. Some values of their moment W_1 do even exceed the theoretical asymptotic value given in Paper I, showing that for large opacities the doublet structure of these lines must absolutely be taken into consideration. This also clearly demonstrates that any determination of mass-loss rates using the first order moment of these resonance lines based on the approximation $W_1 = W_1^0$ will result in meaningless estimates.

In order to avoid the problem of deriving the ionization fractions, we have only considered the oxygen subordinate lines O IV λ 1341 and O V λ 1371, assuming that $\langle n_g \rangle_s$(O IV) + $\langle n_g \rangle_s$(O V) = 1, such that our mass-loss rates will only provide lower limit estimates. We have also assumed that the abundance of oxygen in the stellar wind is the same as in the nebula (the values are taken from Natta et al., 1980).

Our mass-loss rate determinations are reported in Table 4 altogether with the following quantities:

column 3: the first order moment as found in the literature for O IV λ 1341 and O V λ 1371,
column 4: the value of $\log W_1^0$ obtained from the diagrams illustrated in Figs. 1 and 2. A mean value is reported here with an error estimate derived from the dispersion of the curves,
column 5: the calculated value of $\dot{M}\langle n_g \rangle_s$ using Eq. (14) and the data of Natta et al. (1980). The terminal velocity is taken from the same source as the W_1 values,
column 6: the value of $\dot{M}$ obtained assuming that $\langle n_g \rangle_s$(O IV) + $\langle n_g \rangle_s$(O V) = 1.

As we can immediately see from Table 4, our mass-loss rates are greater by more than one order of magnitude than those obtained by CLS and CP.

Before discussing the origin of these discrepancies, we want to point out that we have considered O IV λ 1341 as forming a single line, despite its multiplet structure. Indeed, if the line is not saturated, this approximation is essentially correct (see CLS and Surdej, 1982) and we can therefore assume that the error possibly induced by this approximation is not too large, at least for NGC 6210 and NGC 6826.

5.2. *Discussion of our results*

The large discrepancy existing between the results of CLS, CP and ours is mainly due to the misuse by these authors of the approximation $W_1 = W_1^0$ for non unsaturated lines. It can be readily seen in Figs. 1–2 that for measured values of $\log W_1$ between -1.4 and -0.7 (see Table 4), the use of this linear relation results in a systematic underestimate of the mean value of W_1^0 – and consequently of $\dot{M}$ – by a factor varying between 3 and 20. All the W_1 values measured for the O IV and O V lines observed with IUE in the low resolution mode and reported by CP are too great to justify the use of the linear relation between W_1 and $\dot{M}$. Therefore we conclude that all their mass-loss rates have probably been underestimated. An independent mass-loss rate determination was performed by Adam and Köppen (1985) for the nucleus of NGC 1535 on the basis of the fitting of line profiles observed in IUE high-resolution spectra. Their value of $\dot{M}$ is 2 orders of magnitude greater than that given by CP for

Table 4. $\dot{M}$ for selected central stars of planetary nebulae

PN	Line	W_1	$\log W_1^0$		$\dot{M}\langle n_g \rangle_s$ ($M_\odot$/yr)	$\dot{M}$ ($M_\odot$/yr)	Previous estimates of $\dot{M}$s
NGC 6210	O IV	0.06[a,c]	$-0.74 \pm$	0.39	$6.7\,10^{-9}$	$1.2\,10^{-8}$	$6\,10^{-10}$ [a]
	O V	0.12[a]	-0.03	0.62	$5.3\,10^{-9}$		
NGC 6826	O IV	0.04[a,c]	-1.02	0.30	$1.1\,10^{-7}$	$1.5\,10^{-7}$	$2\,10^{-8}$ [a]
	O V	0.07[a]	-0.58	0.45	$3.7\,10^{-8}$		
NGC 6543	O IV	0.19[b]	0.60	0.88	$1.6\,10^{-6}$	$1.7\,10^{-6}$	$1\,10^{-7}$ [b]
	O V	0.13[b]	0.03	0.64	$5.3\,10^{-8}$		

Notes

[a] Value from Cerruti-Sola and Perinotto (1985)

[b] Value from Castor, Lutz and Seaton (1981)

[c] As suggested by Cerruti-Sola and Perinotto (1985), the value of W_1 has already been divided by three in order to take into account the contamination by the C II λ 1335 interstellar line

the same object but similar to the value that we have obtained on the basis of "$\log W_1 - \log W_1^0$" diagrams (we also obtained $\dot{M} = 10^{-7} M_\odot$/yr for NGC 1535 using the same data and approximations as Adam and Köppen, 1985. The W_1 measurements are from CP). It is interesting to note that an increase of the majority of the mass-loss rates derived by CP will reinforce their suggestion that there might exist a difference between the winds of planetary nebulae nuclei and Population I OB stars since the same order of mass-loss rates are observed in stars having very different luminosities. But it could also be due to an abundance problem in the wind of PN nuclei.

Another source of discrepancy is the assumption by CP that the central star radiates as a black-body at the He II Zanstra temperature when they derive the intensity $I^c_{\nu_{g1}}$ of the far UV stellar continuum. Indeed, first, the slope of the flux distribution cannot be adequately fitted by a black-body distribution (Natta et al., 1980) and, secondly, the Zanstra temperature T_Z(He II) does probably overestimate the effective temperature of the star when a stellar wind is present (Adam and Köppen, 1983, 1985). If we accept the fact that the three planetary nebulae nuclei are surrounded by a wind as that modelized by Adam and Köppen (1985) for the case of NGC 1535, the energy distribution – only increased below λ 228 Å – will slightly affect the fluxes determined by Natta et al. (1980) near λ 600 Å which enter Eq. (14). Therefore, the use of the black-body approximation with $T = T_Z$(He II) for the three investigated central stars of PN, rather than the UV flux determinations of Natta et al. (1980), results in underestimates of $\dot{M}$ by a factor of about 3.

It should also be pointed out that in the method first developed by CLS, the quantity n(level) does not represent an average value across the envelope as in our calculations but is evaluated at a distance $L = 2$. At such a distance, the photoexcited levels are unfortunately largely depopulated such that the mass-loss rates they derive are systematically overestimated by a factor 2.3.

We can finally give an estimate of the total uncertainty which affects our mass-loss rates. The error on $\log W_1^0$ due to the dispersion of the curves is on the average 0.5 dex (see Table 4). Natta et al. (1980) have reported an uncertainty of about 0.3 dex on their flux values. For planetary nebulae in general, the quantities R^* and A(el) are both only known within a factor 2 (0.3 dex). Therefore, taking into account the error due to the possible presence of an underlying photospheric absorption line (20%), the total uncertainty affecting $\log \dot{M}$ is approximately 0.7 dex. Whereas the uncertainties due to departures from the classical radiative transfer model used here are thought to be small when compared to the others, it should be kept in mind that large systematic errors may be introduced in assuming that the oxygen abundance in the wind is the same as in the nebula and that only two ionization stages are present.

6. Discussion and conclusions

The first order moment method, applied to P Cygni line profiles of resonance lines, has been shown to be a very powerful method for deriving mass-loss rates (CLS, Surdej 1982, 1983a,b). Since the resonance lines are often saturated while unsaturated P Cygni profiles are available from subordinate lines, we have generalized this method to lines originating from excited levels.

In the framework of the Sobolev approximation, we have shown that for unsaturated subordinate lines, there exists a unique relation between the first order moment W_1 and the quantity $\dot{M}\langle n_g \rangle_s$ independently of any velocity and/or opacity distributions. If the line is not sufficiently unsaturated, the use of "$\log W_1 - \log W_1^0$" diagrams still allows one to determine mass-loss rates of early-type stars, PNN, BAL quasars, etc. as well as an estimate of the error. This method is the only useful one for the analysis of unresolved P Cygni profiles (CLS) and is complementary to the "line profile fitting" technique (Olson, 1981) used when analyzing unsaturated and sufficiently well resolved P Cygni profiles, as already pointed out in Paper I.

From the comparison of mass-loss rates obtained with similar methods, we wish to warn the reader against some possible misuse of the relation $W_1 = W_1^0$ whenever the line is not sufficiently unsaturated. We conclude that the mass-loss rates previously derived for the central nuclei of the planetaries NGC 6210 (CP), NGC 6826 (CP), NCG 6543 (CLS) and NGC 1535 (CP) have been systematically underestimated by more than one order of magnitude when compared to those determined in the present work.

We also conclude that high signal to noise spectroscopic data of faint, i.e. unsaturated, P Cygni profiles are very necessary in order to provide accurate but still inexistent mass-loss rate estimates for a wide variety of celestial objects (early-type stars, PN nuclei, BAL quasars, etc.).

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Appendix

Table A1. For selected values of $\log_{10}(W_1)$, we have listed hereafter the smallest and greatest values of $\log_{10}(W_1^0)$ as obtained from the eighteen model calculations illustrated in Figs. 1 and 2

Results referring to Fig. 1			Results referring to Fig. 2		
$\log_{10}(W_1)$	$\log_{10}(W_1^0)_s$ –	$\log_{10}(W_1^0)_g$	$\log_{10}(W_1)$	$\log_{10}(W_1^0)_s$ –	$\log_{10}(W_1^0)_g$
−3.091	−3.096	−3.049	−3.089	−3.090	−3.049
−3.002	−3.004	−2.954	−2.999	−3.004	−2.948
−2.894	−2.901	−2.831	−2.889	−2.889	−2.825
−2.800	−2.806	−2.725	−2.798	−2.795	−2.725
−2.694	−2.691	−2.600	−2.695	−2.691	−2.599
−2.602	−2.599	−2.499	−2.600	−2.596	−2.493
−2.498	−2.496	−2.382	−2.493	−2.487	−2.362
−2.400	−2.396	−2.265	−2.398	−2.398	−2.245
−2.299	−2.292	−2.148	−2.292	−2.281	−2.111
−2.205	−2.200	−2.025	−2.202	−2.192	−1.994
−2.104	−2.089	−1.891	−2.106	−2.085	−1.871
−2.001	−1.983	−1.757	−2.003	−1.979	−1.726
−1.905	−1.879	−1.629	−1.901	−1.865	−1.578
−1.803	−1.771	−1.487	−1.804	−1.762	−1.425
−1.700	−1.662	−1.333	−1.694	−1.644	−1.255
−1.600	−1.550	−1.183	−1.602	−1.530	−1.099
−1.500	−1.436	−1.016	−1.500	−1.404	−0.909
−1.404	−1.321	−0.857	−1.401	−1.287	−0.709
−1.303	−1.207	−0.673	−1.302	−1.159	−0.500
−1.201	−1.068	−0.458	−1.200	−1.022	−0.266
−1.098	−0.937	−0.232	−1.094	−0.858	0.024
−1.003	−0.800	0.002	−1.004	−0.702	0.308
−0.901	−0.633	0.303	−0.903	−0.507	0.664
−0.799	−0.460	0.640	−0.800	−0.284	1.077
−0.695	−0.256	1.099	−0.700	−0.008	1.608
−0.600	−0.000	1.625	−0.599	0.365	2.312
−0.493	0.393	2.444	−0.493	0.986	3.206
−0.400	1.078	–	−0.399	1.938	–

References

Adam, J., Köppen, J.: 1983, *Planetary nebulae*, IAU Symp. Nr 103, p. 338

Adam, J., Köppen, J.: 1985, *Astron. Astrophys.* **142**, 461

Allen, C.W.: 1973, Astrophysical Quantities, Athlone Press, Univ. of London

Castor, J.I., Abott, D.C., Klein, R.I.: 1975, *Astrophys. J.* **195**, 157

Castor, J.I., Lutz, J.H., Seaton, M.J.: 1981, *Monthly Notices Roy. Astron. Soc.* **194**, 547 (CLS)

Cerruti-Sola, M., Perinotto, M.: 1985, *Astrophys. J.* **291**, 237 (CP)

Natta, A., Pottasch, S.R., Preite-Martinez, A.: 1980, *Astron. Astrophys.* **84**, 284

Pottasch, S.R.: 1984, Planetary Nebulae, Reidel, Dordrecht

Olson, G.L.: 1981, *Astrophys. J.* **248**, 1021

Schönberg, K.: 1985, *Astron. Astrophys.* **148**, 405

Sobolev, V.V.: 1947, Dnižuščiesja Oboločki Zvezd, Leningrad, transl. Moving Envelopes of Stars; translated from Russian by S. Gaposchkin, Harvard University Press, Cambridge, Mass., 1960

Sobolev, V.V.: 1957, *Astron. Zh.* **34**, 694 (trans. Soviet Astron. *1*, 678)

Sobolev, V.V.: 1958, in V.A. Ambartsumyan (ed.), Theoretical Astrophysics, Chapter 28, Pergamon Press Ltd., London

Surdej, J.: 1982, *Astrophys. Space Sci.* **88**, 31

Surdej, J.: 1983a, *Astron. Astrophys.* **127**, 304 (Paper I)

Surdej, J.: 1983b, *Astrophys. Space Sci.* **90**, 299

Surdej, J.: 1985, *Astron. Astrophys.* **152**, 361