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On the Relations Between Nonlinear Resonances and Nonlinear Normal Modes

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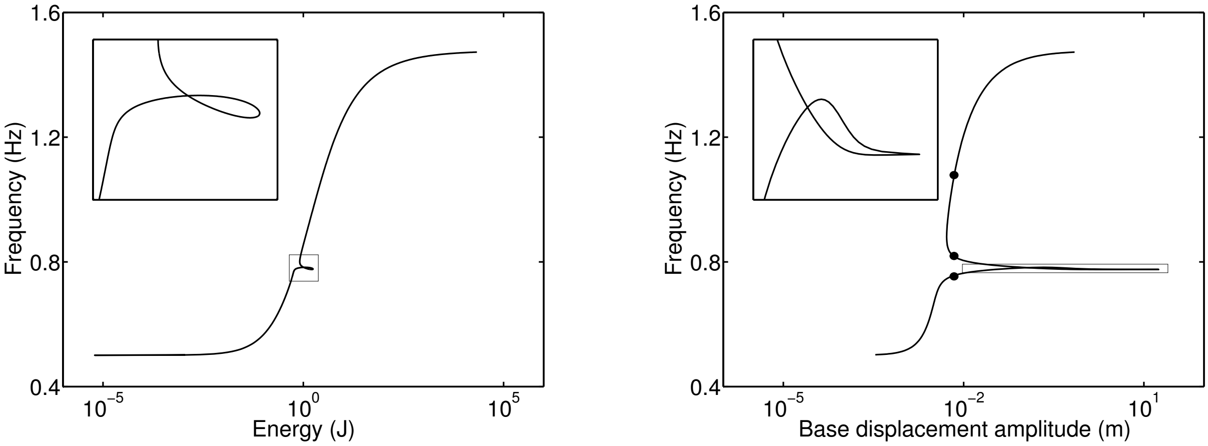
Abstract The objective of the present study is to investigate fundamental and isolated resonances of nonlinear structures using the nonlinear normal modes (NNMs). In particular, an energy balance approach is exploited to explore the connections between isolated response curves (IRCs) and NNMs. In this contribution, IRCs are characterized numerically and experimentally in the case of a discrete system consisting of two masses grounded through cubic springs and sliding on an horizontal guide.

Similarly to their linear counterpart, nonlinear normal modes (NNMs) reflect the deformation at resonance of a structure [1]. Formally, a damped system driven harmonically vibrates according to an undamped NNM denoted $\mathbf{x}(t) \in \mathbb{R}^n$, and so undergoes phase resonance, if the energy dissipated by damping forces over one cycle E_{out} balances the energy provided by the external force E_{in}. Denoting $\mathbf{C} \in \mathbb{R}^{n\times n}$ the linear damping matrix of the system and $\mathbf{p}(t) = A\, e^{i\,\omega\, t}\, \mathbf{e_j} \in \mathbb{R}^n$ a single-point, harmonic forcing applied to degree of freedom (DOF) j with amplitude A and frequency ω, this translates mathematically into

$$E_{in} = A \int_0^T e^{i\,\omega\, t}\, \dot{\mathbf{x}}^{\mathrm{T}}(t)\, \mathbf{e_j}\, dt = \int_0^T \dot{\mathbf{x}}^{\mathrm{T}}(t)\, \mathbf{C}\, \dot{\mathbf{x}}(t)\, dt = E_{out}, \tag{1}$$

where T is the cycle period, and T is the transpose operation. The practical implications of Equation (1) are very important, as this equation establishes a direct link between the undamped, unforced system response, and forced resonance conditions [2]. Specifically, it allows to calculate the amplitude A driving the system to resonance, based only on an undamped NNM motion $\mathbf{x}(t)$ and the knowledge of the damping matrix $\mathbf{C}$. In [3], the authors demonstrated from Equation (1) that isolated resonances, which involve the presence of isolated response curves (IRCs), are closely related to interactions between NNMs. IRCs correspond to closed loops of solutions emerging in nonlinear frequency responses and which are, by definition, detached from the main response branch. An increase in forcing amplitude, however, may cause the merging of the main branch and the IRC, resulting in dramatic frequency and amplitude shifts of the resonance location.

In this study, the relations between nonlinear resonances and NNMs are validated through the numerical and experimental realization of IRCs on a base-excited two-DOF system featuring a 3:1 internal resonance. Figure 1 compares the frequency-energy plot (FEP) of the first NNM of the undamped system in (a) with the base displacement amplitude required to initiate a vibration along this NNM in (b), and calculated using Equation (1). The latter curve exhibits a topology similar to the FEP, including a nonmonotonic increase of amplitude versus frequency. As a result, multiple resonance frequencies exist for forcing amplitudes exceeding 5.2 mm. For instance, the three black dots in Figure 1(b) indicate that the system may resonate at 0.76, 0.81, and 1.08 Hz for a base displacement of 7 mm. It is possible to demonstrate that 5.2 mm is the minimum forcing amplitude responsible for the creation of an IRC, and that the resonance points at frequencies of 0.81 Hz and 1.08 Hz for a 7 mm excitation are located on this IRC.



(a) (b)

Figure 1: Energy balance applied to a base-excited two-DOF system. (a) First NNM of the undamped, unforced system; (b) amplitude of the base displacement required to initiate a vibration of the damped system along its first NNM. Inset close-ups provide details of the behavior of the system close to a 3:1 internal resonance.

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