Simulations of composite laminates inter- and intra-laminar failure using on a non-local mean-field damage-enhanced multi-scale method

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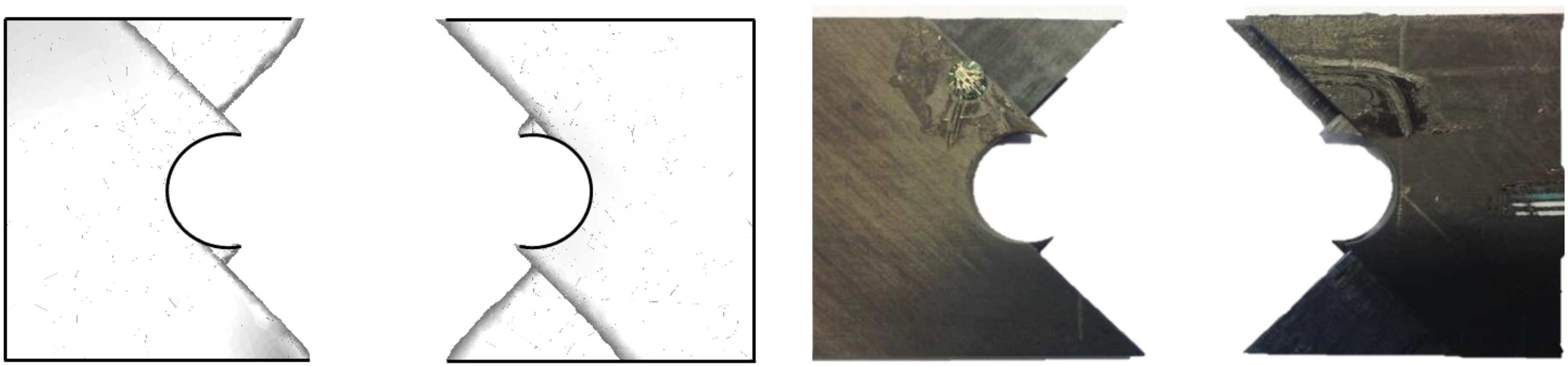
Experiments: F. Sket (IMDEA), J.M. Molina (IMDEA), A. Makradi (List)



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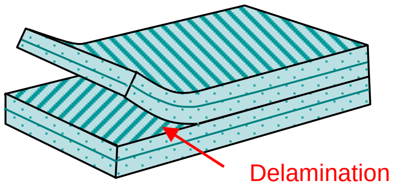
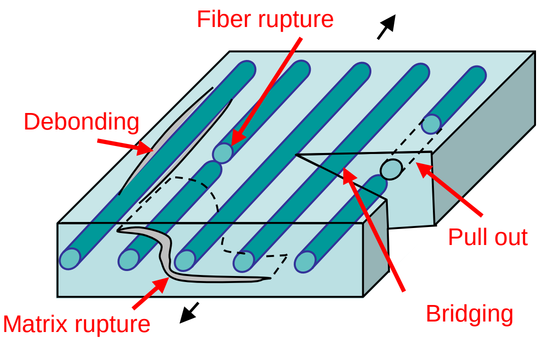
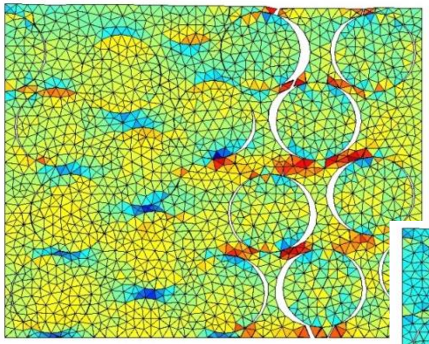
Failure of composite laminates

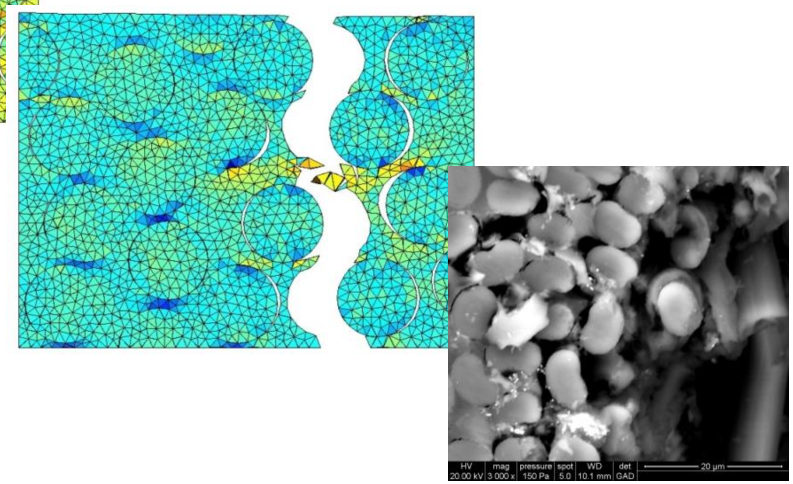
- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation

    On Micro-scale volume

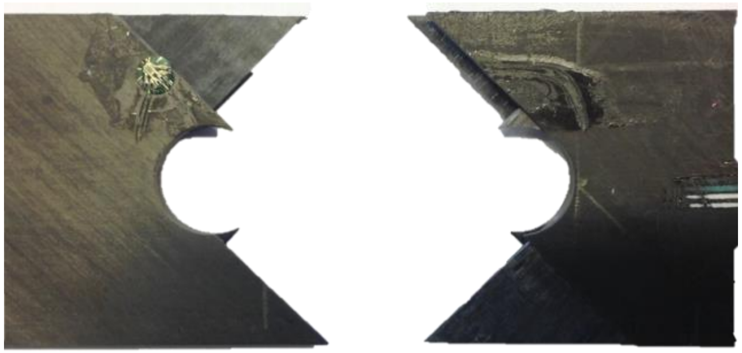






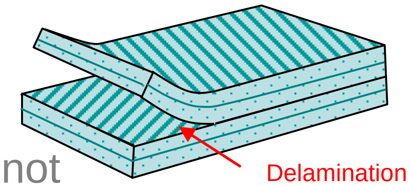


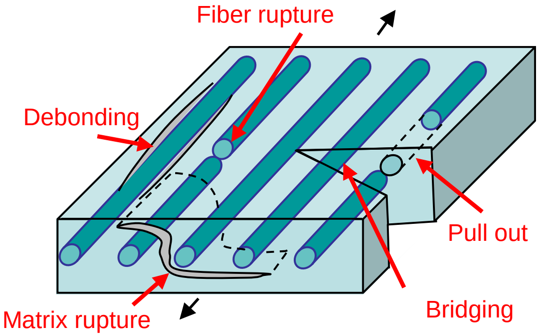
Not possible at structural scale

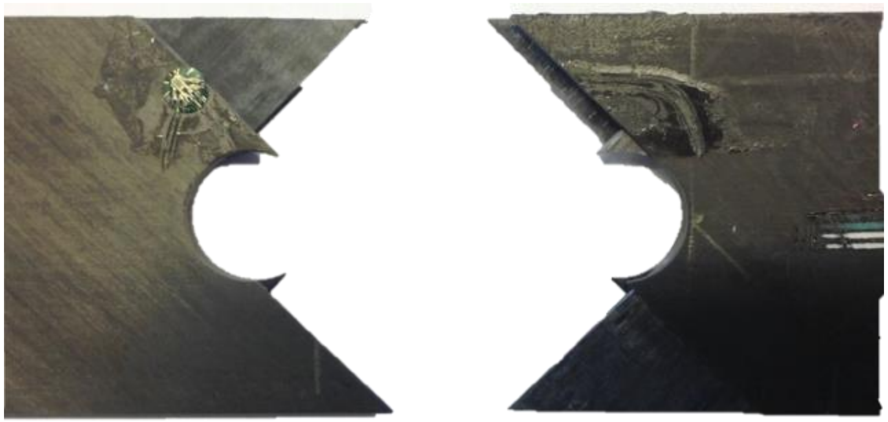


Failure of composite laminates

- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation is not possible at structural scale
  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments

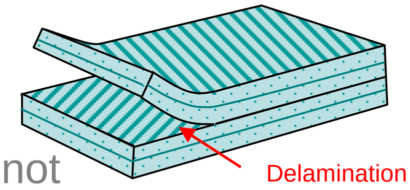


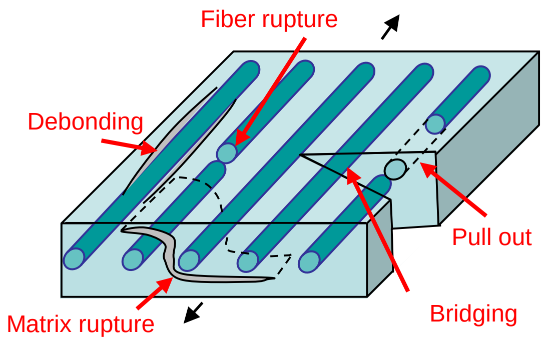


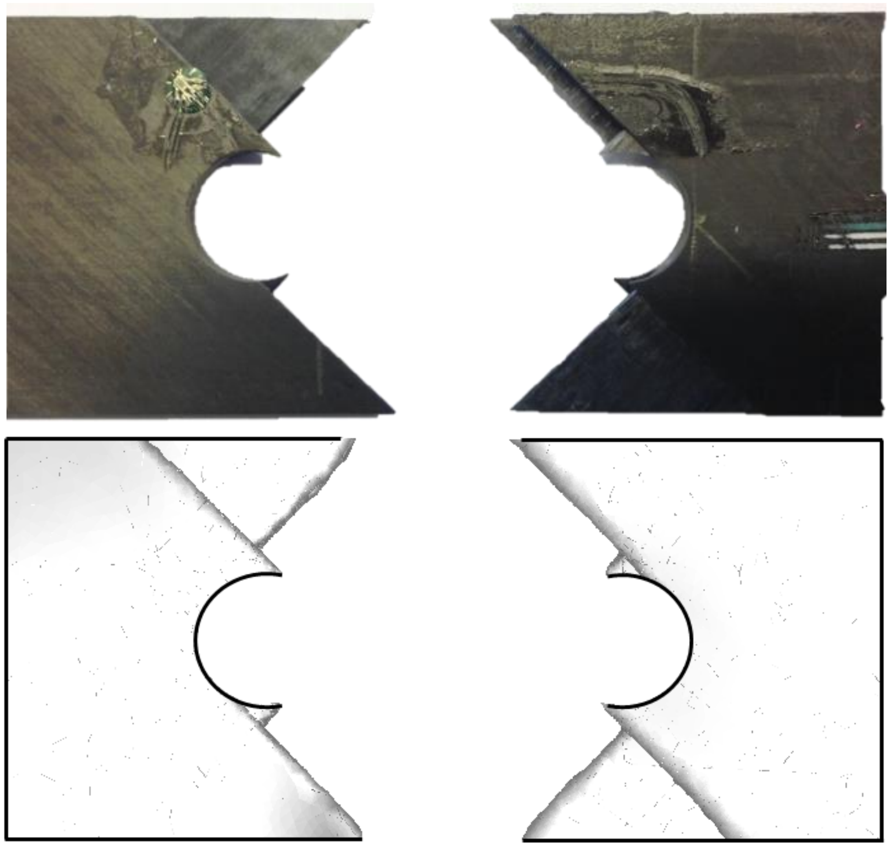


Failure of composite laminates

- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation is not possible at structural scale
  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments
- Solution:
  - Embed damage model in a multi-scale formulation
  - For computational efficiency: use of mean-field-homogenization
  - For macro cracks: using hybrid DG/Cohesive zone model

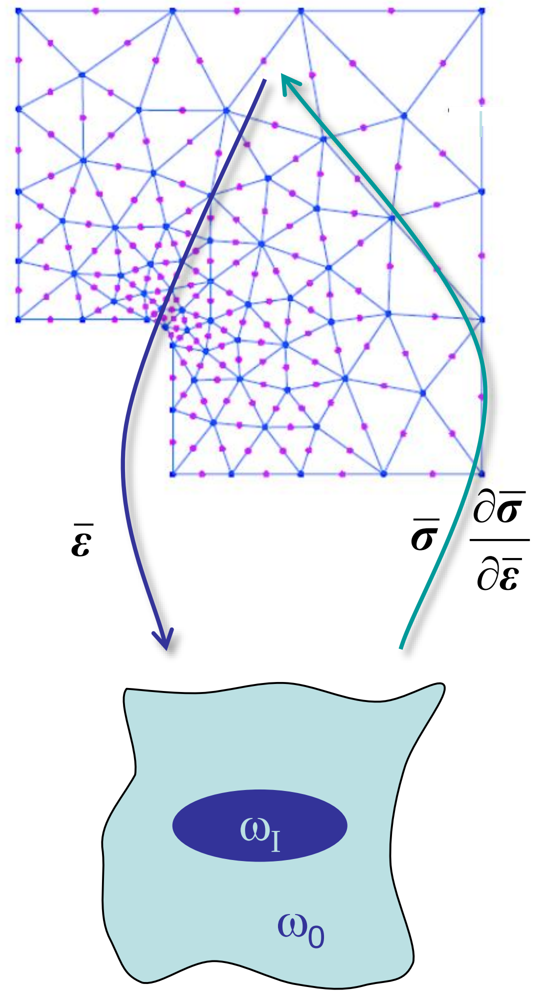






Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\boldsymbol{\varepsilon}}$ is know, $\bar{\boldsymbol{\sigma}}$ is sought
  - Transition
    - Downscaling: $\bar{\boldsymbol{\varepsilon}}$ is used as input of the MFH model
    - Upscaling: $\bar{\boldsymbol{\sigma}}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models

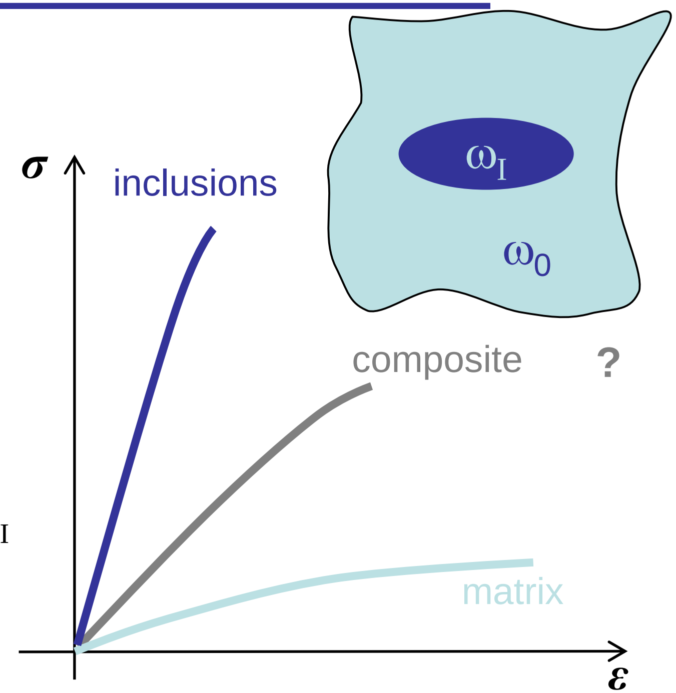


Mori and Tanaka 73, Hill 65, Ponte Castañeda 91, Suquet 95, Doghri et al 03, Lahellec et al. 11, Brassart et al. 12, …

- Key principles
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)



$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$

    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon}\ ?$$

Mean-Field-Homogenization

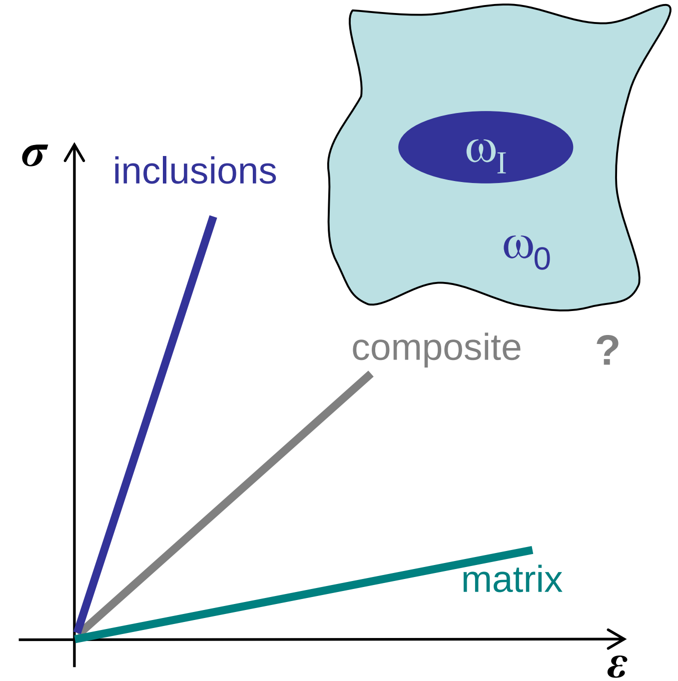
- Key principles (2)
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{0} = \overline{\boldsymbol{C}}_{0} : \boldsymbol{\varepsilon}_{0} \end{cases}$$

    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby tensor

$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$



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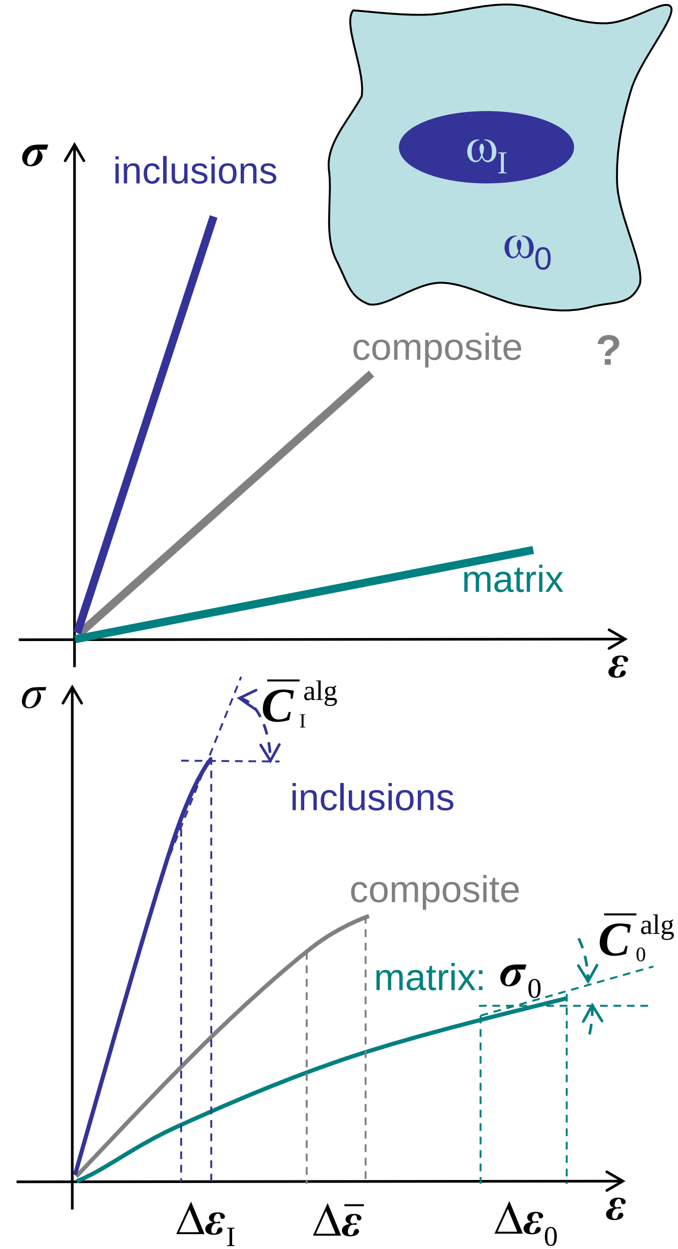
with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

  - Non-linear materials
    - Define a Linear Comparison Composite (LCC)
    - Common approach: incremental tangent

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_{0}^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$

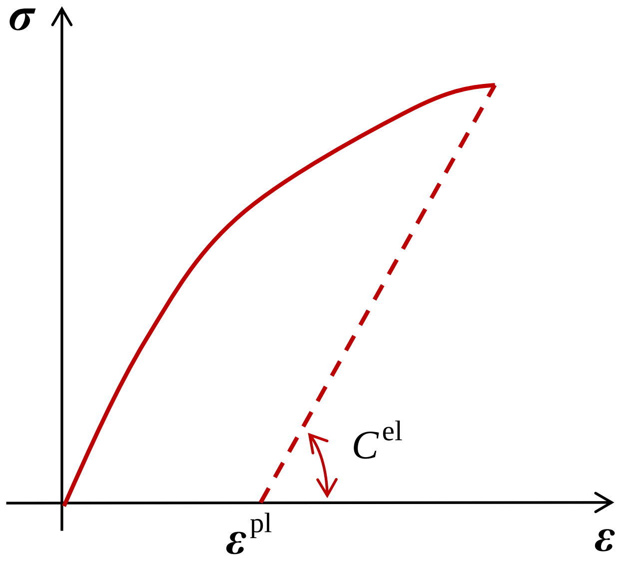


Content

- Micro-scale modelling
  - Incremental-Secant Mean-Field-Homogenization (MFH)
  - Damage-enhanced incremental-secant MFH

Incremental-secant mean-field-homogenization

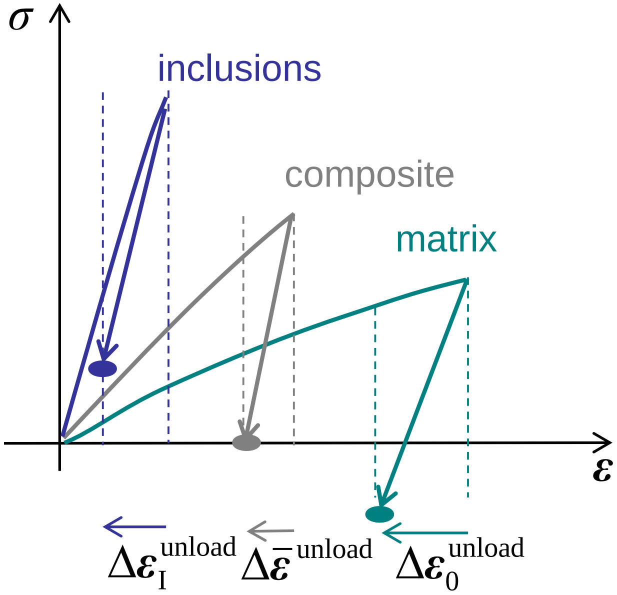
- Material model
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$



Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

New Linear Comparison Composite (LCC)



Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

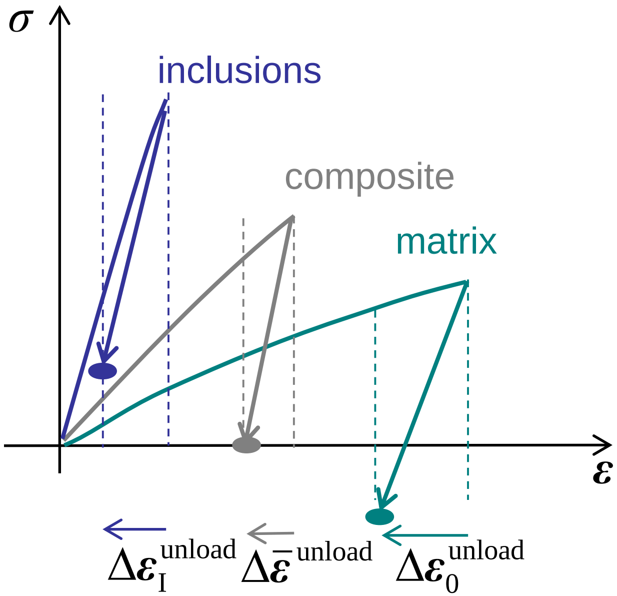
    New Linear Comparison Composite (LCC)

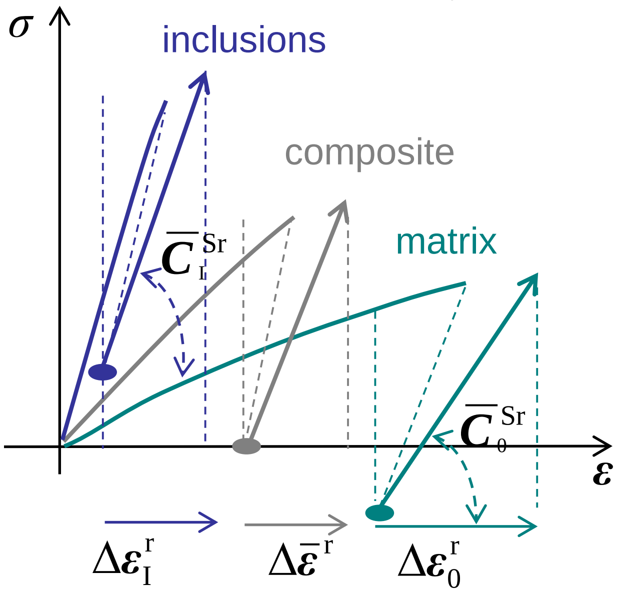
  - Apply MFH from unloaded state
    - New strain increments (>0)

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I/0}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I/0}} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I/0}}$$

    - Use of secant operators

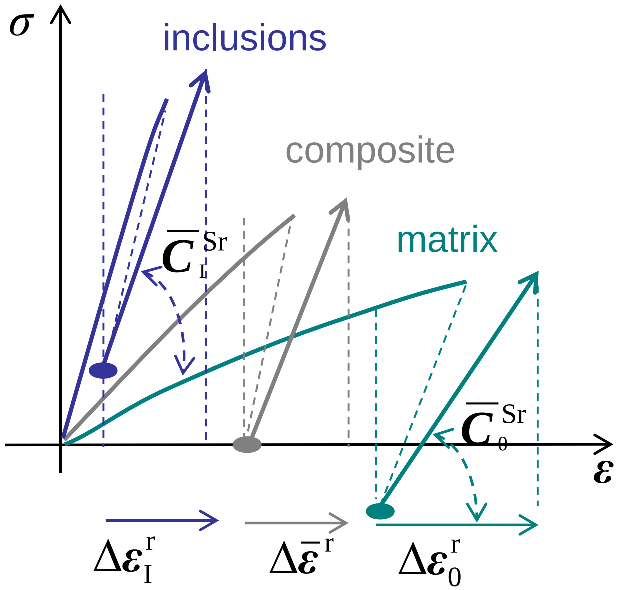
$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$



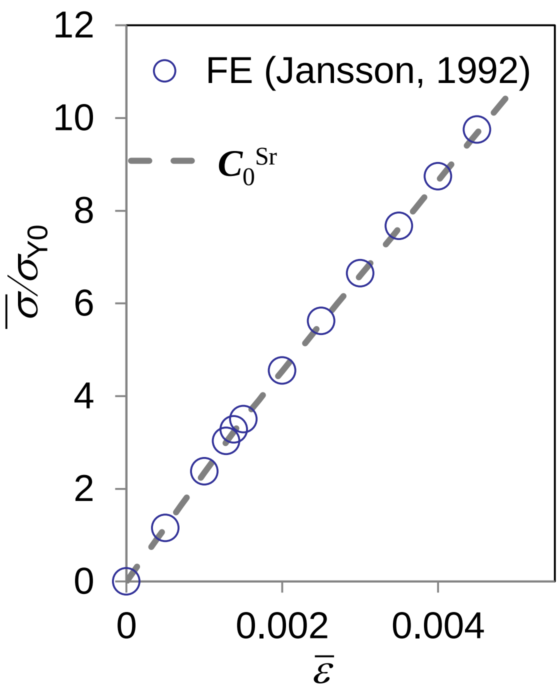


Incremental-secant mean-field-homogenization

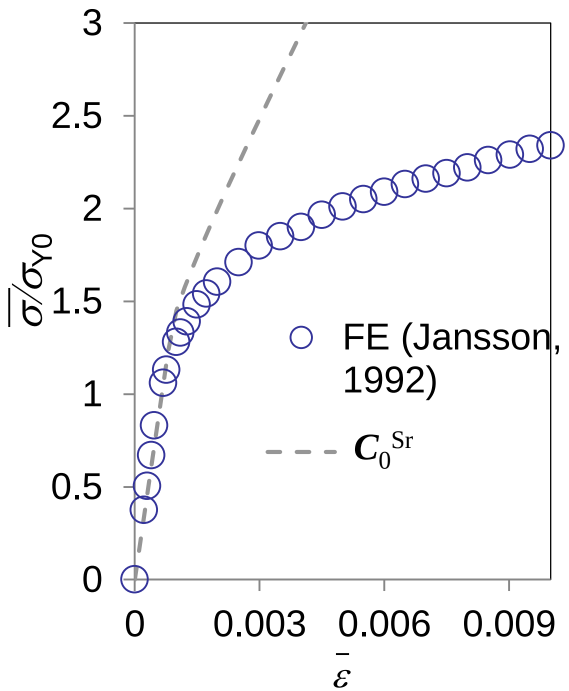
- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - For inclusions with high hardening (elastic)
    - Model is too stiff



Longitudinal tension



Transverse loading



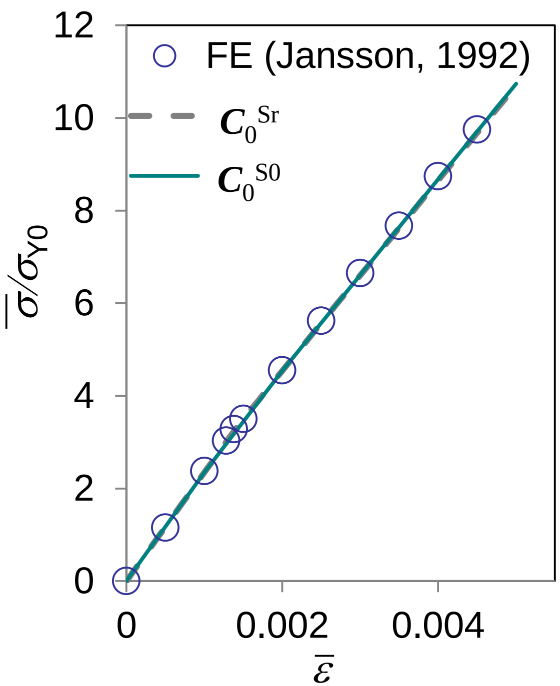
$$f(\boldsymbol{\sigma}, p) = \overline{\boldsymbol{\sigma}}^{\mathrm{eq}} - \sigma^{Y} - R(\overline{p}) \leq 0$$

$\overline{\boldsymbol{\sigma}}^{\mathrm{eq}}$ is <u>underestimated</u>

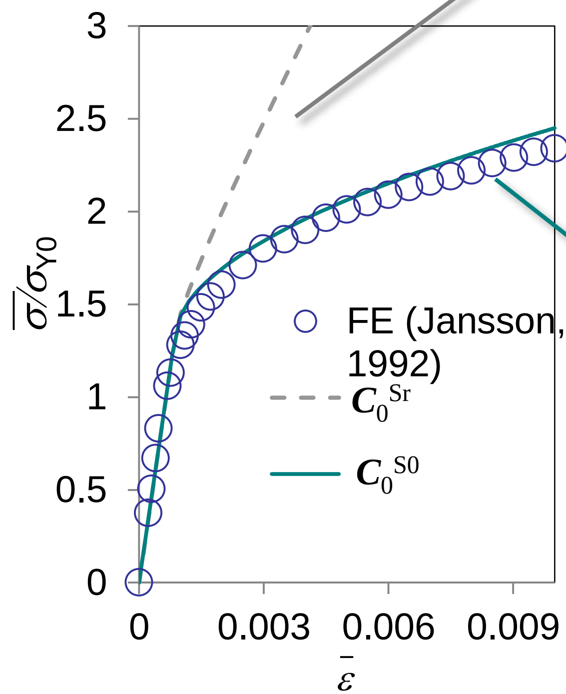
Incremental-secant mean-field-homogenization

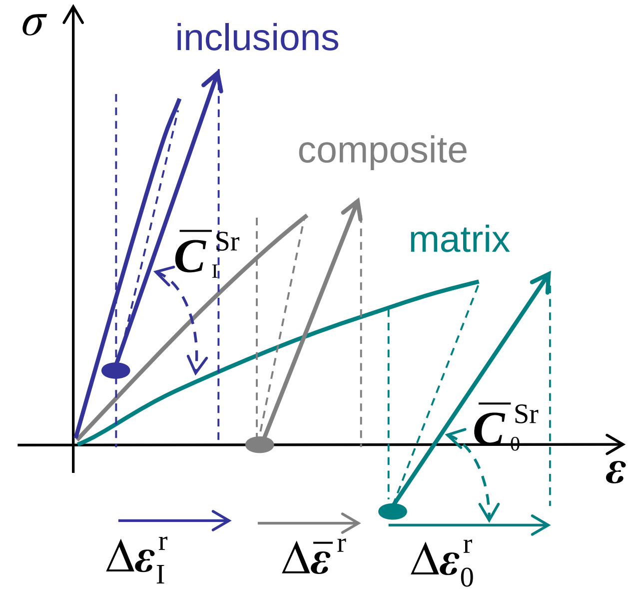
- Zero-incremental-secant method (2)
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if negative residual stress

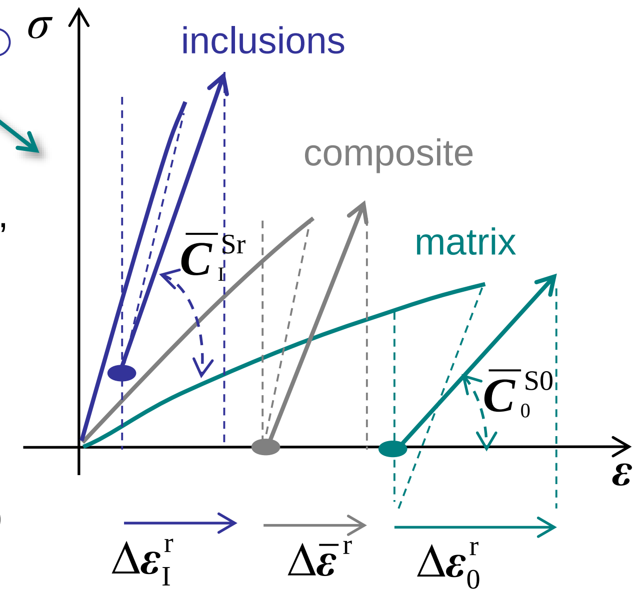
Longitudinal tension



Transverse loading

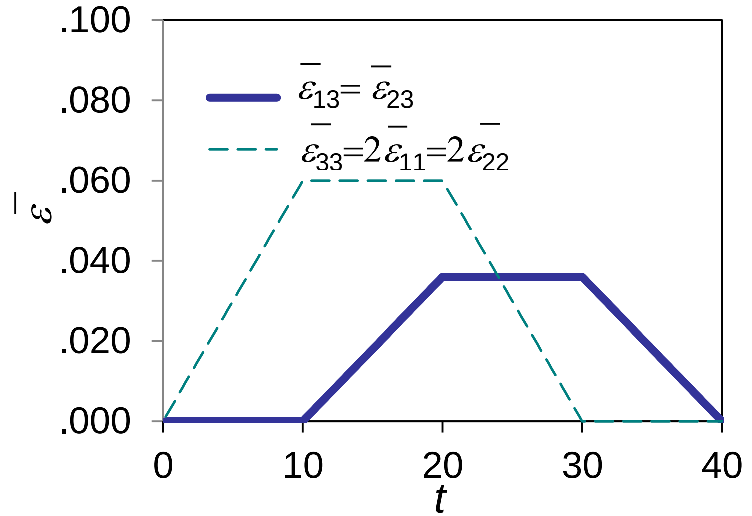


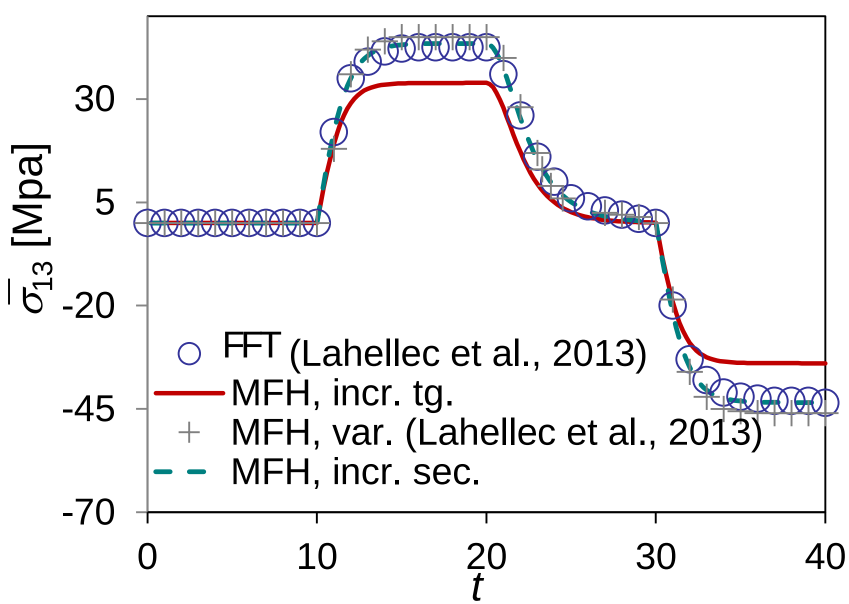


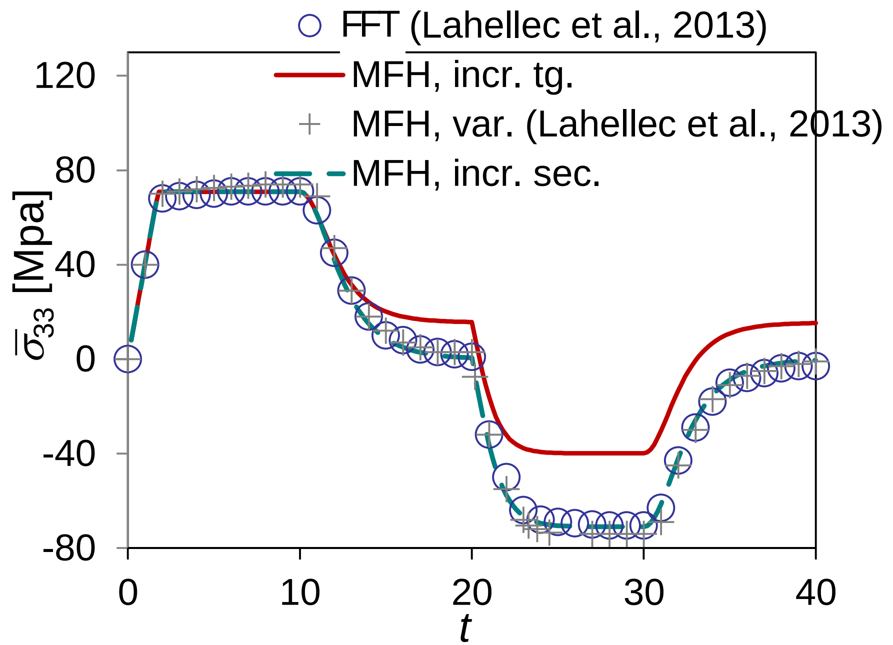


Incremental-secant mean-field-homogenization

- Verification of the method
  - Spherical inclusions
    - 17 % volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-proportional loading

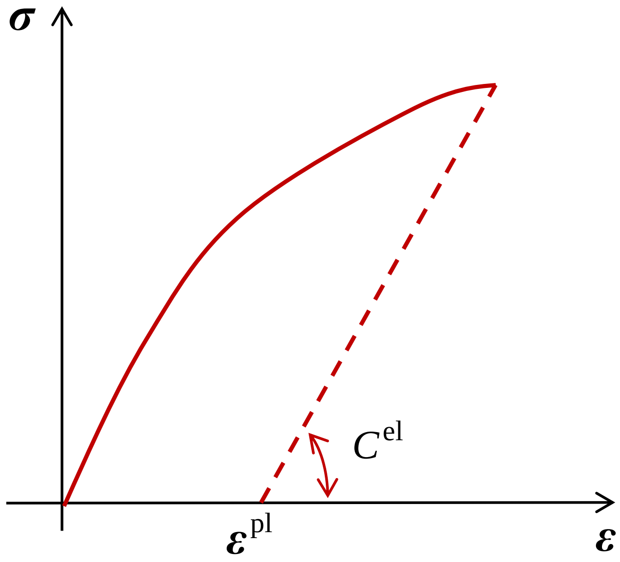






Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$



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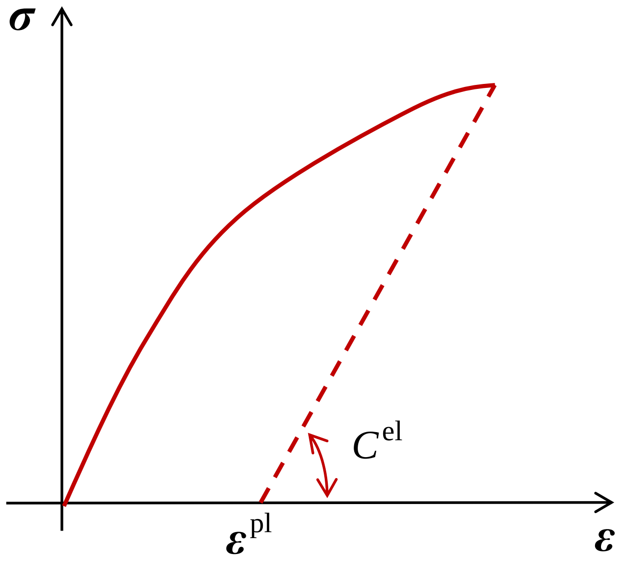
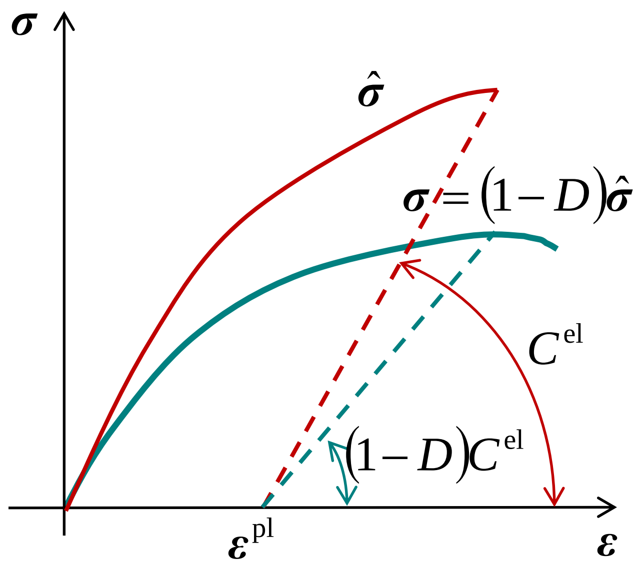
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$





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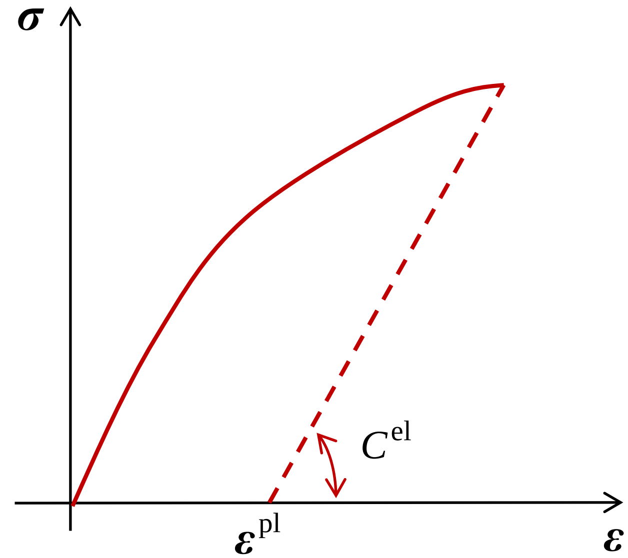
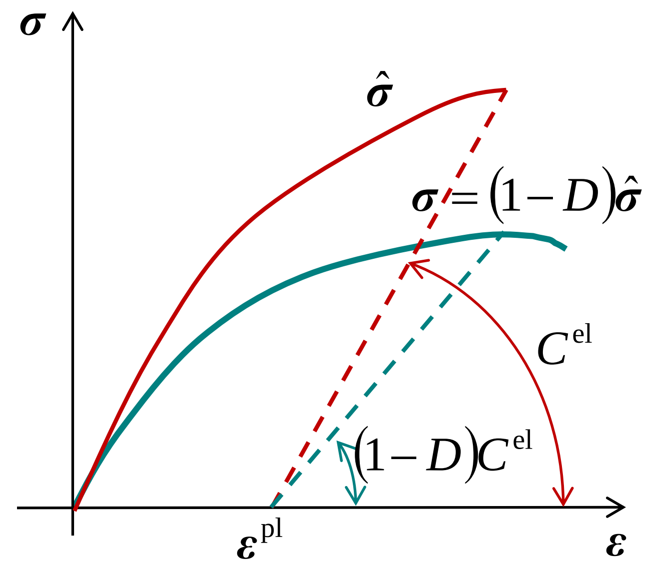
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$
  - Non-Local damage model
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta \tilde{p})$
    - Anisotropic governing equation $\tilde{p} - \nabla \cdot (\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p}) = p$
    - Linearization

$$\delta\boldsymbol{\sigma} = \left[(1 - D)\boldsymbol{C}^{\mathrm{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}}\right] : \delta\boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta\tilde{p}$$





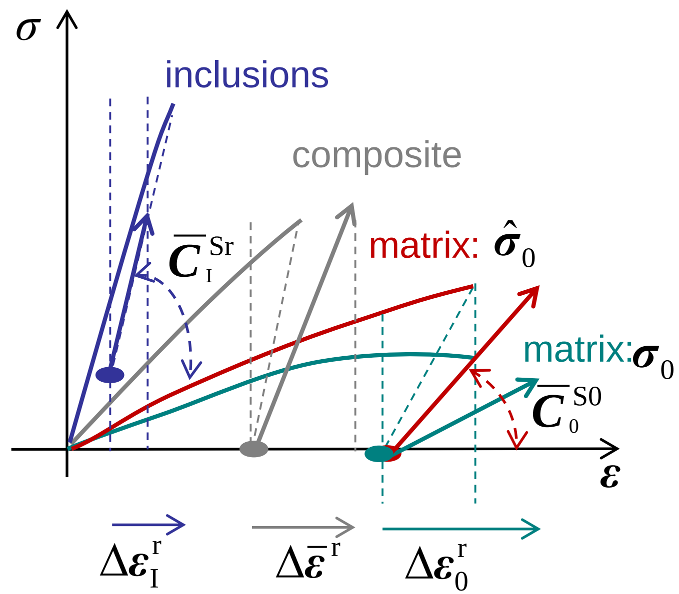
Non-local damage-enhanced MFH

- Equations summary: zero-approach
  - For soft matrix response
    - Remove residual stress in matrix
    - Avoid adding spurious internal energy
  - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_\mathrm{I} \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} = \Delta\boldsymbol{\varepsilon}_\mathrm{I} + \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{unload} \\ \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^\mathrm{unload} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} = \boldsymbol{B}^\varepsilon\left(\mathrm{I}, (1-D)\overline{\boldsymbol{C}}_0^\mathrm{S0}, \overline{\boldsymbol{C}}_\mathrm{I}^\mathrm{Sr}\right) : \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} \end{cases}$$

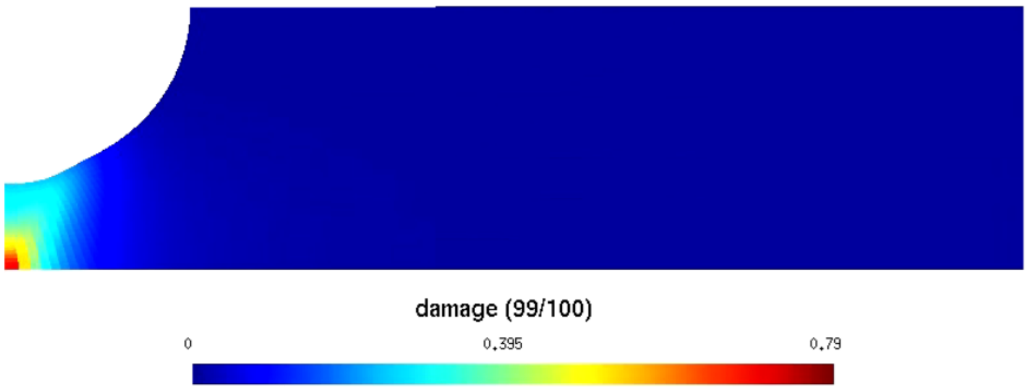
  - With the stress tensors

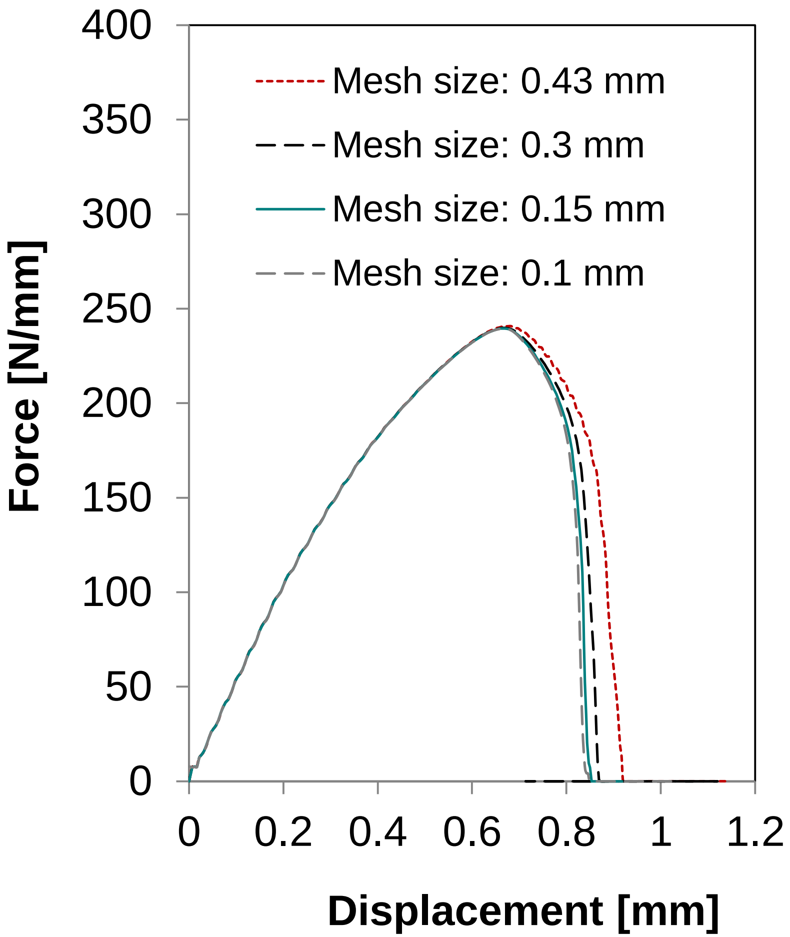
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\sigma}_\mathrm{I} = \boldsymbol{\sigma}_\mathrm{I}^\mathrm{res} + \overline{\boldsymbol{C}}_\mathrm{I}^\mathrm{Sr} : \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} \\ \boldsymbol{\sigma}_0 = (1-D)\overline{\boldsymbol{C}}_0^\mathrm{S0} : \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} \end{cases}$$



Non-local damage-enhanced MFH

- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply





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Content

- Multi-scale method for the failure analysis of composite laminates
  - Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  - Inter-laminar failure: Hybrid DG/cohesive zone model
  - Experimental validation

- Weak formulation of a composite laminate
  - Strong form

$$\begin{cases} \nabla \cdot \bar{\boldsymbol{\sigma}}^T + \boldsymbol{f} = \boldsymbol{0} & \text{for each homogenized ply } \Omega_i \\ \tilde{p} - \nabla \cdot \left(\boldsymbol{c}_g \cdot \nabla \tilde{p} \right) = p & \text{for the matrix phase} \end{cases}$$

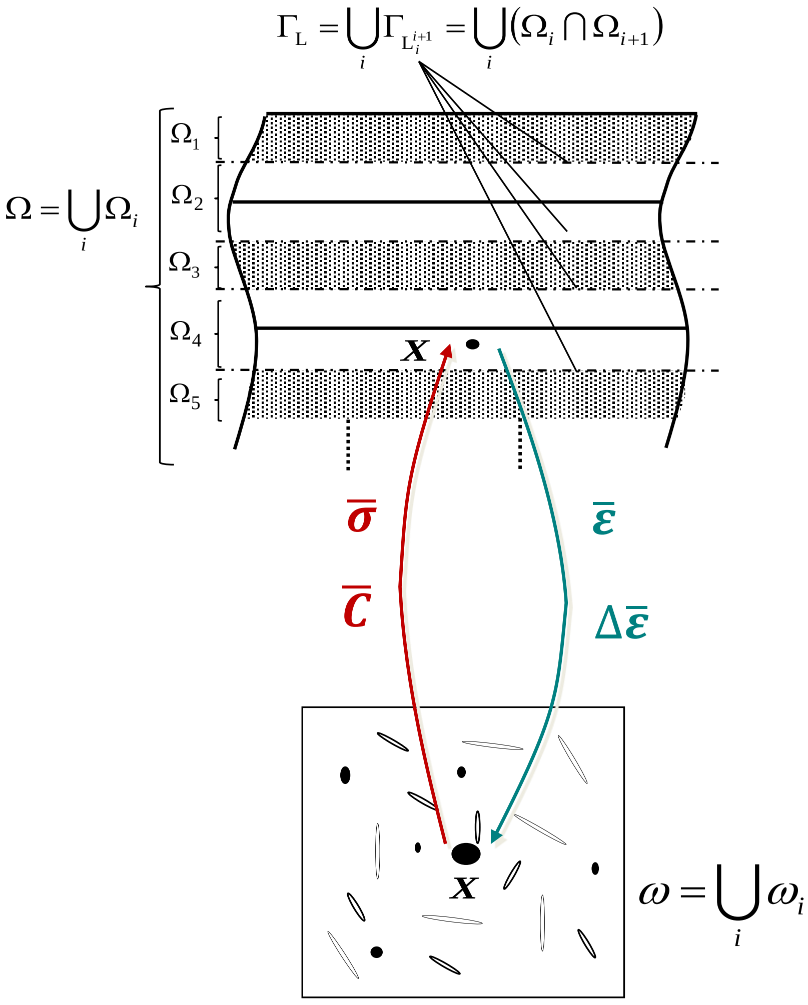
  - Boundary conditions

$$\begin{cases} \bar{\boldsymbol{\sigma}} \cdot \bar{\boldsymbol{n}} = \bar{\boldsymbol{T}} \\ \bar{\boldsymbol{n}} \cdot \left(\boldsymbol{c}_g \cdot \nabla \tilde{p} \right) = 0 \end{cases}$$

  - Macro-scale finite-element discretization

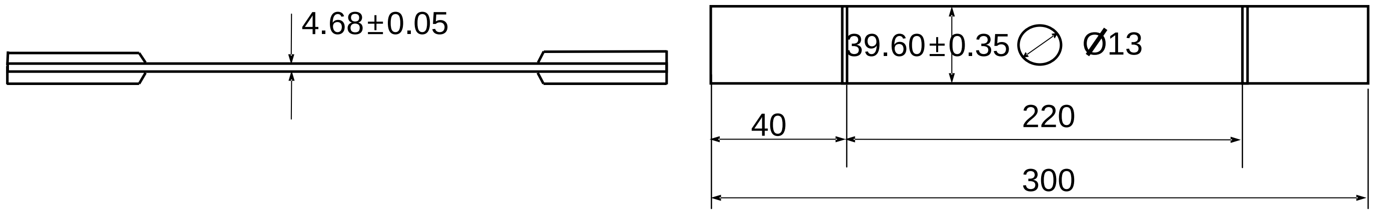
$$\begin{cases} \tilde{p} = N_{\tilde{p}}^a \tilde{\mathbf{p}}^a \\ \bar{\boldsymbol{u}} = N_u^a \bar{\boldsymbol{u}}^a \end{cases}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{K}_{\bar{u}\bar{u}} & \boldsymbol{K}_{\bar{u}\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}\bar{u}} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\bar{\boldsymbol{u}} \\ d\tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\text{ext}} - \boldsymbol{F}_{\text{int}} \\ \boldsymbol{F}_p - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$

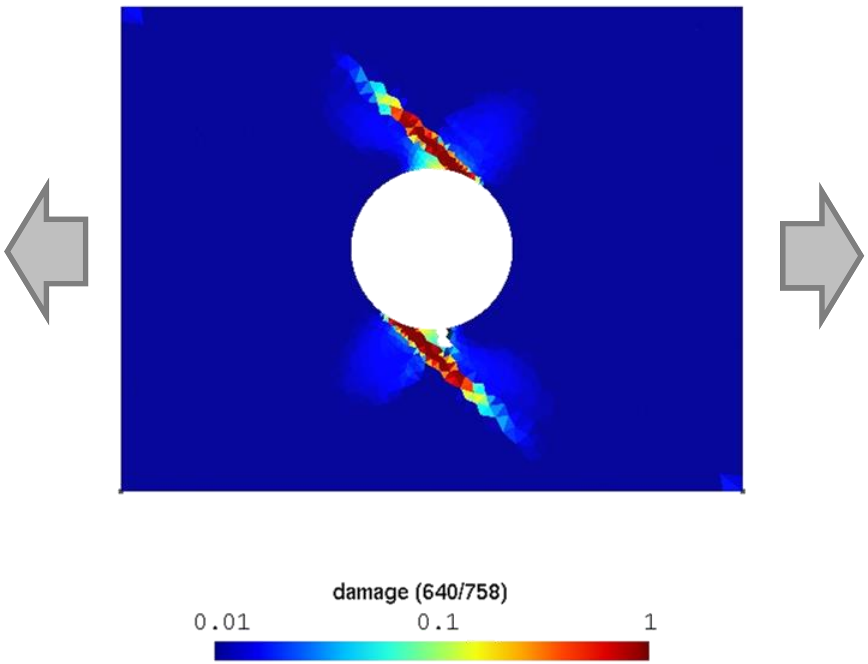


Experimental validation

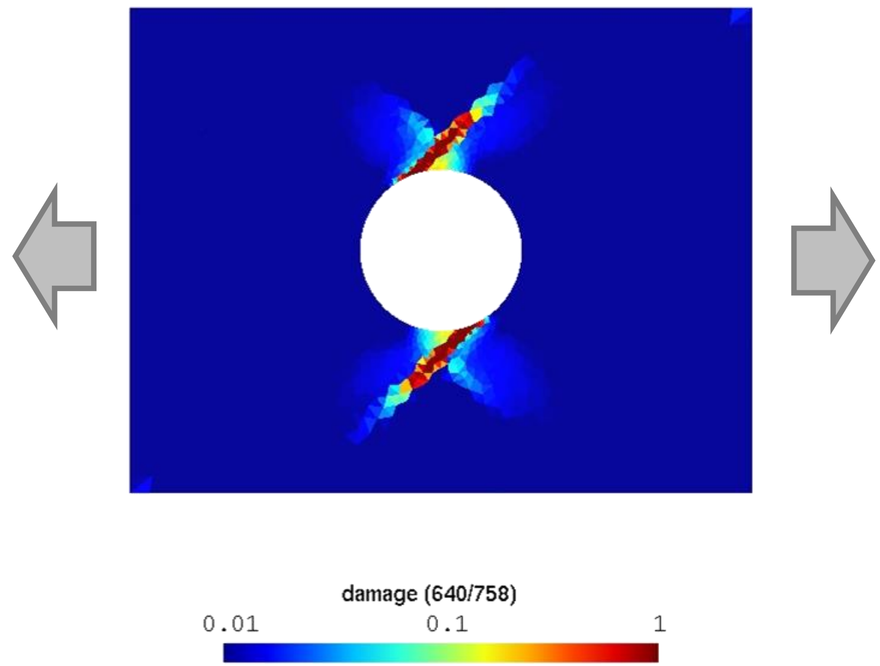
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate
  - Tensile test on several coupons



  - Propagation of the damaged zones in agreement with the fibre direction



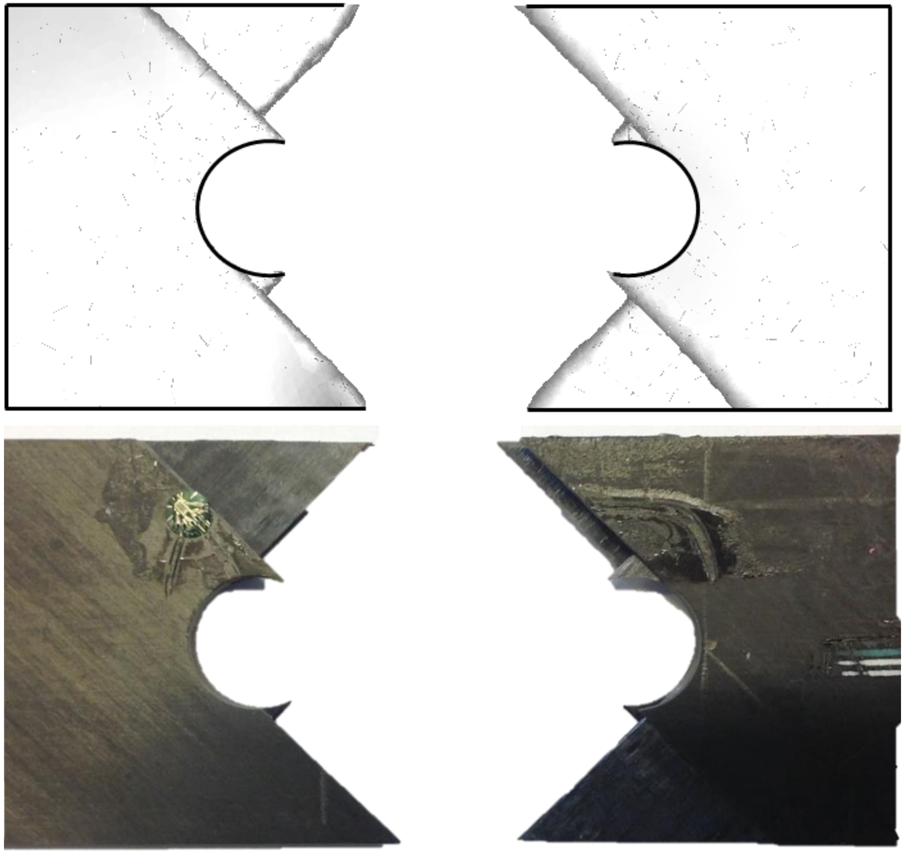
-45^o-ply

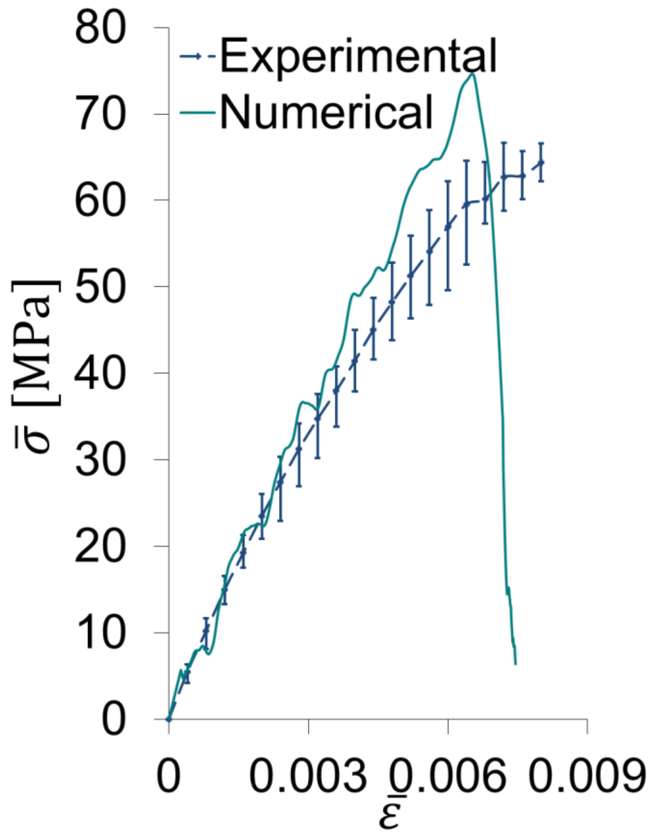


45^o-ply

Experimental validation

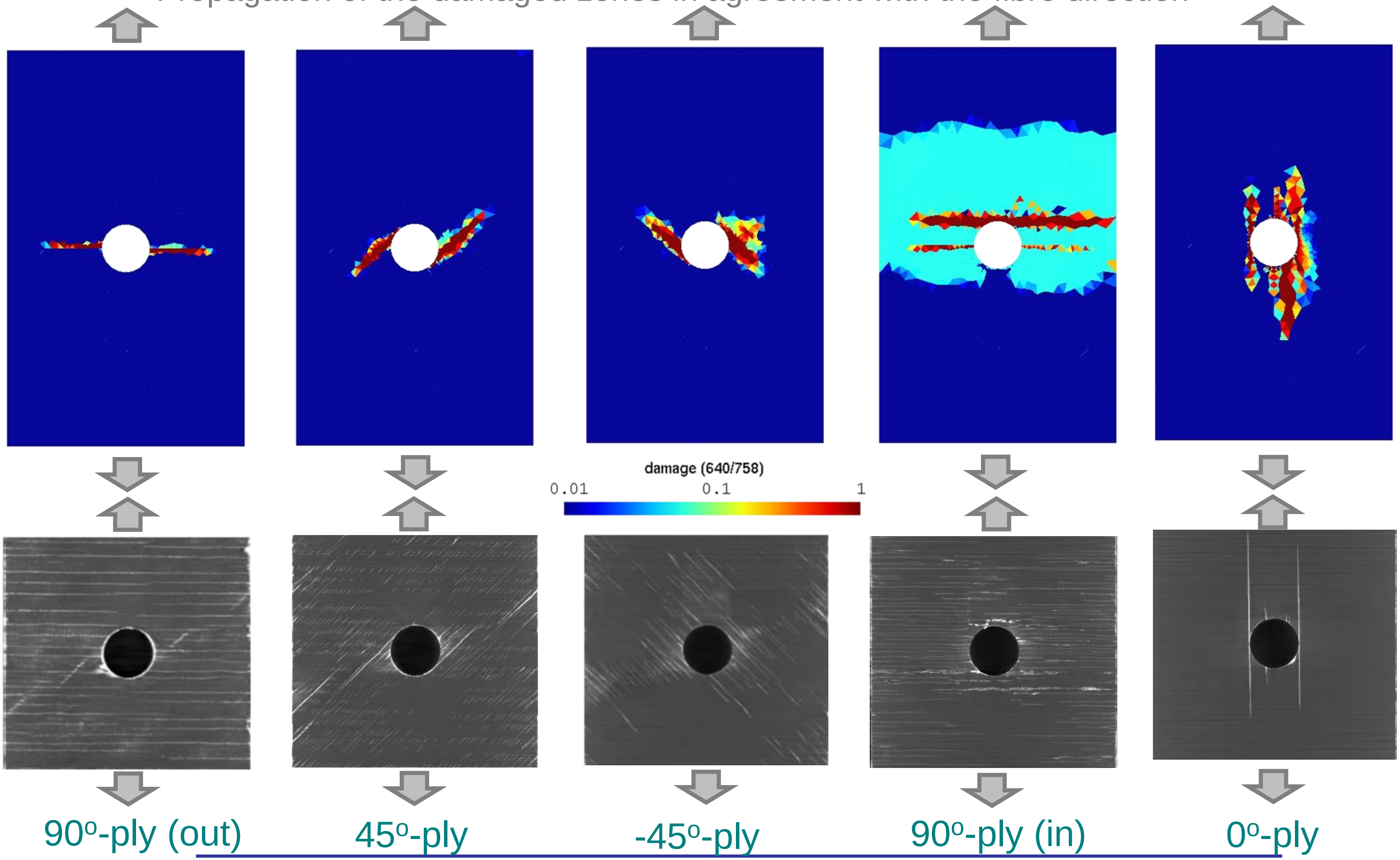
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate (2)
  - Predicted delamination zones in agreement with experiments
  - Tensile stress within 15 %





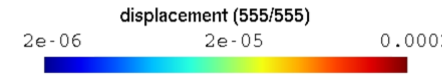
Experimental validation

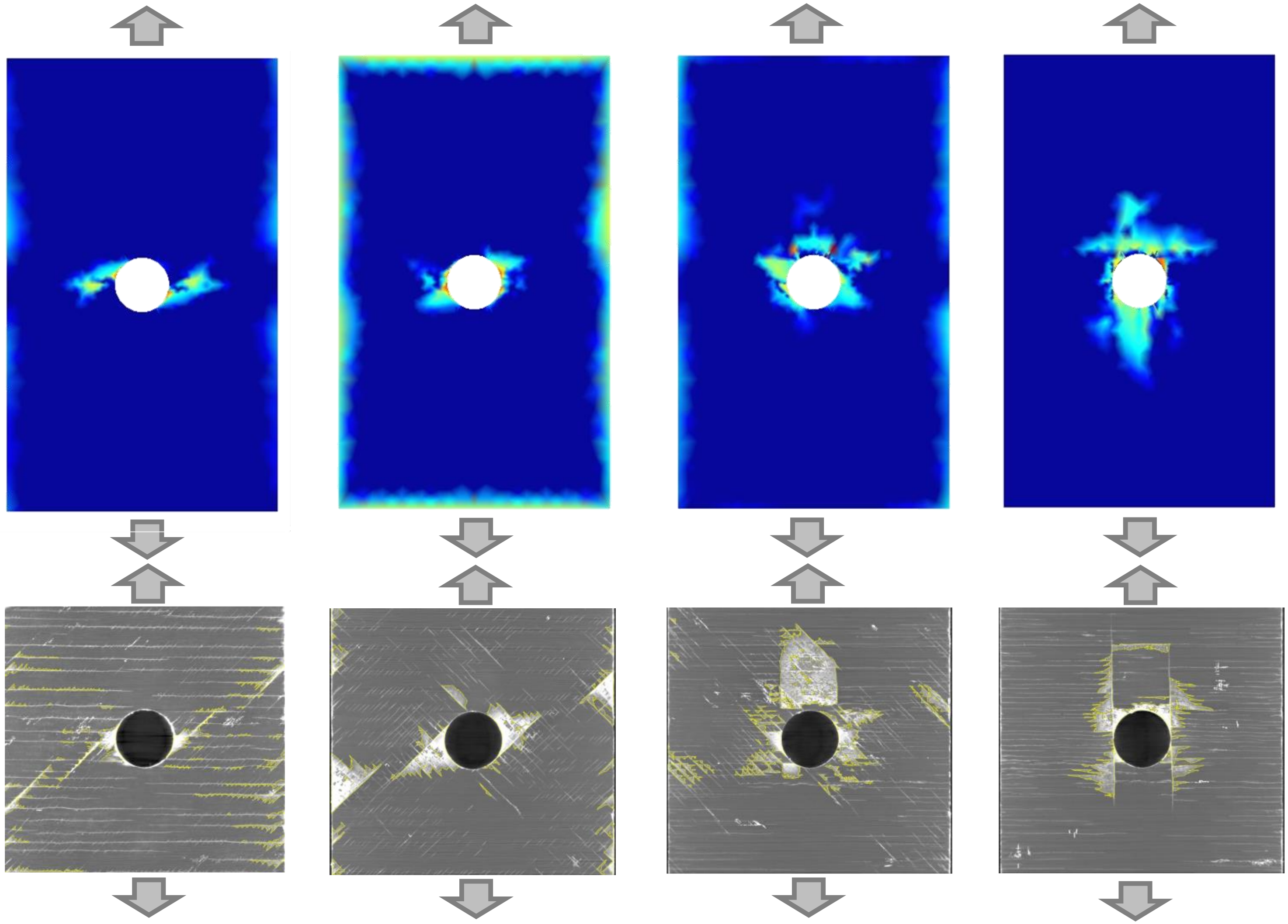
- [90º / 45º / -45º / 90º / 0º]$_S$- open hole laminate (2)
  - Propagation of the damaged zones in agreement with the fibre direction



Experimental validation

- $[90^{o} / 45^{o} / -45^{o} / 90^{o} / 0^{o}]_{S}$- open hole laminate (3)
  - Predicted delamination zones in agreement with experiments

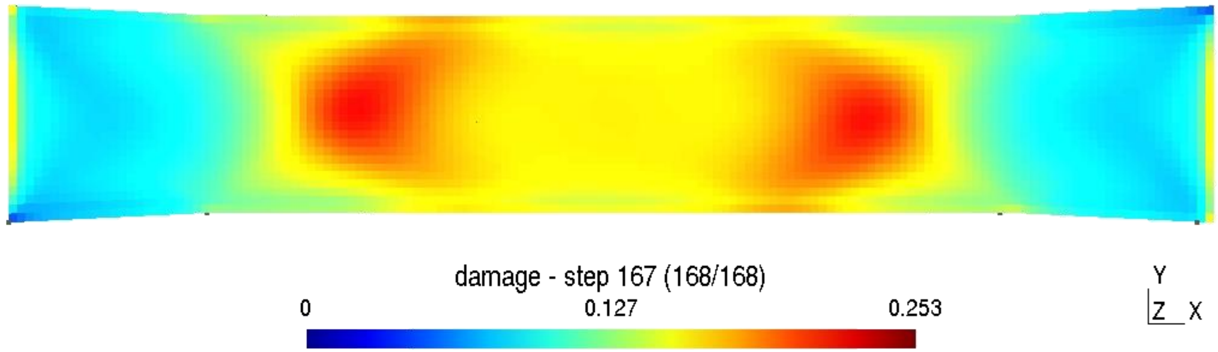




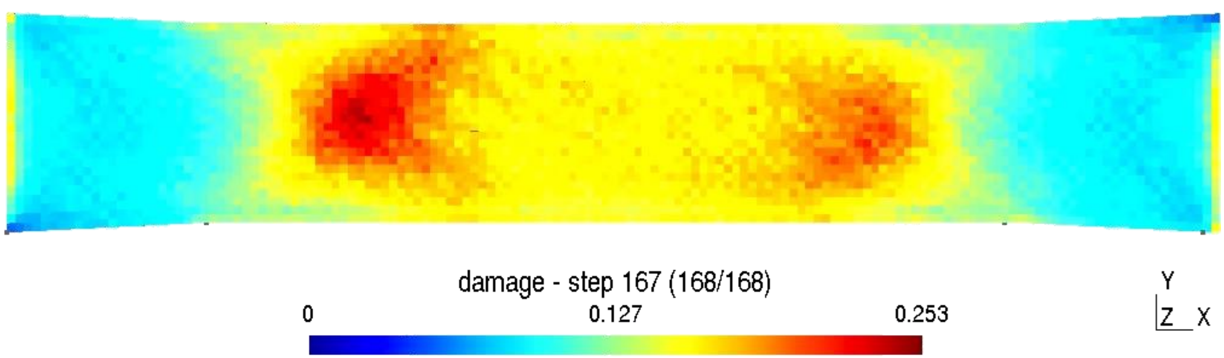
90^{o} (out) / 45^{o} | 45^{o} / -45^{o} | -45^{o} / 90^{o} (in) | 90^{o} (in) / 0^{o}

Introduction of uncertainties

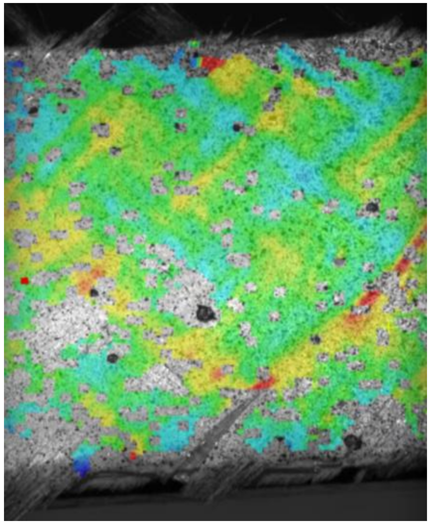
- [45º / -45º]$_S$ laminate under uniform tension
  - No hole to trigger localization
  - Material defects trigger localization



Uniform volume fraction



5% variation in volume fraction



GFRP [45/-45]

https://m.youtube.com/watch?v=PotKTduzTxg

Conclusions

- Multi-scale method for the failure analysis of composite laminates
  - Damage-enhanced MFH
  - Non-local implicit formulation
  - Hybrid DG/CZM for delamination

- Experimental validation
  - Open-hole laminates
  - Different stacking sequences

- Introduction of material uncertainties in the model
  - First simulations