Critics Workshop on Critical Transitions in Complex Systems

Instability and abrupt changes in marine ice sheet behaviour

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Motivation

- **Ice streams** (narrow corridors of fast-flowing ice) drain over 90% of the Antarctic mass flux. Ice stream dynamics and stability are key factors for Antarctic mass balance and future contribution to sea-level rise.

![Map of Antarctic ice flow deduced from satellite data](image)

**Figure:** Full map of Antarctic ice flow deduced from satellite data [NASA/JPL-Caltech/UCI].
Motivation

- In this presentation, I will focus on two physical models that have been proposed to explain current and past behaviours of ice streams:

  - Marine ice sheet instability: Marine ice streams resting on a retrograde bedrock could exhibit a rapid retreat leading to a sudden and important loss of ice (Pine Island and Thwaites glaciers) [Schoof, 2007, 2012].

  - Thermally induced oscillations: Ice streams can show decadal to multi-millennial variability through a thermal feedback between ice mass and bedrock sediments (Siple Coast glaciers) [Robel et al., 2013, 2014].

- It is possible to develop a coupled model of marine ice sheet instability and thermally induced oscillations [Robel et al., 2016].
Outline

■ Motivation.

■ Marine ice sheet instability.

■ Thermally induced oscillations.

■ Coupled model of marine ice sheet instability and thermally induced oscillations.

■ Conclusion and outlook.

■ References.
Marine ice sheet instability (MISI)
Marine ice sheet instability mechanism

- Step 1: Steady state on an upward sloping bed \((q_{\text{in}} = q_{\text{out}})\).
Step 2: Initiation of grounding line retreat \((q_{in} < q_{out})\).
Step 3: Self-sustained grounding line retreat ($q_{in} \ll q_{out}$).
A simple geometrical model for MISI

- We consider an ice stream sliding on an overdeepened bed. Ice flow is described as a gravity-driven viscous flow subject to basal friction. Viscous stresses can be neglected in the ice sheet except in a narrow transition zone near the grounding line.
A mathematical model for MISI

- Continuity equation (nonlinear diffusion equation):

\[
\frac{\partial h}{\partial t} - \frac{\partial}{\partial x} \left[ \left( \frac{\rho_i g}{C} \right)^{1/m} h^{1 + \frac{1}{m}} \left| \frac{\partial (h - b)}{\partial x} \right|^{\frac{1}{m} - 1} \frac{\partial (h - b)}{\partial x} \right] = a.
\]

- Symmetry condition at the ice divide:

\[
\left. \frac{\partial (h - b)}{\partial x} \right|_{x=0} = 0.
\]

- Flotation condition at the grounding line:

\[
\rho_i h(x_g) = \rho_w b(x_g).
\]

- Stress continuity at the grounding line (from boundary layer theory):

\[
q(x_g) = \left( \frac{\overline{A}(\rho_i g)^{n+1}(1 - \frac{\rho_i}{\rho_w})^n}{4^n C} \right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}}.
\]
Steady grounding line positions: graphical approach

Steady grounding line positions are given by

\[
\left( \frac{\bar{A}(\rho_i g)^{n+1}(1 - \frac{\rho_i}{\rho_w})^n}{4^n C} \right)^{\frac{1}{m+1}} \left( \frac{\rho_w}{\rho_i} b(x_g) \right)^{\frac{m+n+3}{m+1}} = ax_g
\]
Stability analysis of steady states

Graphical analysis:

Linear stability analysis: Schoof [Schoof, 2012] has shown that marine ice sheets are unstable if

\[ a(x_g) - q'(x_g) > 0. \]
The system is **bistable** for some values of the parameters. The appearance or disappearance of two steady state solution branches is associated with a **saddle-node bifurcation**. The system can undergo **hysteresis** under variations of parameters.

\[
\Delta h_w \text{ (m)} \quad x_g \text{ (km)} \quad a(C/\bar{A})^{1/(m+1)} \left(10^{15} \text{ Pa}^3 \text{ m}^{3/4}\right)
\]
Conclusions about MISI

- Marine ice sheets have a **discrete number of equilibrium profiles**.

- Marine ice sheets are inherently **unstable on upward-sloping bed**.

- Marine ice sheets can undergo **hysteresis** under variations of physical parameters (sea level, accumulation rate, basal slipperiness and ice viscosity).

- MISI mechanism has been presented for a 2D model. For 3D models, **buttressing effects could stabilise** marine ice sheets.
Thermally induced oscillations
Heinrich events: a thermally oscillating event

- Heinrich events are quasi-periodic episodes of massive ice discharges during the last glacial period. These episodes led to a climatic cooling and high ice-rafted detritus concentrations in the North Atlantic Ocean.

![Graph showing DSDP609 core depth (cm) and δ¹⁸O‰ (NGRIP) with Heinrich Events H1-H6 marked.

% IRD

DSDP609 core depth (cm) (data from [Bond, 1996])

δ¹⁸O‰ (NGRIP)

ky before 2000 AD (data from [Andersen et al., 1996])

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Thermal induced oscillations mechanism

- Step 1: Ice sheet build-up on a frozen bed (binge phase).
Thermal induced oscillations mechanism

- Step 2: Binge/Purge transition.
Thermal induced oscillations mechanism

- Step 3: Rapid basal motion (purge phase).
Thermal induced oscillations mechanism

- Step 4: Purge/Binge transition.
A simple model for thermal oscillations

- The system is described by a set of four dynamical variables:
  1. $h$: Ice thickness;
  2. $w$: Water content of the till ($0 \leq w \leq w_s$);
  3. $Z_s$: Thickness of unfrozen till with zero porosity ($0 \leq Z_s \leq Z_0$);
  4. $T_b$: Basal temperature ($T_b \leq T_m$).

$w$ and $Z_s$ are related through $w = eZ_s$ where $e$ is till void ratio ($e \geq e_c$).

- The system has three main configurations:

\[
T_b = T_m, e > e_c \\
\]

\[
T_b = T_m, e = e_c \\
\]

\[
T_b < T_m \\
\]
A mathematical model for thermal oscillations (1)

■ Equation for $h$:

$$\frac{dh}{dt} = a_c - \frac{u_b}{h} L$$  (continuity equation).

■ Equation for $w$ ($T_b = T_m$):

$$\frac{dw}{dt} = m - Q_w$$  (melt water budget)

with

$$\rho_i L_f m = G + \frac{k_i (T_s - T_b)}{h} + \tau_b u_b,$$

geothermal flux  vertical heat conduction  frictional heating

$$Q_w = \begin{cases} 0 & \text{if } w < w_s \text{ or } m < 0 \\ m & \text{otherwise} \end{cases}.$$
A mathematical model for thermal oscillations (2)

- **Equation for** $Z_s$ ($T_b = T_m$):

\[
e \frac{dZ_s}{dt} = \begin{cases} 
  m & \text{if } e = e_c \text{ and } 0 < Z_s < Z_0 \\
  m & \text{if } e = e_c \text{ and } Z_s = Z_0 \text{ and } m < 0 \\
  m & \text{if } e = e_c \text{ and } Z_s = 0 \text{ and } m > 0 \\
  0 & \text{otherwise}
\end{cases}
\]

- **Equation for** $T_b$:

\[
\frac{dT_b}{dt} = \begin{cases} 
  0 & \text{if } w > 0 \text{ or } (T_b = T_m, w = 0 \text{ and } m > 0) \\
  \frac{\rho_i L_f}{C_i h_b} m & \text{otherwise} \quad \text{(basal cooling)}
\end{cases}
\]
A mathematical model for thermal oscillations (3)

- Equation for $u_b$:

$$u_b = \frac{A g \mathcal{W}^{n+1}}{4^n(n+1)h^n} \max[\tau_d - \tau_b, 0]^n$$

(from force balance)

where

$$\tau_d = \rho i g \frac{h^2}{L},$$

$$\tau_b = \begin{cases} a' \exp(-b(e - e_c)) & \text{if } w > 0 \\ \infty & \text{otherwise} \end{cases}.$$
Characteristic modes of the ice stream

- **Mode 1:** Steady-streaming mode with drainage \((T_s = -15^\circ C)\).
Mode 2: Steady-streaming mode without drainage ($T_s = -20^\circ C$).
Characteristic modes of the ice stream

- **Mode 3**: Weak binge-purge mode ($T_s = -22^\circ C$).
Characteristic modes of the ice stream

- Mode 4: Strong binge-purge mode ($T_s = -35^\circ C$).
Bifurcation diagram

- $w = w_s$
- $0 < w < w_s$
- $w = 0$

$T_d \quad T_s \quad T_f$

$\Delta h (m)$

$w = w_s$

$T_{saddle} \quad T_{hopf}$

$T_s (^\circ C)$

- steady-streaming mode
- binge-purge mode

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Ice streams dynamics
Conclusions about thermal oscillations

- Ice streams on a till with thermomechanically evolving properties have four potential modes of behaviour (steady-streaming modes with and without drainage and weak and strong binge-purge modes).

- Oscillations in ice flow are caused by internal ice stream dynamics.

- Ice streams can undergo a transition between their different modes when environmental conditions (surface temperature and geothermal flux) are changed. The transition between the steady-streaming mode without drainage and the weak binge-purge mode is a subcritical Hopf bifurcation.
Coupled model of marine ice sheet instability and thermally induced oscillations
Classical theories for grounding line stability and thermal oscillations

- Classical theories of grounding line stability:
  - Bed properties are supposed to be static in time.
  - Ice streams tend toward a steady state.
  - Grounding line can not persist on a retrograde slope.

- Classical theories of thermal oscillations:
  - Bed properties evolve dynamically.
  - Ice streams rest on purely downward-sloping beds.
  - Ice streams tend toward a steady state or an oscillatory behaviour.
A coupled model for ice stream dynamics

- Coupled model:
  - Bed section of retrograde slope [Schoof, 2007; Tsai et al., 2015].
  - Bed properties evolve dynamically [Robel et al., 2013, 2014].
  - Two main questions:
    - Is grounding line stability affected by evolving bed properties?
    - Are thermal oscillations affected by a section of retrograde slope?
New ice stream behaviours for the coupled model (1)

- Grounding line can persist on a retrograde slope during stagnation phase.

![Graph showing ice stream dynamics over kiloyears](image-url)
The grounding line of an active ice stream can reverse its direction of migration on a retrograde slope.

![Graph showing ice stream behaviours](graph_image)
New ice stream behaviours for the coupled model (3)

- A retrograde slope can suppress thermal oscillations.

![Graph showing ice stream dynamics over kiloyears](image-url)
Conclusions about coupled model

- Ice streams can exhibit behaviours unexplained by classical theories for grounding line stability and thermal oscillations:

  ◆ **Persistence** of the grounding line on a retrograde slope for centuries (Siple Coast glaciers).
  
  ◆ **Reversal** of the direction of grounding line migration on a retrograde slope (Siple Coast glaciers).
  
  ◆ **Suppression** of thermal oscillations.

- Ice stream behaviour is affected by *environmental conditions* (accumulation rate, geothermal flux, surface temperature, . . .) and *bed topography*. 
Conclusion and outlook
Conclusion and outlook

- Understanding ice stream dynamics is essential to predict future mass balance of ice sheets.

- Ice streams exhibit complex behaviours. Environmental conditions as well as bed topography and properties play a key role in ice stream behaviour. Changes in these parameters can lead to abrupt transitions in ice sheet behaviour.

Future work:
- Investigation of other physical processes (buttressing, 3D model, ...).
- Ice stream dynamics with stochastic forcing.
- Investigation of uncertain parameters and their influence on ice stream dynamics.
References
References

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