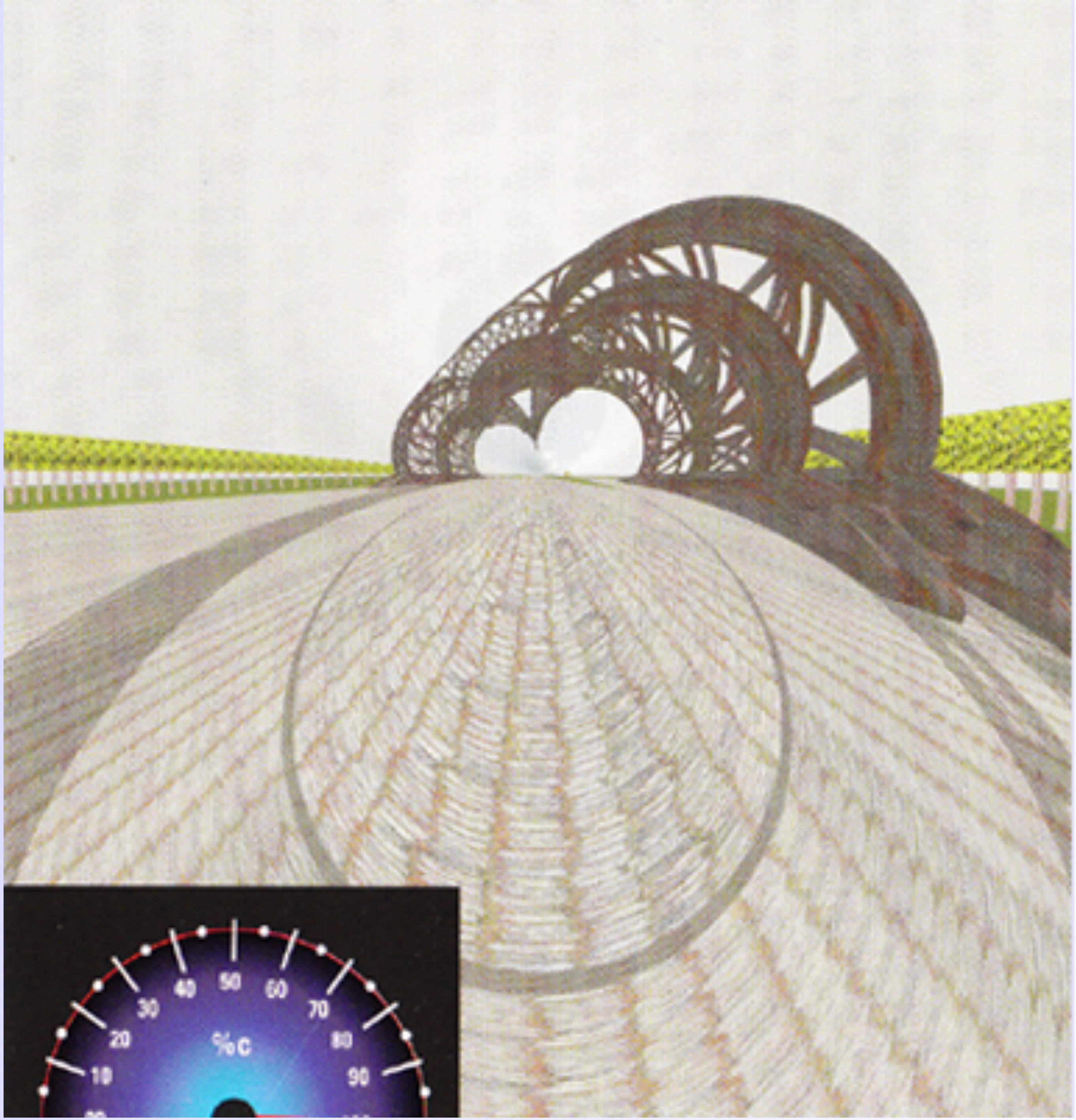
7ème cours de Mécanique Analytique (3/11/2016)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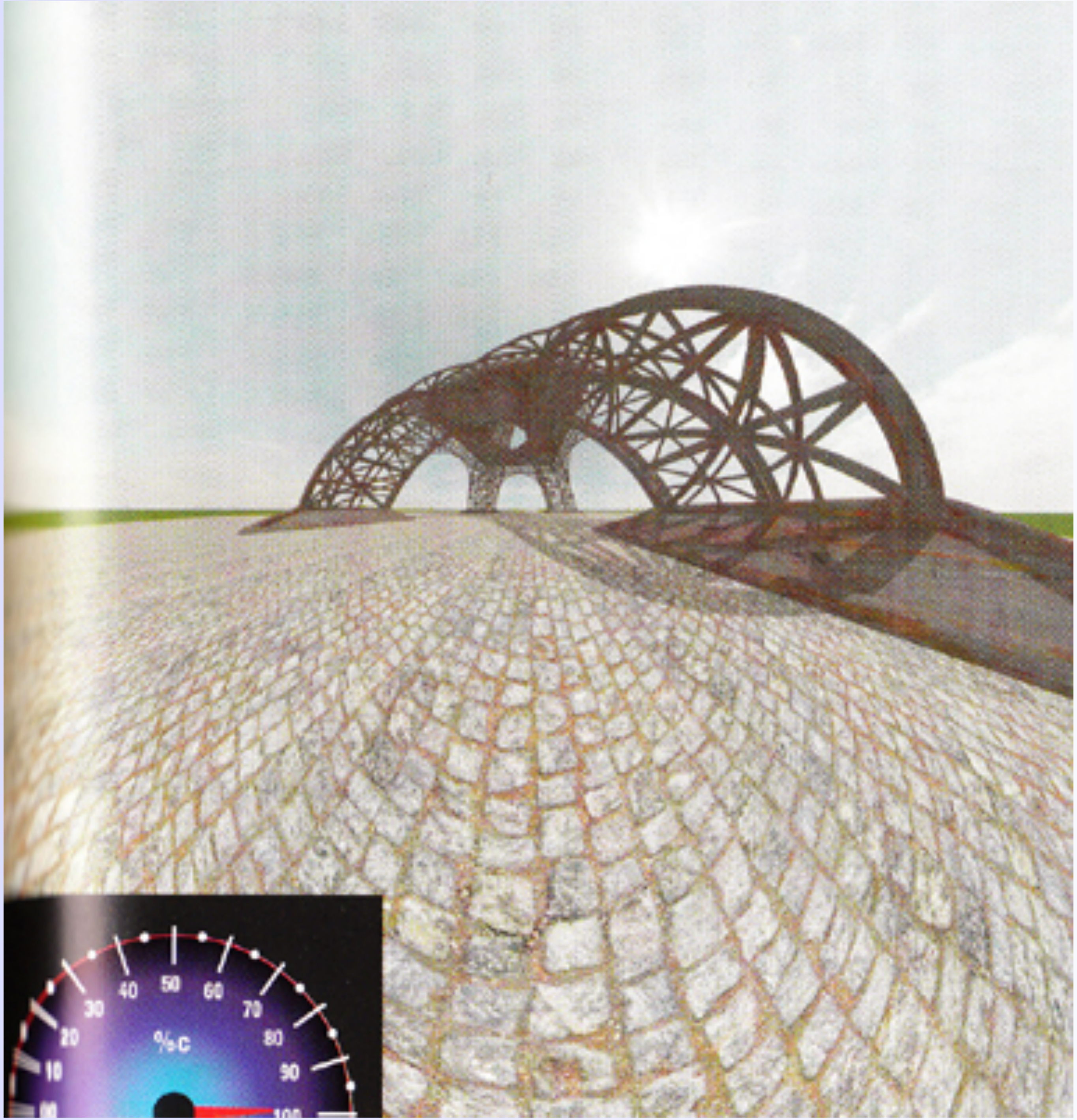

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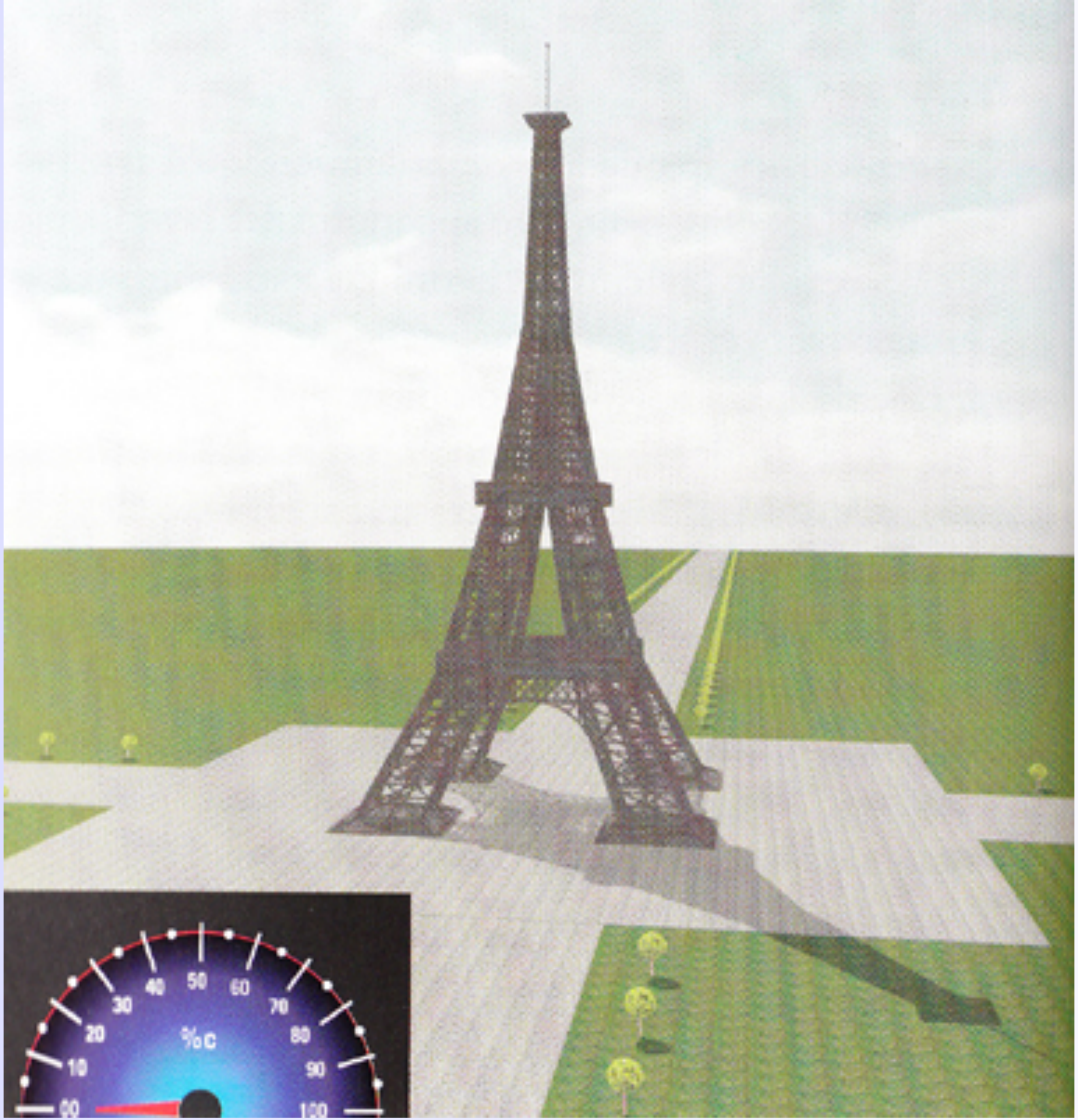
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- 3.3 La transformation de Lorentz : approche standard

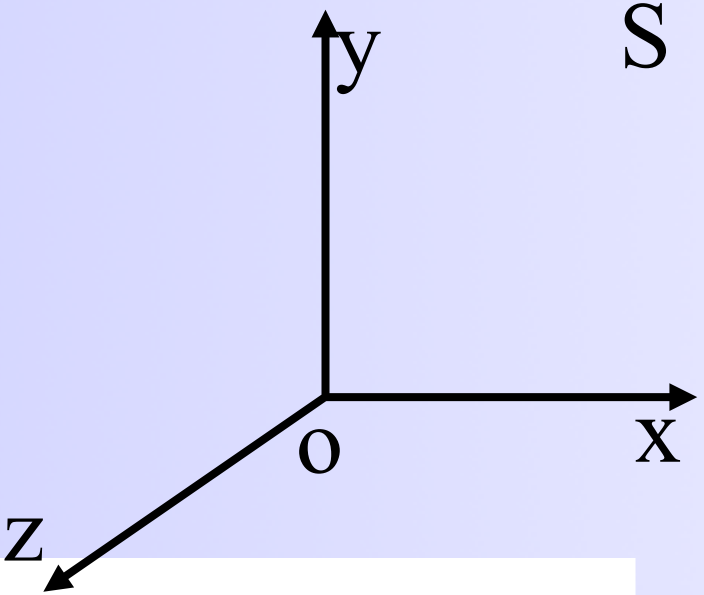
$$\begin{cases} x = \dfrac{\bar{x} + V\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y = \bar{y} \\ z = \bar{z} \\ t = \dfrac{\frac{V}{c^2}\bar{x} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{cases} \quad (3.6)$$

$\vec{r} = \vec{OP}$ et $\vec{\bar{r}} = \vec{\bar{O}P}$

$$\vec{r} = \vec{\bar{r}} + \frac{\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right) \frac{\vec{V} \cdot \vec{\bar{r}}}{V^2} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \vec{V}$$

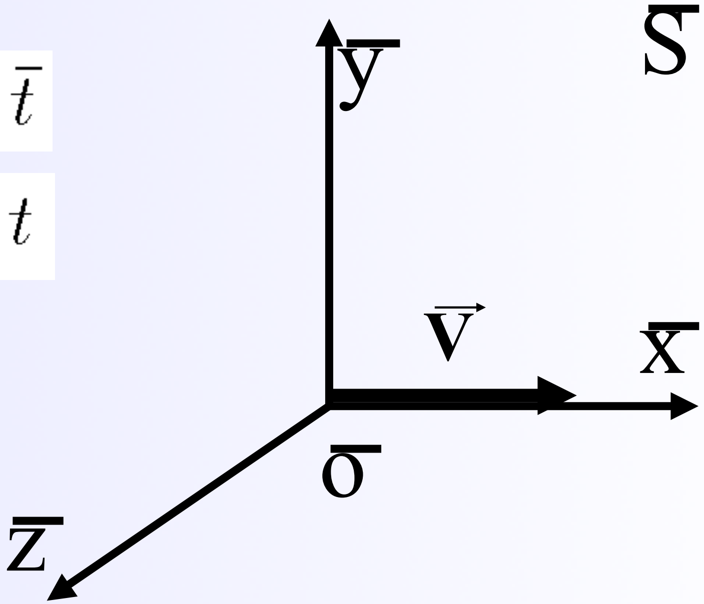
$$t = \frac{\frac{\vec{V} \cdot \vec{\bar{r}}}{c^2} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

• 3.3 La transformation de Lorentz



• $\bar{x},\ \bar{y},\ \bar{z},\ \bar{t}$

$x,\ y,\ z,\ t$



$$\begin{cases} x = \dfrac{\bar{x} + V\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y = \bar{y} \\ z = \bar{z} \\ t = \dfrac{\frac{V}{c^2}\bar{x} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{cases} \quad (3.6)$$

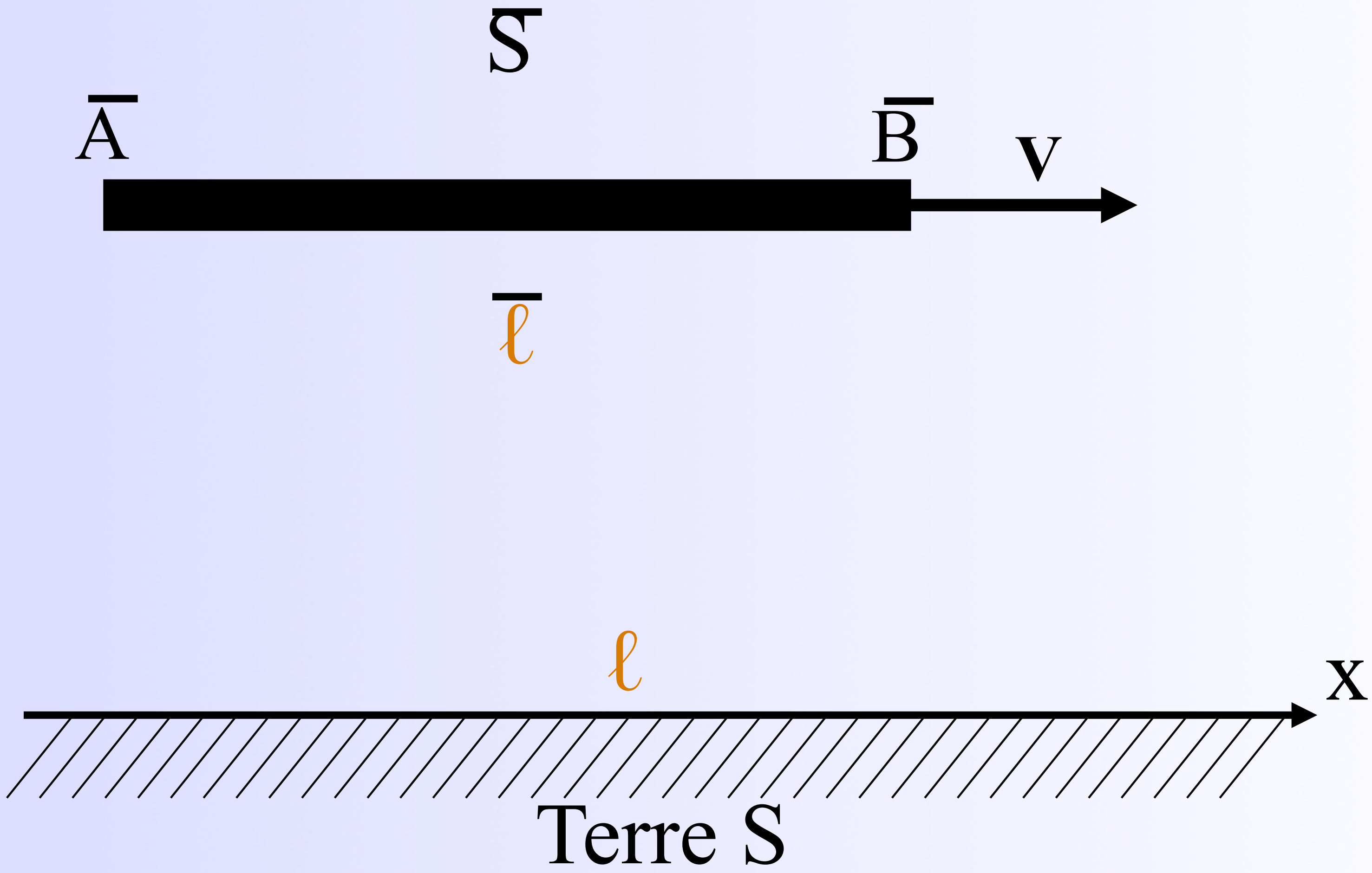
$$v = \frac{\bar{v} + V}{1 + \frac{\bar{v}V}{c^2}} \quad (3.12)$$

$$\vec{\gamma} \neq \vec{\bar{\gamma}}$$

$$\bar{\gamma} = \gamma\left(\frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{vV}{c^2}}\right)^3$$

• 3.4 Dilatation du temps et contraction des longueurs

b. La contraction des longueurs



• 3.4 Dilatation du temps et contraction des longueurs

$$\Delta x = \frac{\Delta \bar{x} + V \Delta \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}, \qquad \Delta t = \frac{\frac{V}{c^2}\Delta \bar{x} + \Delta \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\Delta t = 0$$

$$\Delta x = \sqrt{1 - \frac{V^2}{c^2}}\Delta \bar{x} \qquad (3.14)$$

$$l = \sqrt{1 - \frac{V^2}{c^2}}\bar{l}. \qquad (3.15)$$

longueur relative < longueur propre

• 3.4 Dilatation du temps et contraction des longueurs

$$\Delta\bar{t} = 2 \times 10^{-6} s.$$

$$l = 60km$$

$$\bar{l} = \sqrt{(1 - (v/c)^2} l \simeq 600m$$

$$c\Delta\bar{t} = 600m$$

• 3.5 L'espace de Minkowski et la distance spatio-temporelle

$$x^\alpha = (ct, x, y, z) \quad (3.16)$$

Evénement : point de E_4

Observateur : système de coordonnées de E_4

$(x^0 = ct)$ $\quad$ $x^i \, (x^1 = x, x^2 = y, x^3 = z)$

un événement E et deux observateurs S et $\bar{S}$

x^α et $x^{\bar{\beta}}$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^{\alpha} = \sum_{\bar{\beta}=0}^{3} \Lambda^{\alpha}_{\ \bar{\beta}} x^{\bar{\beta}} \qquad (3.17)$$

$$x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}} x^{\bar{\beta}} \qquad (3.18)$$

$$\Lambda^{\alpha}_{\ \bar{\beta}} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (3.19)$$

$$\beta = \frac{V}{c} \qquad (3.20)$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}} x^{\bar{\beta}}$$

$$x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}}\, x^{\bar{\beta}} + M^{\alpha} \qquad (3.21)$$

$$x_i = A_{ij}\left[\bar{x}_j + \left[(\gamma - 1)\frac{V_k \bar{x}_k}{V^2} + \gamma \bar{t}\right] V_j + a_j\right]$$

$$t = \gamma\left[\frac{V_k \bar{x}_k}{c^2} + \bar{t}\right] + b \qquad (3.9)$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad (3.10)$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}}\, x^{\bar{\beta}} + M^{\alpha} \qquad (3.21)$$

$$\Lambda^{\alpha}_{\ \bar{\beta}} = \left(\begin{array}{c|c} \gamma & \gamma \dfrac{V_j}{c} \\ \hline \gamma A_{ik} \dfrac{V_k}{c} & A_{ij} + (\gamma - 1) A_{ik} \dfrac{V_k V_j}{V^2} \end{array} \right) \qquad (3.22)$$

$\alpha = 0, i$
$\bar{\beta} = 0, j$
$i, j = 1 \ldots 3$

$$M^{\alpha} = \left(\begin{array}{c} b\, c \\ \hline A_{ik} a_k \end{array} \right) \qquad (3.23)$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$E_1 \ : x^{\alpha}(1),\ x^{\bar{\alpha}}(1)$ $\qquad$ $E_2 \ : x^{\alpha}(2),\ x^{\bar{\alpha}}(2)$

$$\Delta x^{\alpha} = x^{\alpha}(2) - x^{\alpha}(1)$$
$$\Delta x^{\bar{\alpha}} = x^{\bar{\alpha}}(2) - x^{\bar{\alpha}}(1) \qquad (3.24)$$

4-vecteur $\overrightarrow{E_1 E_2}$

$$x^{\alpha}(2) = \Lambda^{\alpha}_{\ \bar{\beta}}\, x^{\bar{\beta}}(2) + M^{\alpha}$$

$$x^{\alpha}(1) = \Lambda^{\alpha}_{\ \bar{\beta}}\, x^{\bar{\beta}}(1) + M^{\alpha}$$

$$\Delta x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}} \Delta x^{\bar{\beta}} \qquad (3.25)$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta x^{\alpha} = \Lambda^{\alpha}_{\ \bar{\beta}} \Delta x^{\bar{\beta}} \qquad (3.25)$$

$$\Delta s^2 = \eta_{\alpha\beta} \Delta x^{\alpha} \Delta x^{\beta} = \eta_{\alpha\beta} \Lambda^{\alpha}_{\ \bar{\mu}} \Lambda^{\beta}_{\ \bar{\nu}} \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} \qquad (3.26)$$

$$\begin{aligned} \eta_{oo} &= -1 \\ \eta_{oi} &= 0 \\ \eta_{ij} &= \delta_{ij} \end{aligned} \qquad (3.27)$$

$$\Delta s^2 = -c^2 (\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \qquad (3.28)$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\eta_{\alpha\beta}\,\Lambda^{\alpha}{}_{\bar\mu}\,\Lambda^{\beta}{}_{\bar\nu}\,\Delta x^{\bar\mu}\Delta x^{\bar\nu}$$

$$-c^2(\Delta\bar t)^2+(\Delta\bar x)^2+(\Delta\bar y)^2+(\Delta\bar z)^2$$

$$\begin{aligned}
\eta_{\alpha\beta}\Lambda^{\alpha}{}_{\bar\mu}\Lambda^{\beta}{}_{\bar\nu}\Delta x^{\bar\mu}\Delta x^{\bar\nu} &= \eta_{00}\Lambda^{0}{}_{\bar\mu}\Lambda^{0}{}_{\bar\nu}\Delta x^{\bar\mu}\Delta x^{\bar\nu}+\eta_{i0}\Lambda^{i}{}_{\bar\mu}\Lambda^{0}{}_{\bar\nu}\Delta x^{\bar\mu}\Delta x^{\bar\nu}\\
&+ \eta_{0j}\Lambda^{0}{}_{\bar\mu}\Lambda^{j}{}_{\bar\nu}\Delta x^{\bar\mu}\Delta x^{\bar\nu}+\eta_{ij}\Lambda^{i}{}_{\bar\mu}\Lambda^{j}{}_{\bar\nu}\Delta x^{\bar\mu}\Delta x^{\bar\nu}\\
&= - [\gamma^2c^2(\Delta\bar t)^2+2\gamma^2\beta c\Delta\bar t\Delta\bar x+\gamma^2\beta^2(\Delta\bar x)^2]\\
&+ [\gamma^2\beta^2c^2(\Delta\bar t)^2+2\gamma^2\beta c\Delta\bar t\Delta\bar x+\gamma^2(\Delta\bar x)^2]\\
&+ (\Delta\bar y)^2+(\Delta\bar z)^2\\
&= - c^2(\Delta\bar t)^2+(\Delta\bar x)^2+(\Delta\bar y)^2+(\Delta\bar z)^2
\end{aligned}$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$(\Delta t)^2$ $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2.$

$$\Delta s^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$$

$$\boxed{-c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = -c^2(\Delta \bar{t})^2 + (\Delta \bar{x})^2 + (\Delta \bar{y})^2 + (\Delta \bar{z})^2} \quad (3.29)$$

$$\Delta s^2 = \delta_{ij} \Delta x^i \Delta x^j = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \quad (3.30)$$

$\Delta \vec{x}$ $(\Delta x, \Delta y, \Delta z)$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

1) $\Delta s^2 < 0$ (intervalle Δs temporel)

le quadri-vecteur $\overrightarrow{E_1 E_2}$ est du genre "temps"

$$\Delta s^2 = |\Delta \vec{r}|^2 - c^2 \Delta t^2 < 0 \qquad (3.31)$$

$$\Delta \vec{r} = 0$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta\vec{r} = 0$$

$$\vec{r} = \vec{\bar{r}} + \left[(\gamma - 1)\frac{\vec{V}\cdot\vec{\bar{r}}}{V^2} + \gamma\bar{t}\right]\vec{V} + \vec{a}$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad (3.10)$$

$$t = \gamma\left(\frac{\vec{V}\cdot\vec{\bar{r}}}{c^2} + \bar{t}\right) + b \quad (3.8)$$

$$\Delta\vec{\bar{r}} + \left[(\gamma - 1)\frac{\vec{V}\cdot\Delta\vec{\bar{r}}}{V^2} + \gamma\Delta\bar{t}\right]\vec{V} = 0$$

$$V < c$$

$$\vec{V} = -\frac{\Delta\vec{\bar{r}}}{\Delta\bar{t}} \quad (3.32)$$

$$V^2 = \frac{|\Delta\vec{\bar{r}}|^2}{\Delta\bar{t}^2} < c^2$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\vec{r} = \vec{\bar{r}} + \left[(\gamma - 1)\frac{\vec{V}\cdot\vec{\bar{r}}}{V^2} + \gamma\bar{t}\right]\vec{V} + \vec{a}$$

$$t = \gamma\left(\frac{\vec{V}\cdot\vec{\bar{r}}}{c^2} + \bar{t}\right) + b \qquad (3.8)$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad (3.10)$$

$$\vec{V} = -\frac{\Delta\vec{\bar{r}}}{\Delta\bar{t}} \qquad (3.32)$$

$$\Delta t = \gamma\left[\frac{\vec{V}\cdot\Delta\vec{\bar{r}}}{c^2} + \Delta\bar{t}\right] = \frac{\Delta\bar{t}}{\gamma} \qquad (3.33)$$

l'ordre temporel de ces événements est donc absolu.

deux événements en relation causale ont un Δs^2 négatif.

• 3.5 L'espace de Minkowski et la distance sp.-temp.

2) $\Delta s^2 = 0$ (intervalle Δs lumineux ou nul)

3) $\Delta s^2 > 0$ (intervalle Δs spatial)

Le quadri-vecteur $\overrightarrow{E_1 E_2}$ est dit du "genre espace"

$$\Delta s^2 = |\Delta\vec{\bar{r}}|^2 - c^2 \Delta \bar{t}^2 > 0$$

$$\Delta t = \gamma \left(\frac{\vec{V} \cdot \Delta\vec{\bar{r}}}{c^2} + \Delta \bar{t} \right) = 0$$

$$\frac{\vec{V} \cdot \Delta\vec{\bar{r}}}{c \cdot c\Delta\bar{t}} = -1$$

$$V < c$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta t = \gamma \left(\frac{\vec{V} \cdot \Delta\vec{\bar{r}}}{c^2} + \Delta\bar{t} \right) = 0$$

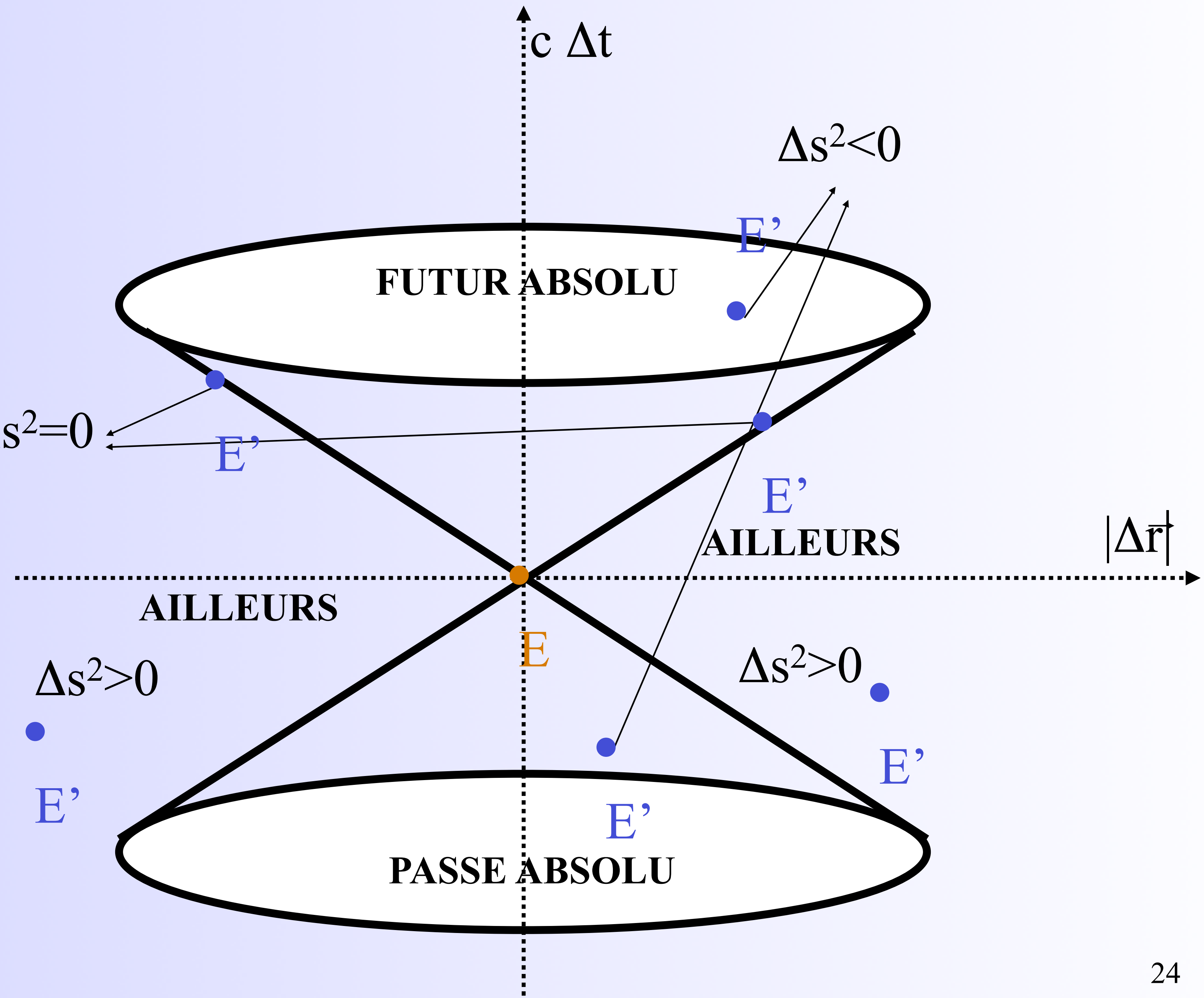
$$\Delta\bar{t} = \gamma \left(\Delta t - \frac{\vec{V}\Delta\vec{r}}{c^2} \right)$$

$$\Delta\bar{t} = -\frac{\vec{V}\Delta\vec{r}}{\sqrt{c^2 - V^2}} \times \frac{1}{c}$$

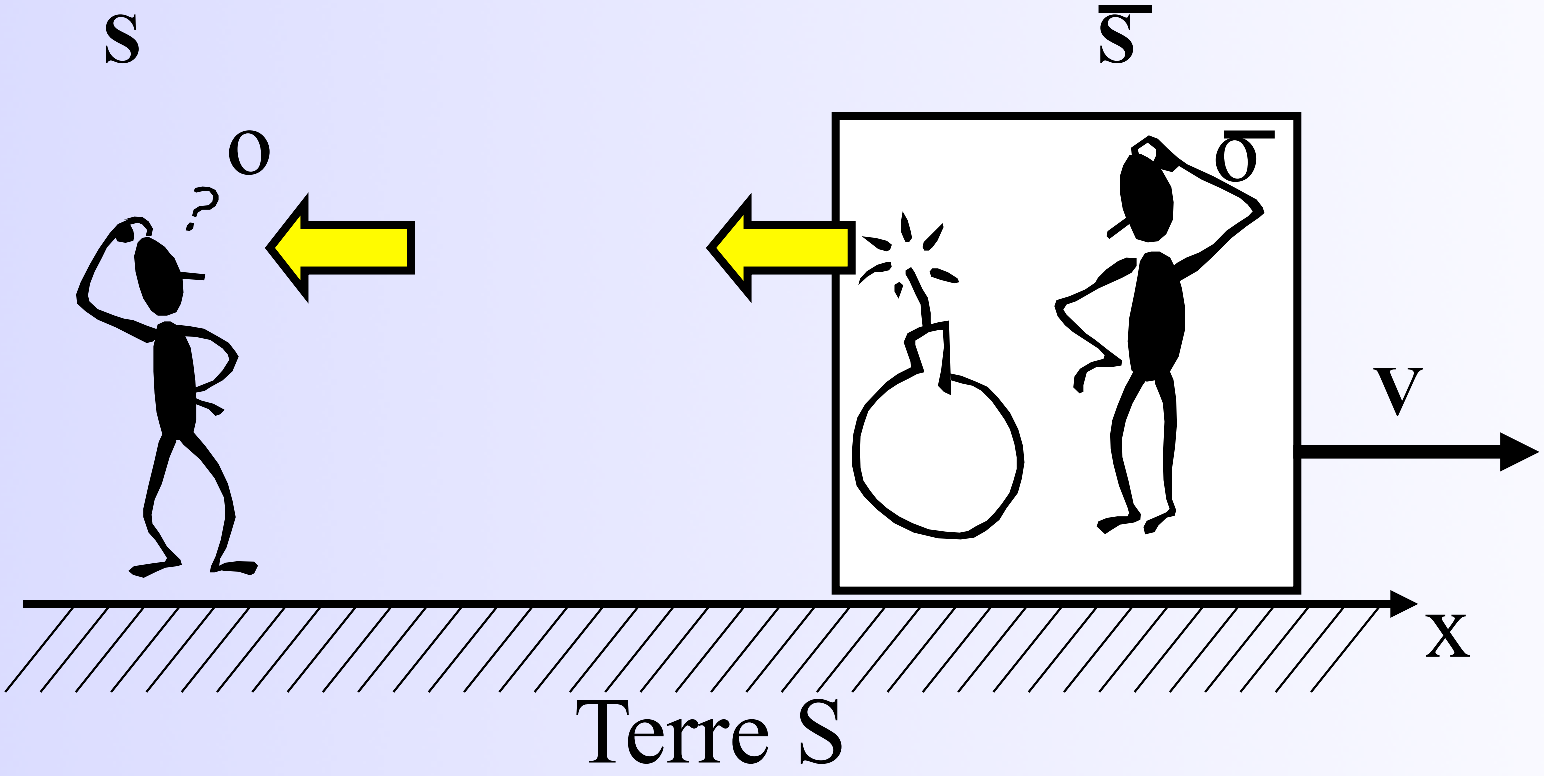
Le cône de lumière

$$(x' - x)^2 + (y' - y)^2 + (z' - z)^2 - c^2(t' - t)^2 = 0 \qquad (3.34)$$

• 3.5 L'espace de Minkowski et la distance sp.-temp.



• 3.6 L'effet Doppler longitudinal



	$\bar{S}$	S
E_1	$\bar{x}_1 = 0\ , \quad \bar{t}_1 = \bar{t}$	
E_2		$x_2 = 0\ , \quad t_2 = t$

- 3.6 L'effet Doppler longitudinal

	$\bar{S}$	S
E_1	$\bar{x}_1 = 0\,, \quad \bar{t}_1 = \bar{t}$	
E_2		$x_2 = 0\,, \quad t_2 = t$

	$\bar{S}$	S
E_1	$\bar{x}_1 = 0\,, \quad \bar{t}_1 = \bar{t}$	$x_1 = \dfrac{V\bar{t}}{\sqrt{1-\frac{V^2}{c^2}}}\,, \quad t_1 = \dfrac{\bar{t}}{\sqrt{1-\frac{V^2}{c^2}}}$
E_2	$\bar{x}_2 = \dfrac{-Vt}{\sqrt{1-\frac{V^2}{c^2}}}\,, \quad \bar{t}_2 = \dfrac{t}{\sqrt{1-\frac{V^2}{c^2}}}$	$x_2 = 0\,, \quad t_2 = t$

- 3.6 L'effet Doppler longitudinal

$$(\bar{x}_2 - \bar{x}_1)^2 = c^2(\bar{t}_2 - \bar{t}_1)^2 \qquad (3.35)$$

$$c(\bar{t}_2 - \bar{t}_1) = -\bar{x}_2$$

$$c\left(\frac{t}{\sqrt{1-\frac{V^2}{c^2}}} - \bar{t}\right) = \frac{Vt}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$t = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\bar{t} \qquad (3.36)$$

$$\Delta t = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\Delta\bar{t}$$

• 3.6 L'effet Doppler longitudinal

$$\Delta t = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\Delta\bar{t}$$

$$\tau = \Delta t \qquad \bar{\tau} = \Delta\bar{t}$$

$$\tau = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\bar{\tau} \quad \Longrightarrow \quad \boxed{\lambda = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\bar{\lambda}} \quad (3.37)$$

$$\lambda = c\tau \qquad \bar{\lambda} = c\bar{\tau}$$

• 3.6 L'effet Doppler longitudinal

$$\boxed{\lambda = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\,\bar{\lambda}}$$

$\lambda > \bar{\lambda}$ (red-shift)

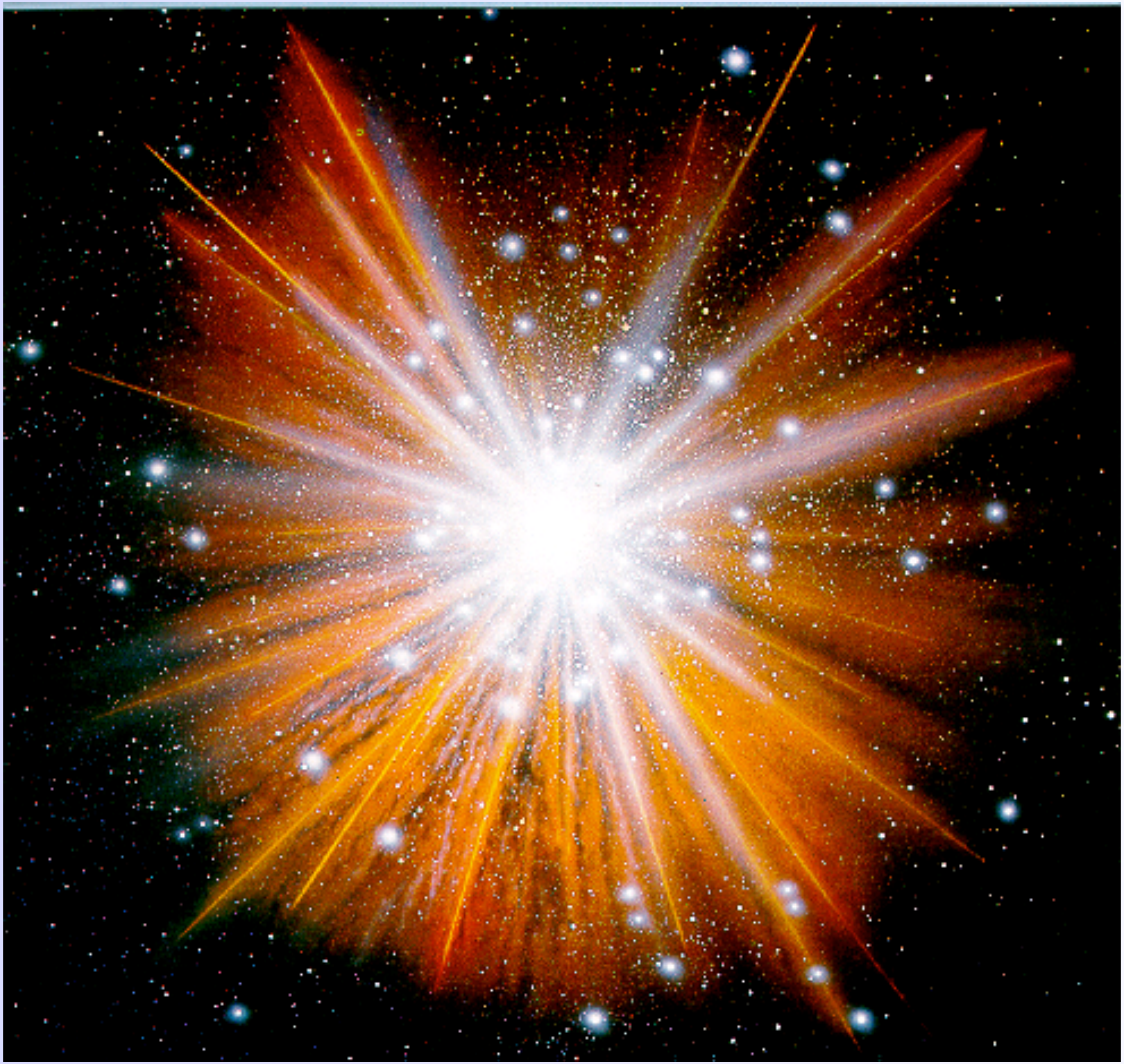
$$z = \frac{\Delta\lambda}{\bar{\lambda}} = \frac{\lambda - \bar{\lambda}}{\bar{\lambda}}$$

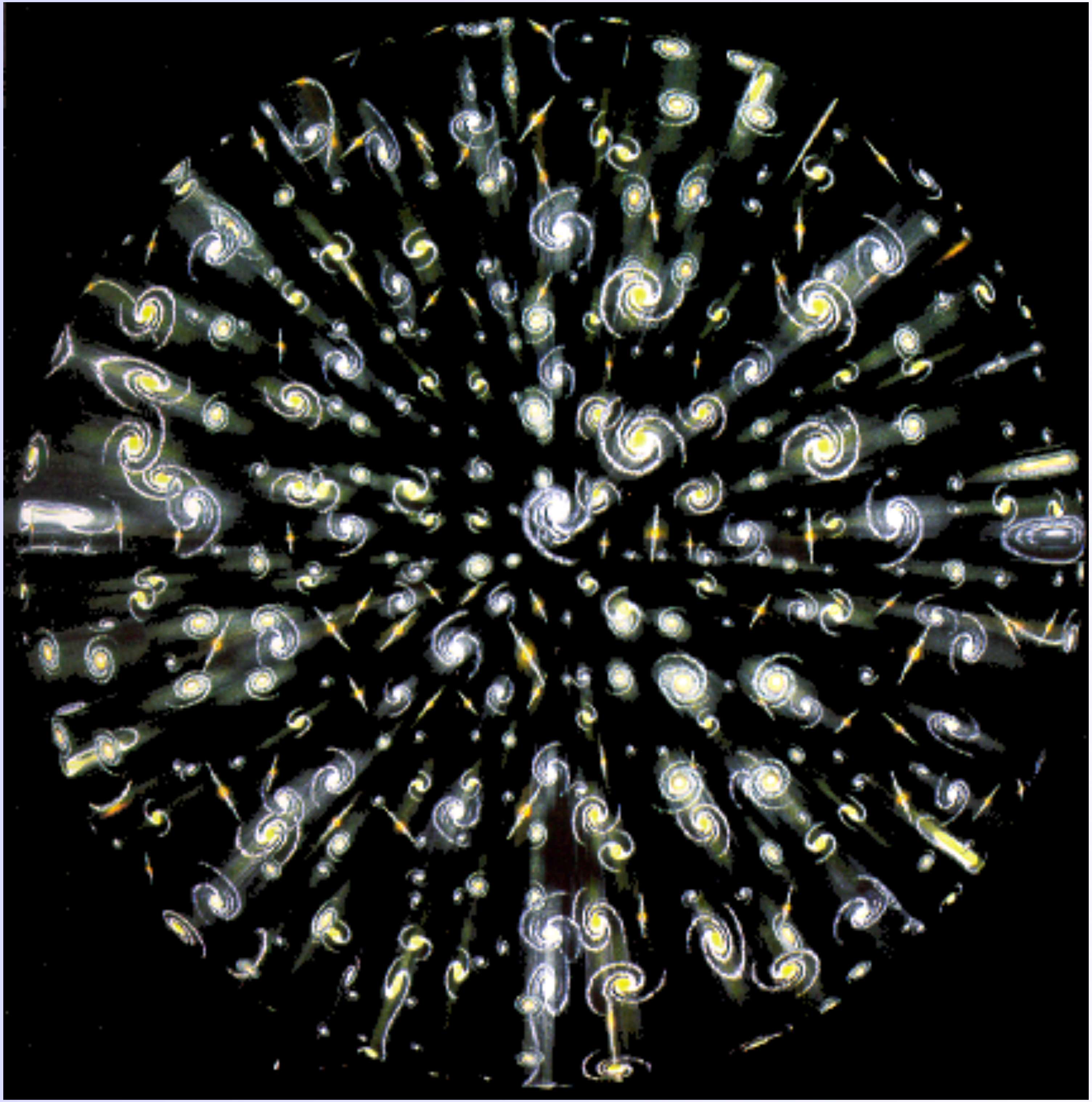
$$\Delta\lambda = \lambda - \bar{\lambda}$$

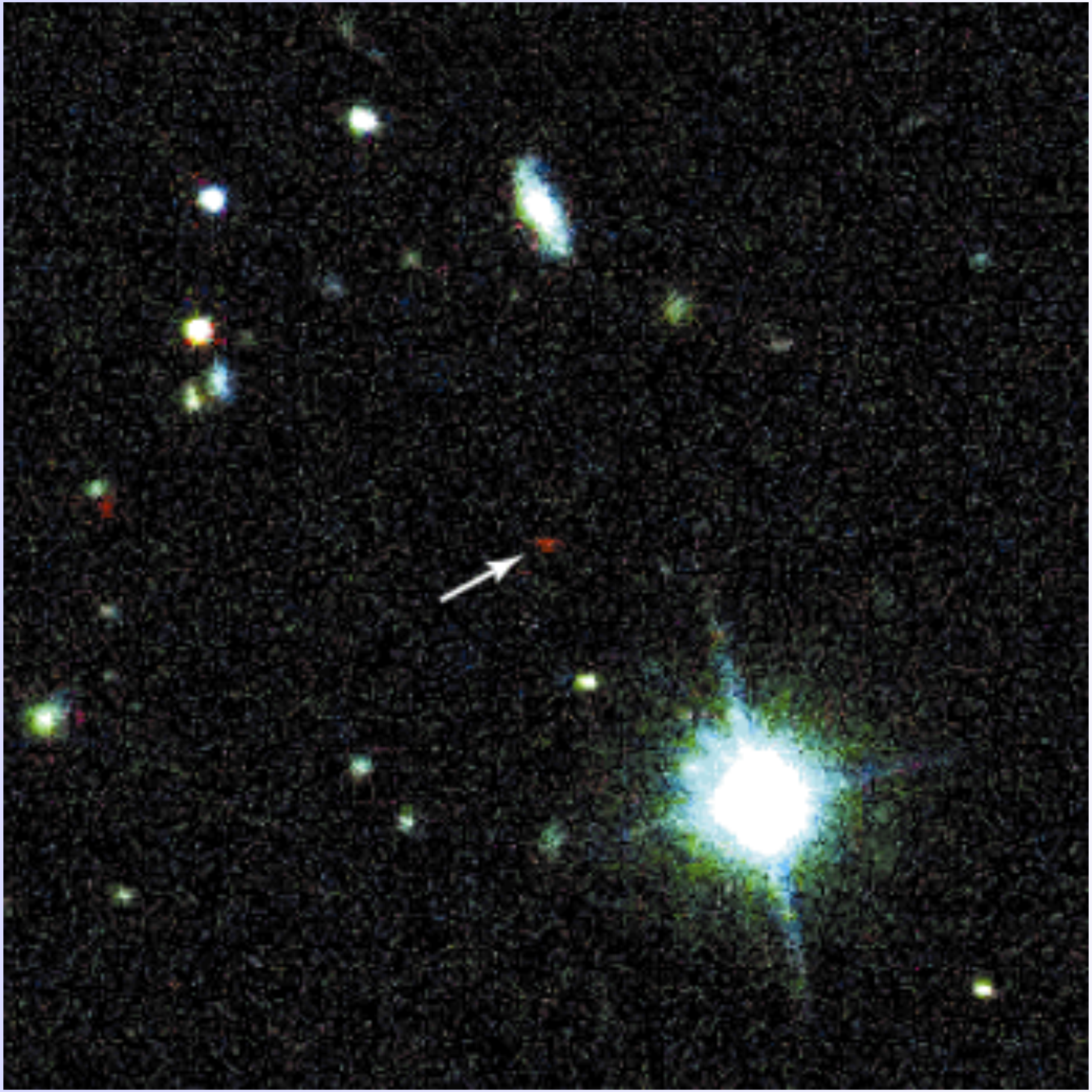
$$z = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}} - 1$$

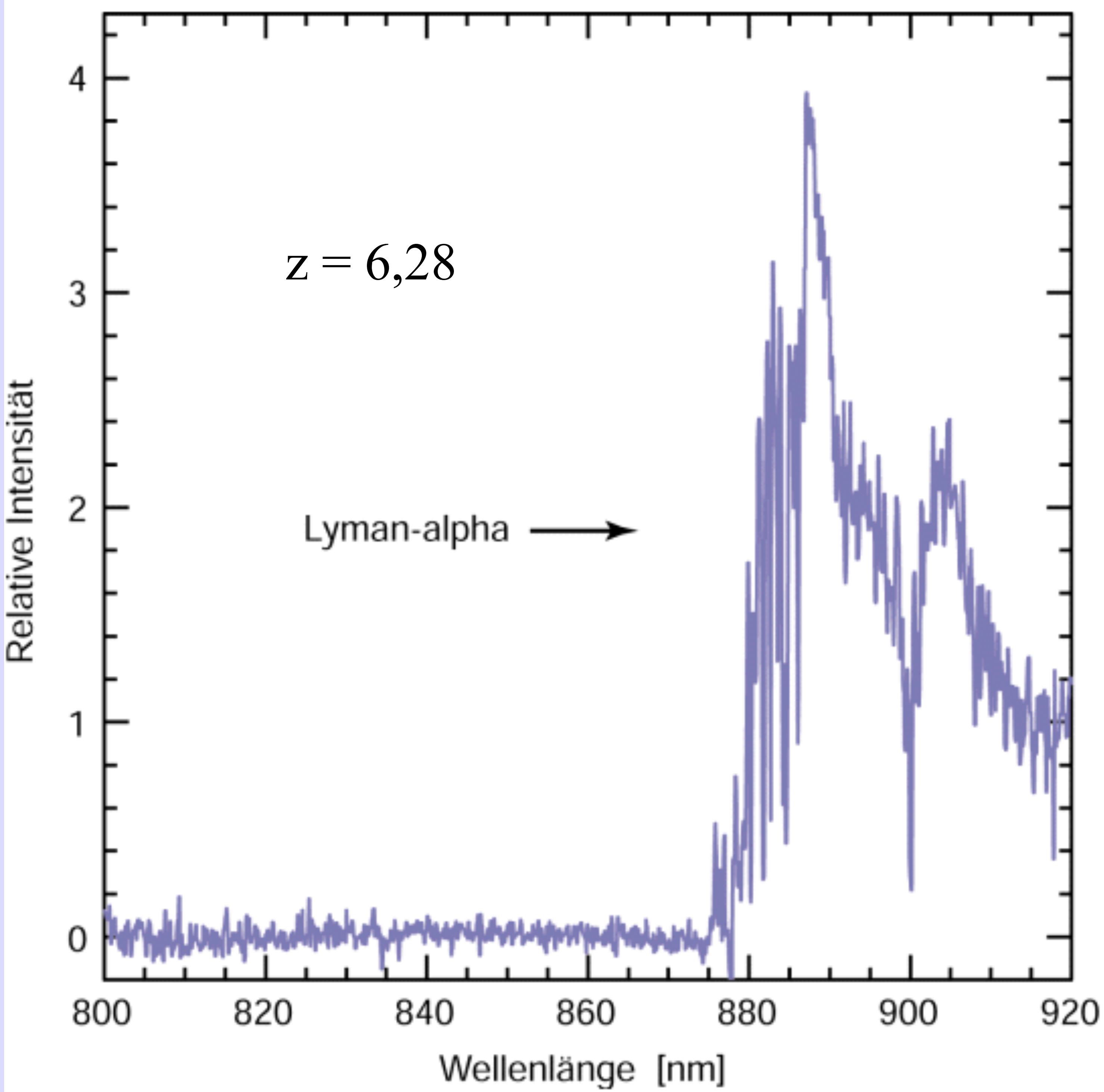
$$z = 6.28$$

V/c = 0,963








z = 6,28
Lyman-alpha
Relative Intensität
Wellenlänge [nm]
0
1
2
3
4
800
820
840
860
880
900
920