Simulations of composite laminates inter- and intra-laminar failure using on a non-local mean-field damage-enhanced multi-scale method

Ling Wu (CM3), L. Adam (e-Xstream), B. Bidaine (e-Xstream), Ludovic Noels. (CM3)
Experiments: F. Sket (IMDEA), J.M. Molina (IMDEA), A. Makradi (List)

STOMMMAC The research has been funded by the Walloon Region under the agreement no 1410246-STOMMMAC (CT-INT 2013-03-28) in the context of M-ERA.NET Joint Call 2014.
SIMUCOMP The research has been funded by the Walloon Region under the agreement no 1017232 (CT-EUC 2010-10-12) in the context of the ERA-NET +, Matera + framework.
Content

• Introduction
  – Failure of composite laminates
  – Multi-scale modelling
  – Mean-Field-Homogenization (MFH)

• Micro-scale modelling
  – Incremental-Secant MFH
  – Damage-enhanced incremental-secant MFH

• Multi-scale method for the failure analysis of composite laminates
  – Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  – Inter-laminar failure: Hybrid DG/cohesive zone model
  – Experimental validation

• Introduction of uncertainties
  – As a random field
Failure of composite laminates

- **Difficulties**
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation

On Micro-scale volume

Not possible at structural scale
Failure of composite laminates

- **Difficulties**
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation is not possible at structural scale
  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments
Failure of composite laminates

• Difficulties
  – Different involved mechanisms at different scales
    • Inter-laminar failure
    • Intra-laminar failure
  – Direct finite element simulation is not possible at structural scale

  – Continuum damage models do not represent accurately the intra-laminar failure
    • Damage propagation direction is not in agreement with experiments

• Solution:
  – Embed damage model in a multi-scale formulation
  – For computational efficiency: use of mean-field-homogenization
  – For macro cracks: using hybrid DG/Cohesive zone model
Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is known, $\bar{\sigma}$ is sought
  - Transition
    - Downscaling: $\bar{\varepsilon}$ is used as input of the MFH model
    - Upscaling: $\bar{\sigma}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models

Mori and Tanaka 73, Hill 65, Ponte Castañeda 91, Suquet 95, Doghri et al. 03, Lahellec et al. 11, Brassart et al. 12, …
Mean-Field-Homogenization

- **Key principles**
  - Based on the averaging of the fields
    \[
    \langle a \rangle = \frac{1}{V} \int_V a(X) dV
    \]
  - Meso-response
    - From the volume ratios (\( v_0 + v_1 = 1 \))
    \[
    \bar{\sigma} = \langle \sigma \rangle = v_0 \langle \sigma \rangle_{\omega_0} + v_1 \langle \sigma \rangle_{\omega_1} = v_0 \sigma_0 + v_1 \sigma_1
    \]
    \[
    \bar{\epsilon} = \langle \epsilon \rangle = v_0 \langle \epsilon \rangle_{\omega_0} + v_1 \langle \epsilon \rangle_{\omega_1} = v_0 \epsilon_0 + v_1 \epsilon_1
    \]
    - One more equation required
      \[
      \epsilon_1 = B^\epsilon : \epsilon_0
      \]
  - Difficulty: find the adequate relations
    \[
    \begin{align*}
    \sigma_1 &= f(\epsilon_1) \\
    \sigma_0 &= f(\epsilon_0) \\
    \epsilon_1 &= B^\epsilon : \epsilon_0
    \end{align*}
    \]
Key principles (2)

- Linear materials
  
  - Materials behaviours
    
    \[
    \begin{align*}
    \sigma_1 &= \overline{C}_1 : \varepsilon_1 \\
    \sigma_0 &= \overline{C}_0 : \varepsilon_0
    \end{align*}
    \]

  - Mori-Tanaka assumption \( \varepsilon^\infty = \varepsilon_0 \)

  - Use Eshelby tensor
    
    \[
    \varepsilon_1 = B^\varepsilon \left( I, \overline{C}_0, \overline{C}_1 \right) : \varepsilon_0
    \]
    
    with \( B^\varepsilon = [ I + S : \overline{C}_0^{-1} : (\overline{C}_1 - \overline{C}_0)]^{-1} \)
Mean-Field-Homogenization

• Key principles (2)
  – Linear materials
    • Materials behaviours
      \[
      \begin{align*}
      \sigma_1 &= \overline{C}_1 : \varepsilon_1 \\
      \sigma_0 &= \overline{C}_0 : \varepsilon_0
      \end{align*}
      \]
    • Mori-Tanaka assumption \( \varepsilon^\infty = \varepsilon_0 \)
    • Use Eshelby tensor
      \[
      \varepsilon_1 = B^\varepsilon(I, \overline{C}_0, \overline{C}_1) : \varepsilon_0
      \]
      with \( B^\varepsilon = [I + S : \overline{C}_0^{-1} : (\overline{C}_1 - \overline{C}_0)]^{-1} \)
  – Non-linear materials
    • Define a Linear Comparison Composite (LCC)
    • Common approach: incremental tangent
      \[
      \Delta \varepsilon_1 = B^\varepsilon(I, \overline{C}_0^{\text{alg}}, \overline{C}_1^{\text{alg}}) : \Delta \varepsilon_0
      \]
Content

- Micro-scale modelling
  - Incremental-Secant Mean-Field-Homogenization (MFH)
  - Damage-enhanced incremental-secant MFH
Incremental-secant mean-field-homogenization

- **Material model**
  - Elasto-plastic material
    - Stress tensor \( \sigma = C^{el} : (\varepsilon - \varepsilon^{pl}) \)
    - Yield surface \( f(\sigma, p) = \sigma^{eq} - \sigma^Y - R(p) \leq 0 \)
    - Plastic flow \( \Delta \varepsilon^{pl} = \Delta p N \quad & \quad N = \frac{\partial f}{\partial \sigma} \)
    - Linearization \( \delta \sigma = C^{alg} : \delta \varepsilon \)
New incremental-secant approach

- Perform a virtual elastic unloading from previous solution
  - Composite material unloaded to reach the stress-free state
  - Residual stress in components

New Linear Comparison Composite (LCC)
New incremental-secant approach

- Perform a virtual elastic unloading from previous solution
  - Composite material unloaded to reach the stress-free state
  - Residual stress in components

**New Linear Comparison Composite (LCC)**

- Apply MFH from unloaded state
  - New strain increments (>0)
    \[
    \Delta \varepsilon_{I/0}^r = \Delta \varepsilon_{I/0} + \Delta \varepsilon_{I/0}^{unload}
    \]
  - Use of secant operators
    \[
    \Delta \varepsilon_1^r = B^s \left( I, \overline{C}_{0}^{Sr}, \overline{C}_{1}^{Sr} \right): \Delta \varepsilon_0^r
    \]
• Zero-incremental-secant method
  – Continuous fibres
    • 55% volume fraction
    • Elastic
  – Elasto-plastic matrix
  – For inclusions with high hardening (elastic)
    • Model is too stiff

Longitudinal tension

Transverse loading

\[ f(\sigma, p) = \overline{\sigma}^{eq} - \sigma^Y - R(\overline{p}) \leq 0 \]

\( \overline{\sigma}^{eq} \) is underestimated
Incremental-secant mean-field-homogenization

- **Zero-incremental-secant method (2)**
  - Continuous fibres
    - 55% volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if negative residual stress

Longitudinal tension

Transverse loading

\[ \frac{\sigma}{\sigma_0} - \varepsilon \]

\[ \frac{\sigma}{\sigma_0} - \varepsilon^f \]

\[ \Delta\varepsilon^f \]

\[ \Delta\varepsilon^0 \]

\[ \Delta\varepsilon^r \]

\[ \varepsilon \]

\[ \varepsilon^f \]

\[ \varepsilon^0 \]
Incremental-secant mean-field-homogenization

- Verification of the method
  - Spherical inclusions
    - 17% volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-proportional loading

\[ \varepsilon_{13} = \varepsilon_{23} \]
\[ \varepsilon_{33} = 2 \varepsilon_{11} = 2 \varepsilon_{22} \]

\[ \sigma_{13} \text{ [Mpa]} \]
\[ \sigma_{33} \text{ [Mpa]} \]

FF (Lahellec et al., 2013)
MFH, incr. tg.
MFH, var. (Lahellec et al., 2013)
MFH, incr. sec.
Non-local damage-enhanced MFH

- **Material models**
  - Elastoplastic material
    - Stress tensor \( \sigma = C^{el} : (\varepsilon - \varepsilon^{pl}) \)
    - Yield surface \( f(\sigma, p) = \sigma^{eq} - \sigma^Y - R(p) \leq 0 \)
    - Plastic flow \( \Delta \varepsilon^{pl} = \Delta p N \quad & \quad N = \frac{\partial f}{\partial \sigma} \)
    - Linearization \( \delta \sigma = C^{alg} : \delta \varepsilon \)
Non-local damage-enhanced MFH

- **Material models**
  - Elasto-plastic material
    - Stress tensor \( \sigma = C^{el} : (\varepsilon - \varepsilon^{pl}) \)
    - Yield surface \( f(\sigma, p) = \sigma^{eq} - \sigma^Y - R(p) \leq 0 \)
    - Plastic flow \( \Delta \varepsilon^{pl} = \Delta p N \) & \( N = \frac{\partial f}{\partial \sigma} \)
    - Linearization \( \delta \sigma = C^{alg} : \delta \varepsilon \)
  - Local damage model
    - Apparent-effective stress tensors \( \sigma = (1 - D)\hat{\sigma} \)
    - Plastic flow in the effective stress space
    - Damage evolution \( \Delta D = F_D(\varepsilon, \Delta p) \)
Non-local damage-enhanced MFH

• Material models
  – Elasto-plastic material
    • Stress tensor \( \sigma = C^{el} : (\varepsilon - \varepsilon^{pl}) \)
    • Yield surface \( f(\sigma, p) = \sigma^{eq} - \sigma^{Y} - R(p) \leq 0 \)
    • Plastic flow \( \Delta \varepsilon^{pl} = \Delta p N \) & \( N = \frac{\partial f}{\partial \sigma} \)
    • Linearization \( \delta \sigma = C^{alg} : \delta \varepsilon \)
  – Local damage model
    • Apparent-effective stress tensors \( \sigma = (1 - D)\hat{\sigma} \)
    • Plastic flow in the effective stress space
    • Damage evolution \( \Delta D = F_D(\varepsilon, \Delta p) \)
  – Non-Local damage model
    • Damage evolution \( \Delta D = F_D(\varepsilon, \Delta \tilde{p}) \)
    • Anisotropic governing equation \( \tilde{p} - \nabla \cdot (c_g \cdot \nabla \tilde{p}) = p \)
    • Linearization \( \delta \sigma = \left[ (1 - D)C^{alg} - \hat{\sigma} \otimes \frac{\partial F_D}{\partial \varepsilon} \right] : \delta \varepsilon - \hat{\sigma} \frac{\partial F_D}{\partial \tilde{p}} \delta \tilde{p} \)
Non-local damage-enhanced MFH

**Equations summary: zero-approach**

- For soft matrix response
  - Remove residual stress in matrix
  - Avoid adding spurious internal energy

- Solve iteratively the system

\[
\begin{align*}
\Delta \bar{e}^{(r)} &= v_0 \Delta e_0^{(r)} + v_1 \Delta e_1^{(r)} \\
\Delta e_1^r &= \Delta e_1 + \Delta e_1^{unload} \\
\Delta e_0^r &= \Delta e_0 + \Delta e_0^{unload} \\
\Delta e_1^r &= B^\varepsilon \left( I, (1 - D) \overline{C}_0^{S0}, \overline{C}_1^{Sr} \right) : \Delta e_0^r
\end{align*}
\]

- With the stress tensors

\[
\begin{align*}
\bar{\sigma} &= v_0 \sigma_0 + v_1 \sigma_1 \\
\sigma_1 &= \sigma_1^{res} + \overline{C}_1^{Sr} : \Delta e_1^r \\
\sigma_0 &= (1 - D) \overline{C}_0^{S0} : \Delta e_0^r
\end{align*}
\]
- **Mesh-size effect**
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply

---

**Non-local damage-enhanced MFH**

**Force [N/mm]**

**Displacement [mm]**

- Mesh size: 0.43 mm
- Mesh size: 0.3 mm
- Mesh size: 0.15 mm
- Mesh size: 0.1 mm
Content

• Multi-scale method for the failure analysis of composite laminates
  – Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  – Inter-laminar failure: Hybrid DG/cohesive zone model
  – Experimental validation
Intra-laminar failure: Non-local damage-enhanced MFH

- Weak formulation of a composite laminate
  - Strong form
    \[
    \nabla \cdot \bar{\sigma}^T + f = 0 \quad \text{for each homogenized ply } \Omega_i
    \]
    \[
    \tilde{p} - \nabla \cdot (c_g \cdot \nabla \tilde{p}) = p \quad \text{for the matrix phase}
    \]
  - Boundary conditions
    \[
    \bar{\sigma} \cdot \bar{n} = \bar{T}
    \]
    \[
    \bar{n} \cdot (c_g \cdot \nabla \tilde{p}) = 0
    \]
  - Macro-scale finite-element discretization
    \[
    \tilde{p} = N^a_p \tilde{p}^a
    \]
    \[
    \bar{u} = N^a_u \bar{u}^a
    \]
    \[
    \begin{bmatrix}
    K_{\bar{u}u} & K_{\bar{u}p} \\
    K_{\bar{p}u} & K_{\bar{p}p}
    \end{bmatrix}
    \begin{bmatrix}
    d\bar{u} \\
    d\tilde{p}
    \end{bmatrix}
    =
    \begin{bmatrix}
    F_{\text{ext}} - F_{\text{int}} \\
    \bar{F}_p - \tilde{F}_p
    \end{bmatrix}
    \]
    \[
    \Gamma_L = \bigcup \Gamma_{L,i} = \bigcup (\Omega_i \cap \Omega_{i+1})
    \]
    \[
    \omega = \bigcup \omega_i
    \]
Experimental validation

- \([45^\circ_4 / -45^\circ_4]_S\)- open hole laminate
  - Tensile test on several coupons

  - Propagation of the damaged zones in agreement with the fibre direction

\[ \begin{array}{c}
\begin{array}{c}
4.68 \pm 0.05 \\
\end{array}
\end{array} \]

\[ \begin{array}{ccc}
\text{39.60} & \pm & 0.35 \\
\text{Ø13} & & \\
\hline
40 & & 220 \\
& & 300
\end{array} \]

-45\(^\circ\)-ply

- 45\(^\circ\)-ply
Experimental validation

- $[45^\circ_4 / -45^\circ_4]_S$ - open hole laminate (2)
  - Predicted delamination zones in agreement with experiments
  - Tensile stress within 15 %
Experimental validation

- $[90^\circ / 45^\circ / -45^\circ / 90^\circ / 0^\circ]_S$- open hole laminate (2)
  - Propagation of the damaged zones in agreement with the fibre direction
Experimental validation

- \([90^\circ / 45^\circ / -45^\circ / 90^\circ / 0^\circ]_s\) - open hole laminate (3)
  - Predicted delamination zones in agreement with experiments
Introduction of uncertainties

- $[45^\circ / -45^\circ]_s$ laminate under uniform tension
  - No hole to trigger localization
  - Material defects trigger localization

5% variation in volume fraction

GFRP [$45/-45$]

https://m.youtube.com/watch?v=PotKTduzTxg
Conclusions

• Multi-scale method for the failure analysis of composite laminates
  – Damage-enhanced MFH
  – Non-local implicit formulation
  – Hybrid DG/CZM for delamination

• Experimental validation
  – Open-hole laminates
  – Different stacking sequences

• Introduction of material uncertainties in the model
  – First simulations