Probabilistic prediction of the quality factor of micro-resonator using a stochastic thermo-mechanical multi-scale approach

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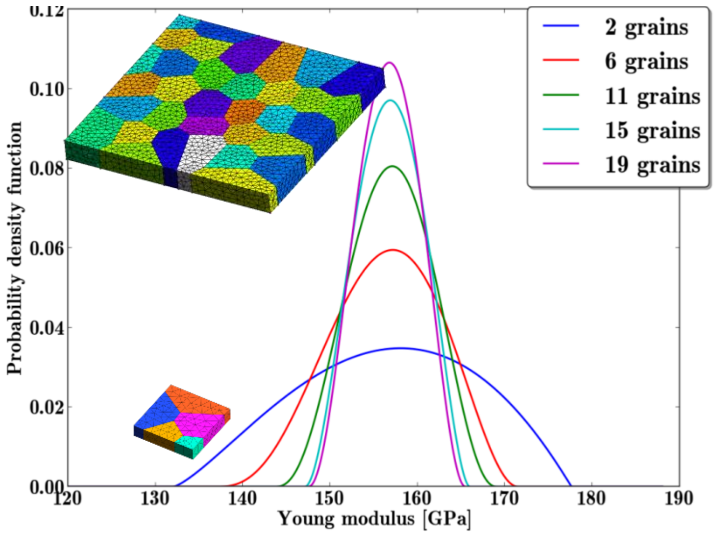
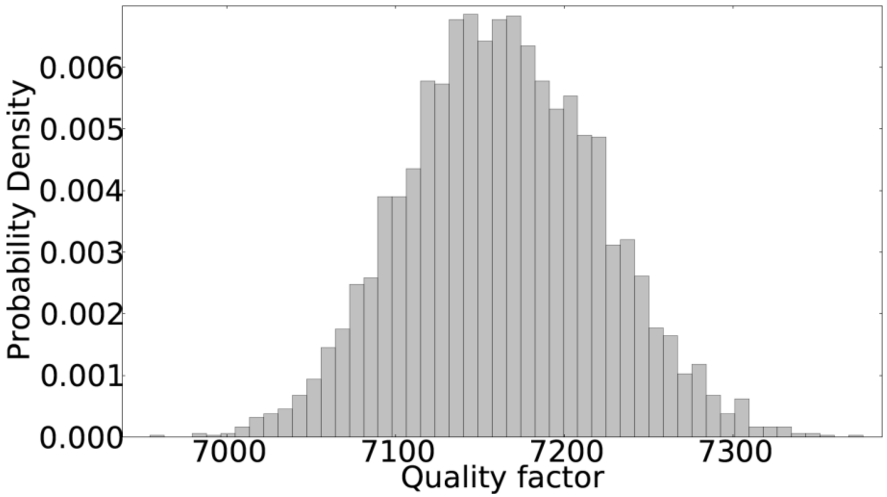
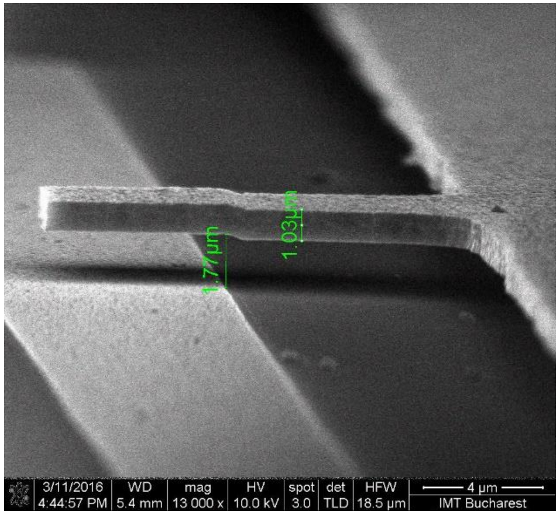




3SMVIB: The research has been funded by the Walloon Region under the agreement no 1117477 (CT-INT 2011-11-14) in the context of the ERA-NET MNT framework. Experimental measurements provided by IMT Bucharest (Voicu Rodica, Baracu Angela, Muller Raluca)

The problem

- MEMS structures
  - Are not several orders larger than their micro-structure size
  - Parameters-dependent manufacturing process
    - Low Pressure Chemical Vapor Deposition (LPCVD)
    - Properties depend on the temperature, time process, and flow gas conditions
  - As a result, their macroscopic properties can exhibit a **scatter**
    - Due to the fabrication process (photolithography, wet and dry etching)
    - Due to uncertainties of the material
    - …



→ The objective of this work is to estimate this scatter

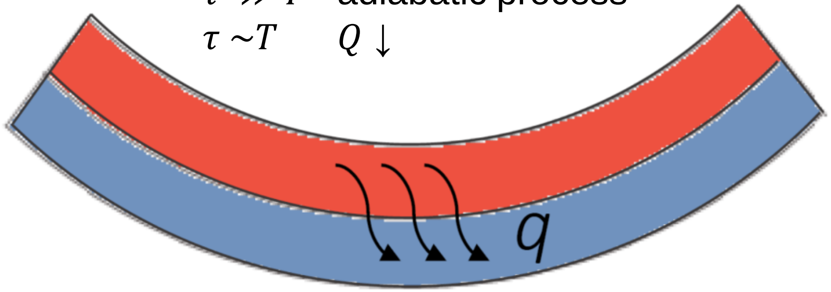
The problem

- Application example
  - Poly-silicon resonators
  - Quantities of interest
    - Eigen frequency
    - Quality factor due to thermo-elastic damping Q ~ $W/\Delta W$
    - Thermoelastic damping is a source of intrinsic material damping present in almost all materials

$\tau \ll T$ isothermal process

$\tau \gg T$ adiabatic process

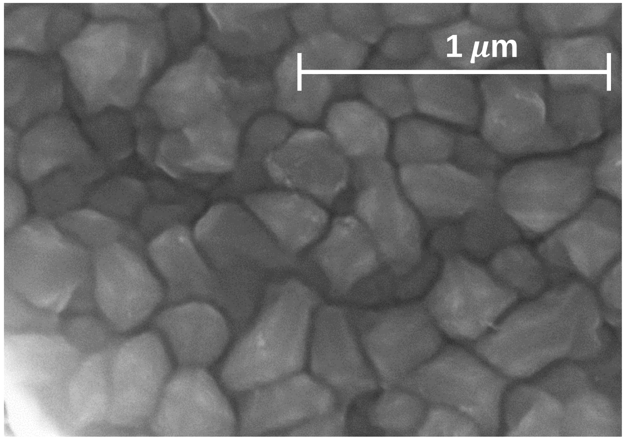
$\tau \sim T$ $Q \downarrow$



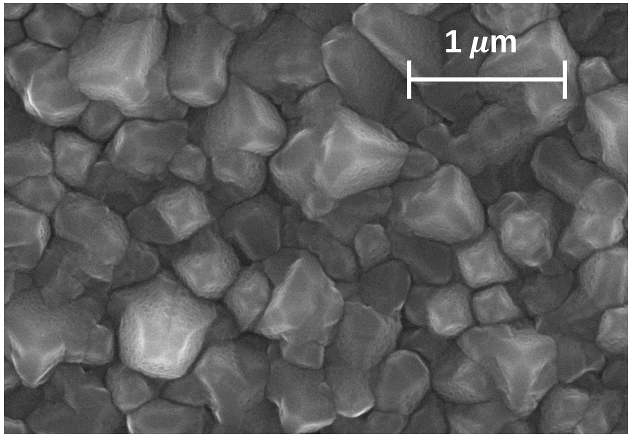
The problem

- Material structure: grain size distribution

  SEM Measurements (Scanning Electron Microscope)
  - Grain size dependent on the LPCVD temperature process
  - 2 µm-thick poly-silicon films



Deposition temperature: 580 °C



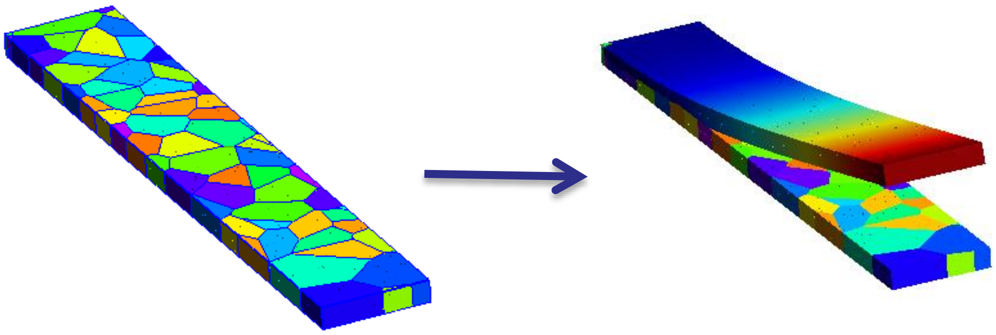
Deposition temperature: 650 °C

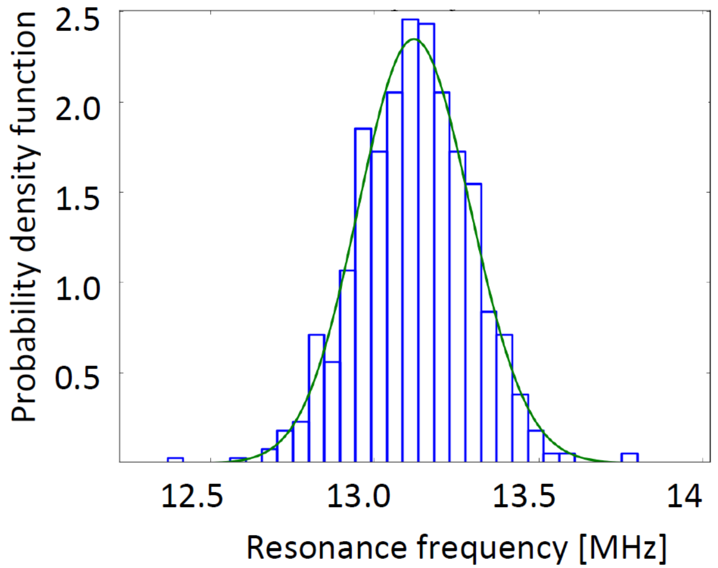
Deposition temperature [°C]	580	610	630	650
Average grain diameter [µm]	0.21	0.45	0.72	0.83

SEM images provided by IMT Bucharest, Rodica Voicu, Angela Baracu, Raluca Muller

Monte-Carlo for a fully modelled beam

- The first mode frequency distribution can be obtained with
  - A 3D beam with each grain modelled
  - Grains distribution according to experimental measurements
  - Monte-Carlo simulations



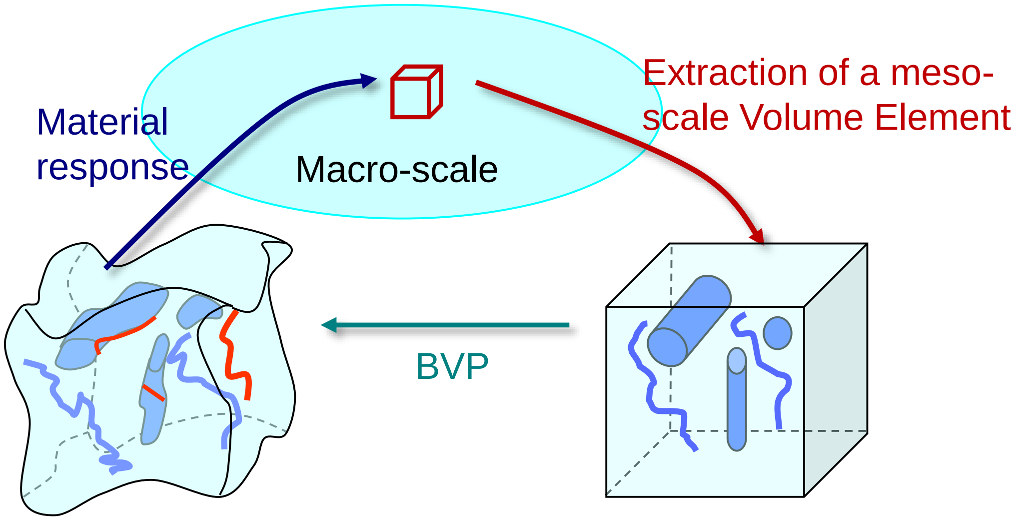


- Considering each grain is expensive and time consuming

  ↳ **Motivation for stochastic multi-scale methods**

Motivations

- Multi-scale modelling
  - 2 problems are solved concurrently
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)



- Length-scales separation

$$L_{macro} >> L_{VE} >> L_{micro}$$

For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading

To be statistically representative: Size of the meso-scale volume element larger than the characteristic length of the micro-structure

Motivations

- For structures not several orders larger than the micro-structure size

$$L_{macro} >> L_{VE} >\sim L_{micro}$$

For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading

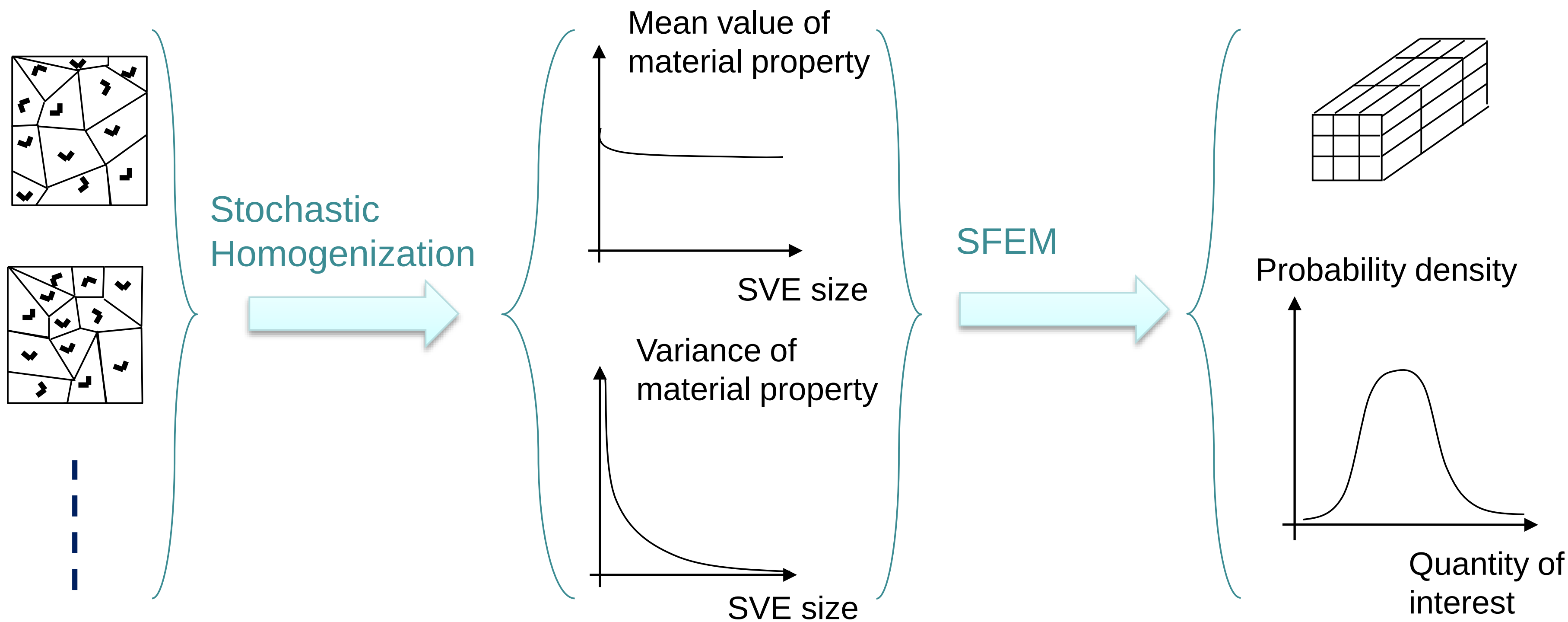
Meso-scale volume element no longer statistically representative: Stochastic Volume Elements*

- Possibility to propagate the uncertainties from the micro-scale to the macro-scale

**M Ostoja-Starzewski, X Wang, 1999*
P Trovalusci, M Ostoja-Starzewski, M L De Bellis, A Murrali, 2015
X. Yin, W. Chen, A. To, C. McVeigh, 2008
J. Guilleminot, A. Noshadravan, C. Soize, R. Ghanem, 2011
….

A 3-scale process

Grain-scale or micro-scale	Meso-scale	Macro-scale
➢ Samples of the microstructure (volume elements) are generated ➢ Each grain has a random orientation	➢ Intermediate scale ➢ The distribution of the material property $\mathbb{P}(C)$ is defined	➢ Uncertainty quantification of the macro-scale quantity ➢ E.g. the first mode frequency $\mathbb{P}(f_1)$ /Quality factor $\mathbb{P}(Q)$



Content

- Thermo-mechanical problems
  - Governing equations
  - Macro-scale stochastic finite element
  - Meso-scale volume elements
- From the micro-scale to the meso-scale
  - Thermo-mechanical homogenization
  - Definition of Stochastic Volume Elements (SVEs) & Stochastic homogenization
  - Need for a meso-scale random field
- The meso-scale random field
  - Definition of the thermo-mechanical meso-scale random field
  - Stochastic model of the random field: Spectral generator & non-Gaussian mapping
- From the meso-scale to the macro-scale
  - 3-Scale approach verification
  - Application to extract the quality factor

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- Macro-scale stochastic finite element method
  - Meso-scale material properties subjected to uncertainties
    - Elasticity tensor $\mathbb{C}_M(\boldsymbol{\theta})$,
    - Heat conductivity tensor $\boldsymbol{\kappa}_M(\boldsymbol{\theta})$, and
    - Thermal expansion tensors $\boldsymbol{\alpha}_M(\boldsymbol{\theta})$

    in the sample space $\boldsymbol{\theta} \in \boldsymbol{\Omega}$

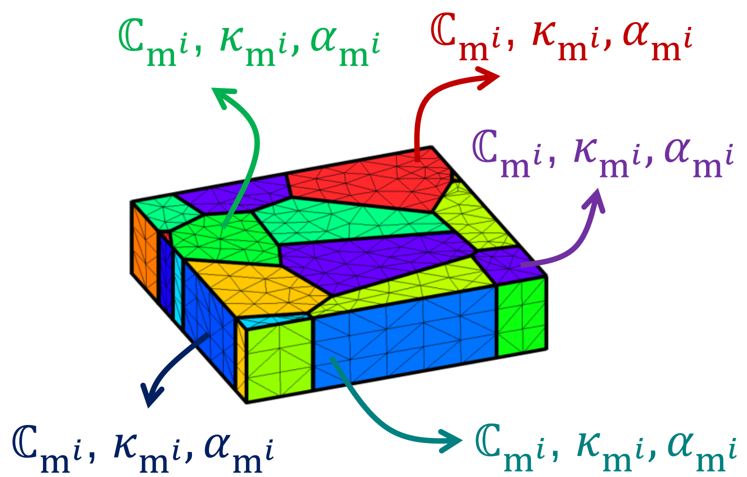
$$\begin{bmatrix} \mathbf{M}(\rho_\mathrm{M}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{\vartheta \mathrm{u}}(\boldsymbol{\alpha}_\mathrm{M}, \mathbb{C}_\mathrm{M}) & \mathbf{D}_{\vartheta\vartheta}(\rho_\mathrm{M} C_{v\mathrm{M}}) \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{uu}}(\mathbb{C}_\mathrm{M}) & \mathbf{K}_{\mathrm{u}\vartheta}(\boldsymbol{\alpha}_\mathrm{M}, \mathbb{C}_\mathrm{M}) \\ \mathbf{0} & \mathbf{K}_{\vartheta\vartheta}(\boldsymbol{\kappa}_\mathrm{M}) \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_\mathrm{u} \\ \boldsymbol{F}_\vartheta \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{\vartheta \mathrm{u}}(\boldsymbol{\theta}) & \mathbf{D}_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{uu}}(\boldsymbol{\theta}) & \mathbf{K}_{\mathrm{u}\vartheta}(\boldsymbol{\theta}) \\ \mathbf{0} & \mathbf{K}_{\vartheta\vartheta}(\boldsymbol{\theta}) \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_\mathrm{u} \\ \boldsymbol{F}_\vartheta \end{bmatrix}$$

- Defining the random properties at the meso-scale by
  - Using micro-scale information (SEM, XRD, images)
  - Homogenization method

- Meso-scale Volume Elements (VE)
  - Micro-scale material properties
    - Elasticity tensor $\mathbb{C}_m$,
    - Heat conductivity tensor $\boldsymbol{\kappa}_m$, and
    - Thermal expansion tensors $\boldsymbol{\alpha}_m$

    defined on each phase/heterogeneity



  - Length scales separation assumptions
    - VE small enough for the time for strain wave to propagate in the SVE to remain negligible
    - VE small enough for the time variation of heat storage to remain negligible

$$\begin{bmatrix} \mathbf{M}(\rho_{\mathrm{m}}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{\vartheta \mathrm{u}}(\boldsymbol{\alpha}_{\mathrm{m}}, \mathbb{C}_{\mathrm{m}}) & \mathbf{D}_{\vartheta\vartheta}(\rho_{\mathrm{m}} C_{v\mathrm{m}}) \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{uu}}(\mathbb{C}_{\mathrm{m}}) & \mathbf{K}_{\mathrm{u}\vartheta}(\boldsymbol{\alpha}_{\mathrm{m}}, \mathbb{C}_{\mathrm{m}}) \\ \mathbf{0} & \mathbf{K}_{\vartheta\vartheta}(\boldsymbol{\kappa}_{\mathrm{m}}) \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{u}} \\ \boldsymbol{F}_{\vartheta} \end{bmatrix}$$

$$\hookrightarrow \quad \begin{bmatrix} \mathbf{K}_{\mathrm{uu}} & \mathbf{K}_{\mathrm{u}\vartheta} \\ \mathbf{0} & \mathbf{K}_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{u}} \\ \boldsymbol{F}_{\vartheta} \end{bmatrix}$$

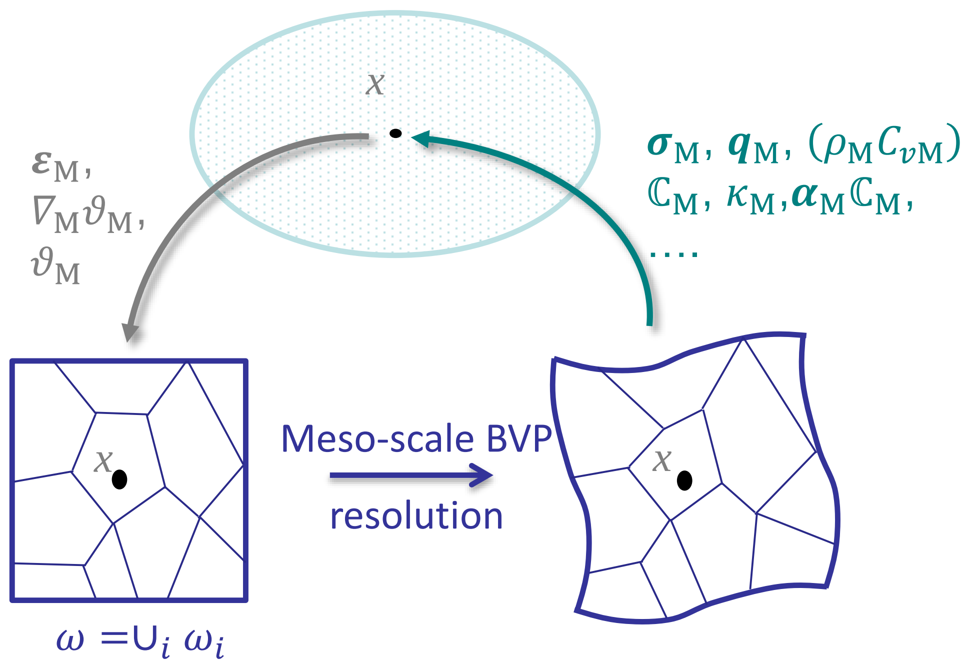
- Transition from meso-scale BVP realizations to the meso-scale random properties

→ Stochastic thermo-mechanical homogenization

Content

- Thermo-mechanical homogenization
  - Down-scaling

$$\begin{cases} \boldsymbol{\varepsilon}_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\varepsilon}_{\mathrm{m}} d\omega \\ \nabla_{\mathrm{M}}\vartheta_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \nabla_{\mathrm{m}}\vartheta_{\mathrm{m}} d\omega \\ \vartheta_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \dfrac{\rho_{\mathrm{m}} C_{v\mathrm{m}}}{\rho_{\mathrm{M}} C_{v\mathrm{M}}} \vartheta_{\mathrm{m}} d\omega \end{cases}$$



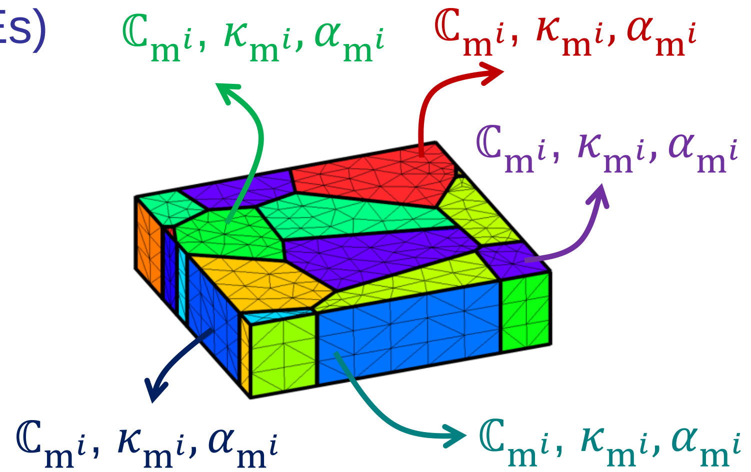
  - Up-scaling

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\sigma}_{\mathrm{m}} d\omega \\ \boldsymbol{q}_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{q}_{\mathrm{m}} d\omega \\ \rho_{\mathrm{M}} C_{v\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int \rho_{\mathrm{m}} C_{v\mathrm{m}} dV \end{cases} \longrightarrow \begin{cases} \mathbb{C}_{\mathrm{M}} = \dfrac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \boldsymbol{u}_{\mathrm{M}} \otimes \boldsymbol{\nabla}_{\mathrm{M}}} \quad \& \quad \boldsymbol{\alpha}_{\mathrm{M}} : \mathbb{C}_{\mathrm{M}} = -\dfrac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \vartheta_{\mathrm{M}}} \\ \boldsymbol{\kappa}_{\mathrm{M}} = -\dfrac{\partial \boldsymbol{q}_{\mathrm{M}}}{\partial \nabla_{\mathrm{M}}\vartheta_{\mathrm{M}}} \end{cases}$$

  - Consistency → Satisfied by periodic boundary conditions

From the micro-scale to the meso-scale

- Definition of Stochastic Volume Elements (SVEs)
  - Poisson Voronoï tessellation realizations
    - SVE realization ω_j
  - Each grain ω_i is assigned material properties
    - $\mathbb{C}_{\mathrm{m}^i}, \kappa_{\mathrm{m}^i}, \alpha_{\mathrm{m}^i},$
    - Defined from silicon crystal properties
  - Each set $\mathbb{C}_{m^i}, \kappa_{\mathrm{m}^i}, \alpha_{\mathrm{m}^i}$ is assigned a random orientation
    - Following XRD distributions



$\omega_j = \cup_i \, \omega_i$

- Stochastic homogenization
  - Several SVE realizations
  - For each SVE $\omega_j = \cup_i \, \omega_i$

$\mathbb{C}_{\mathrm{m}^i}, \kappa_{\mathrm{m}^i}, \alpha_{\mathrm{m}^i} \quad \forall i \longrightarrow \mathbb{C}_{\mathrm{M}^j}, \kappa_{\mathrm{M}^j}, \alpha_{\mathrm{M}^j}$

Computational homogenization

Samples of the meso-scale homogenized elasticity tensors

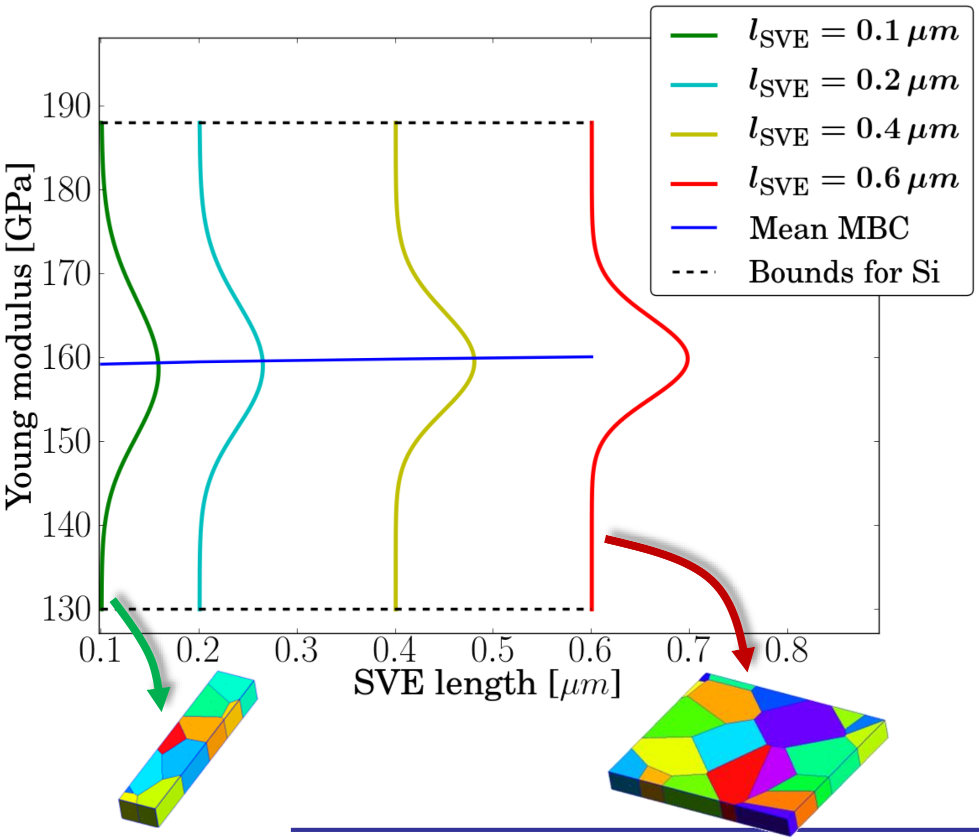
  - Homogenized material tensors not unique as statistical representativeness is lost*

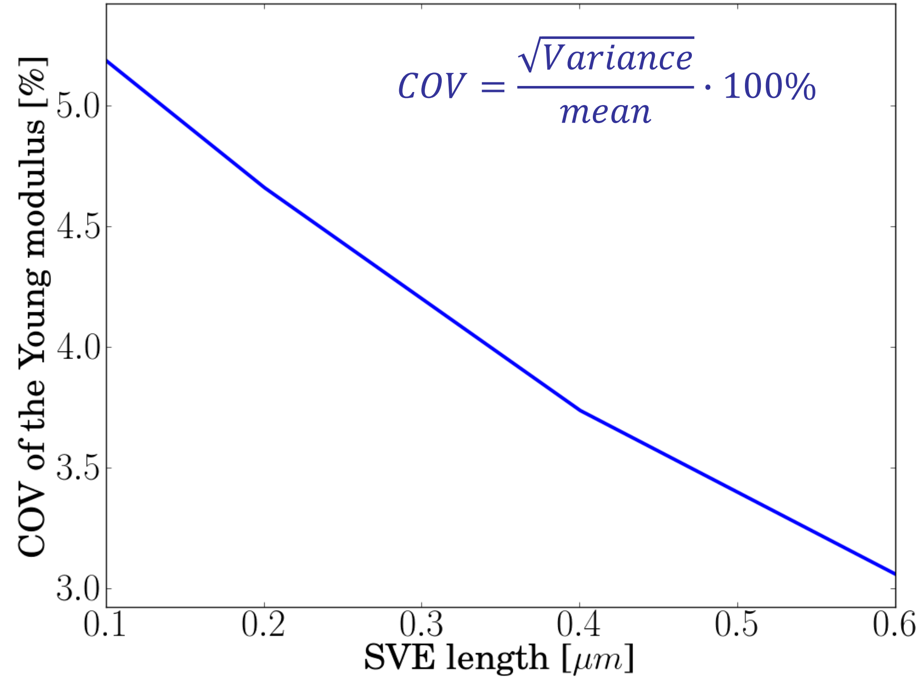
*"C. Huet, 1990

From the micro-scale to the meso-scale

- Distribution of the apparent meso-scale elasticity tensor $\mathbb{C}_M$

  - For large SVEs, the apparent tensor tends to the effective (and unique) one



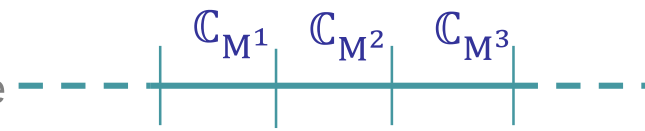


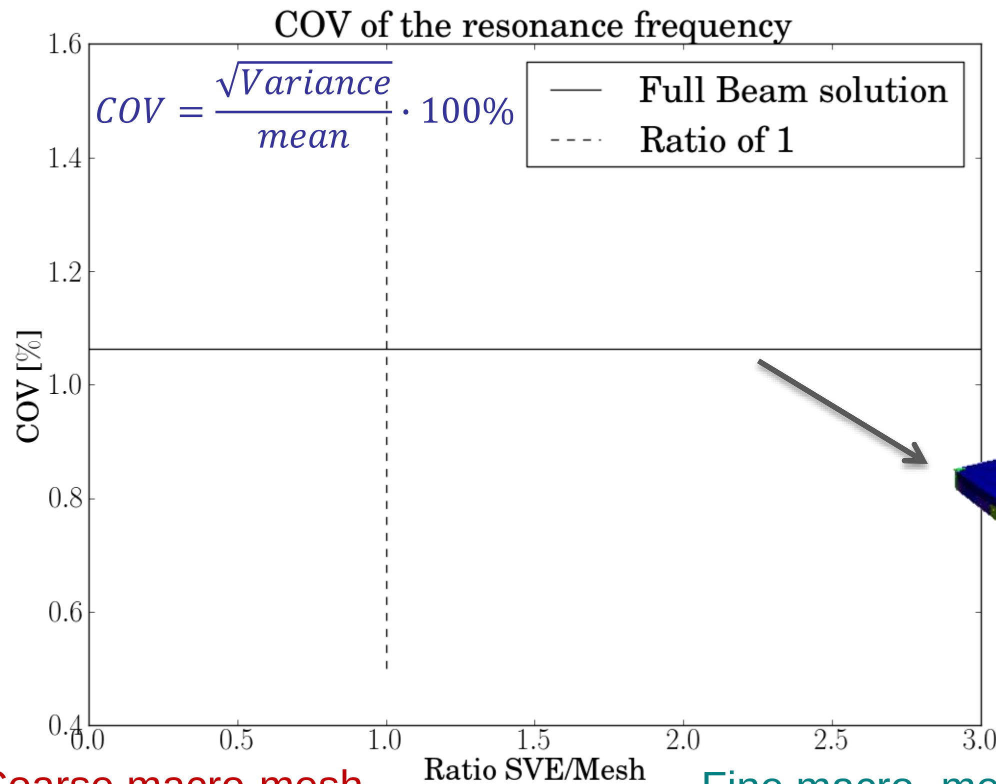
  - The bounds do not depend on the SVE size but on the silicon elasticity tensor

  - However, the larger the SVE, the lower the probability to be close to the bounds

From the micro-scale to the meso-scale

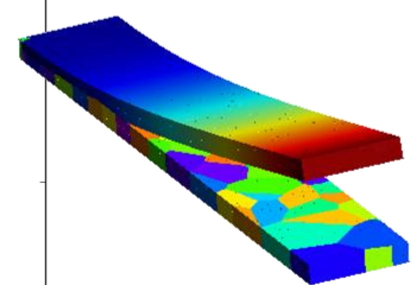
- Use of the meso-scale distribution with macro-scale finite elements
  - Beam macro-scale finite elements
  - Use of the meso-scale distribution as a random variable
  - Monte-Carlo simulations





First bending mode of

a 3.2 μm-long beam



Coarse macro-mesh

Fine macro- mesh

From the micro-scale to the meso-scale

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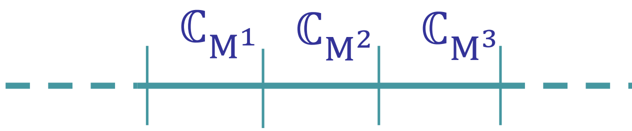
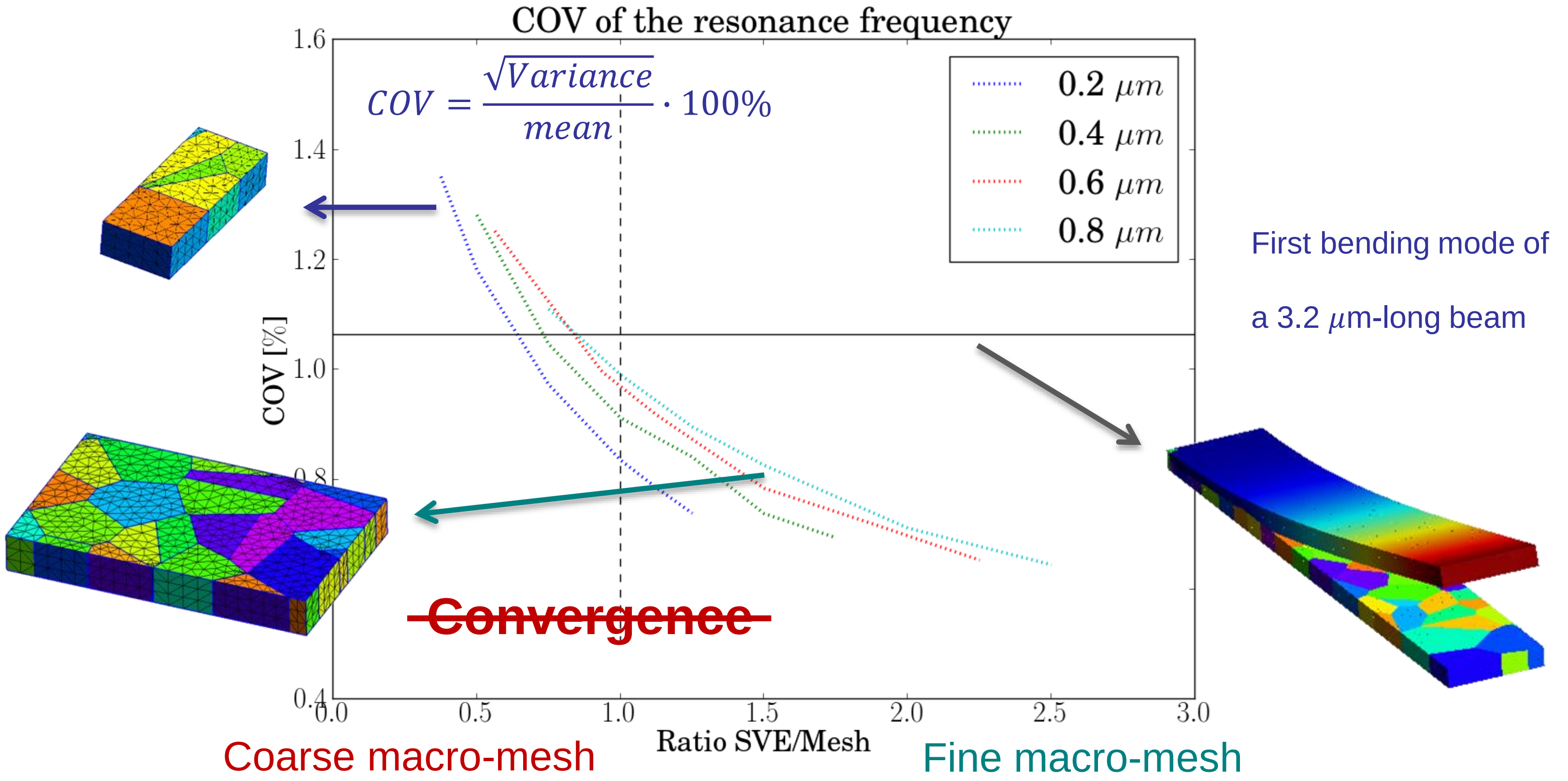




- No convergence: the macro-scale distribution (first resonance frequency) depends on SVE and mesh sizes

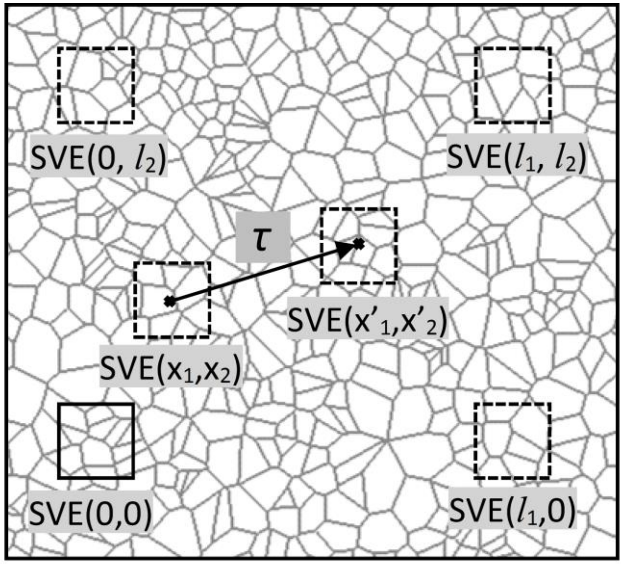
From the micro-scale to the meso-scale

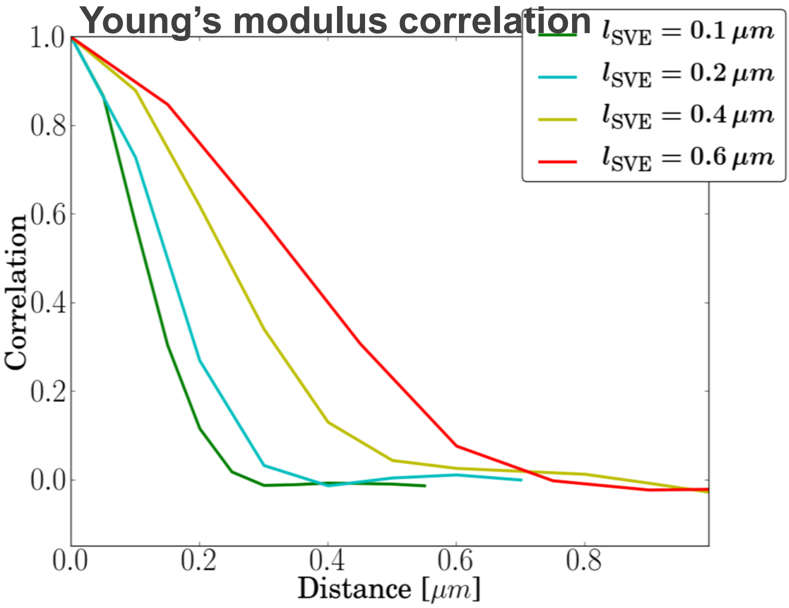
- Need for a meso-scale random field
  - Introduction of the (meso-scale) spatial correlation
    - Define large tessellations
    - SVEs extracted at different distances in each tessellation
  - Evaluate the spatial correlation between the components of the meso-scale material operators
  - For example, in 1D-elasticity
    - Young's modulus correlation

$$R_{E_x}(\boldsymbol{\tau}) = \frac{\mathbb{E}\left[\left(\boldsymbol{E}_x(\boldsymbol{x}) - \mathbb{E}(\boldsymbol{E}_x)\right)\left(\boldsymbol{E}_x(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}(\boldsymbol{E}_x)\right)\right]}{\mathbb{E}\left[\left(\boldsymbol{E}_x - \mathbb{E}(\boldsymbol{E}_x)\right)^2\right]}$$

    - Correlation length

$$L_{E_x} = \frac{\int_{-\infty}^{\infty} R_{E_x}(\tau)d\tau}{R_{E_x}(0)}$$

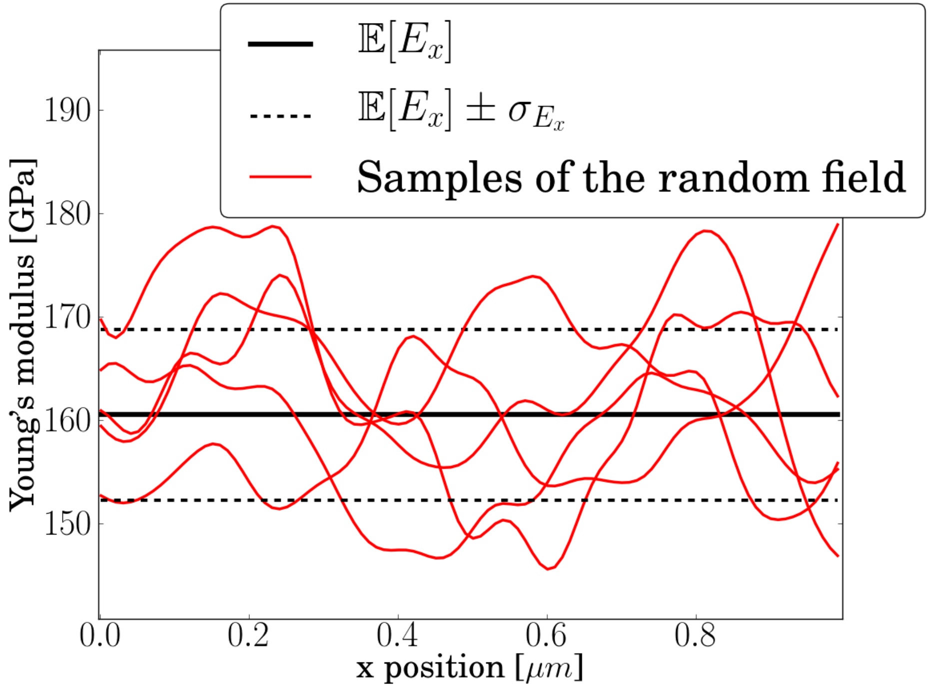




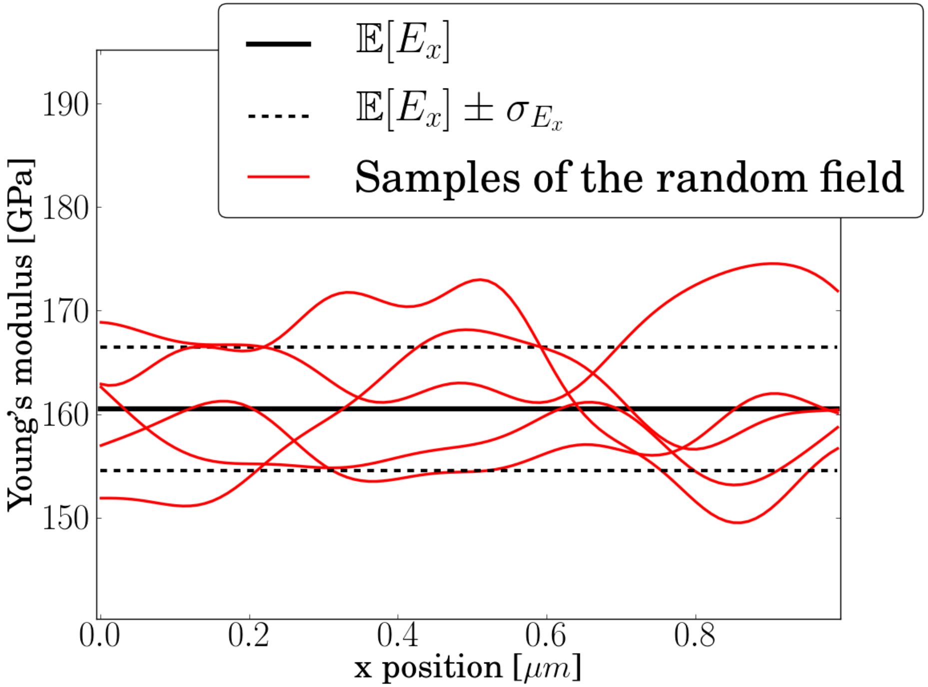
- Need for a meso-scale random field (2)
  - The meso-scale random field is characterized by the correlation length L_{E_x}
  - The correlation length L_{E_x} depends on the SVE size

Random field with different SVEs sizes

$l_{\mathrm{SVE}} = 0.1\ \mu m$



$l_{\mathrm{SVE}} = 0.4\ \mu m$



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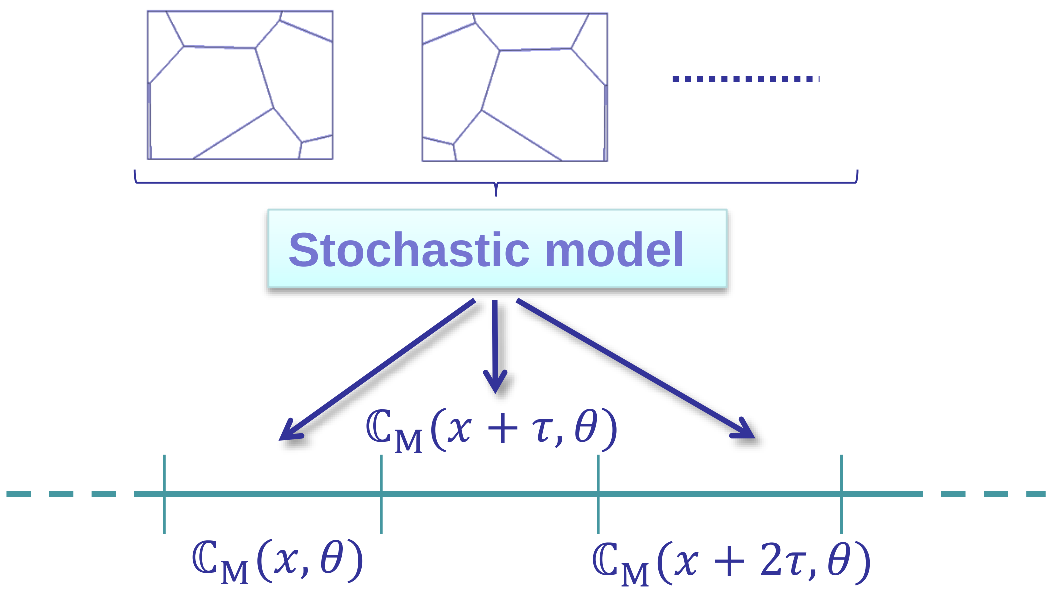
The meso-scale random field

- Use of the meso-scale distribution with stochastic (macro-scale) finite elements
  - Use of the meso-scale random field

    → Monte-Carlo simulations at the macro-scale

  - BUT we do not want to evaluate the random field from the stochastic homogenization for each simulation → Meso-scale random field from a generator

Stochastic model of meso-scale
elasticity tensors



The meso-scale random field

- Definition of the thermo-mechanical meso-scale random field
  - Elasticity tensor $\mathbb{C}_{\mathrm{M}}(x,\theta)$ (matrix form $\boldsymbol{C}_M$) & thermal conductivity $\boldsymbol{\kappa}_{\mathrm{M}}$ are bounded
    - Ensure existence of their inverse
    - Define lower bounds $\mathbb{C}_{\mathrm{L}}$ and $\boldsymbol{\kappa}_{\mathrm{L}}$ such that

$$\begin{cases} \boldsymbol{\varepsilon} : (\mathbb{C}_{\mathrm{M}} - \mathbb{C}_{\mathrm{L}}) : \boldsymbol{\varepsilon} > 0 & \forall \boldsymbol{\varepsilon} \\ \nabla\vartheta \cdot (\boldsymbol{\kappa}_{\mathrm{M}} - \boldsymbol{\kappa}_{\mathrm{L}}) \cdot \nabla\vartheta > 0 & \forall \nabla\vartheta \end{cases}$$

  - Use a Cholesky decomposition when semi-positive definite matrices are required

$$\begin{cases} \boldsymbol{C}_{\mathrm{M}}(\boldsymbol{x},\boldsymbol{\theta}) = \boldsymbol{C}_{\mathrm{L}} + \left(\bar{\boldsymbol{\mathcal{A}}} + \boldsymbol{\mathcal{A}}'(\boldsymbol{x},\boldsymbol{\theta})\right)^T \left(\bar{\boldsymbol{\mathcal{A}}} + \boldsymbol{\mathcal{A}}'(\boldsymbol{x},\boldsymbol{\theta})\right) \\ \boldsymbol{\kappa}_{\mathrm{M}}(\boldsymbol{x},\boldsymbol{\theta}) = \boldsymbol{\kappa}_{\mathrm{L}} + \left(\bar{\boldsymbol{\mathcal{B}}} + \boldsymbol{\mathcal{B}}'(\boldsymbol{x},\boldsymbol{\theta})\right)^T \left(\bar{\boldsymbol{\mathcal{B}}} + \boldsymbol{\mathcal{B}}'(\boldsymbol{x},\boldsymbol{\theta})\right) \\ \boldsymbol{\alpha}_{\mathrm{M}_{ij}}(\boldsymbol{x},\boldsymbol{\theta}) = \bar{\boldsymbol{\mathcal{V}}}^{(t)} + \boldsymbol{\mathcal{V}}'^{(t)}(\boldsymbol{x},\boldsymbol{\theta}) \end{cases}$$

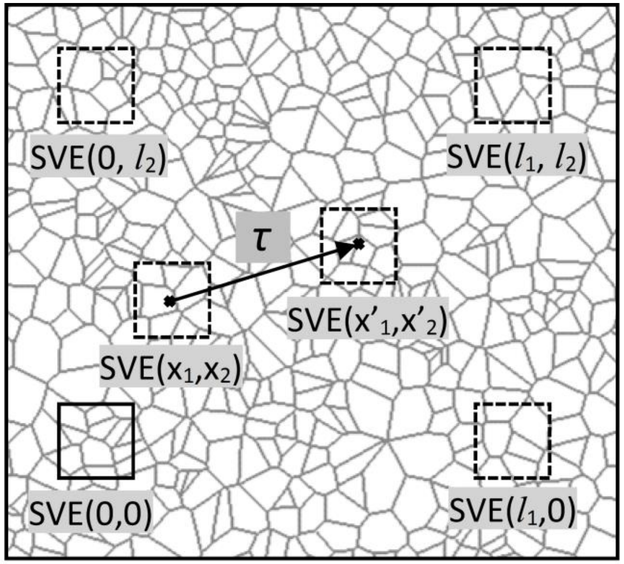
  - We define the homogenous zero-mean random field $\boldsymbol{\mathcal{V}}'(\boldsymbol{x},\boldsymbol{\theta})$, with as entries
    - Elasticity tensor $\mathcal{A}'(\boldsymbol{x},\boldsymbol{\theta}) \Rightarrow \boldsymbol{\mathcal{V}}'^{(1)} \dots \boldsymbol{\mathcal{V}}'^{(21)}$,
    - Heat conductivity tensor $\mathcal{B}'(\boldsymbol{x},\boldsymbol{\theta}) \Rightarrow \boldsymbol{\mathcal{V}}'^{(22)} \dots \boldsymbol{\mathcal{V}}'^{(27)}$
    - Thermal expansion tensors $\boldsymbol{\mathcal{V}}'^{(t)} \Rightarrow \boldsymbol{\mathcal{V}}'^{(28)} \dots \boldsymbol{\mathcal{V}}'^{(33)}$

The meso-scale random field

- Characterization of the meso-scale random field
  - Generate large tessellation realizations
  - For each tessellation realization
    - Extract SVEs centred on $\boldsymbol{x}+\boldsymbol{\tau}$
    - For each SVE evaluate $\mathbb{C}_{\mathrm{M}}(\boldsymbol{x}+\boldsymbol{\tau})$, $\kappa_{\mathrm{M}}(\boldsymbol{x}+\boldsymbol{\tau}), \alpha_{\mathrm{M}}(\boldsymbol{x}+\boldsymbol{\tau})$
  - From the set of realizations $\mathbb{C}_{\mathrm{M}}(\boldsymbol{x},\boldsymbol{\theta})$, $\kappa_{\mathrm{M}}(\boldsymbol{x},\boldsymbol{\theta}), \alpha_{\mathrm{M}}(\boldsymbol{x},\boldsymbol{\theta})$
    - Evaluate the bounds $\mathbb{C}_{\mathrm{L}}$ and $\boldsymbol{\kappa}_{\mathrm{L}}$
    - Apply the Cholesky decomposition ⇒ $\boldsymbol{\mathcal{A}}'(\boldsymbol{x},\boldsymbol{\theta})$, $\boldsymbol{\mathcal{B}}'(\boldsymbol{x},\boldsymbol{\theta})$
    - Fill the 33 entries of the zero-mean homogenous field $\boldsymbol{\mathcal{V}}'(\boldsymbol{x},\boldsymbol{\theta})$
  - Compute the auto-/cross-correlation matrix

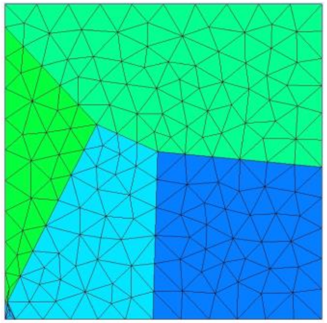
$$R_{\boldsymbol{\mathcal{V}}'}^{(rs)}(\boldsymbol{\tau}) = \frac{\mathbb{E}\left[\left(\boldsymbol{\mathcal{V}}'^{(r)}(\boldsymbol{x}) - \mathbb{E}\left(\boldsymbol{\mathcal{V}}'^{(r)}\right)\right)\left(\boldsymbol{\mathcal{V}}'^{(s)}(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}\left(\boldsymbol{\mathcal{V}}'^{(s)}\right)\right)\right]}{\sqrt{\mathbb{E}\left[\left(\boldsymbol{\mathcal{V}}'^{(r)} - \mathbb{E}(\boldsymbol{\mathcal{V}}'^{(r)})\right)^2\right]\mathbb{E}\left[\left(\boldsymbol{\mathcal{V}}'^{(s)} - \mathbb{E}(\boldsymbol{\mathcal{V}}'^{(s)})\right)^2\right]}}$$

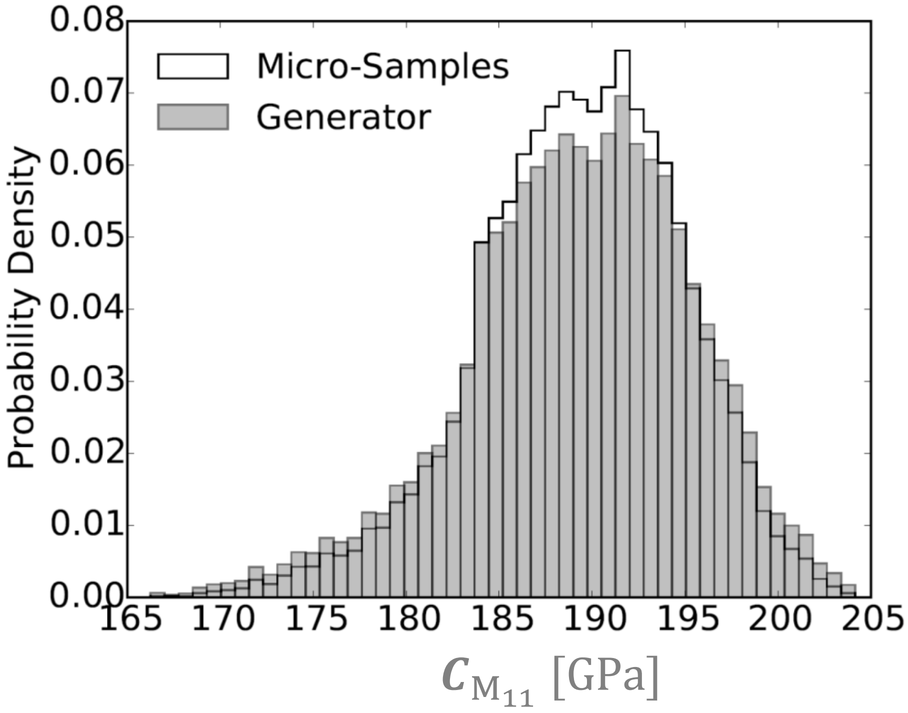
  - Generate zero-mean random field $\boldsymbol{\mathcal{V}}'(\boldsymbol{x},\boldsymbol{\theta})$
    - Spectral generator & non-Gaussian mapping

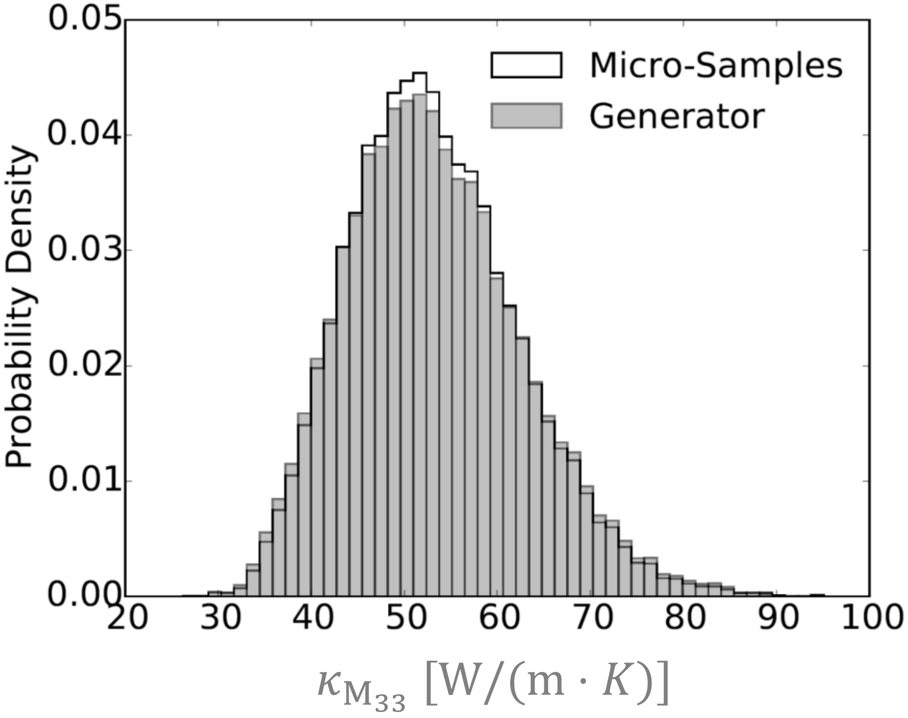


The meso-scale random field

- Polysilicon film deposited at 610 ºC
  - SVE size of 0.5 x 0.5 μm^2
  - Comparison between micro-samples and generated field PDFs

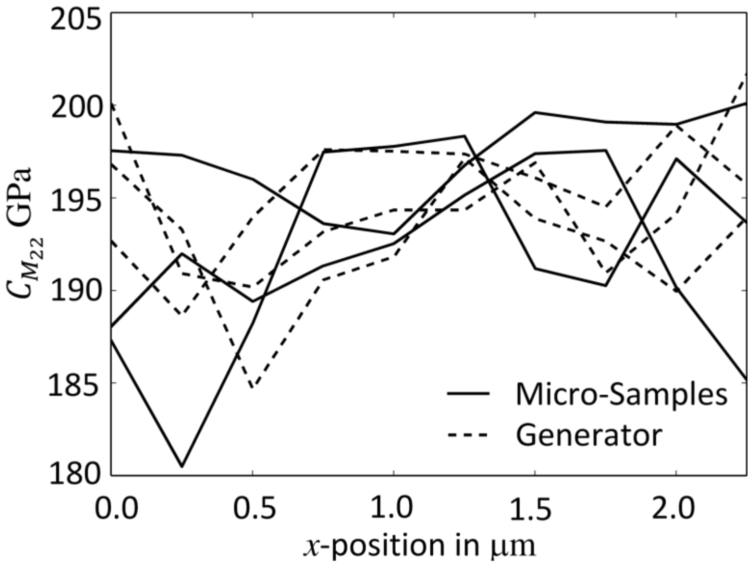


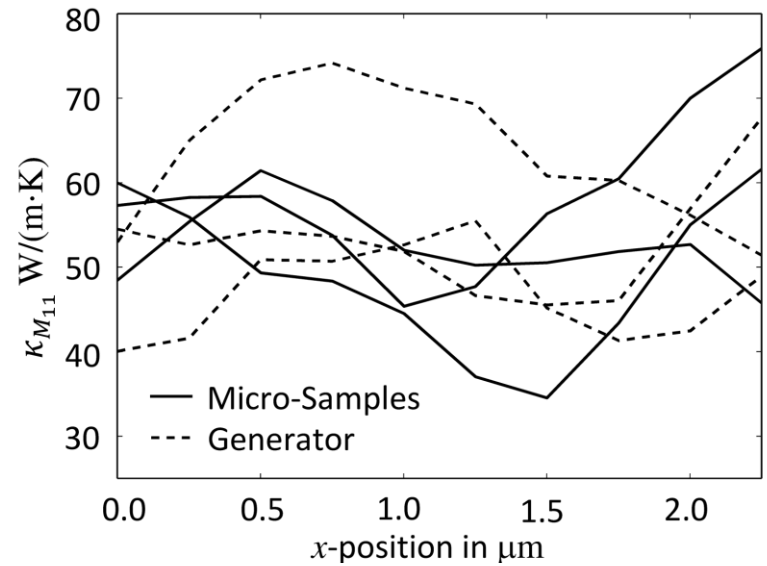


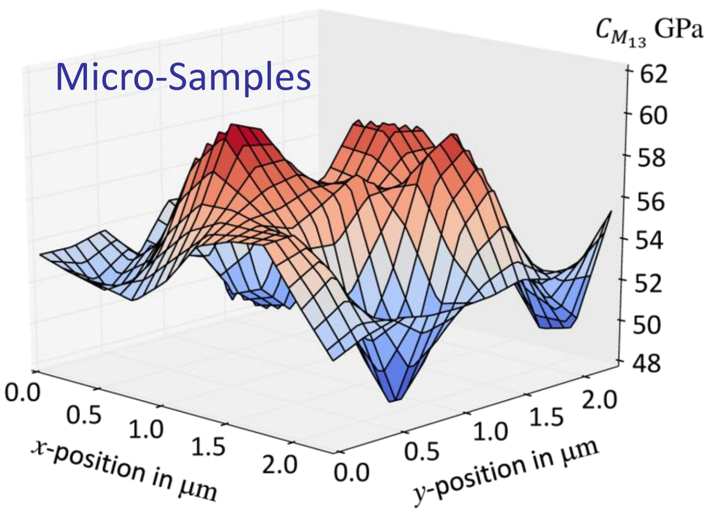


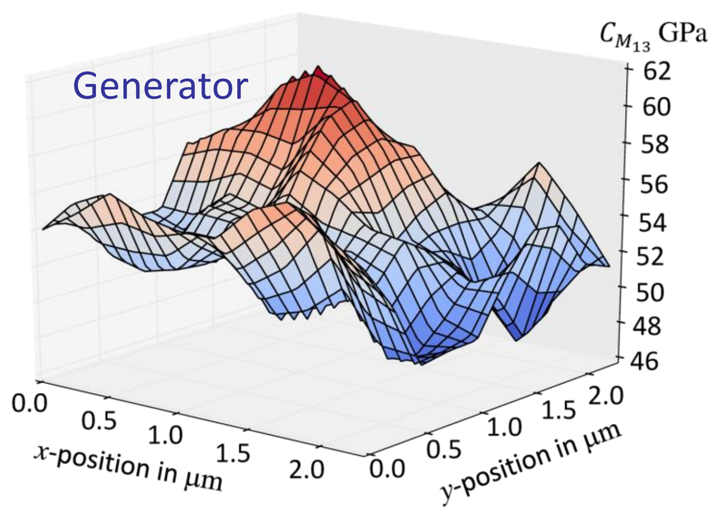
The meso-scale random field

- Polysilicon film deposited at 610 ºC (3)
  - Comparison between micro-samples and generated random field realizations







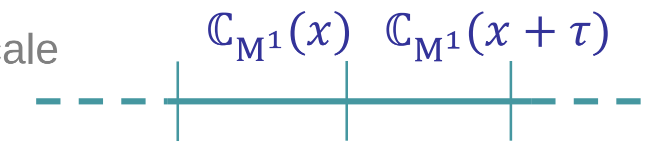


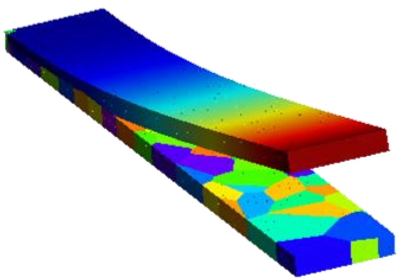
Content

- Thermo-mechanical problems
  - Governing equations
  - Macro-scale stochastic finite element
  - Meso-scale volume elements
- From the micro-scale to the meso-scale
  - Thermo-mechanical homogenization
  - Definition of Stochastic Volume Elements (SVEs) & Stochastic homogenization
  - Need for a meso-scale random field
- The meso-scale random field
  - Definition of the thermo-mechanical meso-scale random field
  - Stochastic model of the random field: Spectral generator & non-Gaussian mapping
- From the meso-scale to the macro-scale
  - 3-Scale approach verification
  - Application to extract the quality factor

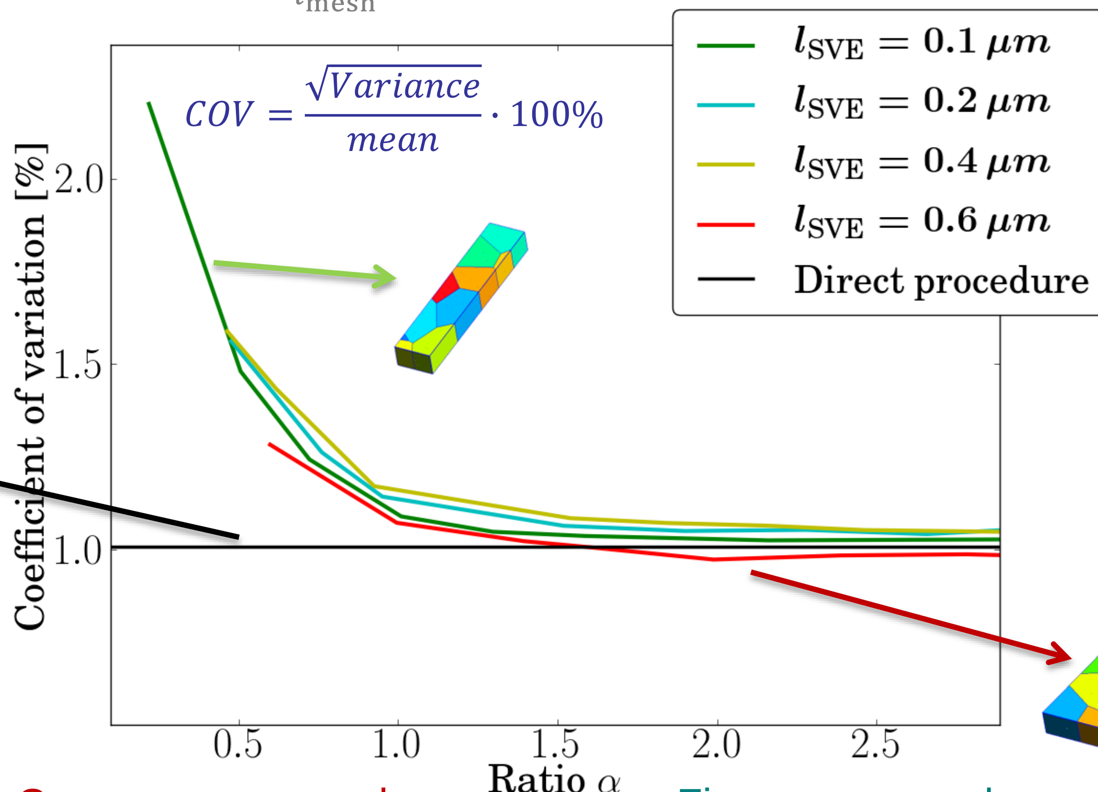
- 3-Scale approach verification with direct Monte-Carlo simulations
  - Use of the meso-scale random field

    → Monte-Carlo simulations at the macro-scale
  - Macro-scale beam elements of size l_{mesh}
  - Convergence in terms of $\alpha = \frac{l_{E_x}}{l_{\text{mesh}}}$



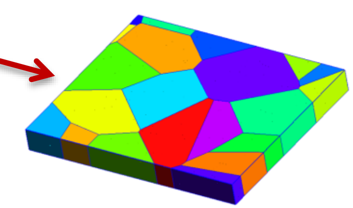


First bending mode of a 3.2 μm-long beam



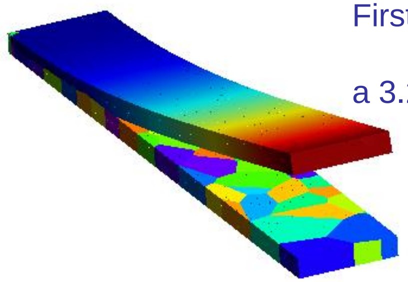
Coarse macro-mesh

Fine macro-mesh



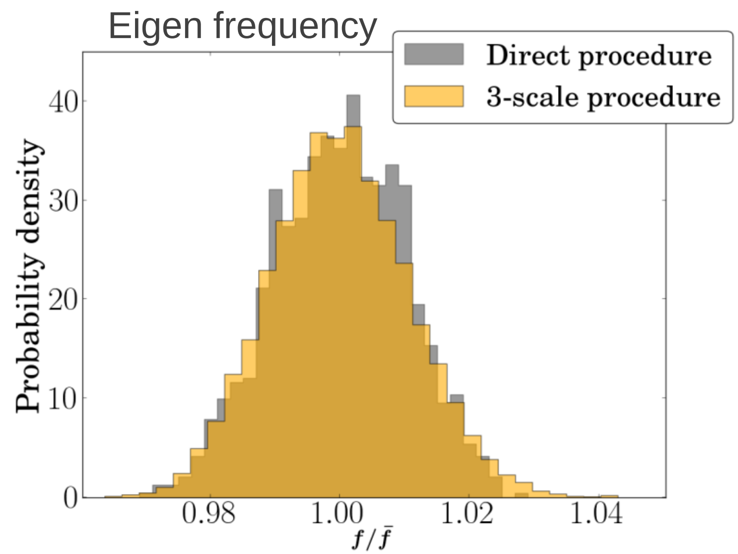
From the meso-scale to the macro-scale

- 3-Scale approach verification ($\alpha \sim 2$) with direct Monte-Carlo simulations
  - First bending mode

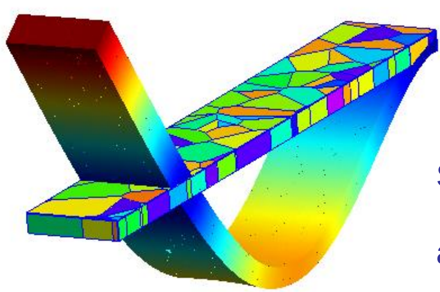


First bending mode of

a 3.2 μm-long beam

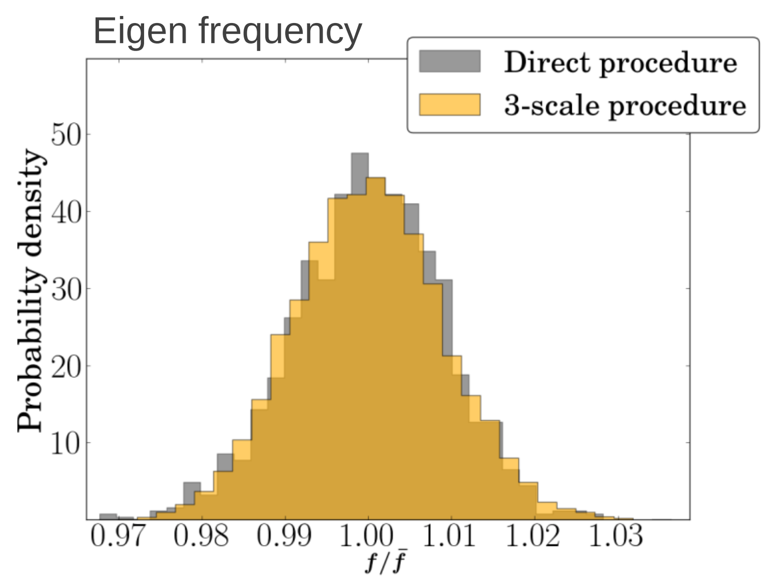


  - Second bending mode

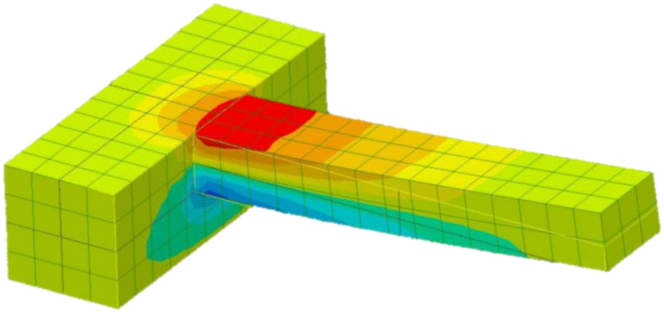


Second bending mode of

a 3.2 μm-long beam



- Quality factor
  - Micro-resonators
    - Temperature changes with compression/traction
    - Energy dissipation



  - Eigen values problem
    - Governing equations

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{\mathrm{u}\vartheta}(\boldsymbol{\theta}) & \mathbf{D}_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\vartheta}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{uu}}(\boldsymbol{\theta}) & \mathbf{K}_{\mathrm{u}\vartheta}(\boldsymbol{\theta}) \\ \mathbf{0} & \mathbf{K}_{\vartheta\vartheta}(\boldsymbol{\theta}) \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{u}} \\ \boldsymbol{F}_{\vartheta} \end{bmatrix}$$

    - Free vibrating problem

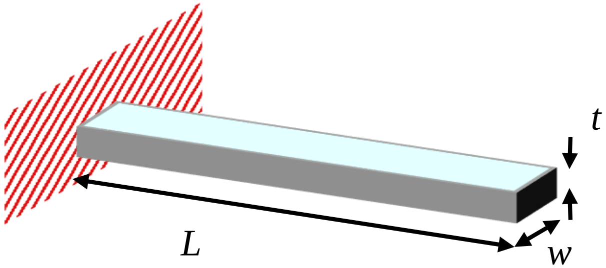
$$\begin{bmatrix} \mathbf{u}(t) \\ \boldsymbol{\vartheta}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{u}_0 \\ \boldsymbol{\vartheta}_0 \end{bmatrix} e^{i\omega t}$$

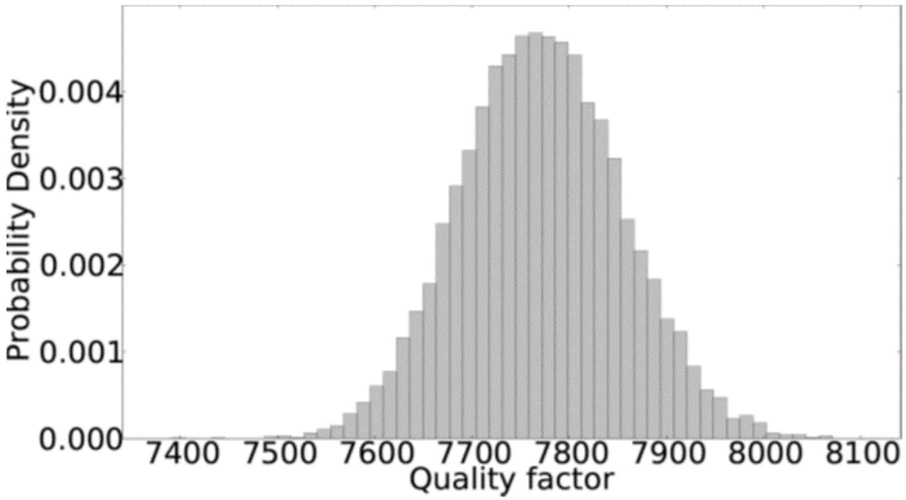
$$\begin{bmatrix} -\mathbf{K}_{\mathrm{uu}}(\boldsymbol{\theta}) & -\mathbf{K}_{\mathrm{u}\vartheta}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{0} & -\mathbf{K}_{\vartheta\vartheta}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \\ \dot{\mathbf{u}} \end{bmatrix} = i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{M} \\ \mathbf{D}_{\vartheta\mathrm{u}}(\boldsymbol{\theta}) & \mathbf{D}_{\vartheta\vartheta} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\vartheta} \\ \dot{\mathbf{u}} \end{bmatrix}$$

  - Quality factor
    - From the dissipated energy per cycle
    - $Q^{-1} = \dfrac{2|\Im\omega|}{\sqrt{(\Im\omega)^2 + (\Re\omega)^2}}$

From the meso-scale to the macro-scale

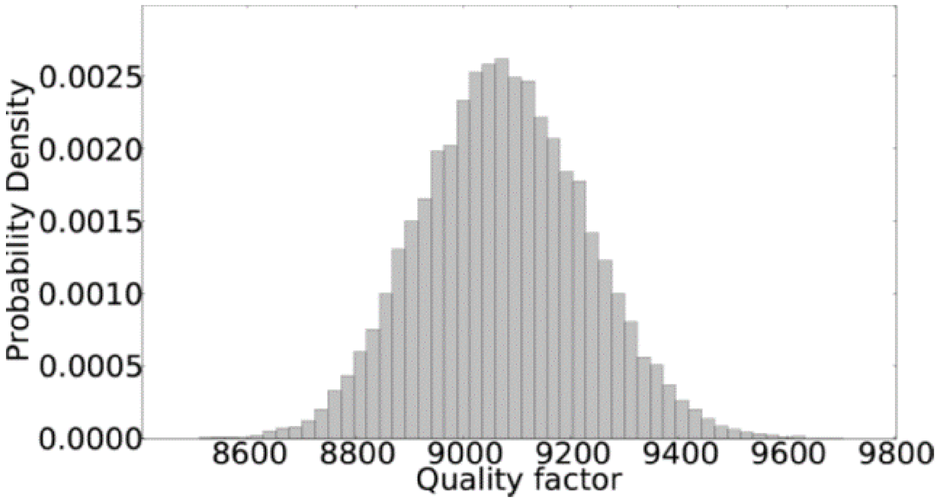
- Application of the 3-Scale method to extract the quality factor distribution
  - Perfectly clamped micro-resonator
    - Different sizes easily considered
  - Meso-scale random fields
    - From stochastic homogenization
    - Generated for different deposition temperatures
  - Effect of the deposition temperature





15 x 3 x 2 μm^3-beam,

deposited at 610 ºC

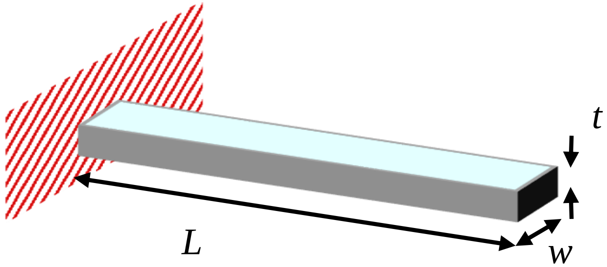


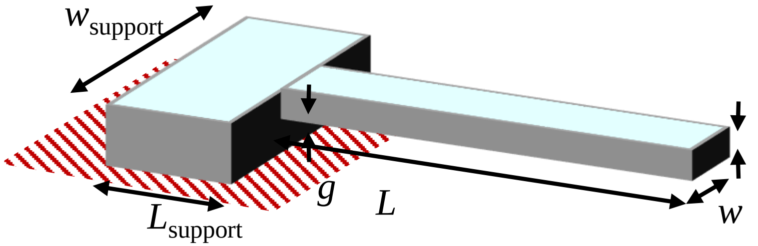
15 x 3 x 2 μm^3-beam,

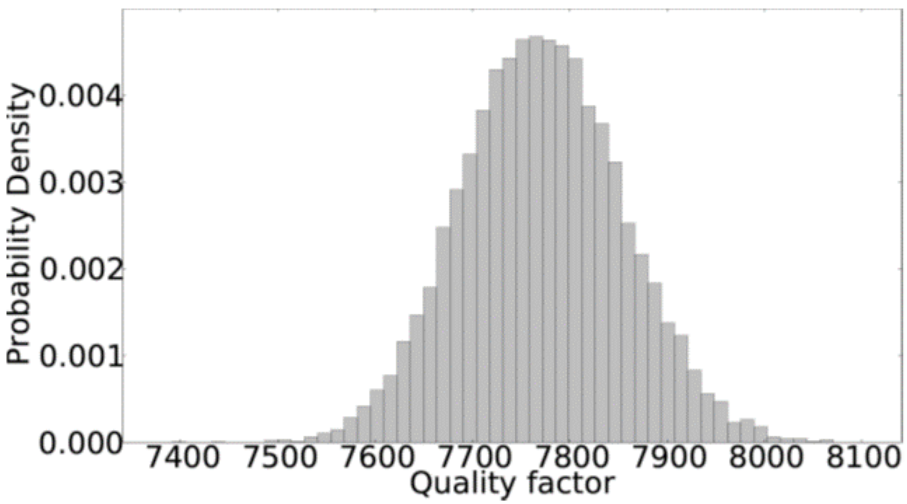
deposited at 630 ºC

From the meso-scale to the macro-scale

- Application of the 3-Scale method to extract the quality factor distribution (3)
  - 3D models readily available
  - The effect of the anchor can be studied

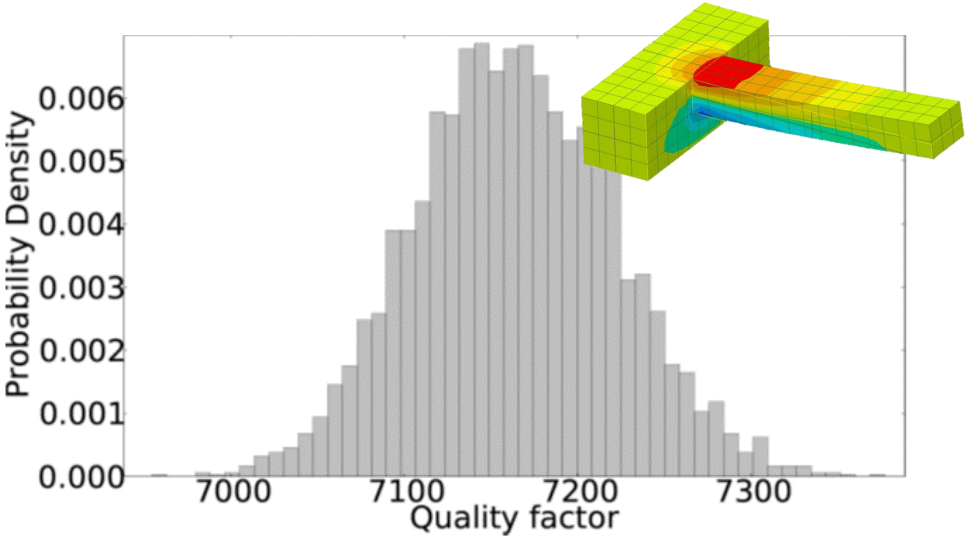






15 x 3 x 2 μm^3-beam,

deposited at 610 ºC



15 x 3 x 2 μm^3-beam & anchor,

deposited at 610 ºC

Conclusions & Perspectives

- Efficient stochastic multi-scale method
  - Micro-structure based on experimental measurements
  - Computational efficiency relies on the meso-scale random field generator
  - Used to study probabilistic behaviors

- Perspectives
  - Other material systems
  - Non-linear behaviors
  - Non-homogenous random fields

Thank you for your attention !