Critics Workshop on Critical Transitions in Complex Systems

Instability and abrupt changes in marine ice sheet behaviour

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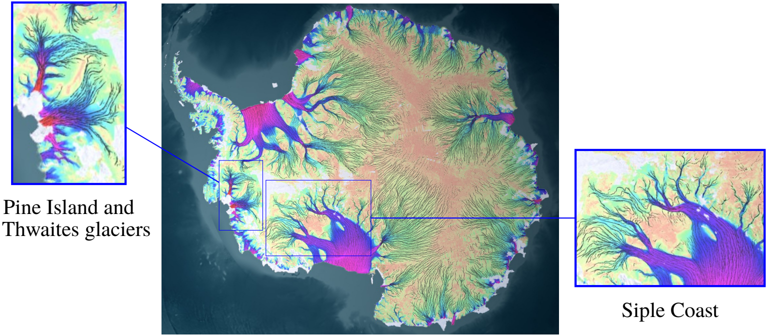
Monday 5 September 2016

Motivation

- Ice streams (narrow corridors of fast-flowing ice) drain over 90% of the Antarctic mass flux. Ice stream dynamics and stability are key factors for Antarctic mass balance and future contribution to sea-level rise.



Pine Island and Thwaites glaciers

Siple Coast

Figure: Full map of Antarctic ice flow deduced from satellite data [NASA/JPL-Caltech/UCI].

Motivation

- In this presentation, I will focus on two physical models that have been proposed to explain current and past behaviours of ice streams:
  - Marine ice sheet instability: Marine ice streams resting on a retrograde bedrock could exhibit a rapid retreat leading to a sudden and important loss of ice (Pine Island and Thwaites glaciers) [Schoof, 2007, 2012].
  - Thermally induced oscillations: Ice streams can show decadal to multi-millennial variability through a thermal feedback between ice mass and bedrock sediments (Siple Coast glaciers) [Robel et al., 2013, 2014].
- It is possible to develop a coupled model of marine ice sheet instability and thermally induced oscillations [Robel et al., 2016].

Outline

- Motivation.

- Marine ice sheet instability.

- Thermally induced oscillations.

- Coupled model of marine ice sheet instability and thermally induced oscillations.

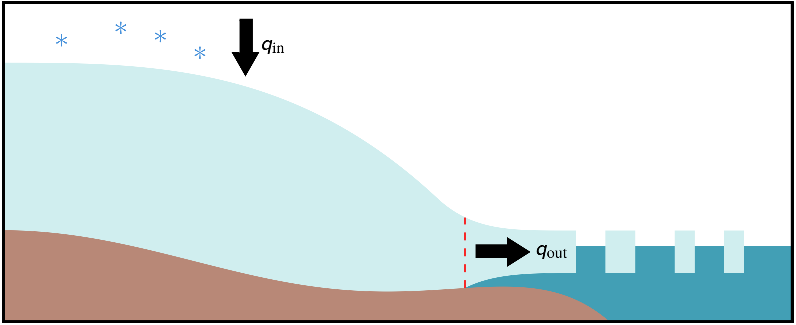
- Conclusion and outlook.

- References.

Marine ice sheet instability (MISI)

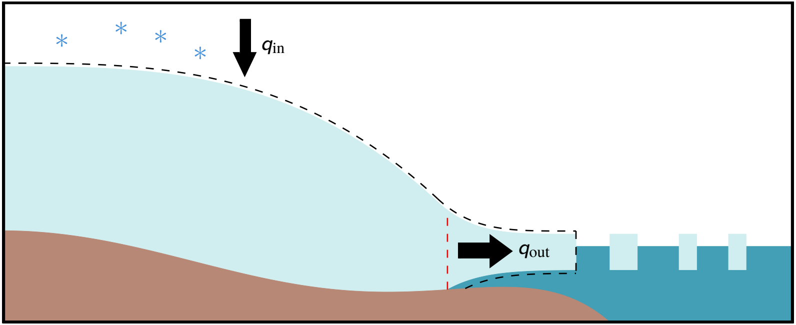
Marine ice sheet instability mechanism

- Step 1: Steady state on an upward sloping bed ($q_{\text{in}} = q_{\text{out}}$).



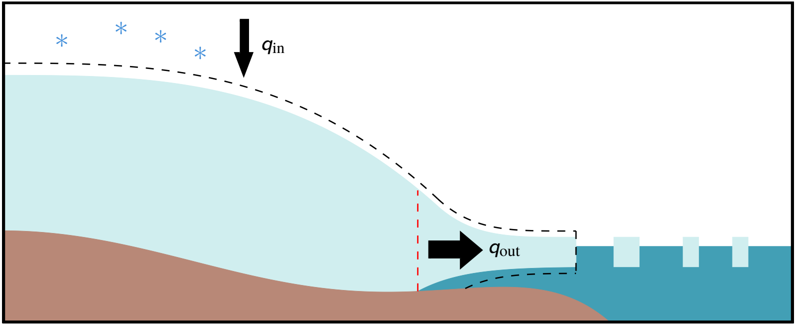
Marine ice sheet instability mechanism

- Step 2: Initiation of grounding line retreat ($q_{in} < q_{out}$).



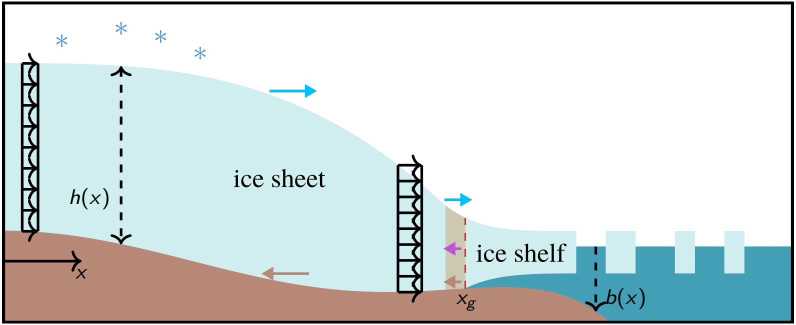
Marine ice sheet instability mechanism

- Step 3: Self-sustained grounding line retreat ($q_{\text{in}} \ll q_{\text{out}}$).



A simple geometrical model for MISI

- We consider an ice stream sliding on an overdeepened bed. Ice flow is described as a gravity-driven viscous flow subject to basal friction. Viscous stresses can be neglected in the ice sheet except in a narrow transition zone near the grounding line.



A mathematical model for MISI

- Continuity equation (nonlinear diffusion equation):

$$\frac{\partial h}{\partial t} - \frac{\partial}{\partial x}\left[\left(\frac{\rho_i g}{C}\right)^{1/m} h^{1+\frac{1}{m}} \left|\frac{\partial (h-b)}{\partial x}\right|^{\frac{1}{m}-1} \frac{\partial (h-b)}{\partial x}\right] = a.$$

- Symmetry condition at the ice divide:

$$\left.\frac{\partial (h-b)}{\partial x}\right|_{x=0} = 0.$$

- Flotation condition at the grounding line:

$$\rho_i h(x_g) = \rho_w b(x_g).$$

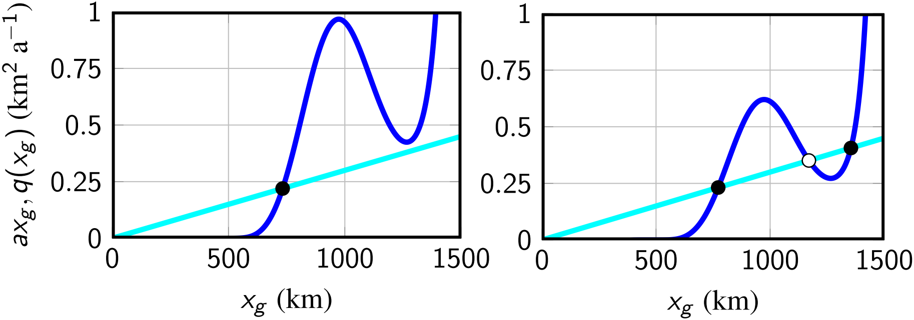
- Stress continuity at the grounding line (from boundary layer theory):

$$q(x_g) = \left(\frac{\overline{A}(\rho_i g)^{n+1}(1-\frac{\rho_i}{\rho_w})^n}{4^n C}\right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}}.$$

Steady grounding line positions: graphical approach

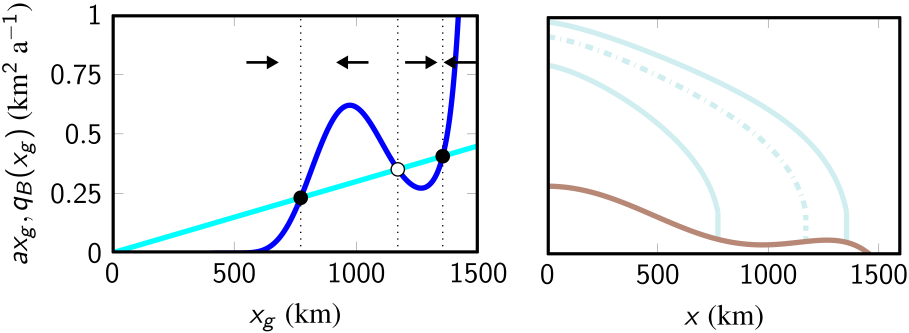
- Steady grounding line positions are given by

$$\left(\frac{\overline{A}(\rho_i g)^{n+1}(1-\frac{\rho_i}{\rho_w})^n}{4^n C}\right)^{\frac{1}{m+1}} \left(\frac{\rho_w}{\rho_i} b(x_g)\right)^{\frac{m+n+3}{m+1}} = a x_g$$



Stability analysis of steady states

- Graphical analysis:

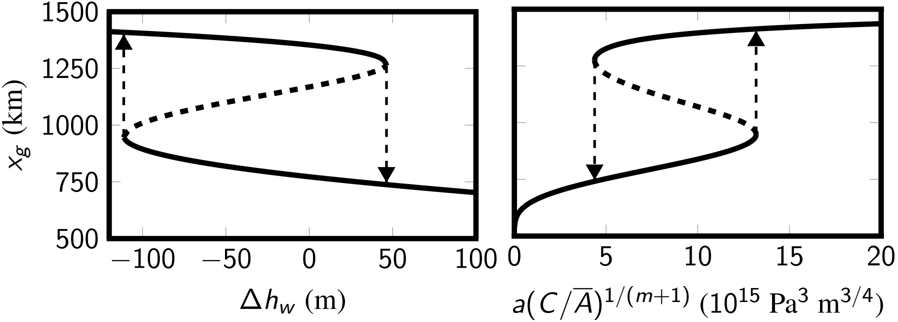


- Linear stability analysis: Schoof [Schoof, 2012] has shown that marine ice sheets are unstable if

$$\boxed{a(x_g) - q'(x_g) > 0.}$$

Bifurcation diagram

- The system is bistable for some values of the parameters. The appearance or disappearance of two steady state solution branches is associated with a saddle-node bifurcation. The system can undergo hysteresis under variations of parameters.



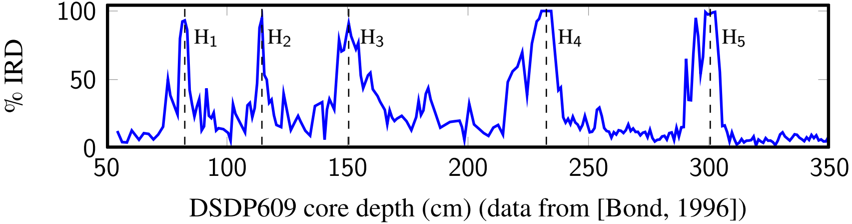
Conclusions about MISI

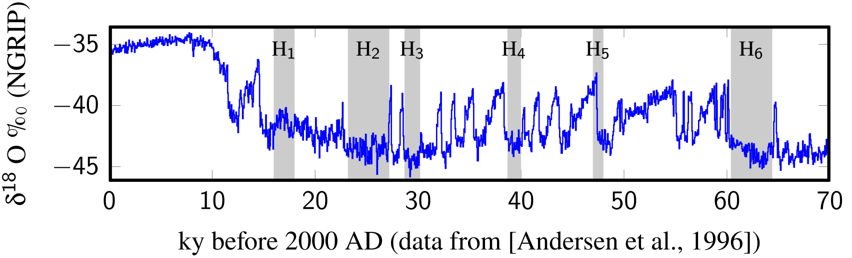
- Marine ice sheets have a discrete number of equilibrium profiles.
- Marine ice sheets are inherently unstable on upward-sloping bed.
- Marine ice sheets can undergo hysteresis under variations of physical parameters (sea level, accumulation rate, basal slipperiness and ice viscosity).
- MISI mechanism has been presented for a 2D model. For 3D models, buttressing effects could stabilise marine ice sheets.

Thermally induced oscillations

Heinrich events: a thermally oscillating event

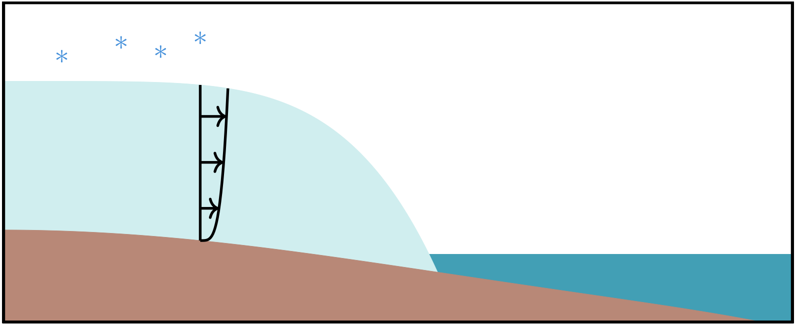
- Heinrich events are quasi-periodic episodes of massive ice discharges during the last glacial period. These episodes led to a climatic cooling and high ice-rafted detritus concentrations in the North Atlantic Ocean.





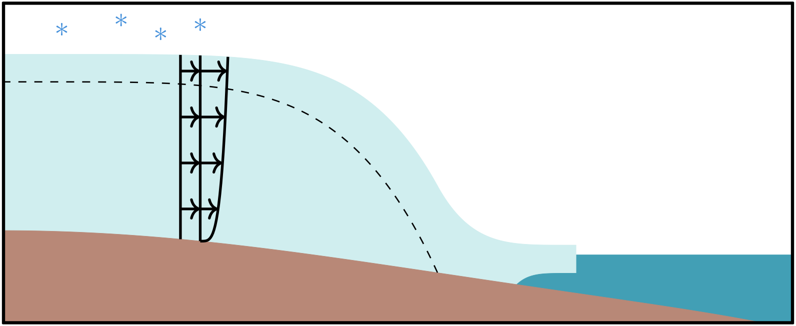
Thermal induced oscillations mechanism

- Step 1: Ice sheet build-up on a frozen bed (binge phase).



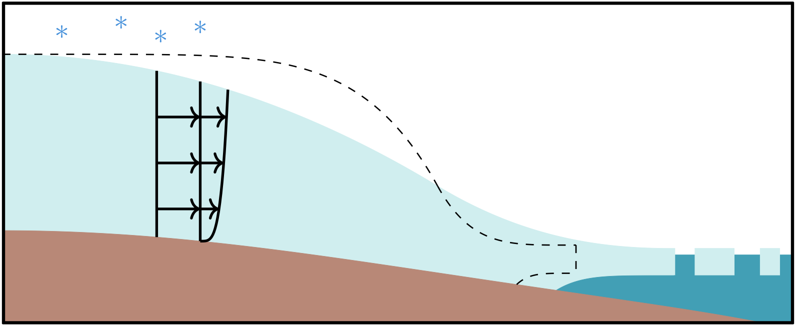
Thermal induced oscillations mechanism

- Step 2: Binge/Purge transition.



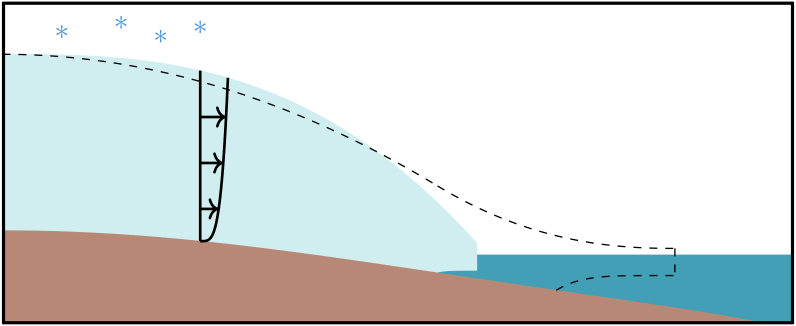
Thermal induced oscillations mechanism

- Step 3: Rapid basal motion (purge phase).



Thermal induced oscillations mechanism

- Step 4: Purge/Binge transition.



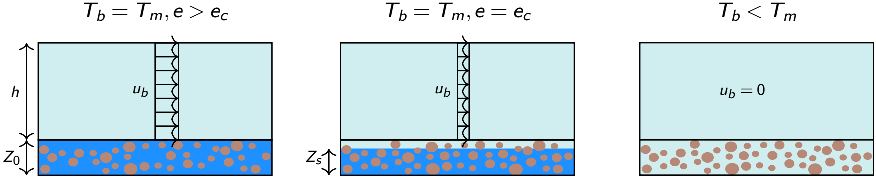
A simple model for thermal oscillations

- The system is described by a set of four dynamical variables:

  1. h: Ice thickness;
  2. w: Water content of the till ($0 \leqslant w \leqslant w_s$);
  3. Z_s: Thickness of unfrozen till with zero porosity ($0 \leqslant Z_s \leqslant Z_0$);
  4. T_b: Basal temperature ($T_b \leqslant T_m$).

  w et Z_s are related trough $w = eZ_s$ where e is till void ratio ($e \geqslant e_c$).

- The system has three main configurations:


$T_b = T_m, e > e_c$ $T_b = T_m, e = e_c$ $T_b < T_m$

A mathematical model for thermal oscillations (1)

- Equation for h:

$$\frac{dh}{dt} = a_c - \frac{u_b}{h} L \quad \text{(continuity equation).}$$

- Equation for w ($T_b = T_m$):

$$\frac{dw}{dt} = m - Q_w \quad \text{(melt water budget)}$$

with

$$\rho_i L_f m = \underbrace{G}_{\text{geothermal flux}} + \underbrace{\frac{k_i(T_s - T_b)}{h}}_{\text{vertical heat conduction}} + \underbrace{\tau_b u_b}_{\text{frictional heating}},$$

$$Q_w = \begin{cases} 0 & \text{if } w < w_s \text{ or } m < 0 \\ m & \text{otherwise} \end{cases}.$$

A mathematical model for thermal oscillations (2)

- Equation for Z_s ($T_b = T_m$):

$$e\frac{dZ_s}{dt} = \begin{cases} m & \text{if } e = e_c \text{ and } 0 < Z_s < Z_0 \qquad \text{(ice fringe)} \\ m & \text{if } e = e_c \text{ and } Z_s = Z_0 \text{ and } m < 0 \qquad \text{(ice fringe)} \\ m & \text{if } e = e_c \text{ and } Z_s = 0 \text{ and } m > 0 \qquad \text{(ice fringe)} \\ 0 & \text{otherwise} \end{cases}.$$

- Equation for T_b:

$$\frac{dT_b}{dt} = \begin{cases} 0 & \text{if } w > 0 \text{ or } (T_b = T_m, w = 0 \text{ and } m > 0) \\ \frac{\rho_i L_f}{C_i h_b} m & \text{otherwise} \quad \text{(basal cooling)} \end{cases}.$$

A mathematical model for thermal oscillations (3)

- Equation for u_b:

$$u_b = \frac{A_g W^{n+1}}{4^n (n+1) h^n} \max[\tau_d - \tau_b, 0]^n \quad \text{(from force balance)}$$

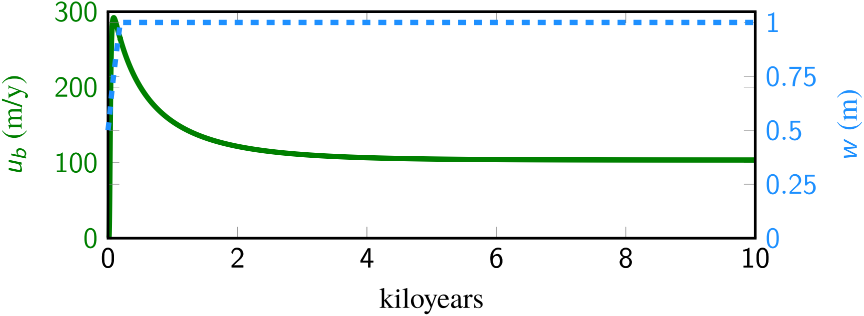
where

$$\tau_d = \rho_i g \frac{h^2}{L},$$

$$\tau_b = \begin{cases} a' \exp(-b(e - e_c)) & \text{if } w > 0 \\ \infty & \text{otherwise} \end{cases}.$$

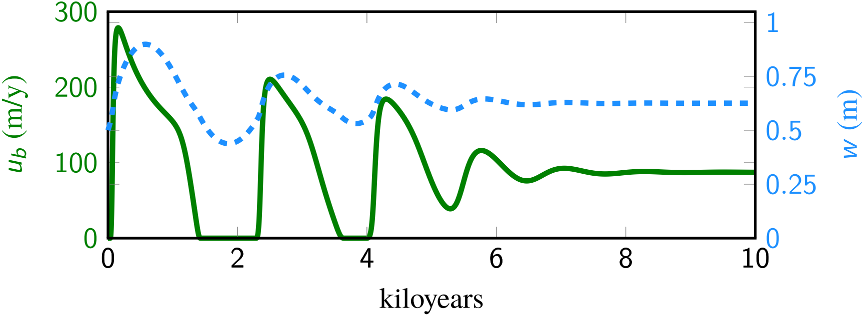
Characteristic modes of the ice stream

- Mode 1: Steady-streaming mode with drainage ($T_s = -15°\text{C}$).



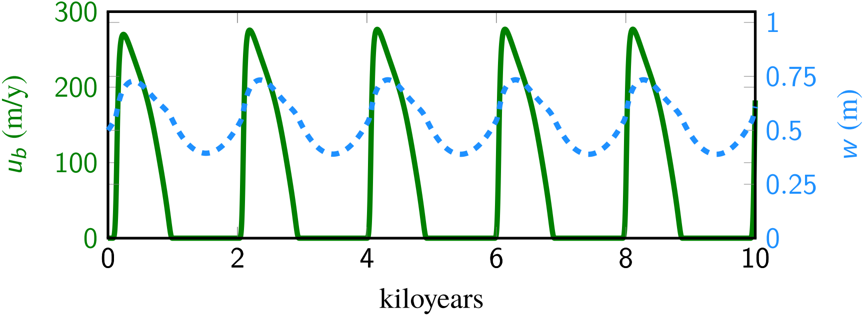
Characteristic modes of the ice stream

- Mode 2: Steady-streaming mode without drainage ($T_s = -20°\mathrm{C}$).



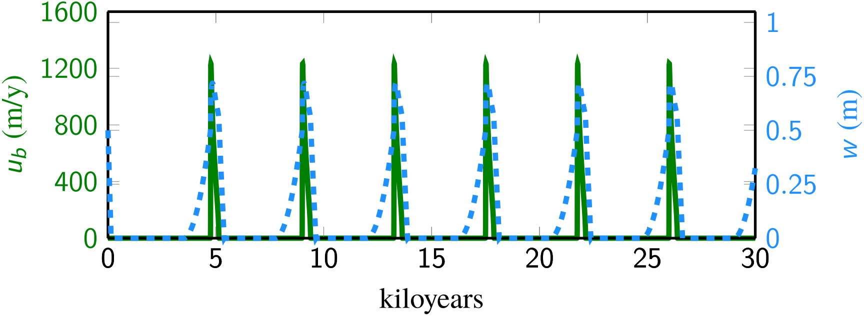
Characteristic modes of the ice stream

- Mode 3: Weak binge-purge mode ($T_s = -22°\mathrm{C}$).

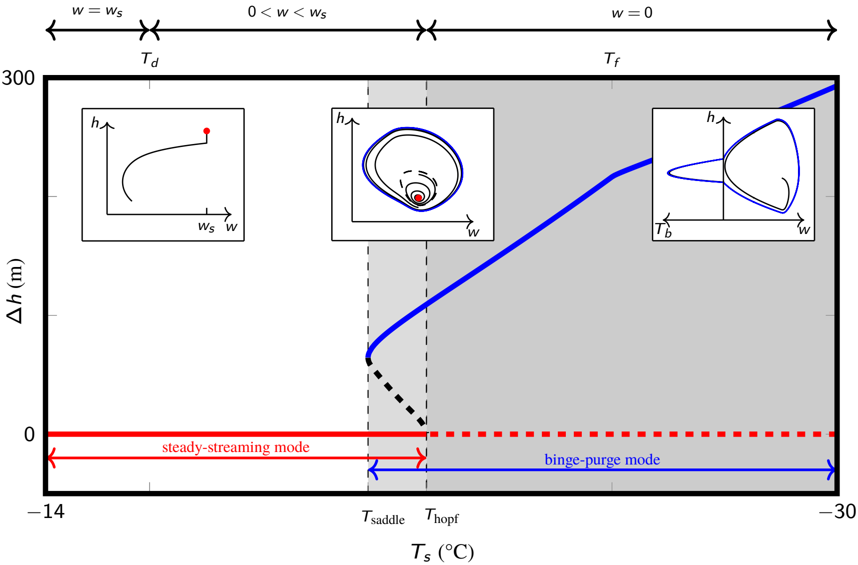


Characteristic modes of the ice stream

- Mode 4: Strong binge-purge mode ($T_s = -35°\text{C}$).



Bifurcation diagram



Conclusions about thermal oscillations

- Ice streams on a till with thermomechanically evolving properties have four potential modes of behaviour (steady-streaming modes with and without drainage and weak and strong binge-purge modes).

- Oscillations in ice flow are caused by internal ice stream dynamics.

- Ice streams can undergo a transition between their different modes when environmental conditions (surface temperature and geothermal flux) are changed. The transition between the steady-streaming mode without drainage and the weak binge-purge mode is a subcritical Hopf bifurcation.

Coupled model of marine ice sheet instability and thermally induced oscillations

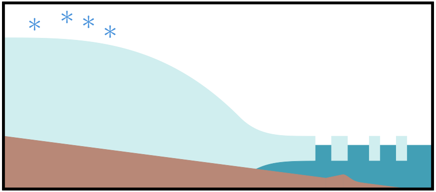
Classical theories for grounding line stability and thermal oscillations

- Classical theories of grounding line stability:
  - Bed properties are supposed to be static in time.
  - Ice streams tend toward a steady state.
  - Grounding line can not persist on a retrograde slope.

- Classical theories of thermal oscillations:
  - Bed properties evolve dynamically.
  - Ice streams rest on purely downward-sloping beds.
  - Ice streams tend toward a steady state or an oscillatory behaviour.

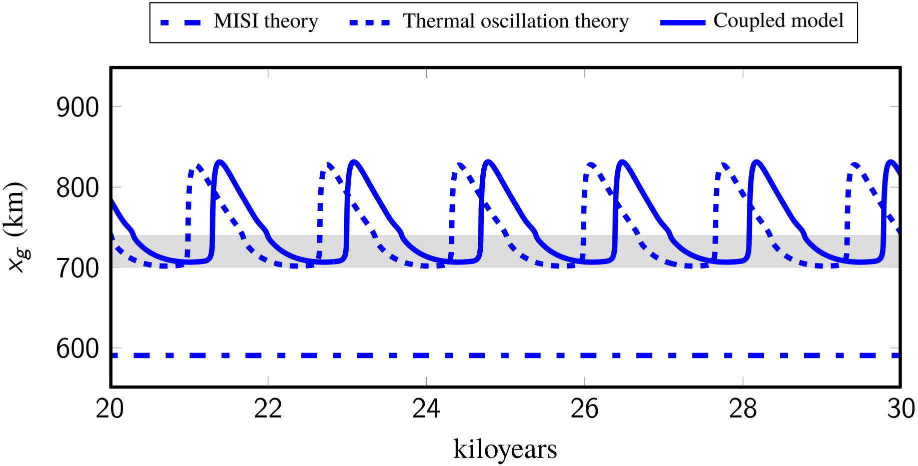
A coupled model for ice stream dynamics

- Coupled model:
  - Bed section of retrograde slope [Schoof, 2007; Tsai et al., 2015].
  - Bed properties evolve dynamically [Robel et al., 2013, 2014].
  - Two main questions:
    - → Is grounding line stability affected by evolving bed properties ?
    - → Are thermal oscillations affected by a section of retrograde slope ?



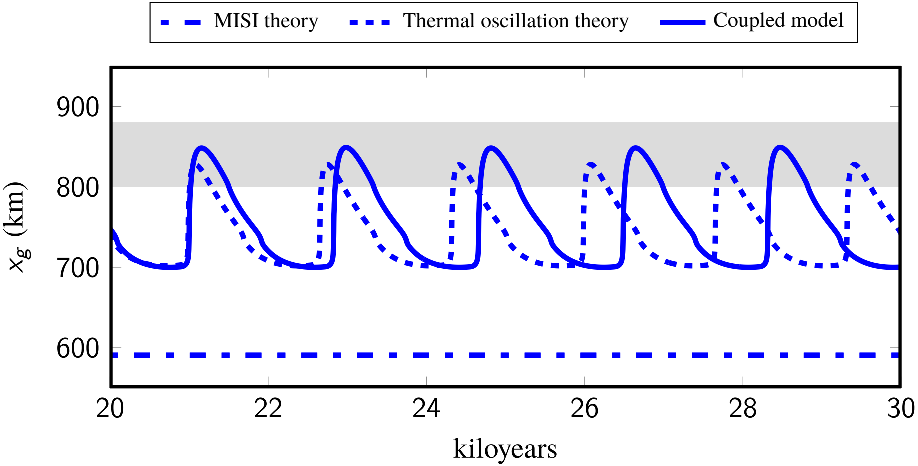
New ice stream behaviours for the coupled model (1)

- Grounding line can persist on a retrograde slope during stagnation phase.



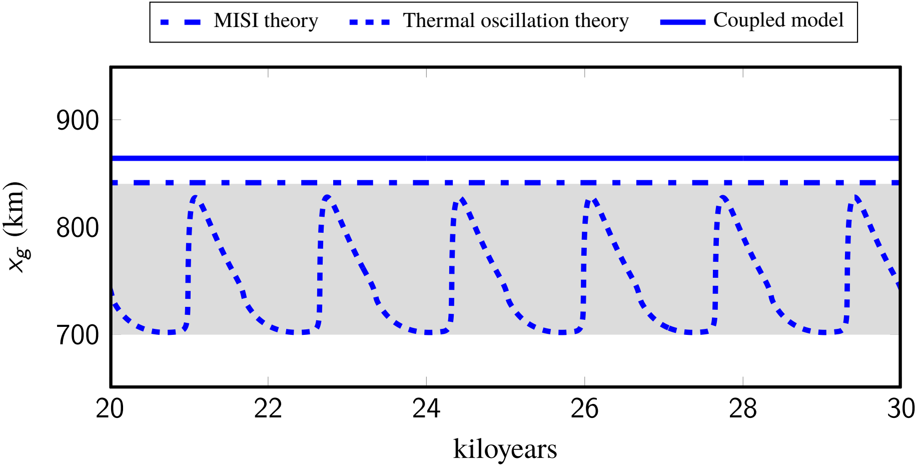
New ice stream behaviours for the coupled model (2)

- The grounding line of an active ice stream can reverse its direction of migration on a retrograde slope.



New ice stream behaviours for the coupled model (3)

- A retrograde slope can suppress thermal oscillations.



Conclusions about coupled model

- Ice streams can exhibit behaviours unexplained by classical theories for grounding line stability and thermal oscillations:
    - Persistence of the grounding line on a retrograde slope for centuries (Siple Coast glaciers).
    - Reversal of the direction of grounding line migration on a retrograde slope (Siple Coast glaciers).
    - Suppression of thermal oscillations.
- Ice stream behaviour is affected by environmental conditions (accumulation rate, geothermal flux, surface temperature, . . .) and bed topography.

Conclusion and outlook

Conclusion and outlook

- Understanding ice stream dynamics is essential to predict future mass balance of ice sheets.
- Ice streams exhibit complex behaviours. Environmental conditions as well as bed topography and properties play a key role in ice stream behaviour. Changes in these parameters can lead to abrupt transitions in ice sheet behaviour.
- Future work:
  - Investigation of other physical processes (buttressing, 3D model,. . .).
  - Ice stream dynamics with stochastic forcing.
  - Investigation of uncertain parameters and their influence on ice stream dynamics.

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Acknowledgement

The first author, Kevin Bulthuis, would like to acknowledge the Belgian National Fund for Scientific Research (F.R.S.-FNRS) for their financial support (F.R.S-FNRS Research Fellowship).

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