BIFURCATION OF SPACE FRAMES

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FOREWORD

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This report has been reviewed and is approved.

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ABSTRACT

A finite element for a prismatic member subjected to axial loads is derived for the stability analysis of elastic space frames. The derivation is based on the fact that in a stable state of equilibrium the total energy is positive definite. A feature of the element developed is that the reduction of torsional rigidity due to the presence of axial stresses is taken into account. Several examples are presented. An excellent agreement with analytical results is obtained, where closed form solutions are available.

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LIST OF ABBREVIATIONS AND SYMBOLS

A = the sectional area

E, G = the elastic constants

e_{xx} = the axial strain

I_y , I_z = the moments of inertia about the y and the z axes respectively

I_p = the polar moment of inertia

J = the torsion constant of a section

K = the stiffness matrix

M_y , M_z = the bending moments about the y and the z axes respectively

N = the axial force

$n = \frac{N}{P}$

P = the load intensity, or when specifically stated in the text, the potential energy

q = the generalized displacement, a vector

S = the stability ("geometric stiffness") matrix

T = the torsional moment

U = the strain energy

$(U + P)$ = the total energy

$u^{(o)}$, $v^{(o)}$, $w^{(o)}$, $\phi_x^{(o)}$ = the displacements in the x, y, z directions and the angle of twist in a stable (initial) configuration of a structure

εu , εv , εw , $\varepsilon \phi_x$ = small disturbances from the stable configuration, where ε is a small quantity

$\hat{u}$, $\hat{v}$, $\hat{w}$, $\hat{\phi}_x$ = the total displacements and the total angle of twist, including the disturbance

Superscript (1) indicates first order terms w.r.t. ε in the expansion of $\hat{u}$, $\hat{v}$, $\hat{w}$, $\hat{\phi}_x$

Superscript (2) indicates second order terms w.r.t. ε in the expansion of $\hat{u}$, $\hat{v}$, $\hat{w}$, $\hat{\phi}_x$

x, y, z = the coordinates of a point before the deformation

ξ, η, ζ = the coordinates of a point after the deformation

σ_{xx} = the axial stress

θ = a unit angle of twist

κ_y , κ_z = the curvatures about the y and the z axes respectively

$\Delta(U + P)$ = the complete increment of the total energy

$\delta^n(U + P)$ = the n-th variation of the total energy

Summary.

A finite element procedure is developed for the determination of critical loads of elastic space frames with members subjected to axial forces. A stability criterion of positive definiteness of the second variation of total energy is used. Several examples of space frames are presented. By comparison with analytical solutions, where such solutions are readily obtainable, the method is shown to yield satisfactory results.

I. Introduction.

In the vast literature on structural stability there is an increasing emphasis on numerical solutions, which is, no doubt, prompted by the developments in computer technology. Numerical solutions for the determination of critical loads of space structures, the objective of this paper, have been discussed in several other publications (12, 8, 10). It is therefore desirable to point out briefly those features of the present paper which are different from the material presented elsewhere.

In this paper the conditions of stability are derived in a different manner, namely from considerations of positive definiteness of the second variation of total energy, thereby achieving generality and clarity. Examples of solutions of space frames are presented, a feature which is for some reason often omitted elsewhere. The reduction of torsional rigidity due to the presence of axial stresses is taken into account. Whilst this effect has been investigated both analytically and experimentally (2, 16, 15, 14, 9, 7, 1), it has apparently been ignored in finite element solutions of stability problems. It can be significant when the members of a frame are of thin walled open sections.

An introduction to a paper is not complete without a discussion of its known limitations : the derivation of stiffness and stability matrices is based on the assumption that sections, which are subjected to a torque, are free to warp. This assumption is often implicitly made in structural analysis of space frames, though it can contradict the boundary conditions of some structures. Another assumption which is tacitly made here, and for which above comments still apply, is that the center of twist coincides with the center of gravity of the sections. Finally, the present paper is limited to structures which are perfectly elastic and in which in a state of stable equilibrium all members are subjected to axial forces only. Thus, the paper is limited to one type of bifurcation problem.

As a starting point for the criterion of stability, we consider a change of the total energy due to a small disturbance from an equilibrium state. The total energy is the sum of the strain energy, U, and the potential energy, P. The change of the total energy, $\Delta(U + P)$, can be expanded in terms of the first and higher variations as follows

$$\Delta(U + P) = \varepsilon\, \delta\, (U + P) + \frac{1}{2}\, \varepsilon^2\, \delta^2(U + P) + \ldots + \frac{1}{n!}\, \varepsilon^n\, \delta^n\, (U + P) + \ldots \qquad (I)$$

Here $\delta^n(U + P)$ is the n-th variation and ε is an arbitrarily small scaling factor. The condition of equilibrium (6.4) can be stated as

$$\delta(U + P) = 0 \qquad (2)$$

The condition of stability can be written as (12, 40, 37)

$$\delta^2(U + P) \geqslant 0 \qquad (3)$$

Equations (2) and (3) must be satisfied for any kinematically admissible mode of disturbance if the structure is in a stable state of equilibrium. Thus, to investigate the stability of a given state of equilibrium, one must investigate the least value of $\delta^2(U + P)$ for all kinematically admissible disturbances of the equilibrium configuration. If the least value of $\delta^2(U + P)$ is positive, then the equilibrium is stable, if it is negative, then the equilibrium is unstable and if it is equal to zero, then the equilibrium is neutral. This minimum of the second variation of the total energy can be found by considering a variation of the disturbance itself [11].

Thus, to investigate the stability of a structure, we will first seek an expression for the second variation of the total energy, $\delta^2(U + P)$.

2. Derivation of an expression for $\delta^2(U + P)$.

Let an equilibrium configuration be defined by three orthogonal displacement components, $u^{(o)}$, $v^{(o)}$, $w^{(o)}$, and the angle of twist $\Phi_x^{(o)}$ throughout the structure. Similarly, let a kinematically admissible disturbance be defined by εu , εv , εw , $\varepsilon \Phi_x$, where ε is an arbitrarily small scaling factor. The change of total energy, $\Delta(U + P)$, due to this disturbance, is equal to

$$\Delta(\bar{U} + P) = (U + P)_{(\hat{u},\hat{v},\hat{w},\hat{\theta})} - (U + P)_{(u^{(o)},v^{(o)},w^{(o)},\theta^{(o)})} \tag{4}$$

where

$$\begin{aligned} \hat{u} &= u^{(o)} + \varepsilon u \\ \hat{v} &= v^{(o)} + \varepsilon v \\ \hat{w} &= w^{(o)} + \varepsilon w \\ \hat{\Phi}_x &= \Phi_x^{(o)} + \varepsilon \Phi_x \end{aligned} \tag{5}$$

If the right-hand side of equation (4) is evaluated and the terms are collected according to the order of ε , the terms containing ε^2 yield the second variation of the strain energy, $\delta^2(U + P)$. This evaluation is carried-out in two steps.

First, the deformation components at $\hat{u}$, $\hat{v}$, $\hat{w}$, $\hat{\theta}$ are expanded in terms of ε up to the second order terms. This can be written as

$$\hat{e}_{xx} = e_{xx}^{(o)} + \varepsilon\, e_{xx}^{(1)} + \frac{1}{2}\varepsilon^2\, e_{xx}^{(2)}$$

$$\hat{\kappa}_y = \kappa_y^{(o)} + \varepsilon\, \kappa_y^{(1)} + \frac{1}{2}\varepsilon^2\, \kappa_y^{(2)}$$

$$\hat{\kappa}_z = \kappa_z^{(o)} + \varepsilon\, \kappa_z^{(1)} + \frac{1}{2}\varepsilon^2\, \kappa_z^{(2)} \qquad (6)$$

$$\hat{\theta} = \theta^{(o)} + \varepsilon\, \theta^{(1)} + \frac{1}{2}\varepsilon^2\, \theta^{(2)}$$

In the above expression $\hat{e}_{xx}$ is the axial strain at the centre of gravity of a section, $\hat{\kappa}_y$ and $\hat{\kappa}_z$ are the curvatures about the y and the z axes respectively and $\hat{\theta}$ is the twist per unit length. The right-hand sides of equations (6) will be evaluated in terms of $\hat{u}$, $\hat{v}$, $\hat{w}$ and $\hat{\Phi}_x$ later.

Second, equations (6) are substituted into an expression for the change of total energy, eq. (4). In the first step, the strain energy per unit length of a member is considered. In matrix terms this can be written as

$$\Delta(U) = \{\, \hat{e}_{xx}\ \hat{\kappa}_y\ \hat{\kappa}_z\ \hat{\theta} \,\}^T\ D\ \{\, \hat{e}_{xx}\ \hat{\kappa}_y\ \hat{\kappa}_z\ \hat{\theta} \,\}$$

$$- \{\, e_{xx}^{(o)}\ \kappa_y^{(o)}\ \kappa_z^{(o)}\ \theta^{(o)} \,\}^T\ D\ \{\, e_{xx}^{(o)}\ \kappa_y^{(o)}\ \kappa_z^{(o)}\ \theta^{(o)} \,\} \qquad (7)$$

where $\{\ \ldots\ \}$ indicates a vector and where

$$D = \begin{vmatrix} EA & & & \\ & EI_y & & \\ & & EI_z & \\ & & & (GJ + \sigma_{xx}^{(o)}\ I_p) \end{vmatrix} \qquad (8)$$

All elements of matrix D are self explanatory, except the last one, namely, $GJ + \sigma_{xx}^{(o)}\ I_p$. In this term G is the shear modulus, J is the torsion constant, $\sigma_{xx}^{(o)}$ is the initial axial stress (with tension as positive) and I_p is the polar moment of inertia, which is equal to $I_x + I_y$. Thus, GJ is the usual expression for the torsional rigidity of a bar of unit length, whilst $\sigma_{xx}^{(o)}\ I_p$ is its reduction (Wagner effect) due to the axial compressive stress [2]. It has been shown [16] that an axially loaded column can buckle in pure torsion when $GJ + \sigma_{xx}^{(o)} I_p = 0$. When the initial stress $\sigma_{xx}^{(o)}$ is non uniform, the statement of the stability of a member is more involved [13]. We restrict ourselves here to the limitations outlined in the introduction.

Substituting the right-hand sides of equations (6) into equation (7) and collecting terms with ε^2, one obtains

$$\frac{1}{2}\varepsilon^2\, \delta^2\,(U) = \frac{1}{2}\varepsilon^2 \left\{ e_{xx}^{(1)}\ \kappa_y^{(1)}\ \kappa_z^{(1)}\ \theta^{(1)} \right\}^T D \left\{ e_{xx}^{(1)}\ \kappa_y^{(1)}\ \kappa_z^{(1)}\ \theta^{(1)} \right\}$$
$$+ \frac{1}{2}\varepsilon^2 \left\{ e_{xx}^{(2)}\ \kappa_y^{(2)}\ \kappa_z^{(2)}\ \theta^{(2)} \right\}^T D \left\{ e_{xx}^{(o)}\ \kappa_y^{(o)}\ \kappa_z^{(o)}\ \theta^{(o)} \right\} \tag{9}$$

The second term of the equation (9) can be readily simplified because, in general,

$$D \left\{ e_{xx}^{(o)}\ \kappa_y^{(o)}\ \kappa_z^{(o)}\ \theta^{(o)} \right\} = \left\{ N\ M_y\ M_z\ T \right\} \tag{10}$$

where N, M_y, M_z and T are respectively the axial force, the bending moment about the y axis, the bending moment about the z axis and the torsion in the initial equilibrium configuration, defined by the displacements $u^{(o)}$, $v^{(o)}$, $w^{(o)}$, $\theta^{(o)}$, before the disturbances are introduced. But, since our discussion is limited to frames with members subjected to axial forces only, $M_y = M_z = T = 0$.

Thus, after cancelling ε^2, equation (9) can be written as

$$\frac{1}{2}\delta^2\,(U) = \frac{1}{2}\left\{ e_{xx}^{(1)}\ \kappa_y^{(1)}\ \kappa_z^{(1)}\ \theta^{(1)} \right\}^T D \left\{ e_{xx}^{(1)}\ \kappa_y^{(1)}\ \kappa_z^{(1)}\ \theta^{(1)} \right\}$$
$$+ \frac{1}{2} e_{xx}^{(2)}\ N \tag{11}$$

The second variation of the potential energy vanishes, since it is here a linear function of $\hat{u}$, $\hat{v}$, $\hat{w}$. Thus, after integration, the second variation of the total energy of the structure can be written as

$$I_2 = \frac{1}{2}\int_R \delta^2(U + P)\, d\,R =$$
$$= \frac{1}{2}\int EA\,(e_{xx}^{(1)})^2\,dx + \frac{1}{2}\int EI_y\,(\kappa_y^{(1)})^2 dx + \frac{1}{2}\int EI_z\,(\kappa_z^{(1)})^2\,dx$$
$$+ \frac{1}{2}\int GJ\,(\theta^{(1)})^2\,dx + \frac{1}{2}\int \frac{N}{A}\, I_p\,(\theta^{(1)})^2\,dx + \frac{1}{2}\int N\,(e_{xx}^{(2)})^2\,dx \tag{12}$$

In the above equation we always take the x axis in the direction of a member and the integration is over the whole of the structure. Thus, for an analytical expression of the second variation in terms of the disturbances u , v , w and θ , it remains to find expressions for $e_{xx}^{(1)}$, $e_{xx}^{(2)}$, $\kappa_y^{(1)}$, $\kappa_z^{(1)}$ and $\theta^{(1)}$.

Since $\hat{e}_{xx}$ expansion up to the second order of ε^2 is required, terms of one higher order than $\frac{d\hat{u}}{dx}$ must be retained in the expression for the axial strain. Consider a segment $\overline{AB}$, which lies in the direction of axis x and has before the deformation a length of dx , i.e. $A(x,o,o)$ and $B(x+dx,o,o)$, ref. fig. 1 . A Lagrange'ian System of coordinates is used, where the displacements of a point are regarded as functions of the coordinates of that point before the deformation. After the displacement $\hat{u}$, $\hat{v}$, $\hat{w}$, the points A and B move over to the positions A' and B' respectively. The position of A' and B' is defined by the coordinates of the deformed state ξ , η , ζ , i.e. $A'(\xi,\eta,\zeta)$ and $B'(\xi+d\xi$, $\eta+d\eta$, $\zeta+d\zeta)$. By the definition of Lagrange'ian coordinates,

$$\begin{aligned} \xi &= x + \hat{u}(x) \\ \eta &= y + \hat{v}(x) \\ \zeta &= z + \hat{w}(x) \end{aligned} \tag{I3}$$

Hence, by differentiation

$$\begin{aligned} d\,\xi &= \left(1 + \frac{d\hat{u}}{dx}\right) dx \\ d\,\eta &= \left(\frac{d\hat{v}}{dx}\right) dx \\ d\,\zeta &= \left(\frac{d\hat{w}}{dx}\right) dx \end{aligned} \tag{I4}$$

since $y \equiv z \equiv 0$. Strain $\hat{e}_{xx}$ is given by definition as

$$\hat{e}_{xx} = \frac{\overline{A'B'}}{\overline{AB}} - \frac{\overline{AB}}{\overline{AB}}$$

or

$$1 + \hat{e}_{xx} = \frac{\overline{A'B'}}{\overline{AB}} \tag{I5}$$

Both sides of eq. (I5) can be squared :

$$1 + 2\,\hat{e}_{xx} + \hat{e}_{xx}^2 = \left(\frac{\overline{A'B'}}{\overline{AB}}\right)^2 \tag{I6}$$

Since the strain is small, $\hat{e}_{xx}^2$ can be neglected, so that

$$\hat{e}_{xx} = \frac{1}{2}\left(\frac{\overline{A'B'}}{\overline{AB}}\right)^2 - 1 \tag{I7}$$

The advantage of the expression for $\hat{e}_{xx}$ of equation (I7), over equation (I4), is that $\left(\frac{\overline{A'B'}}{\overline{AB}}\right)^2$ can be evaluated exactly.

This is the same idea which underlies the Green's strain tensor. In fact, the derivation of an expression for e_{xx} could be dispensed with if Green's strain tensor were adopted [24]. Now,

$$\left(\frac{\overline{A'B'}}{\overline{AB}}\right)^2 = \frac{(d\xi)^2 + (d\eta)^2 + (d\zeta)^2}{(dx)^2} = \left(1 + \frac{d\hat{u}}{dx}\right)^2 + \left(\frac{d\hat{v}}{dx}\right)^2 + \left(\frac{d\hat{w}}{dx}\right)^2 \tag{I8}$$

After the substitution into eq. (I7), the axial stress, $\hat{e}_{xx}$, is found as

$$\hat{e}_{xx} = \frac{d\hat{u}}{dx} + \frac{1}{2}\left(\frac{d\hat{u}}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\hat{v}}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\hat{w}}{dx}\right)^2 \tag{I9}$$

For small disturbances from an equilibrium state the rates of displacements associated with flexure are greater than those in the direction of the member, so that $\left(\frac{d\hat{u}}{dx}\right)^2 << \left(\frac{d\hat{v}}{dx}\right)^2$, $\left(\frac{d\hat{w}}{dx}\right)^2$. Thus, neglecting $\left(\frac{d\hat{u}}{dx}\right)^2$ we have

$$\hat{e}_{xx} = \frac{d\hat{u}}{dx} + \frac{1}{2}\left(\frac{d\hat{v}}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\hat{w}}{dx}\right)^2 \tag{20}$$

Equation (20) gives the adopted expression for the axial strain, $\hat{e}_{xx}$.
Substitution of (5) into (20) yields :

$$\hat{e}_{xx} = \frac{du^{(o)}}{dx} + \varepsilon\frac{du}{dx} + \frac{1}{2}\left(\frac{dv^{(o)}}{dx} + \varepsilon\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw^{(o)}}{dx} + \varepsilon\frac{dw}{dx}\right)^2 \tag{2I}$$

Since $v^{(o)} \equiv w^{(o)} \equiv 0$ for the types of structures under consideration,

$$\hat{e}_{xx} = \frac{du^{(o)}}{dx} + \varepsilon\frac{du}{dx} + \frac{1}{2}\varepsilon^2\left(\left(\frac{dv}{dx}\right)^2 + \left(\frac{dw}{dx}\right)^2\right) \tag{22}$$

Hence, bearing in mind equation (6),

$$e_{xx}^{(1)} = \frac{du}{dx}$$

$$e_{xx}^{(2)} = \left(\frac{dv}{dx}\right)^2 + \left(\frac{dw}{dx}\right)^2 \tag{23}$$

The displacements $\hat{v}$ and $\hat{w}$ are taken in the direction of the principal axes of members in their undeformed state. Since the disturbances include a twisting of the sections, the curvatures about the principal axes are affected by v , w and θ . Figure 2 shows the cross-section of a member subjected to a twist $\hat{\Phi}_x$. The moments of inertia about the y and the z axes (in the initial state of the member) are denoted by I_y and I_z respectively. The curvatures of a twisted section, associated with these moments of inertia, are after the twist equal to

$$\kappa_y = \frac{d^2 \hat{w}^*}{dx^2}$$

$$\kappa_z = \frac{d^2 \hat{v}^*}{dx^2} \tag{24}$$

where $\hat{w}^*$ and $\hat{v}^*$ are the deflection components shown in figure 2 b. From geometry,

$$\hat{v}^* = \hat{v} \cos \hat{\Phi}_x + \hat{w} \sin \hat{\Phi}_x$$

$$\hat{w}^* = - \hat{v} \sin \hat{\Phi}_x + \hat{w} \cos \hat{\Phi}_x \tag{25}$$

Bearing in mind that $\Phi_x^{(o)} = 0$, following expansions (up to ε^2) can be written down :

$$\cos \hat{\Phi}_x = \cos(\Phi_x^{(o)} + \varepsilon\, \Phi_x) = 1 - \frac{1}{2} \varepsilon^2\, \Phi_x^2 + \ldots$$

$$\sin \hat{\Phi}_x = \sin(\Phi_x^{(o)} + \varepsilon\, \Phi_x) = \varepsilon\, \Phi_x + \ldots \tag{26}$$

Substitution of (5), (25) and (26) into equation (24), together with the condition that $w^{(o)} = v^{(o)} = 0$, yields

$$\kappa_y = \varepsilon \frac{d^2 w}{dx^2} + \frac{1}{2} \varepsilon^2\; 2 \frac{d^2 v}{dx^2} \Phi_x$$

$$\kappa_z = \varepsilon \frac{d^2 v}{dx^2} + \frac{1}{2} \varepsilon^2\; 2 \frac{d^2 w}{dx^2} \Phi_x \tag{27}$$

From (27) and noting that $\hat{\theta} = \frac{d\hat{\phi}_x}{dx} = \frac{d\phi_x^{(o)}}{dx} + \varepsilon \frac{d\phi_x}{dx}$, we obtain

$$\kappa_y^{(1)} = \frac{d^2w}{dx^2}$$

$$\kappa_z^{(1)} = \frac{d^2v}{dx^2} \qquad (28)$$

$$\theta^{(1)} = \frac{d\phi_x}{dx}$$

Finally, substituting (23) and (28) into (II) one obtains the equation for the second variation of the total energy

$$I_2 = \{ \frac{1}{2} \int EA \left(\frac{du}{dx}\right)^2 dx + \frac{1}{2} \int EI_y \left(\frac{d^2w}{dx^2}\right)^2 dx + \frac{1}{2} \int EI_z \left(\frac{d^2v}{dx^2}\right)^2 dx$$
$$+ \frac{1}{2} \int GJ \left(\frac{d\phi_x}{dx}\right)^2 dx \} + P \{ \frac{1}{2} \int \frac{N}{P} \left(\left(\frac{dv}{dx}\right)^2 + \left(\frac{dw}{dx}\right)^2 \right) dx$$
$$+ \frac{1}{2} \int \frac{N}{P} \frac{I_p}{A} \left(\frac{d\phi_x}{dx}\right)^2 dx \} = I_3 + P\, I_4 \qquad (29)$$

where I_3 is the first bracketed term $\{ \dots \}$, P is a parameter of load intensity, and I_4 is the second bracketed term $\{ \dots \}$. As in eq. (II), the integration is over the whole of the structure.

3. A Finite Element.

Instead of integrating over the whole of the structure, one can integrate over a number of finite elements and obtain the overall integral by a summation of individual element integrations.

The integration over each element is made in an approximate way, assuming a polynomial form of the deformed shape. The variation of I_2 of eq. (29) w.r.t. disturbances u , v , w , ϕ_x , then yields a criterion of stability. This, in a nutshell, is the finite element approach to the instability problems under consideration. The method follows closely the well established pattern of structural analysis by finite elements.

An element shown in figure 3 will be considered. We start by the determination of term I_4 for each element in terms of a "stability matrix". The displacements are assumed to have the following form :

$$\begin{vmatrix} u \\ v \\ w \\ \Phi_x \end{vmatrix} = \begin{vmatrix} 1 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & x & x^2 & x^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & x^2 & x^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x \end{vmatrix} \{ \alpha \} \tag{30}$$

Here the vector $\{ \alpha \}$ is equal to

$$\{ \alpha \} = \{ \alpha_1 \quad \alpha_2 \ \ldots \ \alpha_{12} \} \tag{31}$$

For the stability matrix the values of $\frac{dv}{dx}$, $\frac{dw}{dx}$ and $\frac{d\phi_x}{dx}$ are required. We can write

$$\{ \frac{dv}{dx} \quad \frac{dw}{dx} \quad \frac{d\phi_x}{dx} \} = B \{ \alpha \} \tag{32}$$

where

$$B = \begin{vmatrix} 0 & 0 & 0 & 1 & 2x & 3x^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2x & 3x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \tag{33}$$

Hence, for an element

$$I_{4i} = \frac{1}{2} \{ \alpha \}^T \left(\int_0^\ell n \, B^T F B \, dx \right) \{ \alpha \} \tag{34}$$

where $n = \frac{N}{P}$ and where

$$F = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{I_P}{A} \end{vmatrix} \tag{35}$$

We choose in the local (element) axes the following generalized displacements at end points 1 and 2 : the translations u, v, w in the direction of local x, y, z axes, and the rotations about the local x, y and z axes, namely Φ_x $(\frac{d\Phi_x}{dx} = \theta)$, $\Phi_y = -\frac{dw}{dx}$, $\Phi_z = \frac{dv}{dx}$ respectively.

The generalized displacement vector in the local axes is denoted by $\{ q_\ell \}$ and is equal to

$$\{ q_\ell \} = \{ u_1 \quad v_1 \quad w_1 \quad \Phi_{x1} \quad \Phi_{y1} \quad \Phi_{z1} \quad u_2 \quad v_2 \quad w_2 \quad \Phi_{x2} \quad \Phi_{y2} \quad \Phi_{z2} \} \tag{36}$$

From (30), by differentiation where appropriate and substitution of $x = 0$ for point 1 and $x = \ell$ for point 2, $\{ q_\ell \}$ can be related to $\{ \alpha \}$ as follows :

$$\{ q_\ell \} = T^{-1} \{ \alpha \} \tag{37}$$

where

$$T^{-1} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \ell & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ell & \ell^2 & \ell^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ell & \ell^2 & \ell^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ell \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2\ell & -3\ell^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2\ell & 3\ell^2 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \tag{38}$$

Substitution of (37) into (34) yields

$$\begin{aligned} I_{4i} &= \frac{1}{2} \{ q_\ell \}^T \; T^T \left(\int_0^\ell n \, B^T F \, B \, dx \right) T \{ q_\ell \} \\ &= \frac{1}{2} \{ q_\ell \}^T \; S_\ell \{ q_\ell \} \end{aligned} \tag{39}$$

where the stability matrix of an element in local axes, S_ℓ, is defined by

$$S_\ell = T^T \left(\int_0^\ell n \, B^T F \, B \, dx \right) T \tag{40}$$

Matrix T is obtained by inverting the matrix of eq. (38) analytically. The integration of equation (40) and the matrix multiplications are also performed analytically. The resulting stability matrix, S_ℓ, (in local axes) is shown in table 1.

Table 1.

Stability matrix, S_ℓ, in local axes for the generalized displacements $\{u_1 \; v_1 \; w_1 \; \Phi_{x1} \; \Phi_{y1} \; \Phi_{z1} \; u_2 \; v_2 \; w_2 \; \Phi_{x2} \; \Phi_{y2} \; \Phi_{z2}\}$:

$$\frac{n}{30\ell}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 36 & 0 & 0 & 0 & 3\ell & 0 & -36 & 0 & 0 & 0 & 3\ell \\
 & & 36 & 0 & -3\ell & 0 & 0 & 0 & -36 & 0 & -3\ell & 0 \\
 & & & 30\frac{I_p}{A} & 0 & 0 & 0 & 0 & 0 & -30\frac{I_p}{A} & 0 & 0 \\
 & & & & 4\ell^2 & 0 & 0 & 0 & 3\ell & 0 & -\ell^2 & 0 \\
 & & & & & 4\ell^2 & 0 & -3\ell & 0 & 0 & 0 & -\ell^2 \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 36 & 0 & 0 & 0 & -3\ell \\
 & & \text{SYMMETRY} & & & & & & 36 & 0 & 3\ell & 0 \\
 & & & & & & & & & 30\frac{I_p}{A} & 0 & 0 \\
 & & & & & & & & & & 4\ell^2 & 0 \\
 & & & & & & & & & & & 4\ell^2
\end{bmatrix}$$

Similar operations on I_3 term of eq. (29) lead to

$$I_{3i} = \frac{1}{2}\{q_\ell\}^T K_\ell \{q_\ell\} \qquad (41)$$

where K_ℓ is the usual form of the stiffness matrix of a prismatic member of length ℓ. From the element stiffnesses, the gross stiffness matrix, K, is assembled in the usual way : the generalized displacements of an element in local axes are transformed to global axes by axes rotation,

$$\{q_\ell\} = H\{q_g\} \qquad (42)$$

and the element stiffness matrices are added together by addressing their terms to their proper place in the gross stiffness matrix, K. In the same way the gross stability matrix, S, is assembled from the element stability matrices, thus yielding

$$I_2 = \Sigma I_{3i} + P \Sigma I_{4i} = \frac{1}{2} \Sigma \{ q_g \}^T H^T K_\ell H \{ q_g \} + P \frac{1}{2} \Sigma \{ q_g \}^T S_\ell \{ q_g \}$$

$$= \frac{1}{2} \{ q \}^T K \{ q \} + P \frac{1}{2} \{ q \}^T S \{ q \} > 0 \qquad (43)$$

where $\{ q \}$ is a vector of all generalized displacements of a structure, where each "displacement" is, in fact, a disturbance from an equilibrium configuration.

Equation (43) is a condition of stability in a finite form. We can employ it in order to answer two different, but related questions :

(a) Is the structure stable for a specified load intensity P?

To answer it we must find the minimum value of I_2 w.r.t. $\{ q \}$ and see if this value of I_2 is positive. The equation for I_2 (eq. 43) is quadratic and homogeneous in $\{ q \}$, with a trivial minimum at $\{ q \} \equiv 0$. In order to obtain information about the relative values of I_2 for the various modes of disturbances, the disturbance scale should be constrained by a norm [3].

We choose as a norm

$$\frac{1}{2} \{ q \}^T S \{ q \} = 1 \qquad (44)$$

Hence, the augmented functional of eq. (43) is

$$I_{2a} = \frac{1}{2} \{ q \}^T K \{ q \} + P \frac{1}{2} \{ q \}^T S \{ q \} - \gamma \left(\frac{1}{2} \{ q \}^T S \{ q \} - 1 \right)$$

or

$$I_{2a} = \frac{1}{2} \{ q \}^T K \{ q \} - (\gamma - P) \frac{1}{2} \{ q \}^T S \{ q \} + \gamma = \text{min.} \qquad (45)$$

where γ is a Lagrange'ian multiplier. For an extremum we consider a non-trivial variation of the disturbances, $\delta \{ q \} \neq 0$. Hence, since K and S are symmetric,

$$K \{ q \} - (\gamma - P) S \{ q \} = 0 \tag{46}$$

A compatible system of equations for which a solution of eq. (46) exists with non-trivial $\{ q \} = \{ q_r \}$, is defined by the condition

$$| K - (\gamma_r - P) S | = 0 \tag{47}$$

whence the characteristic values of the Lagrange'ian multiplier, γ_r, and the characteristic vector of the disturbances, $\{ q_r \}$, can be determined. Only the modes of the latter can be found, as the scaling factor of $\{ q_r \}$ must be adjusted to fulfill the constraint of the norm. It can be shown that the Lagrange'ian multiplier, γ_r, is equal to I_2 (min.). To this end, $\{ q_r \}$ and γ_r are substituted into eq. (46)

$$K \{ q_r \} + P S \{ q_r \} - \gamma_r S \{ q_r \} = 0 \tag{48}$$

Premultiplication by $\frac{1}{2} \{ q_r \}^T$ yields

$$\frac{1}{2} \{ q_r \}^T K \{ q_r \} + P \frac{1}{2} \{ q_r \} S \{ q_r \} = \gamma_r \frac{1}{2} \{ q_r \}^T S \{ q_r \} = \gamma_r \tag{49}$$

since by eq. (44), $\frac{1}{2} \{ q_r \}^T S \{ q_r \} = 1$. But the left-hand side of eq. (49) is the same as the right-hand side of eq. (43) with $\{ q_r \}$ substituted for $\{ q \}$. Thus,

$$\gamma_r = I_2 \text{ (min.)} \tag{50}$$

Hence, a structure is stable if $\gamma_r > 0$.
The second question can be frased as :

(b) <u>At what load intensity $P = P_{cr}$ is the structure in a neutral equilibrium</u>?

A neutral equilibrium is defined by

$$I_2 \text{ (min.)} = \gamma_r = 0 \tag{51}$$

Substitution into eq. (46) yields a characteristic equation

$$(K + P_{cr} S) \{ q_r \} = 0 \qquad (52)$$

Thus, the critical load intensity, P_{cr}, can be found from eq. (52). This is the method adopted here. The solution of eq. (52) is done by standard iterative procedure, in which the largest characteristic value of $\lambda = \frac{1}{P_{cr}}$ is determined in the first cycle of iterations.

4. Examples.

To test the rate of convergence of the finite element solution, the critical load of a cantilever column, with a predominant weakness for bending about the zz axis, was calculated. The results are listed below in terms of the effective length, ℓ, and the moment of inertia about the zz axis, I_z.

Ex. 1 : Cantilever column, buckling out of xz plane.

Number of elements :	1	2	3
Computed critical load :	$1.0075 \frac{\pi^2 EI_z}{\ell^2}$;	$1.0005 \frac{\pi^2 EI_z}{\ell^2}$;	$1.0001 \frac{\pi^2 EI_z}{\ell^2}$

Evidently, with as few as two elements in a loaded member, an excellent accuracy is achieved.
To investigate other modes of buckling, we compute several eigenvalues of a cantilever column which is divided into two elements.

Ex. 2 : Cantilever column, 2 elements.

Section properties : Area, $A = 5.0\ in^2$, Moments of Inertia, $I_y = 240\ in^4$, $I_z = 12\ in^4$, Torsion constant, $J = 0.35\ in^4$.
Elastic properties : $E = 30.000\ K/in^2$, $G = 12.000\ K/in^2$.
Actual length of the column, $L = 100$ in.
Applied load, $n = 83.400$ K.

Mode n° :	1,2	3
Computed critical load factor :	0.99920	1.06562

The first and the second modes are torsional, which is evident from inspection of the eigenvectors (not reproduced here). The torsional critical load factor can be evaluated "exactly" in this example from the condition

$$GJ - \frac{P_{cr}}{A} (I_y + I_z) = 0 \tag{53}$$

whence

$$P_{cr} = \frac{GJ\ A}{n(I_y + I_z)} = 0.99920$$

It is not a coincidence that the "exact" answer for the torsional modes is obtained by the finite elements, since the element satisfies fully all the requirements of the beam theory for the torsional buckling. The third mode represents buckling about the z-z axis, and the load factor can be calculated exactly as

$$P_{cr} = \frac{\pi^2 \times 30000 \times 12}{834 \times 4 \times 100^2} = 1.065064$$

which is to within 0.05 % of the value computed by the finite element method.

The same example was solved several times with a different orientation of the column with respect to the global axes. Consistent results were obtained.

Next example is a plane frame shown in figure 4. One element represents the beam (which is not loaded in the stable state of equilibrium), each column is subdivided into two elements.

Ex. 3 : Plane frame of figure 4.

All members have the same sectional and elastic properties, namely :

$A = 5.0\ in^2$, $I_y = 12\ in^4$, $E = 30.000\ K/in^2$.

The applied load, $n = 1.0$.

Torsional buckling and buckling out of $\bar{x}$-$\bar{z}$ plane is prevented.

Computed critical load = 71.050 K .

The members which are not subjected to the axial forces are represented "exactly" by a single element, since the third order polynomial, which is used to represent the deflections of an element, is an exact expression for the deflections of a beam with constant shear. This has been verified on the present example by subdividing the beam into several elements. As expected, identical results were obtained each time.

The eigenvector shows that the first buckling mode is a sidesway buckling, with maximum deflections occuring at the top of the columns. For this mode the critical load can be obtained analytically from the equation

$$\frac{1}{6}\frac{I_c}{I_b}\frac{L_b}{L_c} + \frac{\tan(L_c \sqrt{\frac{P}{EI_c}})}{L_c \sqrt{\frac{P}{EI_c}}} = 0 \qquad (54)$$

where I_c and L_c are the pertinent moment of inertia and the length of the column, whilst I_b and L_b are the corresponding values for the beam. Substitution of the critical load obtained by the finite elements, yields the following results

$$\tan(L_c \sqrt{\frac{P}{EI_c}}) = \tan\ (2.52873^c) = -\ 0.703182$$

and the substitution into equation (54) gives

$$0.27778 - 0.27808 = 0.00030 \simeq 0$$

This indicates that a close approximation of the critical load was obtained by the finite elements.

Ex. 4 : Frame of figure 5.

Point A is free to rotate in any direction; there are no restraints against any mode of buckling.

Section properties (w.r.t. local axes) : $A = 5\ in^2$, $I_y = 240\ in^4$, $I_z = 12\ in^4$, $J = 0.4\ in^4$.

Elastic properties : $E = 30.000\ K/in^2$, $G = 12.000\ K/in^2$.

Applied load, $n = 1.0$.

There are two elements in the column AB and one element in the beam BC .

Computed critical load = 7.28862 K .

The corresponding eigenvector shows that the buckling is out of the global $\bar{x}\,\bar{z}$ plane.

To check the order of magnitude of the numerically computed critical load, a simple approximate solution of this problem is possible. Because of the large torsional flexibility of the member BC (fig. 5), the critical load of the frame, associated with the buckling out of $\bar{x}\,\bar{z}$ plane, is of the same magnitude as the critical load of the mechanism of figure 6, provided that the spring constant, $k = 3\ EI_z/300^3 = 0.04$ K/in . By equilibrium considerations of the mechanism in a deflected state, we find that

$P\ u = 0.04.u.I80$.

Hence, $P = 7.2$ K , and it is evident that the numerical solution is of the right order of magnitude.

Ex. 5 : Tripoid of figure 7.

Section properties : $A = 5\ in^2$, $I_y = 240\ in^4$, $I_z = I2\ in^4$, $J = 0.6\ in^4$.

Elastic properties : $E = 30.000\ K/in^2$, $G = I2.000\ K/in^2$.

Applied loads on all columns, n = I.0 .

There are two elements in each column, and one element in each beam.

Computed critical load = 89.82I2 k .

The structure is axisymmetric, though this information is not used in the formulation of the problem and it is not " known" to the computer.

Inspection of the eigenvector reveals that the structure buckles indeed in an axisymmetric fashion, the largest deflections (scaled down to unity) occuring at the top of the columns in the directions marked by arrows on the plan of figure 7.

Thus, the numerical solution shows a buckling behaviour which can be expected from an inspection of the problem.

Ex. 6 : Tripoid having a plan of figure 8.

This example is similar to the example 5, but the columns B and C are oriented differently, as shown in figure 8.

Computed critical load = I20.495 k .

The increase of the load bearing capacity (I20.5 k as compared with 89.8 k) is due to the additional restraint of the columns B and C by the beams. The beams must now bend about their local y ("strong") axis, in order to allow the columns to undergo rotation about their weak axes at the joint.

An inspection of the eigenvector reveals the displacements of the tops of the columns, shown in figure 8.

It is seen that the column A buckles first, and the columns B and C provide some restraint to it.

5. Conclusion.

A finite element solution of the stability problem of elastic space frames is a convenient method, which can be applied to a variety of problems with ease. Where analytical solutions are available, the numerical results are found to be in an excellent agreement with the analytical ones. Inspection of examples 5 and 6 illustrates that the critical loads can not be assessed by a consideration of a compression member alone, a procedure often employed in older design methods.

The whole of the structure must be considered in its entirety, and the finite element method provides a convenient means to achieve this.

References.

I. Argyris, J.H. "Flexure-torsion failure of panels", Aircraft Engineering, June-July I954.

2. Engel, H.L., Goodier, J.N. "Measurements of torsional stiffness changes and instability due to tension, compression and bending", J. of Applied Mechanics, Dec. I953, pp. 553-560.

3. Fraeijs de Veubeke, B. "The second variation test with algebraic and differential constraints", Advanced Problems and Methods for Space Flight Optimization, pp. 292, Pergamon I969.

4. Fraeijs de Veubeke, B. "Displacement and equilibrium models in the finite element method", Stress Analysis, ed. by O.C. Zienkiewicz and G.S. Holister, Chapter 9, pp. I45-I97, Wiley I965.

5. Fraeijs de Veubeke, B. "Large displacement formulations for elastic bodies", Report SA-I5, Laboratoire de Techniques Aéronautiques et Spatiales, Université de Liège, 75 rue du Val-Benoît, Liège.

6. Fung, Y.C. "Foundations of solid mechanics", Prentice Hall I965, pp. 1-525.

7. Goodier, J.N. "Torsional and flexural buckling of bars of thin-walled open section under compressive and bending loads", Transactions ASME, 64, A-I03 (I942).

8. Zarghamee, M.S., Shah, J.M. "Stability of Space Frames", Journal of the Engineering Mechanics Division, ASCE, V.94, n. EM 2, Proc. Paper 5878, April I968, pp. 37I-384.

9. Hoff, N.J. "A strain energy derivation of the torsional-flexural buckling loads of straight columns of thin walles sections", Quarterly of Applied Mathematics, Vol. I, N. 4, Jan. I944, pp. 34I-345.

IO. Mallett, R.H., Marçal, P.V. "Finite element analysis of non linear structures", Journal of the Structural Division, ASCE, V.94, ST 9, Proc. Paper 6II5, Sept. I968, pp. 208I-2I05.

II. Marguerre, K. "On the application of the energy method to stability problems",
NACA Technical Memorandum n. II38, Oct. I947, pp. 1-47 (translation of "Über die Anwendung der energetischen Methode auf Stabilitätsprobleme", Jahrb. I938, DVL, pp. 252-262).

I2. Martin, H.C. "Large deflection and stability analysis by the direct stiffness method",
Technical Report N° 32-93I, NASA, Jet Prop. Lab., California Institute of Technology, Pasadena, California, August 1, I966.

I3. Pekoz, T.B., Winter, G. "Torsional-flexural buckling of thin walled sections under eccentric load",
Journal ASCE, Vol. 95, No. ST 5, Proc. Paper 657I, May I969, pp. 94I-963.

I4. Wagner, H. "Verdrehung und Knickung von offenen Profilen",
25th Anniversary Volume of the Technische Hochschule, Danzig, I929, p. 329 (Verlag A.W. Kaffeman, Danzig, I929).

I5. Massonnet, M.Ch. "Théorie du flambement par torsion" (Theory of buckling by twisting),
I.A. for Bridge Struct. Eng., Publications, Vol. 6, pp. 2II-23I.

I6. Massonnet, M.Ch. "Flambement spatial des barres droites",
C.E.C.M. notes techniques, B-I0.55, Bruxelles, 1 Septembre I958.

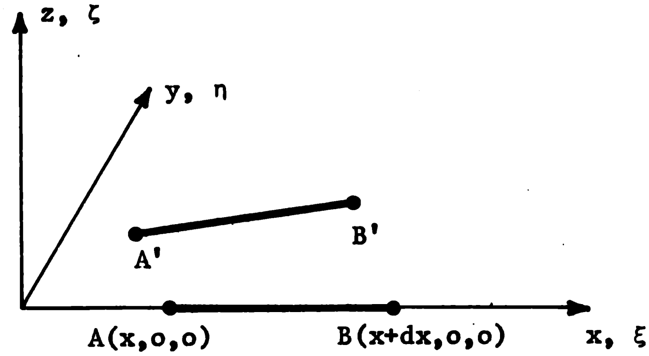


FIGURE 1

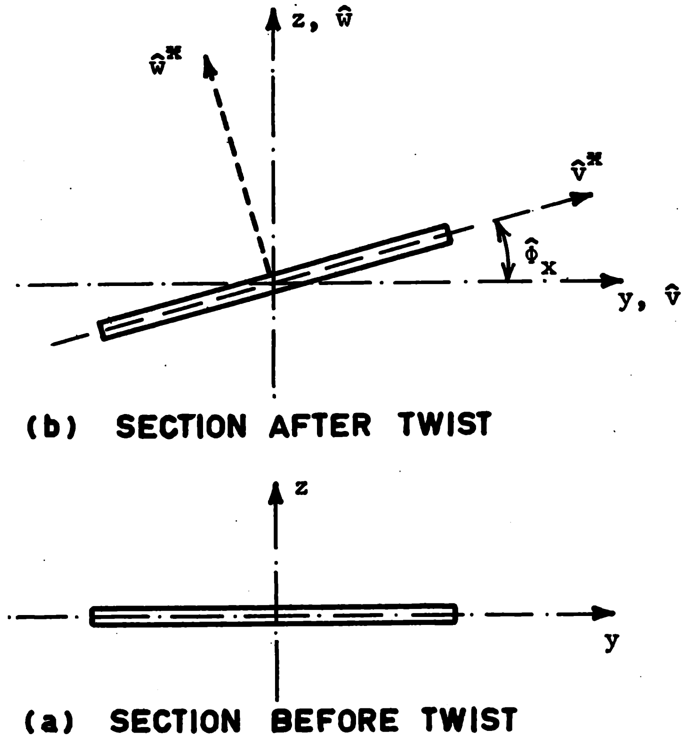
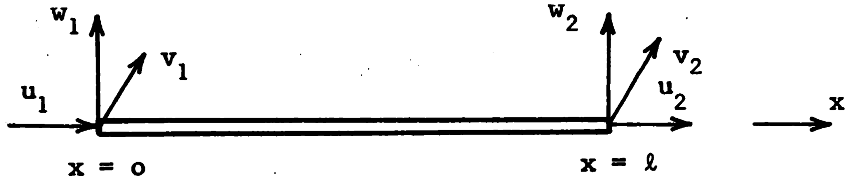
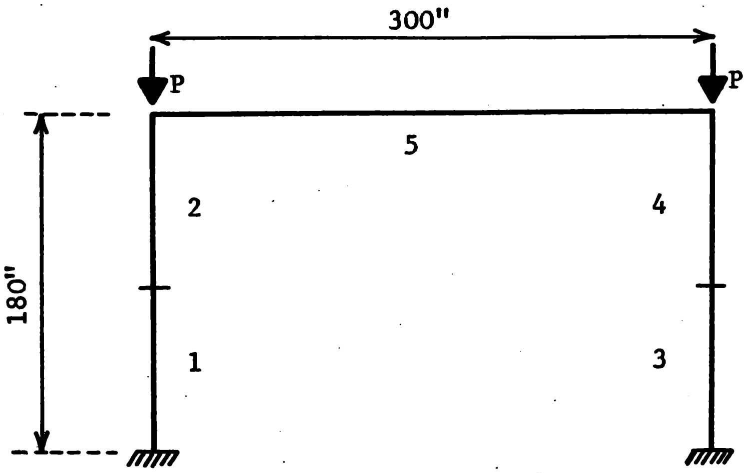


FIGURE 2



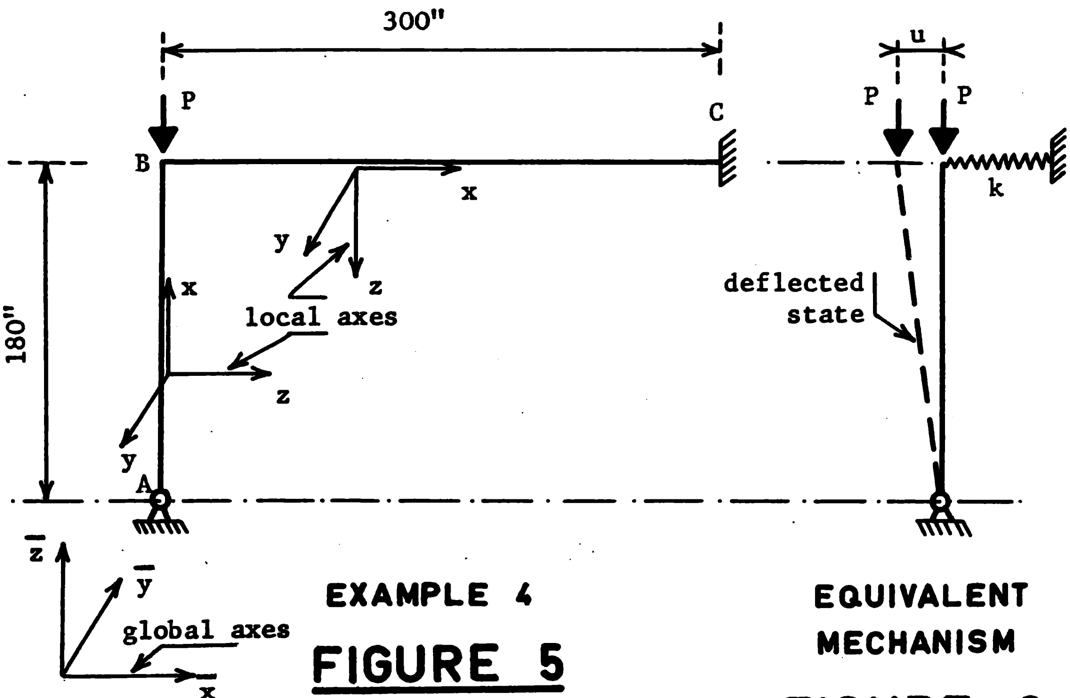
FINITE ELEMENT

FIGURE 3



EXAMPLE 3

FIGURE 4

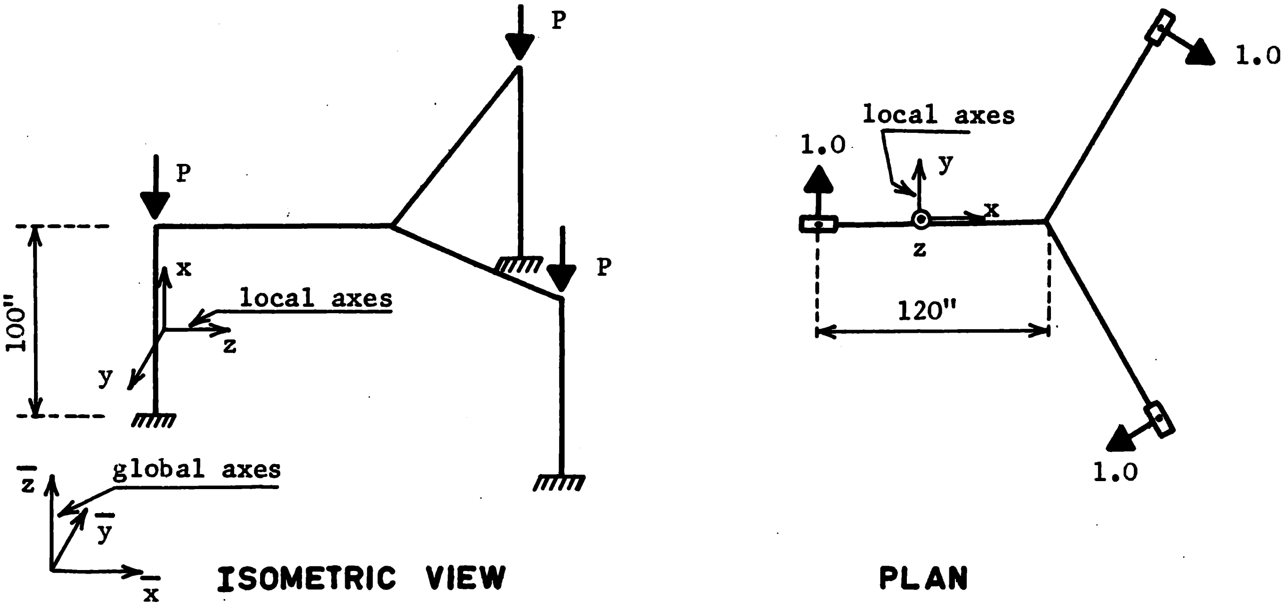


EXAMPLE 4

FIGURE 5

EQUIVALENT MECHANISM

FIGURE 6



ISOMETRIC VIEW

PLAN

FIGURE 7

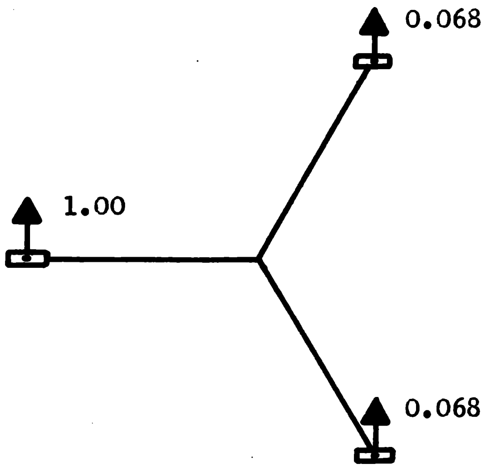


FIGURE 8

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13. ABSTRACT

A finite element for a prismatic member subjected to axial loads is derived for the stability analysis of elastic spare frames. The derivation is based on the fact that in a stable state of equilibrium the second variation of the total energy is positive definite. A feature of the element developed is that the reduction of torsional rigidity due to the presence of axial stresses is taken into account. Several examples are presented. An excellent agreement with analytical results is obtained, where closed form solutions are available.

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Variational Stability Criteria						
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Buckling						
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