10 years of advances in nonlinear system identification in structural dynamics: a review

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Abstract

Nonlinear system identification is a vast research field, today attracting a great deal of attention in the structural dynamics community. Ten years ago, an MSSP paper reviewing the progress achieved until then [1] concluded that the identification of simple continuous structures with localised nonlinearities was within reach. The past decade witnessed a shift in emphasis, accommodating the growing industrial need for a first generation of tools capable of addressing complex nonlinearities in larger-scale structures. The objective of the present paper is to survey the key developments which arose in the field since 2006 towards developing these tools.

1 Introduction

To address the demand for structures and devices with ever-increasing technological and environmental performances, researchers in academia try more and more regularly to take advantage of nonlinear phenomena to outperform linear designs. For instance, Ref. [2] demonstrated a new mechanism for tunable rectification that uses bifurcations and chaos. In Ref. [3], a new strategy for engineering low-frequency noise oscillators was developed through the coupling of modes in internal resonance conditions. Another example is the cascade of parametric resonances proposed by Strachan *et al.* as a basis for the development of passive frequency dividers [4]. Nonlinearity is also increasingly exploited for vibration absorption [5, 6, 7] and energy harvesting [8, 9, 10].

If attempts to utilise nonlinearity are today frequent in the technical literature, current designs and models in industry predominantly remain linear. However, nonlinearity is often encountered during the tests performed on the first prototype of a structure. In addition to distorted resonances and jumps between high- and low-amplitude responses, nonlinearity can generate complex dynamic phenomena, such as subharmonic and superharmonic resonances, modal interactions, quasiperiodicity and chaos, with the consequence that essentially-linear models may fail to predict the structural response within the necessary level of reliability [11].

Two examples taken from the aerospace sector and for which nonlinearities were detected during ground vibration test campaigns are the Cassini-Huygens spacecraft [12] and the Airbus A400M aircraft [13]. In the former example, distorted frequency responses and jump phenomena around a critical mode were reported. Because this nonlinear behaviour was an important concern as for the integrity of the spacecraft, additional tests were conducted, and revealed that nonlinearity was caused by the appearance of gaps in the support of the Huygens probe. In the latter example, structural resonances showing significant peak skewness were incorrectly fitted by a linear modal analysis software. Different sources of nonlinearity, including elastomeric mounts and hydraulic actuators, were ascertained after careful analysis. As evidenced by these two examples, nonlinear phenomena complicate vibration test campaigns, and usually require profound investigations for which nonlinearity must first be identified.

In 1998 and 2000, the developments in nonlinear system identification in structural dynamics conducted until the end of the 20th century were reviewed by Adams and Allemang [14] and Worden [15], respectively. The first book on the topic was published a couple of years later [16]. In 2006, a great many existing methods to tackle nonlinearity detection, characterisation and parameter estimation were surveyed [1]. The need for a new review paper arises from the progress made during the last 10 years, which substantially advanced the available capabilities in the identification of nonlinear mechanical systems. Specifically, even if there are still significant challenges ahead of us, the first methods that can potentially address large-scale structures vibrating in strongly nonlinear regimes were developed. In addition, researchers recognised the importance of quantifying uncertainties in nonlinear system identification, which led to a change of paradigm within the community.

The paper starts in Section 2 with a discussion on the factors which have driven the recent progress achieved in nonlinear system identification in structural dynamics. It is explained that advances in nonlinear theory, computation and testing have largely contributed to this progress. In Section 3, a review of the key developments which arose during the 2006 – 2016 decade is conducted. The main focus of this literature survey is on parameter estimation methods, classified into seven categories as suggested in Ref. [1], namely linearisation, time- and frequency-domain methods, time-frequency analysis, modal methods, black-box modelling and numerical model updating. Finally, concluding remarks are drawn in Section 4 and directions for future research are suggested.

2 A perspective on the global progress in nonlinear mechanical vibrations

Two facts have arguably acted as catalysts for the progress across the nonlinear system identification field. First, nonlinearity manifestations have been increasingly encountered by engineers during vibration tests [11]. For instance, the linear modal analysis of two aircraft of the Airbus family, namely the A400M and the A350XWB, was experimentally confronted with nonlinearities in elastomeric engine mounts and hydraulic actuators [13], in landing gears [17] and in the auxiliary power unit of the airframe tail-cone [18]. Second, the pressure faced in industry to devise environment-friendly structures has greatly escalated. As an illustration, the report of the *High Level Group on Aviation Research in Europe* [19] deems necessary to achieve by 2050 reductions of 75 % in CO_2 emission and 90 % in NO_X emission per passenger kilometre. This ambitious goal necessarily entails the design of lighter aircraft structures featuring new technologies, *e.g.*, composite materials, which inevitably makes nonlinear behaviours more significant [20].

These two facts have motivated researchers in academia to make the first attempts to apply nonlinear system identification to real structures. These contributions mostly feature *ad hoc* approaches derived to solve specific nonlinearity modelling problems. As examples, experimental modal analysis of an engine casing assembly and nonlinear finite element model updating of a complete aircraft engine model were carried out in Refs. [21] and [22], respectively. The nonlinearities of structural prototypes of full-scale satellites were identified in Refs. [23, 24] based on typical qualification test campaign data. Ref. [25] estimated the variation of the natural frequencies and damping ratios of an Agusta-Westland helicopter as a function of the response level. The performance of nonlinear devices embedded in large structures was also examined, as in Ref. [26], where a nonlinear vibration absorber was used to mitigate the high response levels of an eleven-ton, nine-storey building subject to blast events.

Adopting a wider perspective, important advances have been achieved since the beginning of the 2000s in the three facets of the analysis of nonlinear mechanical vibrations, namely theory, computation and testing. We provide in what follows a brief review of this global progress, which has clearly contributed to push the envelope in nonlinear system identification.

Theory

The theory of nonlinear dynamic systems was developed by pure mathematicians based on the seminal work of Poincaré. A couple of decades ago, this theory spread across the engineering field thanks to a series of reference monographs, thoroughly characterising the different phenomena and attractors nonlinear mechanical systems can exhibit. Nayfeh and Mook [27] applied perturbation methods to study nonlinear phenomena in single- and multi-degree-of-freedom systems. Guckenheimer and Holmes [28] adopted a different, geometric viewpoint, appearing as an ideal companion to the perturbation approach, and Kuznetsov [29] published a complete treatise on bifurcations. It is only recently that theories described in there have been fully embraced by the structural dynamics community. For instance, for aircraft ground dynamics in Fig. 1 [30], bifurcation theory provides valuable insight into the overall behaviour and complexity of the system. Another example is isolated response curves, illustrated in Fig. 2 [31], which have received increasing attention during the past few years.

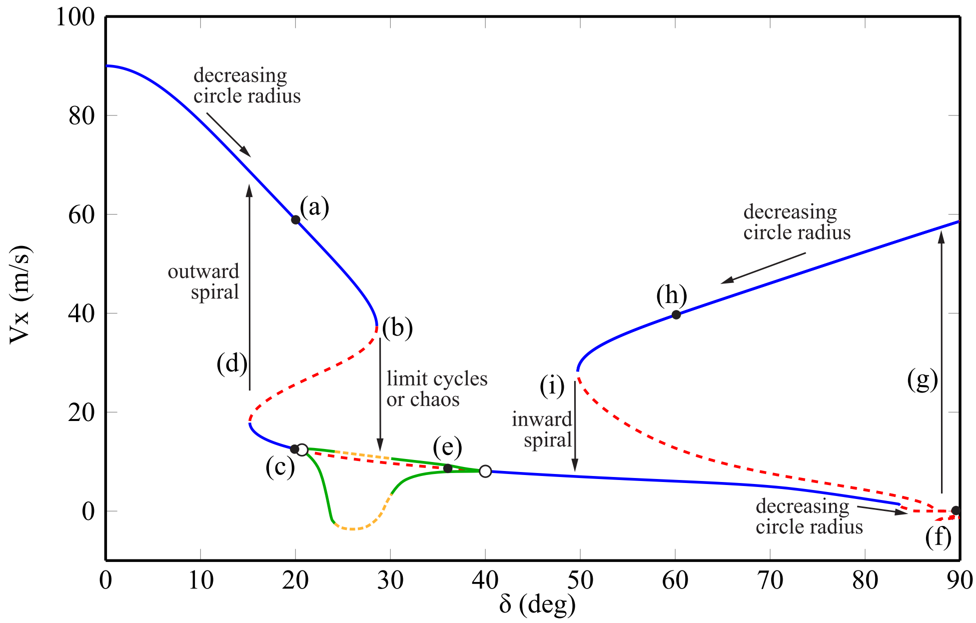


Figure 1: Single-aisle aircraft bifurcation diagram [30]. Courtesy of Bernd Krauskopf, University of Auckland, Auckland, New Zealand.

In view of the importance of modal analysis in structural dynamics, attempts to complete the nonlinear normal mode (NNM) theory, put forward by Rosenberg [32], Vakakis [33] and Shaw and Pierre [34], have also been made since the 2000s. Major developments include the concept of complex nonlinear modes based on a generalized Fourier series, the energy balance criterion linking NNMs and nonlinear frequency responses, and the nonlinear phase lag quadrature criterion which indicates when an NNM vibrates in isolation.

Computation

In the era of computational mechanics, commercial finite element software can effectively integrate in time the governing equations of motion of nonlinear structures using, *e.g.*, the Newmark's method. However, the resulting transient time series convey little information about the underlying structural behaviour and do not provide a global picture of the possible regimes of motion.

By revealing competing attractors together with their nature, bifurcation analysis offers a much better understanding of the dynamics. Two popular software, MATCONT [35] and AUTO [36], have enriched the

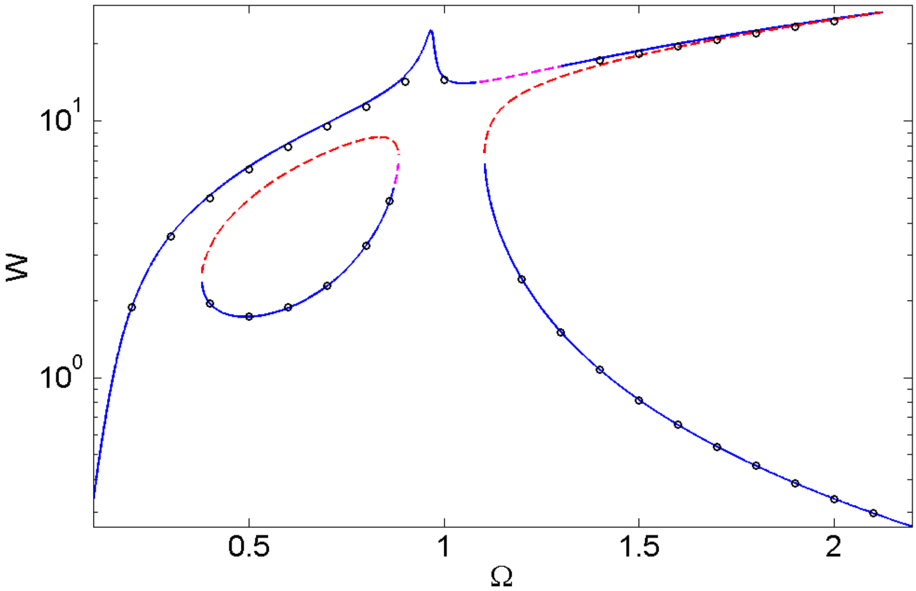


Figure 2: Isolated response curve situated inside the nonlinear frequency response [31]. Courtesy of Gianluca Gatti, University of Calabria, Rende, Italy.

nonlinear structural dynamicist's toolbox, allowing a shift in emphasis from the analytical analysis of low-dimensional systems to the numerical analysis of moderately-complex systems. Further efforts have been undertaken to progress towards large-scale and real-world structures. Specifically, tailored harmonic balance, shooting and collocation algorithms have been proposed for rigorously computing intrinsic dynamic features, in particular NNMs and nonlinear frequency responses. This is illustrated in Figs. 3 and 4, where the dynamics of a vibro-impact system [37] and of a compressor blade [38], respectively, is studied using the harmonic balance method. Both applications gave rise to large computational problems. In the former example, 200 harmonics were utilised to model accurately the nonsmooth nature of the system, whereas, in the latter example, NNMs were calculated from a detailed finite element model.

Recent computational developments concern the calculation of branches of quasiperiodic motion [39], the effective computation of basins of attraction [40, 41], and the tracking of bifurcations [42]. Noteworthy are also the advances in the area of nonlinear model reduction, with the generalisation of the Craig-Bampton substructuring technique to nonlinear systems [43, 44].

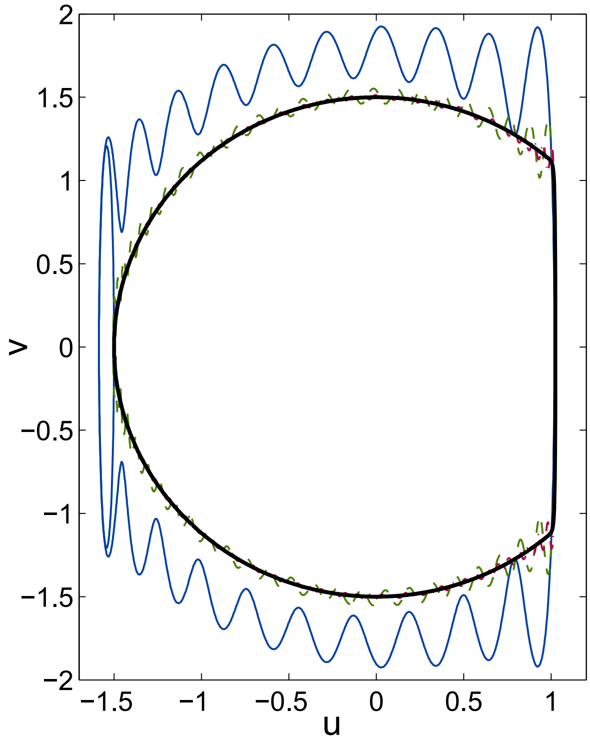


Figure 3: Phase diagram of a vibro-impact system given by the harmonic balance method for different numbers of harmonics N_h (blue: $N_h = 20$; green: $N_h = 50$; red: $N_h = 200$) [37]. Courtesy of Bruno Cochelin, Ecole Centrale de Marseille, Marseille, France.

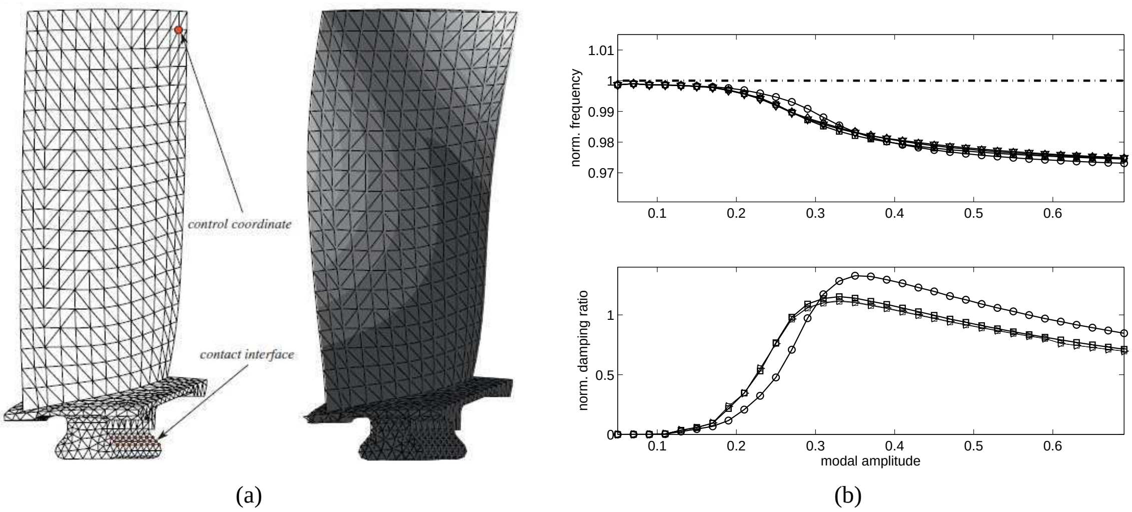


Figure 4: Compressor blade with friction [38]. (a) Finite element model and deformed shape; (b) changes in the natural frequency and modal damping depending on modal amplitude for different numbers of harmonics N_h (circles: $N_h = 1$; squares: $N_h = 3$; triangles: $N_h = 5$; diamonds: $N_h = 7$). Courtesy of Fabrice Thouverez, Ecole Centrale de Lyon, Lyon, France.

Testing

Nonlinear testing demands more data and efforts than linear testing, *e.g.*, the measurement sampling rate should be significantly increased to account for harmonics. Experimentalists have had increasingly recourse to full-field measurement methodologies (*e.g.*, scanning laser vibrometry and digital image correlation) as an aid to rapidly and very accurately capture operational deformation shapes and identify resonances exhibiting nonlinear distortions [21, 45].

However, the challenges brought by nonlinear testing go largely beyond data acquisition issues and instrumentation. Indeed, since solutions of nonlinear systems are nonunique and may be unstable, new methodologies that can experimentally characterise competing attractors, *i.e.* periodic, quasiperiodic and chaotic attractors, and that can cope with stability changes should be devised. In this respect, the stochastic interrogation method, proposed in Ref. [46], offers a systematic approach to map basins of attraction [47, 48], as depicted in Fig. 6. Experimental continuation, the physical realisation of numerical continuation, elegantly addresses instability issues. It exploits feedback control strategies to stabilise the measured response, enabling both stable and unstable branches to be measured, as illustrated in Fig. 5 [49].

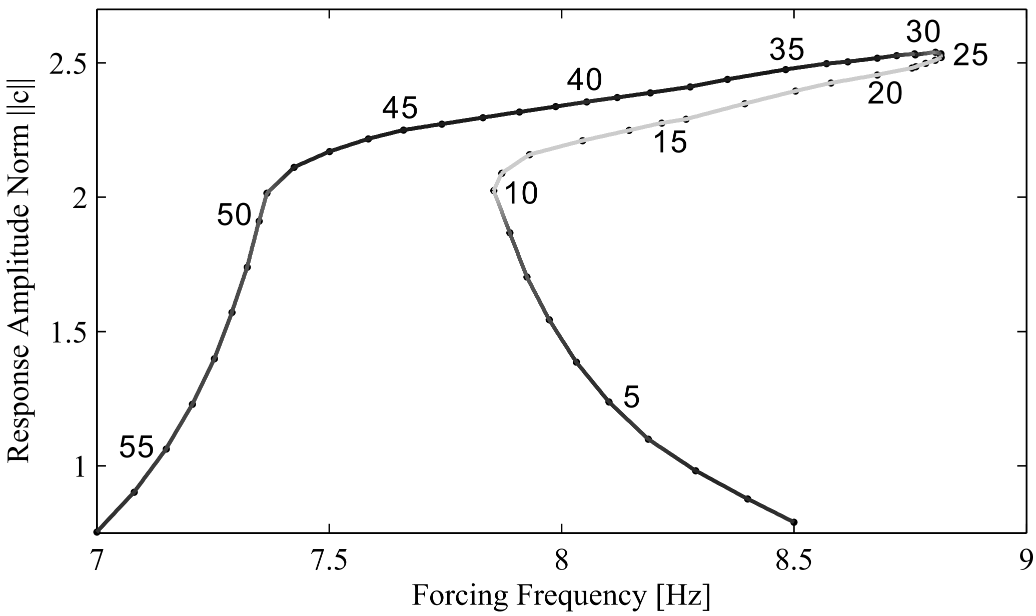


Figure 5: Experimental bifurcation diagram of an impact oscillator [49] with a continuous measure of stability plotted in grey-scale. Dark tones denote a stable state; lighter tones denote an unstable state. Courtesy of Jon-Juel Thomsen, Danmarks Tekniske Universitet, Lyngby, Denmark.

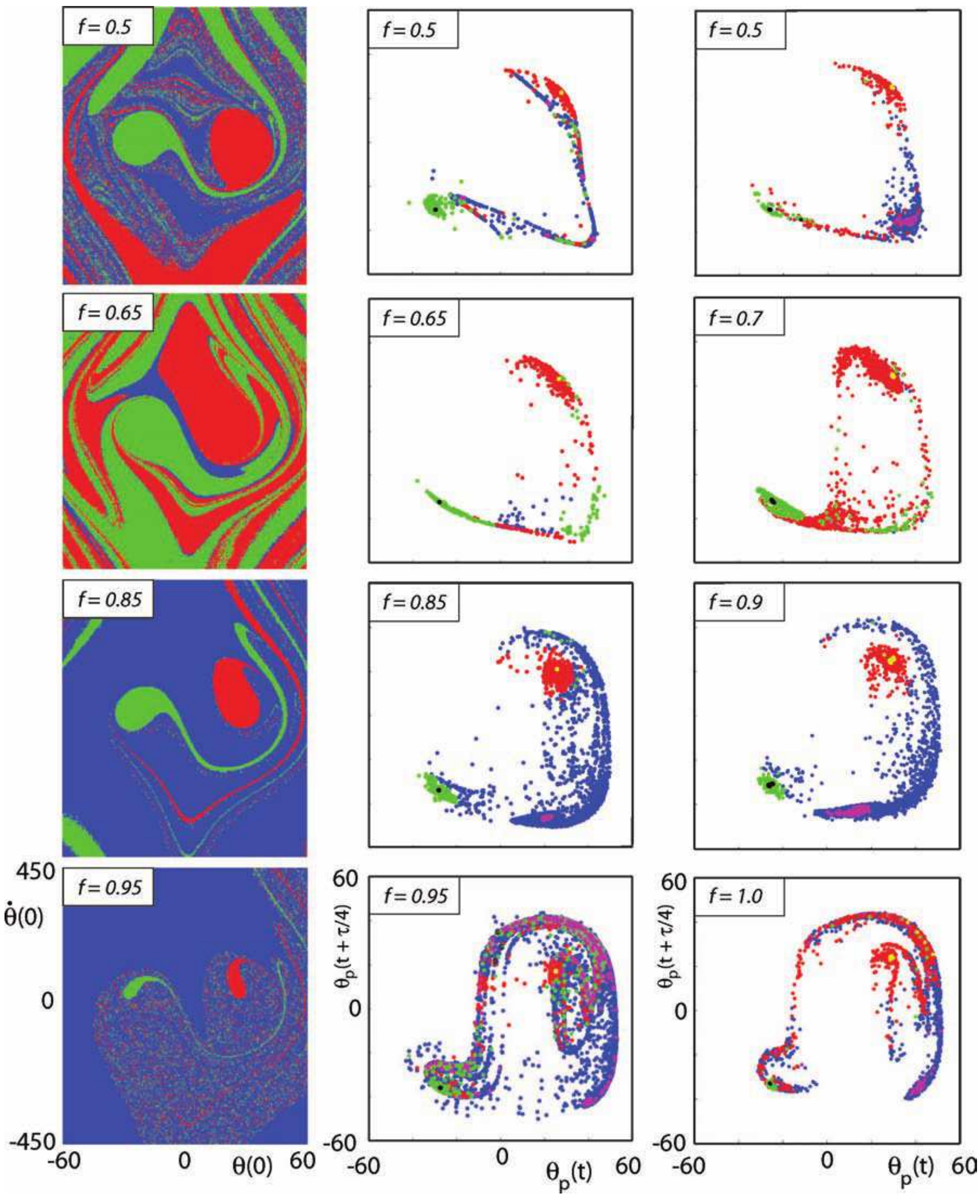


Figure 6: Initial condition plots for a frequency range during the transition into cross-well motion [47]. Left column: numerical simulation from a regular grid; centre column: numerically simulated results mimicking the experimental stochastic interrogation; right column: experimental data. Courtesy of Lawrence N. Virgin, Duke University, Durham, NC, USA.

3 Review of the literature on nonlinear system identification in structural dynamics over the past 10 years

It was proposed in Ref. [1] to regard the identification of nonlinear structural models as a progression through three steps, namely detection, characterisation and parameter estimation, as outlined in Fig. 7. An important change of paradigm has blossomed over the past few years in the third step of this process, as researchers have progressively recognised the importance of quantifying uncertainties in nonlinear system identification. This has given rise to methods estimating parameters together with, *e.g.*, confidence bounds or distributions. In this context, the Bayesian framework put forward by Jim Beck and his collaborators [50, 51] is currently drawing noticeable attention in the community [52, 53, 54]. This class of methods is appealing since it can facilitate the characterisation step by finding the optimum model within a set of competing model structures [55]. Other approaches, including nonparametric probabilistic [56] and nonprobabilistic [57] methods, have also been considered for uncertainty analysis of nonlinear mechanical systems.

1. Detection: *Is there?*

   Ascertain if nonlinearity exists in the structural behaviour, *e.g.*, yes.

2. Characterisation: *Where? What? How?*

   (a) Localise the nonlinearity, *e.g.*, at the joint;
   (b) determine the type of nonlinearity, *e.g.*, Coulomb friction;
   (c) select the functional form of the nonlinearity, *e.g.*, $g(q, \dot{q}) = c\,\text{sign}(\dot{q})$.

3. Parameter estimation: *How much?*

   Calculate the coefficients of the nonlinearity model and quantify their uncertainty, *e.g.*, in a probabilistic sense, $c \sim \mathcal{N}(5.47, 1)$.

Figure 7: Identification process for nonlinear structural models.

The aim of the present section is to review the key developments which arose during the 2006 – 2016 decade towards applying nonlinear system identification to complex structures. The main focus of this literature survey is on parameter estimation methods. Their classification into seven categories, namely linearisation, time- and frequency-domain methods, time-frequency analysis, modal methods, black-box modelling and numerical model updating, is adopted following Ref. [1]. The subject of nonlinear system identification in structural dynamics is vast, and we stress that this paper is inevitably biased towards those areas which the authors are most familiar with, and this of course means those areas which the authors and colleagues have conducted research in.

3.1 Linearisation methods

There are, at least, two good reasons not to linearise the behaviour of a nonlinear vibrating structure around its operating point. First, linearised models are essentially valid for a unique set of excitation parameters, preventing them from being interpolated, *i.e.* used to predict the structural response at lower forcing levels. Second, they fail to predict intrinsically nonlinear phenomena, including harmonics, jumps or modal interactions. Yet, using linear system identification to model nonlinear structures has persisted to be a popular solution. The reason for this is probably the maturity of linear techniques [58, 59, 60, 61], and the fact that most standardised design and certification procedures followed in industry still rely on linear structural models, *e.g.*, flutter prediction of aircraft [17] or load analysis of coupled launcher-satellite systems [62].

In the case of random vibrations, the contributions of Schoukens and co-workers provide a solid theoretical framework to derive the best possible linear model of a nonlinear system, in a least-squares optimality sense, termed best linear approximation (BLA) [63]. The calculation of the BLA is performed in the frequency domain, and most frequently takes advantage of periodic excitations, though nonstationary signals can also be addressed [64]. An appealing asset of this approach is that the level of nonlinear distortions which is not captured by the linear model can be assessed. Using a carefully-selected set of input frequencies, distortions originating from odd and even nonlinearities can also be distinguished. The BLA of a wet-clutch test rig was analysed in Ref. [65]. Fig. 8 shows another application to an F-16 fighter aircraft [66], where the BLA is represented in black. The levels of odd and even nonlinearities plotted in red and blue, respectively, are seen to be substantially larger than the noise level displayed in green. We remark that another frequency-domain linearisation method applicable to random data was introduced in 2009 by Grange *et al.* [67]. In this latter paper, the nonlinear dynamics of break squeal was studied by seeking a linear model which synthesises at best the measured power spectral density of the nonlinear system output.

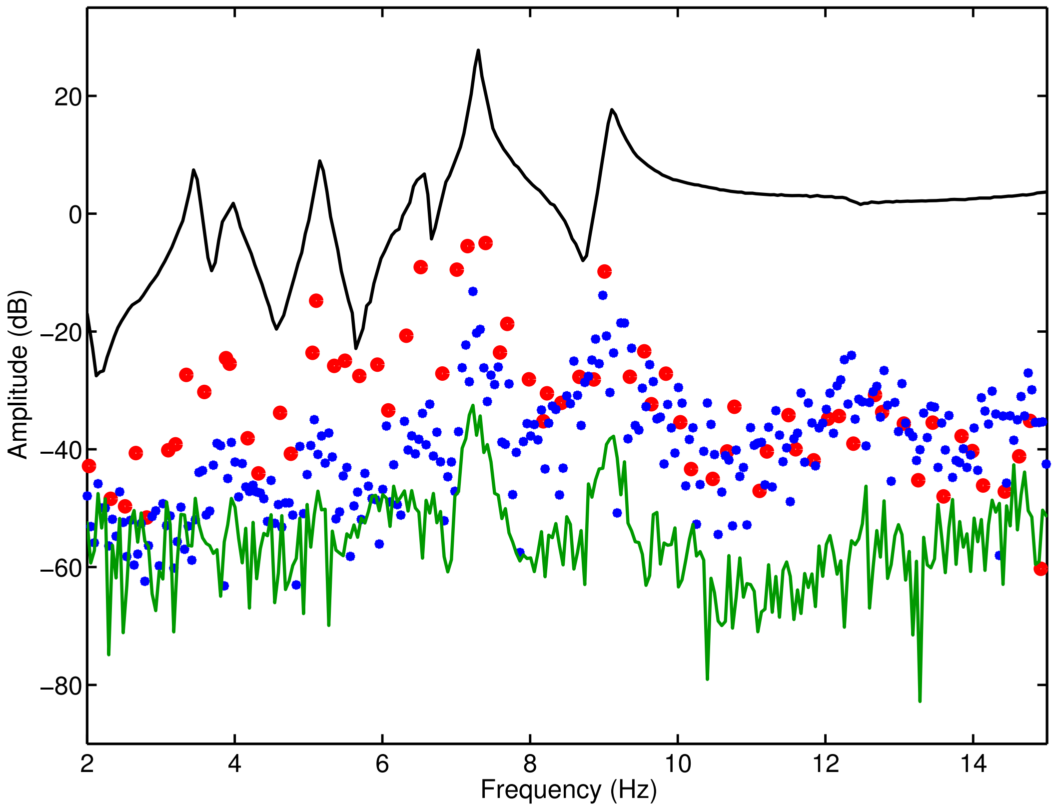


Figure 8: Best linear approximation (BLA) of an F-16 fighter aircraft (in black), odd (in red) and even (in blue) nonlinear distortions, and noise level (in green) [66]. Courtesy of Johan Schoukens, Vrije Universiteit Brussel, Brussels, Belgium.

In the case of harmonic vibrations, a classical linearisation methodology consists in assuming that the system of interest only responds at the excitation frequency. This single-component harmonic balance simplification procedure is referred to as the describing function method in the literature [68], and has enjoyed some progress since 2006 [69, 70]. Recently, Wang followed a similar reasoning to propose the equivalent dynamic stiffness mapping technique [71], and tackled the experimental identification of a metal mesh damper element.

Finally, some authors have considered the use of time-varying models as linearisation tools, suggesting that, by analysing nonlinear vibrations over small portions of time, linear system identification may reasonably well apply. Sracic and Allen pursued that idea by fitting linear time-periodic models to transient data recorded in response to slight perturbations superposed to sustained periodic excitations [72]. The method was originally demonstrated using single-degree-of-freedom systems, but its application to multiple degrees of freedom followed in 2014 [73]. In Ref. [74], linear subspace identification was exploited to represent the time-varying modal properties of various nonlinear systems through an ensemble of linear state-space models. In particular, the behaviour of a seven-storey building subject to three historical earthquake base excitations was studied. Fig. 9 shows the time evolution of the first linearised natural frequency of the structure (left column) and of a measure of its global stiffness (right column). The severe decrease in the two quantities over time resulting from damaged structural members is clearly visible.

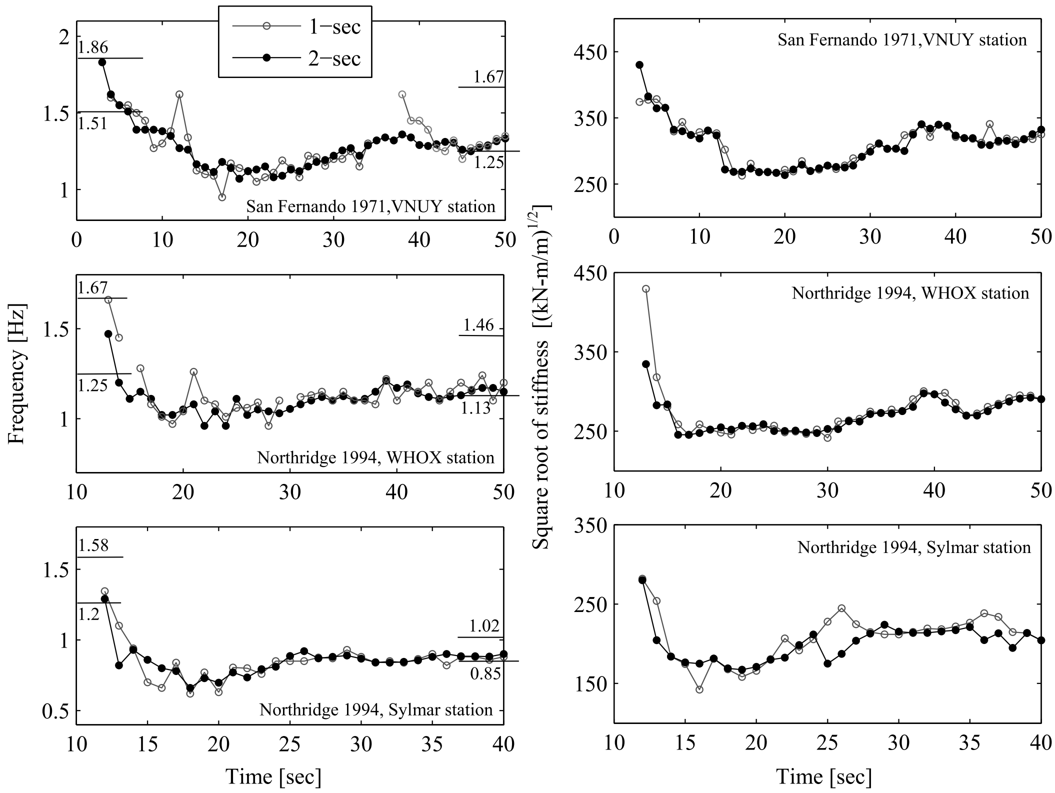


Figure 9: First linearised natural frequency (left column) and global stiffness measure (right column) of a seven-storey building subject to three historical earthquake base excitations [74]. Circle and point markers correspond to divisions of the measured time histories into windows of 1 and 2 s, respectively. Courtesy of Babak Moaveni, Tufts University, Medford, MA, USA.

3.2 Time-domain methods

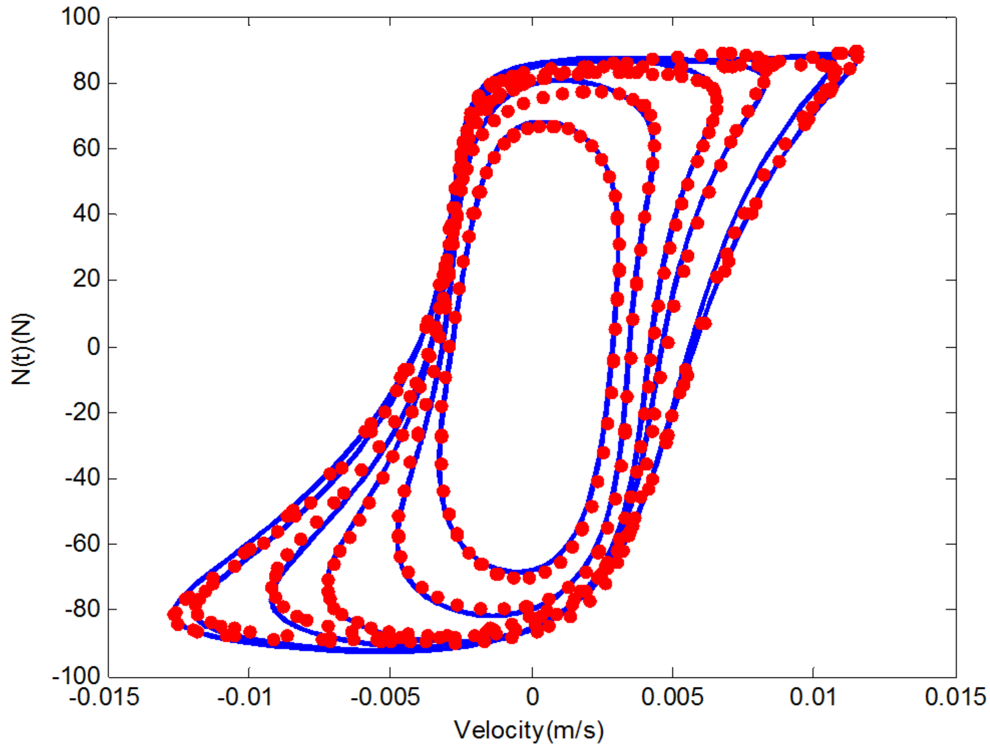
The time-domain identification of nonlinear structural models exclusively relies on processing time series, *e.g.*, impedance head and accelerometer raw records. In 2006, a substantial body of literature concerning time-domain identification was surveyed [1]. Three major identification methods were distinguished, namely restoring force surface (RFS) analysis, nonlinear autoregressive moving average with exogenous inputs (NARMAX) modelling, and Hilbert transform-based data decomposition (progress in this latter research direction is reviewed in Section 3.4). Recent publications identifying NARMAX models are mostly due to Worden and co-workers [75, 76, 77, 78]. In general terms, it is admitted that applying the NARMAX framework to large-scale structures is a difficult endeavour, as it suffers from a rapid explosion in the number of parameters, even in the case of systems with reasonable dimension.

The RFS method, which constitutes one of the earliest identification methodologies [79], has continued to attract attention during the past decade. Parameter estimation based on the RFS method is commonly restricted to systems with a few degrees of freedom since it consists in the direct fitting of Newton's second law. Nevertheless, the method can still be exploited to visualise qualitatively nonlinear restoring forces in complex structures [80]. The simplicity of the RFS method and its intuitive outcome certainly explain its success in the community. Applications of the RFS method to the identification of nonlinear stiffness mechanisms have been numerous. In Ref. [81], the prediction capabilities of a nonlinear restoring force model of two elastomer specimens were compared to rheological equations based on traction-compression and shear test data. The hardening-softening behaviour of an elastomagnetic suspension was studied in Ref. [82], and hardening, smooth nonlinearities in a leaf-spring-based tuned mass damper and in a robot arm were identified in Refs. [83] and [84], respectively. Piecewise characteristics were also estimated experimentally in a micro-beam system [85] and a satellite structural prototype [23].

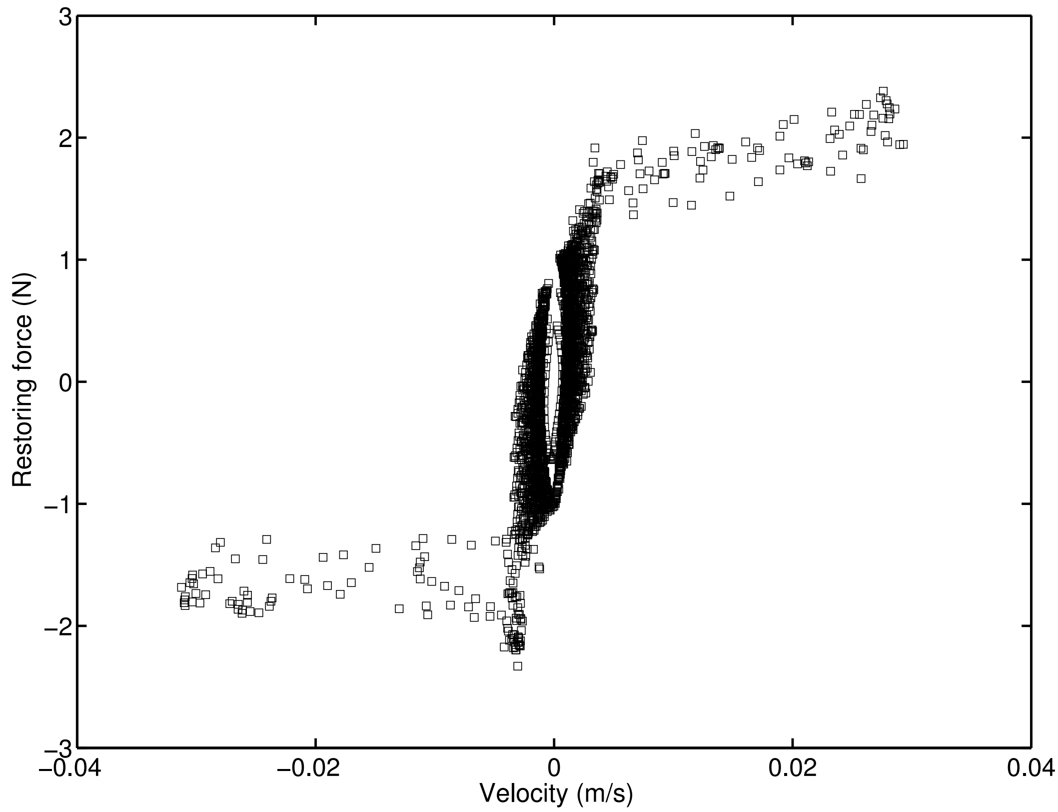
Complex damping nonlinearities have equally been addressed in the technical literature. For example, the RFS and the nonlinear identification through feedback of the outputs (NIFO) methods were assessed in the identification of an automotive damper [86]. The hysteresis loops of a Bouc-Wen oscillator undergoing a stiffness degradation over time and of a shear building structure equipped with a shape memory alloy damper were characterised in Refs. [87] and [88], respectively. Hysteresis dynamics in a frictional contact boundary support was also analysed by Ahmadian and co-authors [89], as seen in Fig. 10 (a), where an hysteretic restoring force calculated experimentally (in red) is compared with the prediction of a Valanis model (in blue). Finally, attention has been given to the identification of combined elastic-dissipative nonlinearities, *e.g.*, in Refs. [80, 90, 91]. In Ref. [80], discontinuous softening stiffness and Coulomb friction were shown to affect the mounting interface of a payload in an F-16 fighter. Fig. 10 (b) depicts the identified restoring force versus the relative velocity measured across the interface, and displays a Coulomb-type of behaviour, including hysteresis at low velocity and stiction for negative force values.

Nonlinear subspace methods, originally proposed in the control literature [92], were first applied to mechanical systems by Marchesiello and Garibaldi in 2008 [93]. The time-domain nonlinear subspace identification (TNSI) method they introduced represents a major advance across the field. TNSI is a nonlinear generalisation of the classical time-domain linear subspace identification algorithms [94]. It relies on the feedback interpretation of nonlinear structural dynamics discussed earlier by Adams and Allemang [95, 96], which views nonlinearities as additional forces applied to the underlying linear structure. The implementation of the TNSI method builds on robust numerical tools, such as the QR and singular value decompositions, yielding superior accuracy compared to competing approaches like the NIFO [96] and the conditioned reverse path (CRP) [97] techniques.

Demonstration of the TNSI method on academic structures possessing smooth and nonsmooth nonlinearities was achieved in Refs. [93] and [98], respectively. The identification of a complete spacecraft structure based on numerical data was also discussed in Ref. [99], where the concept of stabilisation diagram was extended to nonlinear system identification. In a recent effort, the influence of nonphysical poles on the TNSI method was analysed [100]. By interpreting nonlinear coefficients as the ratio of two so-called extended frequency response functions and performing truncated modal expansions of its numerator and denominator, nonphys-



(a)



(b)

Figure 10: Restoring force surface (RFS) method applied to complex damping nonlinearities. (a) Hysteretic restoring force in a frictional contact boundary support calculated experimentally using the RFS method (in red) and corresponding prediction of a Valanis model (in blue) [89]. Courtesy of Hamid Ahmadian, Iran University of Science and Technology, Tehran, Iran. (b) Coulomb-type restoring force, including hysteresis at low velocity and stiction for negative force values, identified in the mounting interface of a payload in an F-16 fighter [80].

ical modes were successfully eliminated. Fig. 11 shows the nonlinear stabilisation diagram computed in the identification of an experimental multi-storey structure affected by a cubic nonlinearity. Nonphysical poles, which appear in black, are discriminated from physical structural modes emerging as stabilised columns of poles.

The TNSI method is, to date, a very promising approach. Assuming an accurate characterisation of nonlinear behaviour, it can potentially tackle complex structures involving multiple inputs and outputs, strong nonlinearities, and closely-spaced and highly-damped modes, as numerically proven in Ref. [99].

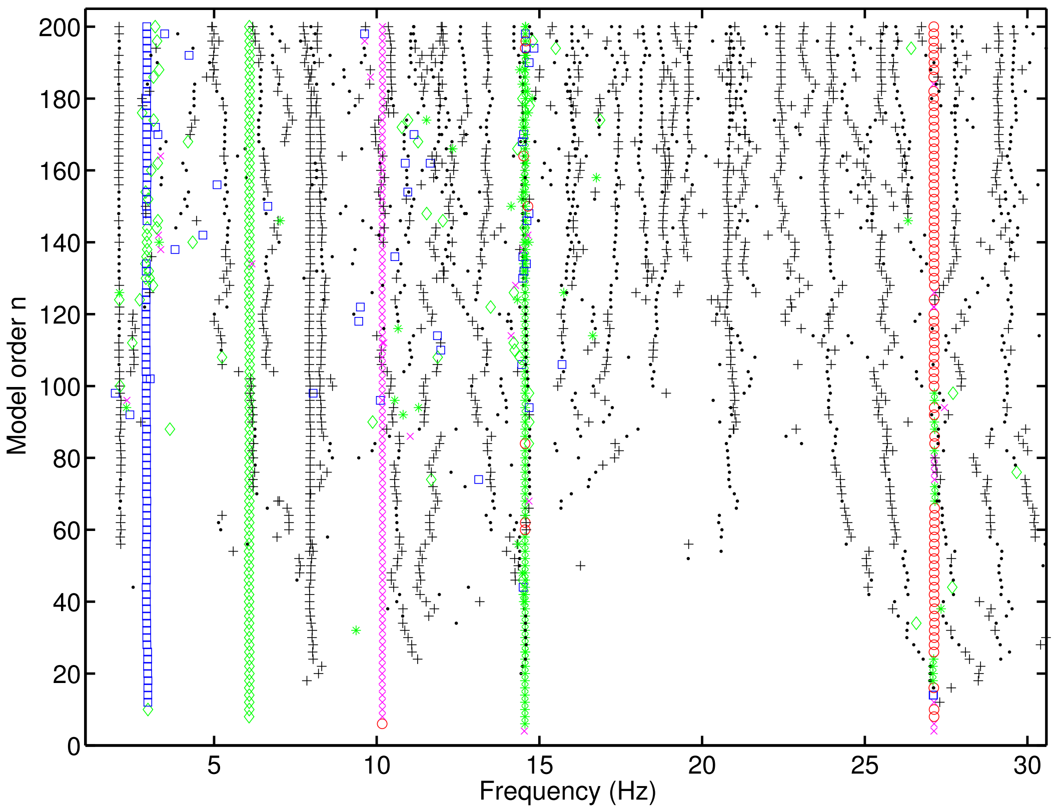


Figure 11: Nonlinear stabilisation diagram computed using the time-domain nonlinear subspace identification (TNSI) method, where nonphysical poles (in black) are discriminated from physical structural modes emerging as stabilised columns (in colour) [100]. Courtesy of Stefano Marchesiello, Politecnico di Torino, Turin, Italy.

3.3 Frequency-domain methods

Data processed in frequency-domain identification are more varied than in the time domain, and can take the form of Fourier spectra, frequency response and transmissibility functions, or power spectral densities. In Ref. [1], two methods were pointed out as promising frequency-domain approaches, namely the nonlinear identification through feedback of the outputs (NIFO) [96] and the conditioned reverse path (CRP) [97] methods. In Ref. [101], a modified H_2 estimator of frequency response functions (FRFs) was introduced to enhance the performance of NIFO in the presence of process and measurement noise. Estimates of underlying linear and nonlinear parameters of different simple numerical systems were shown to benefit from this improved H_2 algorithm. Parameter estimation using the NIFO method in a reduced-order space was also attempted in Ref. [102] and demonstrated experimentally on a clamped-clamped beam structure. Recent developments in reverse path identification include the possibility to use unconditioned spectra in the single-input case [103], and to locate nonlinear degrees of freedom [104, 105]. Besides the NIFO and reverse path techniques, the harmonic balance method was also utilised for nonlinear system identification based on multiharmonic [106] and multiple test [107] data.

The use of functional series, and in particular of the Volterra series [108], is another traditional way of addressing system identification in the frequency domain. The multi-dimensional kernels of Volterra series are the nonlinear generalisation of the classical impulse response of linear systems, and the Fourier transform of the kernels are most often referred to as higher-order FRFs (HOFRFs) or generalised FRFs (GFRFs). The typical hindrance to Volterra identification of high-dimensional systems is the very high number of parameters to be estimated. In Ref. [109], orthogonalised Volterra series associated with Kautz filters were exploited to moderate this number in the numerical identification of a beam. HOFRFs were estimated in Ref. [110] in the special case of a bilinear nonlinearity representing a breathing crack. Volterra series have also formed the basis of the new concept of nonlinear output frequency response functions (NOFRFs) developed during the last 10 years or so by Billings and co-workers [111, 112]. NOFRFs found application in linear [113] and nonlinear [114] system identification, though their complete potential towards identifying real structures has not been fully revealed yet. We finally mention the introduction in 2010 of a multi-degree-of-freedom extension [115] of the associated linear equations models [116], which are related to Volterra operators.

A frequency-domain counterpart to the time-domain nonlinear subspace identification (TNSI) method discussed in Section 3.2 was proposed in 2013 in Ref. [117]. The frequency-domain nonlinear subspace identification (FNSI) method generalises existing linear frequency-domain subspace techniques [118, 119] to nonlinear mechanical systems. It possesses the same foundations as TNSI, *i.e.* the feedback interpretation of nonlinear structural dynamics and the use of linear algebra decompositions. Processing data in the frequency domain offers the possibility to focus on specific frequency ranges. This allows to substantially reduce the computational burden involved in the identification and, in turn, to calculate accurately a great number of nonlinear parameters. The FNSI method was applied to simple numerical and experimental systems in Ref. [117] and to a numerical benchmark beam structure in Ref. [120]. An academic experimental solar panel assembly featuring nonlinear bolted connections and impacts was also addressed in Ref. [121], where cubic splines were utilised as nonlinear basis functions. The TNSI and FNSI methods were compared in the identification of a spacecraft structure in Ref. [99], both in terms of estimation accuracy and parameter dispersion.

3.4 Time-frequency methods

Because nonlinear oscillations are inherently frequency-energy dependent, time-frequency transformations generally offer useful insight into the dynamics of nonlinear systems. Well-established methods, such as the wavelet and Hilbert transforms, have continued to be used during the last 10 years as nonlinear system identification tools, and, in particular, in the identification of backbone curves, as in Refs. [122, 123, 124, 125]. Moreover, two new techniques for the decomposition of multicomponent signals emerged during this period, namely the empirical mode decomposition (EMD) [126] and the Hilbert vibration decomposition (HVD) [127, 128].

The basic idea of EMD is to decompose the original signal into a sum of elemental components, the intrinsic mode functions (IMFs). The extraction process, termed sifting process, relies on a spline approximation of the lower and upper envelopes of the signal based on its extrema. To be amenable to the Hilbert transform, each IMF must satisfy two properties, *i.e.* the number of extrema and zero-crossings can differ by no more than one, and, at any point, the mean value of the envelopes defined by the local maxima and minima should be zero. It follows that an IMF is a monochromatic signal, the amplitude and frequency of which can be modulated. Taken collectively, the Hilbert spectra of the IMFs give a complete characterisation of a multicomponent signal in terms of amplitudes and instantaneous frequencies. A first effort to gain fundamental understanding of EMD in nonlinear structural dynamics was made in Refs. [129, 130]. More specifically, a one-to-one relationship between the analytically-realised slow-flow dynamics of a nonlinear system and the IMFs derived from measured time series was demonstrated. Based on this theoretical link, the slow-flow model identification method, a linear-in-the-parameters identification approach applicable to multi-degree-of-freedom nonlinear systems, was developed [129]. As discussed in Section 3.5, the correspondence between theoretical and empirical slow flow analyses was further utilised for modal identification using the concept of intrinsic modal oscillators (IMOs). EMD was also used in conjunction with perturbation analysis for nonlinear system identification in Ref. [131].

Although similar in spirit to EMD, the HVD method is a distinct approach towards decomposing a vibration signal into a series of monocomponents. HVD is based on the assumptions that the original signal is formed of a superposition of quasiharmonic functions, and that the envelopes of each vibration component differ. Ref. [127] proved that the instantaneous frequency of a multicomponent signal can be split into a slowly-varying part related to the instantaneous frequency of the monocomponent with the greatest amplitude, and a rapidly-varying asymmetric part. Thus, the frequency of the dominant monocomponent can be directly estimated through low-pass filtering of the instantaneous frequency of the complete signal. The other monocomponents can be extracted by recursively applying this process to the residual signal. As an illustration, the HVD analysis of the free response of a Duffing oscillator is shown in Fig. 12. The three extracted components represent the fundamental, third and fifth harmonic components. The HVD method was exploited for the identification of nonlinear systems with two degrees of freedom in Ref. [132], but has not yet been applied to larger-scale structures. It found other applications in structural dynamics, *e.g.*, in the design of nonlinear vibration absorbers [133] and the analysis of linear [134] and nonlinear [135] time-varying systems.

The EMD and HVD methods possess their own limitations and should be used with great care [136, 137], but they nonetheless represent important additions to the nonlinear structural dynamicist's toolbox.

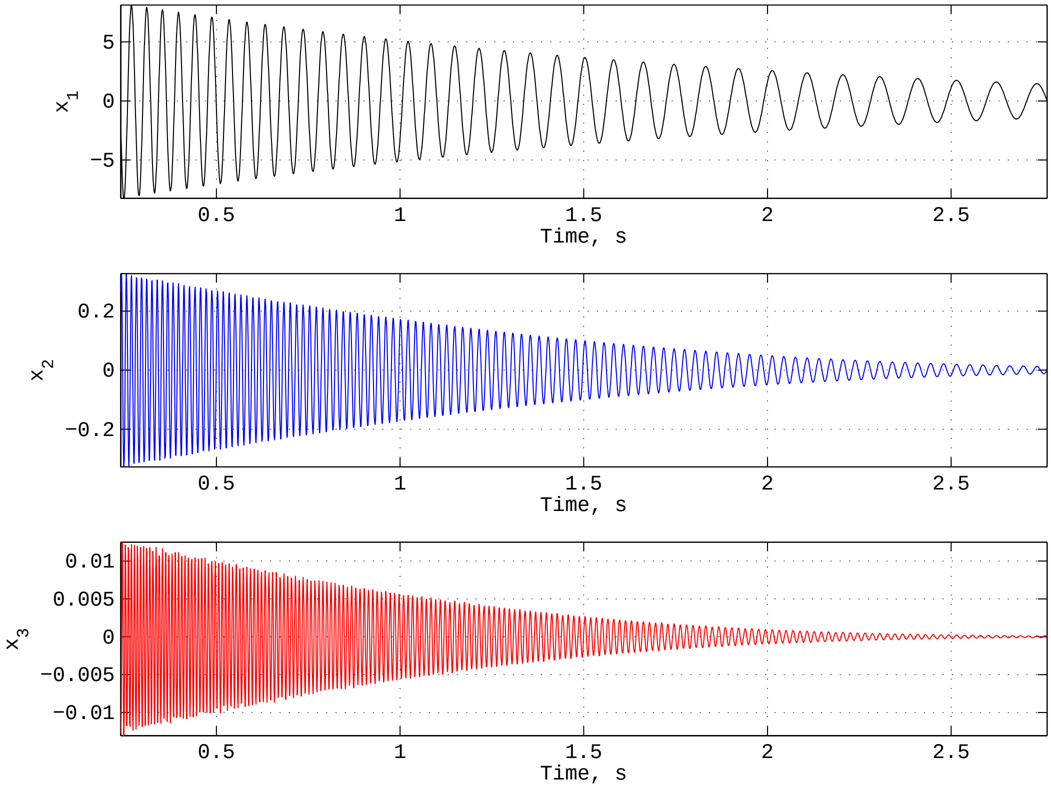


Figure 12: First three harmonic components of the free response of a Duffing oscillator as decomposed by the Hilbert vibration decomposition (HVD) method [127]. Courtesy of Michael Feldman, Technion – Israel Institute of Technology, Haifa, Israel.

3.5 Modal methods

Modal features, *i.e.* natural frequencies, damping ratios and mode shapes, form the basis of classical linear design strategies in engineering dynamics. They provide an effective and intuitive way to study the structural behaviour around resonances, which is one of the prime limiting factors as for integrity and certification. Experimental modal analysis of linear structures is now certainly mature [138]. Conversely, nonlinear modal identification is a quite recent research field, as confirmed by the very few related methods surveyed in 2006 in Ref. [1]. It is today a very active area of investigation, which has witnessed important progress during the last 10 years. This progress has been accompanied by the emergence of efficient algorithms to carry out theoretical nonlinear modal analysis, as reviewed in Ref. [139]. The combination of these experimental and numerical efforts has paved the way for innovative model updating methods, as discussed in Section 3.7.

The code for nonlinear identification from measured response to vibration (CONCERTO), proposed in Ref. [25], identifies an isolated nonlinear resonance based on stepped-sine data. Since CONCERTO relies on a single-harmonic assumption, it adopts a linearised view of nonlinear modal identification, and yields equivalent natural frequencies and damping ratios which vary with the amplitude of motion. The method was applied to an helicopter in Ref. [25] and to anti-vibration mounts using measured transmissibility functions in Ref. [140]. Another single-mode method, closely related to CONCERTO, was presented in Ref. [24]. Inspired by the notching procedures applied in space industry [62], it builds on the idea of keeping the response amplitude constant in the vicinity of a nonlinear resonance to compute equivalent modal properties function of the excitation level.

Unlike CONCERTO, the nonlinear resonant decay method (NLRDM) can perform multiple-mode identification by introducing nonlinear coupling terms in an otherwise linear modal-space model of the tested structure [141]. This method was recognised as one of the most promising nonlinear modal identification approaches in the early 2000s [1]. If no further developments of the theoretical foundations of NLRDM were brought during the last 10 years, the method was successfully validated using structures of increasing complexity, namely a two-degree-of-freedom system with freeplay [142], a single-bay panel structure [143], a wing with two stores connected by means of nonlinear pylons [144], a geometrically nonlinear joined-wing structure [145] and a complete transport aircraft [146].

The solid framework offered by the theory of nonlinear normal modes (NNMs) [32, 33, 34] has motivated throughout the last 10 years the development of rigorous nonlinear modal identification methodologies. Linear phase resonance testing, which consists in exciting the modes of interest one at a time using a multipoint, monoharmonic forcing at the corresponding natural frequency [147], was first extended to nonlinear systems in Ref. [148] following a two-step procedure. During the first step, a multipoint, multiharmonic excitation is applied to isolate an individual NNM with the aid of a nonlinear phase lag quadrature criterion [148] and of a nonlinear mode indicator function [149]. In particular, the quadrature criterion ascertains that a nonlinear structure vibrates according to one of its underlying conservative NNMs if every harmonic of the measured excitations and responses are 90 degree phase-lagged. In a second step, the excitation is turned off, and the complete frequency-energy dependence of the targeted mode is extracted by processing free-decay data. Fig. 13 illustrates the experimental identification of the first NNM of a nonlinear beam structure at five different forcing levels [149]. Modal curves are represented in Fig. 13 (a) in a two-dimensional space spanned by two acceleration signals. The associated modal shapes discretised at seven measurement locations along the beam are plotted in Fig. 13 (b). The highest excitation level in the two plots corresponds to a modification of the natural frequency from around 30 to 40 Hz. Applications of nonlinear phase resonance testing to other moderately complex structures were recently reported in the technical literature, in the case of a steel frame in Ref. [150], of a circular perforated plate in Refs. [45, 151, 152], and of a sliding mass with transverse springs and dry friction in Ref. [153].

An alternative to nonlinear phase resonance, which is potentially more robust to changes of stability and bifurcations, is discussed in Ref. [154]. This paper exploited a control-based continuation scheme to trace out the NNM backbone of the same sliding-mass setup as in Ref. [153]. Stephan *et al.* also performed the tracking of nonlinear modal parameters during free-decay responses [155]. To this end, they estimated discrete-time nonlinear state-space models by combining Bayesian smoother and expectation maximisation algorithms. Other valuable additions to the nonlinear phase resonance approach are the energy balance technique developed in Ref. [156], which allows to calculate the forcing amplitude necessary to excite an NNM at a given frequency, and the concept of phase-locked loop borrowed from the control literature [157]. Finally, we note that exciting nonlinear resonances opens interesting prospects in nonlinear boundary identification [158] and damage detection [159, 160, 161].

The simultaneous identification of multiple NNMs under broadband forcing was first attempted in Ref. [162], providing a generalisation of linear phase separation techniques [94, 163] to nonlinear systems. The proposed methodology integrates nonlinear system identification and numerical continuation. More specifically, acquired input and output data are first processed to derive an experimental state-space model of the tested structure. This state-space model is next converted into a modal-space model from which NNMs are computed using shooting and pseudo-arclength continuation. Ref. [162] compared this nonlinear phase separation technique to nonlinear phase resonance testing [148] based on numerical nonlinear beam data. Clearly, multimodal nonlinear identification enables the experimentalist to save measurement time, which, in view of the current pressure to accelerate testing campaigns, represents a significant advantage.

Multimodal nonlinear identification has also been studied in a number of papers through the direct decomposition of experimental measurements into a reduced set of low-dimensional intrinsic system features. These features do not generally correspond to NNMs because of the absence of superposition principle in non-

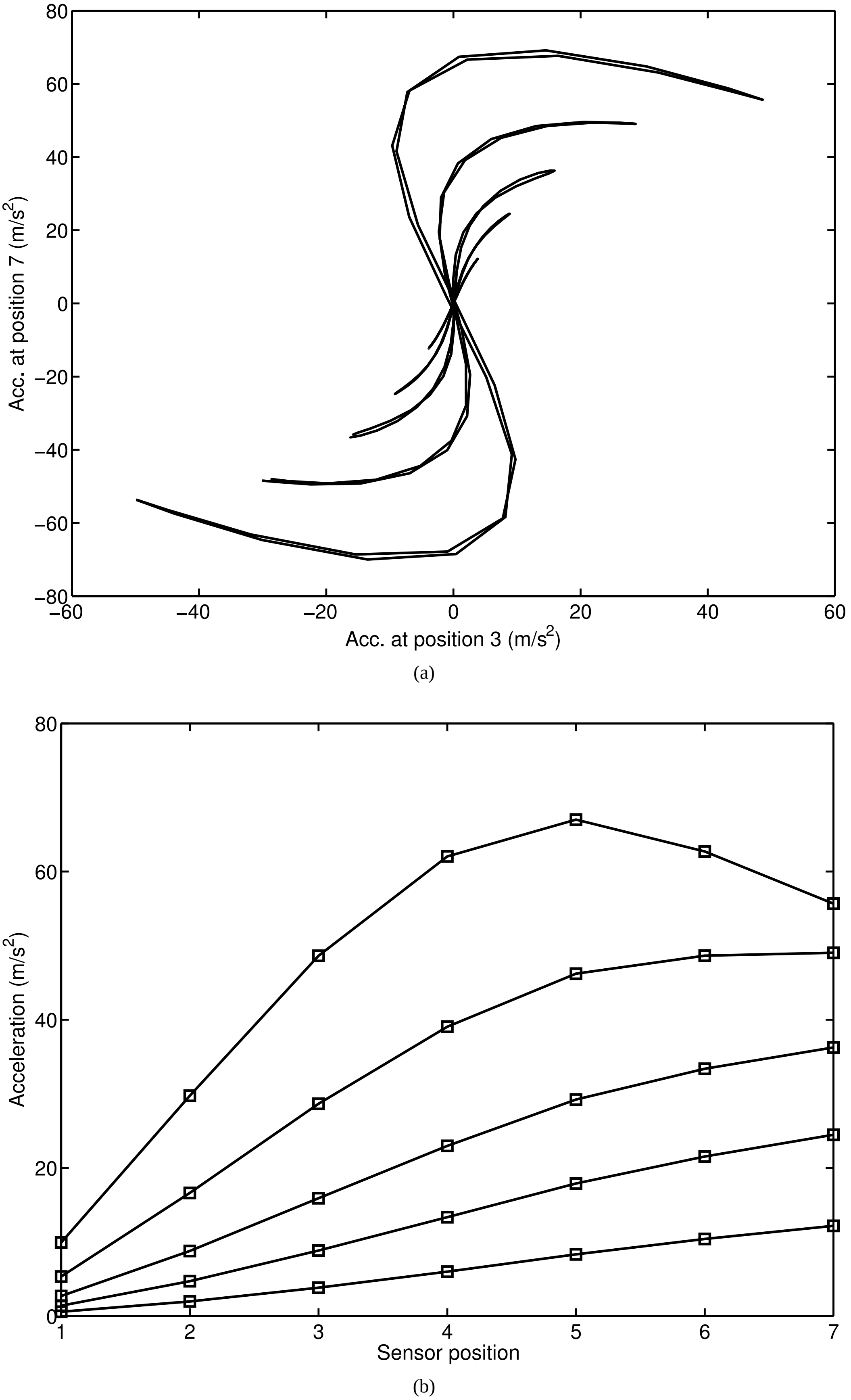


Figure 13: Experimental extraction of the first nonlinear normal mode (NNM) of a nonlinear beam structure at five different forcing levels using nonlinear phase resonance testing [149]. (a) Modal curves represented in a two-dimensional space spanned by two acceleration signals; (b) associated modal shapes featuring seven measurement locations along the beam.

linear dynamics, though they constitute some approximations of them. The strength of these approaches is that they require no characterisation of the observed nonlinearities. In the publications of Vakakis and co-workers, the so-called intrinsic modal oscillators (IMOs) are computed by applying the empirical mode decomposition (EMD) to data [164]. These oscillators synthesise by superposition the measured time series over different time scales, and may account for mode interactions via their forcing terms. The suppression of aeroelastic instabilities was addressed in Ref. [165] using this approach. Beam systems exhibiting friction and vibro-impacts were also successfully identified in Refs. [166] and [167], respectively. Fig. 14 displays the first ten intrinsic mode functions (IMFs) resulting from applying EMD to acceleration data measured on a vibro-impact beam [167]. Impact locations are specified using vertical dashed lines and the frequencies of the different IMFs are obtained by analysing wavelet transform plots.

EMD was similarly exploited by Poon and Chang to numerically identify a two-degree-of-freedom shear-beam building system [168]. A comparison between EMD and the zeroed-early time fast Fourier transform is to be found in Ref. [169]. Other contributions to multimodal nonlinear identification by means of advanced signal processing tools include the use of the Hilbert-Huang transform combined with a conjugate-pair decomposition [170], the so-called smooth decomposition [171], and the sliding window proper orthogonal decomposition (POD) [172]. An attempt to generalise the POD towards nonlinear modal analysis is finally reported in Ref. [173], where the transformation from data to approximations of NNMs is realised in a machine learning context.

3.6 Black-box methods

Solving a nonlinear system identification problem essentially involves selecting a model structure based on available prior knowledge, and processing data to estimate its parameters. Accessing prior knowledge, *i.e.* performing nonlinearity characterisation, may however prove difficult in many circumstances owing to the highly individualistic nature of real nonlinearities. For instance, common joints between substructures most often feature very complex physics, including heterogeneous stick-slip behaviour at the microscopic level, hysteresis, Hertzian contact and local concentrations of stresses and strains. This renders virtually impossible the specification of an accurate, physically-motivated model in terms of macroscopic nonlinear stiffness and damping lumped elements. Black-box models, which incorporate no prior knowledge but take advantage of a sufficiently rich and flexible mathematical structure to capture all relevant physics in measured data, may prove useful in these situations [174]. Typical black-box identification approaches comprise nonlinear autoregressive moving average with exogenous inputs (NARMAX) models (discussed in Section 3.2), artificial neural networks, fuzzy networks, statistical learning theory and kernel methods. The tutorial on natural computing in Ref. [175] provides an overview of most of these different approaches together with an exhaustive list of, or links to, useful references in the field.

Over the past decade, neural network-based identification has clearly remained the most popular black-box modelling technique within the structural dynamics community. Combined use of a nonlinear autoregressive with exogenous inputs (NARX) model and a neural network to identify geometrically nonlinear steel plates was discussed in Ref. [176]. Ref. [177] proposed an efficient strategy for selecting the control points, or knots, in B-spline neural network identification. Initialisation issues of neural networks were also addressed by Pei and Masri in Refs. [178, 179]. In terms of applications, neural networks have been mostly exploited to address dissipative systems. In Ref. [90], an experimental, two-degree-of-freedom joint element was identified using a two-layer feedforward network. A neural network-based output error model of a continuously variable, electrohydraulic semi-active damper for a passenger car was derived in Ref. [180]. Magnetorheological dampers were also studied using feedforward and recurrent networks in Ref. [181] and using radial basis functions networks in Ref. [182]. Finally, we note the identification of an experimental turbojet engine carried out in Ref. [183] considering fuel flow rate and rotational speed as input and output quantities, respectively.

Recently, a new black-box model structure based on a state-space representation of measured data was proposed [184]. It builds on nonlinear model terms constructed as a multivariate polynomial combination of

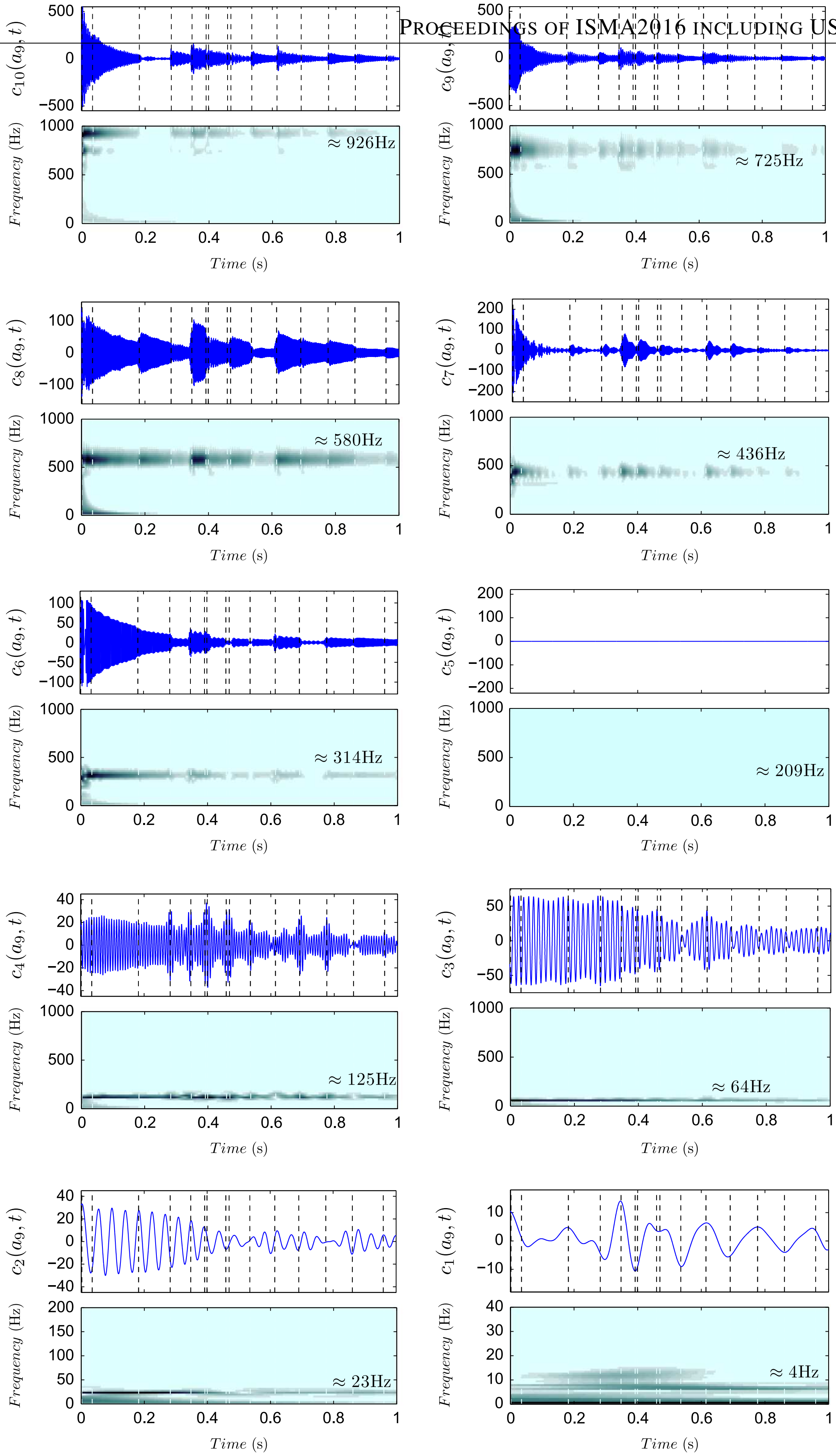


Figure 14: First ten intrinsic mode functions (IMFs) resulting from applying the empirical mode decomposition (EMD) to acceleration data measured on a vibro-impact beam [167]. Impact locations are specified using vertical dashed lines and the frequencies of the different IMFs are obtained by analysing wavelet transform plots. Courtesy of Alexander F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL, USA.

the state and input variables. This approach proved successful in the identification of very diverse nonlinear systems, including a magnetorheological damper [184], a wet-clutch device [65], and a Li-Ion battery [185]. However, as usual in black-box identification, it suffers from a combinatorial increase in the number of parameters because of the multivariate nature of the representation. Attempts to improve the parsimony of nonlinear state-space models by employing tensor decomposition techniques are reported in Ref. [186].

3.7 Model updating methods

Despite the ever-increasing sophistication of computer-aided techniques and, in particular, of the finite element method, numerical models must still be confronted with experimental data and subsequently updated to improve their fidelity. Joints and interfaces between substructures are probably the best example where a purely numerical modelling approach is bound to failure [11].

During the last decade, there has been significant progress in the definition of nonlinearity-sensitive features for model updating. Building upon the work of Meyer and Link [187], who defined single-harmonic frequency residuals, several researchers have developed a multiharmonic comparison between simulation predictions and test results. The multiharmonic balance method, which calculates the response of a nonlinear system to a periodic excitation, was combined with the extended constitutive relation error method for establishing a well-behaved metric for test-analysis correlation in Ref. [188]. The performance of the proposed metric was compared with that of other metrics in Ref. [189]. Multiharmonic balance was also coupled with a signal processing tool which extracts multiharmonic frequency responses from experimental data in Ref. [190]. Fig. 15 illustrates this latter approach by comparing the experimental and simulated harmonic responses of a clamped-clamped beam undergoing large displacements. An alternative methodology to numerically calculate nonlinear frequency response curves exploits shooting and pseudo-arclength continuation, as achieved in Ref. [191]. Finally, the time-domain counterpart of nonlinear frequency responses, *i.e.* Volterra kernels, was exploited for finite element model updating in Ref. [192].

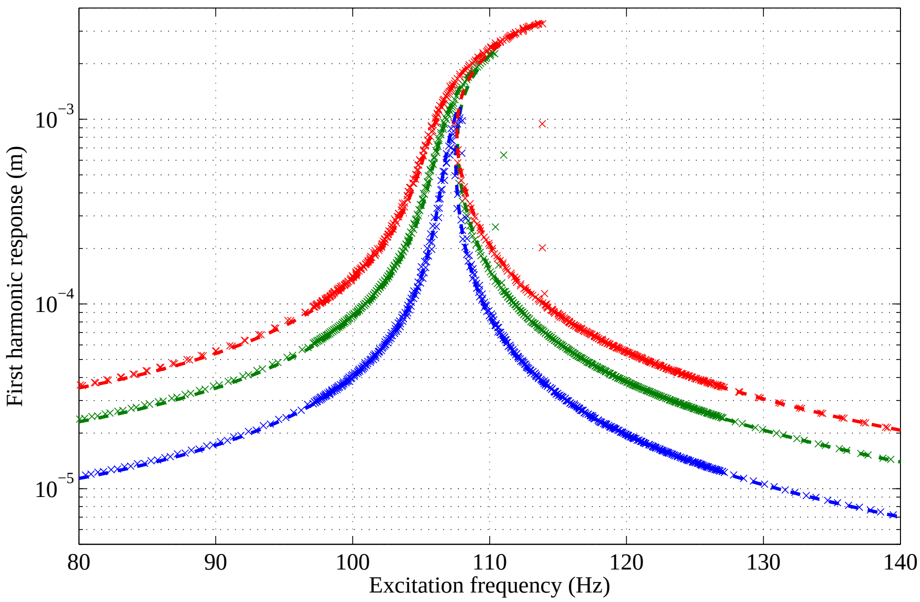


Figure 15: Comparison at three excitation levels between the simulated multiharmonic balance response (dashed curves) of a clamped-clamped beam undergoing large displacements and the corresponding experimental response (cross markers). Courtesy of Jean-Jacques Sinou, Ecole Centrale de Lyon, Lyon, France.

In view of the effectiveness of modal parameters for linear model updating, metrics relying upon nonlinear modes and frequencies have been developed. If empirical modes, such as proper orthogonal modes, were used for test-analysis correlation in the early 2000s [193, 194], nonlinear normal modes (NNMs) have recently been proposed as a more rigorous dynamic feature [195, 196]. In Ref. [195], experimental plots of the frequency-energy dependence of NNMs were extracted from measured multimodal responses using the wavelet transform. Assuming that the system of interest is weakly damped and that the effect of damping is purely parasitic, model updating was performed through the comparison with numerical frequency-energy plots constructed from the periodic orbits of the underlying conservative system. Ref. [196] followed a similar philosophy, but undamped NNMs were excited in isolation using nonlinear phase resonance testing [148]. Other residuals utilising modal information were suggested in the civil engineering community, such as the modal flexibility residual [197] and time-varying modal parameters [198].

Even if it complicates the model updating process significantly, the departure from the traditional paradigm to modelling systems in a deterministic manner has gained a lot of attention during the past 10 years. There exists a myriad of probabilistic and nonprobabilistic methods for characterising and propagating uncertainty in structural dynamics, *e.g.*, Refs. [199, 200, 201]; their description is clearly beyond the scope of this review paper. From our perspective, the stochastic framework based on Bayes' theorem has emerged in the community as the most prevalent approach to performing model updating of nonlinear systems [53]. In Bayesian inference, the probability distribution of the uncertain parameters can be updated using, for example, efficient Markov Chain Monte Carlo (MCMC) simulation techniques [202, 203]. One of the first contributions dealing with Bayesian updating of nonlinear structural models is due to Yuen and Beck [204]. To resolve the limitations inherent to extended Kalman filtering [205] and address strongly nonlinear systems with non-Gaussian uncertainty, unscented Kalman [206] and particle [52, 207, 208, 209, 210] filters were exploited later for nonlinear system identification. Recent developments include the use of Gaussian processes as emulators to greatly accelerate Bayesian sensitivity analysis [211], novel MCMC algorithms [212, 213], model selection [214], online estimation [215], identification under changing ambient conditions [216] and large-scale parallelisation [217].

4 Conclusions and future research directions

This survey paper reviewed the developments in the area of nonlinear system identification in structural dynamics achieved during the past 10 years, and emphasised the progress realised over that period of time. If the overview paper published in 2006 [1] concluded that the identification of simple continuous structures with localised nonlinearity was a reality, we can affirm today that the identification of higher-dimensional models of structures vibrating in strongly nonlinear regimes is within reach. This significant progression in the state of the art is certainly to be attributed to the better understanding of nonlinear vibrations gained by the structural dynamics community, to the utilisation of more rigorous theoretical concepts and tools, to the increased maturity of numerical algorithms, and to the emergence of advanced testing strategies.

New nonlinear system identification methods have been developed, consolidating the progress and addressing some of the challenges raised in 2006:

- building upon the nonlinear resonant decay method (NLRDM), nonlinear modal testing has seen substantial improvement. Although it still relies on careful and time-consuming experimental procedures, the identification of nonlinear modes under stepped-sine and broadband forcing can now be performed.

- new approaches for the decomposition of multicomponent signals, including the empirical mode and Hilbert vibration decompositions, were proposed, facilitating time-frequency analysis of nonlinear systems of moderate to high modal density.

- nonlinear subspace identification was introduced, thereby extending the concept of stabilisation diagram to nonlinear systems. Nonlinear subspace algorithms outperform the nonlinear identification through feedback of the outputs (NIFO) and conditioned reverse path (CRP) methods, which were viewed as two of the most effective nonlinear system identification techniques in 2006.

- model updating based on Bayesian inference has been increasingly exploited to quantify uncertainty in nonlinear mechanical systems.

Interestingly enough, modal testing, time-frequency analysis, subspace identification and Bayesian inference are commonly-used tools for linear system identification. Their nonlinear generalisations have therefore the potential to be understood by practising engineers, which paves the way for their transfer into an industrial context. It is also clear from this discussion that nonlinear system identification in structural dynamics has retained its toolbox philosophy. In view of the highly individualistic nature of nonlinear mechanical systems, we do not see any compelling evidence why this would change in the years to come.

Despite this evident progress, nonlinear system identification remains a difficult exercise, and several important challenges are still ahead of us:

- a number of applications require additional attention and developments. This is, for instance, the case of composite materials and micro-electromechanical systems. However, the most pressing need probably concerns the accurate modelling and robust identification of joints interfacing subcomponents. It is a subject of recent focus in the structural dynamics community [218, 219, 220], not least because of the growing impact of different types of nonlinearities on their dynamics, including friction, micro- and macro-slip, and gaps [166, 221, 222, 223, 224, 225].

- nonlinear structures are rarely identified *in situ*. Practical constraints such as the absence of excitation measurement, the time-varying nature of the problem and the estimation of parameters in real time have barely been addressed in the literature.

- the additional burden brought by nonlinear analysis and identification can be substantial. The resulting efforts should therefore be carefully traded against the impact of nonlinearity in the decision process.

Besides pushing the capabilities of existing methods further and addressing ever more complex applications, future efforts should also consider investigating the state of the art in connected research fields.

The electrical and control community has for a very long time driven the progress in system identification [226]. Traditionally, this community has concentrated its efforts on developing black-box models. For linear systems, models based on impulse responses [61] and transfer functions [58] have become standards. In the presence of nonlinear distortions, the nonlinear state-space modelling framework [184, 186] has shown great promise both in terms of flexibility and interpretability of representation. The adoption of this framework in structural dynamics requires our community to abandon the classical white-box modelling paradigm, undoubtedly successful for modal analysis, but too limiting for nonlinear system identification. An attractive compromise that should certainly deserve attention in the years to come is the emergence of grey-box state-space models, incorporating prior knowledge and engineering insight to moderate the number of parameters to estimate.

Similarly, the reward for venturing into the recent developments of the machine learning community is potentially considerable. The excellent review paper by Worden *et al.* [175] argues for this research investment. Among the different concepts familiar to machine learning researchers, the Gaussian process model structure [227] and the estimation of parameters through Bayesian marginalisation [53] are probably those which should be embraced by our community in the first place.

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