The CLGE Students’ Contest 2015-2016
Galileo, EGNOS, Copernicus

Relative positioning with Galileo E5 AltBOC code measurements

Cécile Deprez
(cecile.deprez@ulg.ac.be)

This paper is an abridged version of the author’s master thesis, defended in September 2015 at the University of Liège in Belgium, under the supervision of Professor René Warnant. The full version is downloadable at http://orbi.ulg.ac.be/handle/2268/194720.
Abstract

For over a decade, Europe has started to develop its own Global Navigation Satellite System (GNSS). Initiated in 1999, the Galileo project finally materialized a few years ago, recently experiencing a prompt expansion with the launch, in 2015 and 2016, of 8 satellites belonging to the Full Operational Capability (FOC) generation. Broadcasting new signals, with new modulations, the first studies addressing this system reveal promising level of precisions on both code and carrier phase observables. Still in test phase but already available for measurements, this recent system can be used to estimate positions.

Among the new signals developed by the Galileo program, the Galileo E5 AltBOC, also known as Galileo E5a+b or Galileo E5, reveals great characteristics. Thanks to its particular AltBOC modulation, it allows more precise code and phase observations besides being less affected by multipath. These innovative performances should lead to more precise position estimations than with any other signal presently in use.

In this master thesis, we compared the positions estimated with GPS and Galileo on their different frequencies (L1, L2, L5 for GPS and E1, E5a, E5b and E5 AltBOC for Galileo). We combined the observations made by the receivers belonging to the University of Liège (2 Trimble NetR9 receivers, 1 Septentrio PolaRxS receiver and one Septentrio PolaRx4 receiver) in double differences (DD) combinations using various configurations (zero baseline (ZB), short baseline (SB) and medium baseline (MB)).

It turns out that Galileo E5 AltBOC outperforms all other signal in terms of observation precision (estimated in a DD SB configuration in order to remove atmospheric and clock error sources affecting the signal). Regarding the precision obtained on the computed positions, we could reach a few decimetres with Galileo E5 code pseudoranges on baselines up to 25 kilometres.
1 Introduction

The state-of-the-art technologies on-board recent satellites gradually lead to a more precise satellite positioning, regardless to the receiver range. In addition to this, the rise in new Global Navigation Satellite Systems (GNSS), such as BeiDou or Galileo, combined to the modernization of former ones (GPS and GLONASS) also improves the quality of position’s estimations. This master’s thesis focuses on the improvements in satellite-based positioning brought by the recent European Galileo satellite system, its new signals and modulation techniques.

1.1 Observables

The signals broadcast by the satellites are composed of carrier phases on which code pseudoranges are modulated. Both codes and phases might be considered as observables in the positioning equations but their intrinsic characteristics lead to different level of precision.

The carrier phase observable is by far the most precise, with position’s estimations ranging from a few centimetres to a few millimetres, depending on the application. However, phase positioning requires to resolve a term called initial ambiguity, which is a tricky problem that may be even more complicated by the occurrence of cycle slips \(^1\). Many GNSS users do not possess initial ambiguity resolution algorithms or even receivers able to deal with these observables.

On the other hand, code pseudoranges are less precise, allowing to reach position estimations of a few metres in best cases. Therefore, precise positioning with code pseudoranges is a real challenge.

However, as the initial phase ambiguities require more complicated algorithms to be solved, code measurements, even if less accurate, are often used in real time applications. Therefore, the code pseudorange is the observable used in the mass market devices and applications (the mobile car GPS, the applications of localization on smart phones and tablets, future autonomous cars guided by satellites, to name but a few). The interest of achieving real-time precise positioning with this observable is thus essential for device manufacturers.

The signal broadcast by satellites is altered during its travel to the receiver. Indeed, two atmospheric layers, the ionosphere and the troposphere, delay the signal and weaken it while reflecting surfaces deviate it from its trajectory (this last effect is known as multipath). Furthermore, the propagation inside satellite and receiver hardware delays the signal arrival to the receiver (hardware delays). In addition, the satellite and receiver clocks are not perfectly synchronized which induces measurement errors and finally, the signal is affected by the observation noise. Other parameters influence and delay this signal in such a way that signal quality is reduced when it is finally tracked by the receiver.

The equations of positioning for code pseudoranges and carrier phases can therefore be written as follows :

Codes:

\[
P_{r,k}(t) = D_{r}^{s} + T_{r}^{s} + i_{r,k} + M_{r,k,m} + c.(\delta t_{s}^{s}(t_{ref}) - \delta t_{r}(t_{r,ref})) + d_{r,k,m} + d_{k,m}^{s} + \epsilon_{r,k,m}^{s} \tag{1}
\]

with \(P_{r,k}(t)\) denoting the code pseudorange measurement in function of the time, \(D_{r}^{s}\) the geometric distance between the receiver \(r\) and the satellite \(s\), \(T_{r}^{s}\) the tropospheric delay, \(i_{r,k}^{s}\) the ionospheric delay, \(M_{r,k,m}^{s}\) the multipath delay, \(c\) the speed of light in the vacuum, \(\delta t_{s}^{s}(t_{ref})\) the satellite clock error, \(\delta t_{r}(t_{r,ref})\) the receiver clock error, \(d_{r,k,m}\) and \(d_{k,m}^{s}\) the hardware delays of the receiver and the satellite, respectively, and \(\epsilon_{r,k,m}^{s}\) the observation noise which encompasses unmodelled errors.

\(^1\)Cycle slip: momentary loss of signal inducing a jump in the phase observable.
The index \( k \) represents the frequency and \( m \) stands for modulation (codes).

Phases:

\[
\Phi_{r,k}^s(t) = D_r^s + T_r^s - I_{r,k}^s + M_{r,k,\phi}^s + c. (\delta t^s(t_{ref}^s) - \delta t_r(t_{ref})) + d_{r,k,\phi}^s + d_{r,k,\phi}^s + \lambda N_{r,k}^s + e_{r,k,\phi}^s \tag{2}
\]

with \( \Phi_{r,k}^s(t) \) the carrier phase measurement in function of the time, \( \lambda \) the wavelength of the signal, \( N_{r,k}^s \) the initial ambiguity, the index \( \phi \) meaning function of the phase and the other error sources remaining the same as in the code pseudorange equation (1).

1.2 Positioning

Satellite-based positioning relies on the measurement of the propagation time of signal between the satellite and the receiver. This time, multiplied by the speed of light, represents the distance between the receiver and the satellite. In a tri-dimensional coordinate system rotating with the Earth, each point situated on the Earth is characterized by three coordinates (X, Y, Z). As three unknowns are present in the equation, at least three satellites must be considered to resolve it. Nevertheless, as the measured time of propagation is affected by clock errors, a fourth unknown must be inserted in the positioning equation to solve the receiver clock error. Therefore, satellite-based positioning implies being able to track at least four satellites simultaneously, in order to be able to solve the fourth unknowns of the positioning equation.

The geometry of the observed satellites also has an influence on the precision obtained on the position estimated. It is characterized by a parameter called the Position Dilution Of Precision (PDOP). Nonetheless, the observation of a great number of satellites usually improves the PDOP factor (a low value of PDOP improves the positioning precision). This factor will therefore degrades the estimated position in a greater extend when a reduced constellation satellite system, such as Galileo, is observed.

1.3 AltBOC modulation

The European satellite system Galileo is at the forefront of the technology with its new satellites and frequencies. Among the signals generated by this system, one stands out from the rest. The Galileo E5a+b signal, also called Galileo AltBOC or Galileo E5 is expected to be revolutionary [Diessongo et al., 2014]. This observable should theoretically reach the centimetre-level precision with code pseudoranges even in challenging environments such as urban environments [Silva et al., 2012]. This exceptional accuracy should lead to precise positioning with codes pseudoranges only, avoiding complex ambiguity resolutions [Junker et al., 2011].

Galileo E5 signal’s particular modulation, the AltBOC modulation, is in major part responsible for its highly improved performances [Shivaramiah & Dempster, 2009]. Composed of two sub-carriers E5a and E5b, the E5 a+b band has a central frequency of 1191.795 MHz. Each sub-carriers is transmitted in different frequency bands [Julien et al., 2015]. This results in a reference bandwidth of at least 51.150 MHz, the largest Radio Navigation Satellite System band [EuropeanUnion, 2010]. Its wide bandwidth and its AltBOC modulation lead to a low observation noise on E5 a+b code measurements [Diessongo et al., 2014].

In addition to this wide bandwidth, the AltBOC modulation is characterized by a sharp autocorrelation function. According to [Silva et al., 2012], observation noise and multipath robustness are closely linked to the autocorrelation function shape. A steep mean peak, as present on the autocorrelation function of Galileo E5 (Fig. 1), indicates low observation noise and high robustness against multipath. The presence of secondary peaks also results in an improvement in robustness against multipath.
Figure 1: Normalized autocorrelation functions for different modulations: BPSK of GPS L1, BOC of Galileo E1 with simplified demodulation⁴, CBOC of Galileo E1 and AltBOC of Galileo E5 signals⁵. Source: [Silva et al., 2012]

The improvement of positioning precision using this signal is the hypothesis tested in this dissertation.

2 Method

Few months ago, Google made a ground-breaking announcement. The global enterprise said that they were ready to make available for users raw GNSS measurements made by devices (smartphones, tablets,...) running under Android ([Cameron, 2016a] and [Cameron, 2016b]). This way, code pseudoranges, and maybe later carrier phases, will be available on the new Android version coming this year.

Developing easy access to satellite data has been for a long time in every GNSS’ specialist thoughts. But, as mentioned earlier, carrier phases data are not tracked by all receivers, contrary to code pseudoranges. Furthermore, the ambiguity resolution is complex and might require great part of the phones’ memory (RAM). Therefore, the development of a high precision positioning technique based on code observable could be at the origin of a revolution in mobile applications.

But using only code observables usually degrades the quality of the computed position. Indeed, the position precision on code pseudoranges is generally close to a few decametres or metres in best case scenarios, which is not sufficient for many applications. Therefore, improving the precision with the code pseudoranges would mean being able to reach a precision of a few decimetres on a position.

In this master’s thesis, we constrained our code pseudorange solution by using only single-frequency methodology. In general, when decimetre-level precision is considered with code pseudoranges, many authors use dual-frequency receivers in order to realize dual-frequency combinations which remove ionospheric effects. However, most of the mass market GNSS receivers are not able to track two frequencies, limiting the use of dual-frequency solutions to highly specialized receivers.
2.1 Double difference

In order to respect these constraints and achieve decimetre precision on position estimation, a method of relative positioning was implemented: the double difference (DD). In a double difference, two receivers \((1, 2)\) simultaneously track two satellites \((i, j)\) (see Fig. 2). Single differences \((P_{12,k}^i(t)\) and \(P_{12,k}^j(t)\)) are first computed on each tracked satellites.

\[
P_{12,k}^i(t) = P_{1,k}^i(t) - P_{2,k}^j(t)
\]

\[
= (D_{1}^{i} - D_{2}^j) + (T_{1}^i - T_{2}^j) + (I_{1,k}^i - I_{2,k}^j) + (M_{1,k,m}^i - M_{2,k,m}^j) + [c(\delta t_1(t_{1,ref}) - \delta t_1(t_{1,ref})) - c(\delta t_2(t_{2,ref}) - \delta t_2(t_{2,ref}))]
\]

\[
+ (d_{1,k,m} + d_{k,m}^i - d_{2,k,m} - d_{k,m}^j) + (\epsilon_{1,k,m}^i - \epsilon_{2,k,m}^j)
\]

\[
= D_{12}^i + T_{12}^i + I_{12,k,m}^i + c(\delta t_2(t_{2,ref}) - \delta t_1(t_{1,ref})) + d_{12,k,m} + \epsilon_{12,k,m}
\]

with the indexes \(s_{12}^i\) representing the single difference terms: \(s_{12}^i = s_{1}^i - s_{2}^j\)

Then the two single differences (between satellites \(i\) and \(j\)) are subtracted to form a double difference \(P_{12,k}^{ij}(t)\).

The double difference equation is:

\[
P_{12,k}^{ij}(t) = P_{12,k}^i(t) - P_{12,k}^j(t)
\]

\[
= (D_{12}^i - D_{12}^j) + (T_{12}^i - T_{12}^j) + (I_{12,k}^i - I_{12,k}^j) + (M_{12,k,m}^i - M_{12,k,m}^j) +
\]

\[
[ c(\delta t_2(t_{2,ref}) - \delta t_1(t_{1,ref})) - c(\delta t_2(t_{2,ref}) - \delta t_1(t_{1,ref})) ] + [d_{12,k,m} - d_{12,k,m}] + (\epsilon_{12,k,m}^i - \epsilon_{12,k,m}^j)
\]

\[
= D_{12}^{ij} + T_{12}^{ij} + I_{12,k,m}^{ij} + M_{12,k,m}^{ij} + \epsilon_{12,k,m}^{ij}
\]

(4)
with the indexes $i_{12}^j$ representing the double difference terms: $i_{12}^j = i_{12}^1 - i_{12}^2$.

The clock and hardware delays disappear from the equation (4) thanks to this combination and only the tropospheric and ionospheric delays as well as the multipath and observation noise are remaining.

2.2 Equipment

Our study is based on the 6 multi-GNSS receivers (2 Septentrio PolaRx4, 1 Septentrio PolaRxS, 1 Septentrio PolaRx5 and 2 Trimble NetR9) installed on the roof of our building in Liege. These receivers are connected by the means of a two-way and a four-way splitter to two Trimble TRM 59800 SCIS choke ring antennae. We also used data from receivers located in Waremme and Brussels (one Septentrio PolaRx4 and one Septentrio PolaRx4TR). They are connected to an Ashtech choke ring antenna and a Javad choke ring antenna DM, respectively.

2.3 Configurations

Baseline length is a crucial parameter as far as relative positioning is concerned. Indeed, the closer the receivers, the more common errors will cancel out in double differences. We distinguished three different configurations.

First, the zero baseline configuration, which consists in two receivers connected to the same antenna thanks to a splitter, has been tested. Not used for practical positioning, this configuration eliminates the tropospheric and ionospheric terms of the equation as well as the multipath, which is common to the two receivers as they are connected to the same antenna. It also removes common part of the observation noise (part due to signal propagation up to the splitter). We implemented this configuration with the aim of estimating the observation noise only due to the receiver.

In practice, when double differences are used in positioning, one user receives code pseudoranges from a reference station located at a given distance but tracking the same satellites simultaneously. For this reason, we created two short baselines of about 5 metres between similar receiver types and two medium baselines of approximately 23 kilometres (Liège-Waremme) and 87 kilometres (Brussels-Waremme). In these two cases, the residuals due to the ionospheric and tropospheric errors increase with the distance. As regards multipath, its value will only depend on the receiver locations: it does not depend on baseline length. The DD observation noise results from observation noise from each individual code measurement.

In the short baselines case, the common part of the troposphere and ionosphere terms will cancel out. With only the observation noise and the multipath remaining, 5 metre baselines are used for estimating the precision of the observables, hereafter called observation precision.

In the medium baseline case, atmospheric errors, multipath depend on the distance between receivers. We used this configuration to estimate up to which distance the decimetre precision could be reached on the estimated position, hereafter called position precision.

3 Results

The precisions of the observables computed on the basis of the short baselines in Liège confirmed the assumptions developed in the introduction on the Galileo E5a+b signal. The Table 3 shows actualized figures obtained on the basis of data collected in June 2016 (DOYs 145 to 154) and the Figure 4 illustrates the results obtained with the different receiver’s types.
In order to obtain such precisions, we computed the mean of satellite’s observations during periods of 10 days. The sampling rate of our receivers is 30 seconds. We did not consider the E14, E18 and E20 satellites of the Galileo constellation in the computation. Indeed, the E14 and E18 were not providing ephemeris at this moment and the E20 satellite was no more emitting on Galileo E5a, E5b and E5AltBOC frequencies. We used a mask of elevation of 10 degrees, to be able to observe as many Galileo satellite as possible without increasing the influence of noise and multipath.

In such conditions, the conclusions regarding the precision of the observables are:

- Differences can be observed between the different receiver’s types
- Galileo E5 a+b outperforms any of the other GPS and Galileo signals
- Galileo E1 and GPS L1, the most commonly used signals, are the worst in term of observable precision
- GPS L2 and GPS L5 show very similar quality, as well as Galileo E5a and Galileo E5b
- In general, the Galileo signals are more precise than the GPS ones
Regarding the **precision obtained on position estimation**, the coordinates of the antennae to which receivers were connected were known. The computation of position precision was therefore obtained by comparing computed positions with the known ones.

The analysis of the results obtained with the position estimation needs to consider two parameters. We computed the positions on the basis of the least squares method. Therefore, the greater the number of observations we had, the greatest the precision on the position. With the reduced Galileo constellation (particularly during the master thesis, were the number of available satellites was limited to 3 commissioned spacecraft (E11, E12 and E19) and four satellites under commissioning (E14, E18, E22 and E26)), the number of observation periods on one day was therefore limited. Secondly, still due to this low number of satellites available, the PDOP parameter was highly degraded (lower number of tracked satellite usually means degraded position due to poor satellite geometry). In order to limit its impact on positioning, we removed from our statistics positioning results with PDOP values larger than 15.

Our expectations of precision regarding the positions (a decimetre to a few decimetre precision) were reached in zero baseline with both GPS and Galileo signals. The short baseline and medium baseline (up to Waremme) lead to mixed results. The high PDOP values paired with the very short observation periods lead to highly degraded Galileo solutions. On a 10-days periods, only 3 days reached our expected precision. From Table 1, all mean values of Galileo position precision in short baseline case are above the metre, apart from Galileo E5 AltBOC signal. But, the minima observed on this 13-days period reached a few decimetre-level precision. This minimum values correspond to favourable geometry days (with low PDOP value). In the Brussels baseline case, such a precision level could not be reached, with both GPS and Galileo.

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{pos}(m) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.462 0.355 0.569</td>
<td>1.794 1.085 1.374 0.604</td>
</tr>
<tr>
<td>Min</td>
<td>0.456 0.342 0.510</td>
<td>0.860 0.601 0.642 0.258</td>
</tr>
<tr>
<td>Max</td>
<td>0.469 0.364 0.725</td>
<td>3.383 1.536 2.278 1.253</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.005 0.008 0.089</td>
<td>1.033 0.342 0.666 0.396</td>
</tr>
</tbody>
</table>

Table 1: **Position precisions**, expressed in meters, obtained with code double differences on the DOYs 180-193 period observed by the Septentrio receivers located in Liège in short baseline mode.

To be able to compare Galileo positions with GPS positions, we decided to reduce the GPS constellation to 4 satellites, with similar elevations than the Galileo satellites in order to observe similar PDOP values in both cases. With this identical constellation configuration, GPS signals showed less precise position estimations than Galileo ones (Table 1).
4 Conclusion

With signals showing a quality highly superior to GPS on both receiver’s type tested, Galileo is a very promising system. We were not able to reach decimetre precision on position in this thesis over large distances with the Galileo signals but the comparison we have made with a reduced GPS constellation offers us good hope for positioning with respect to the future level of precision reachable with the very precise Galileo observations.

References

[Cameron, 2016a] Cameron, A., 2016a, Google opens up GNSS pseudoranges. GPS World, unpublished yet.

[Cameron, 2016b] Cameron, A., 2016b, Google to provide raw GNSS measurements. GPS World, unpublished yet.


