Anomaly-free chiral fermion sets and gauge coupling unification

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Abstract. In this work we search for minimal sets of chiral fermions, with arbitrary quantum numbers, beyond the Standard Model that are anomaly-free and lead to vector-like particles under $\mathsf{SU}(3)$ and $\mathsf{U}(1)_{\mathrm{em}}$ after symmetry breaking. We further study which of these anomaly-free sets lead to unification of the gauge couplings at energy scales higher that 5.0×10^{15} GeV in order to be consistent with proton decay bounds. A similar study is performed in the context of the $\mathsf{SU}(5)$ gauge group; for some of the anomaly-free sets found it is possible to obtain unification of the gauge couplings with the extra fermions decoupling at high intermediate scales.

1. Introduction

Despite its great success and agreement with the experimental data, the standard model (SM) of particle physics, governing quantum chromodynamics and electroweak interactions, provides neither an explanation for the neutrino mass nor a candidate for dark matter. Based on the gauge group $\mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)$, the SM is spontaneously broken down to $\mathsf{SU}(3) \times \mathsf{U}(1)_{\mathrm{em}}$ at electroweak scale by the vacuum expectation value of the neutral component of the Higgs field. At this point the fermions, previously chiral and massless, become massive and the remaining symmetry leads to vector-like particles with respect to $\mathsf{SU}(3)$ and $\mathsf{U}(1)_{\mathrm{em}}$ symmetries.

In the SM there is no symmetry principle relating the three gauge couplings, α_y, α_w and α_s which correspond to hypercharge, weak isospin and strong interactions, respectively. However, if the SM is seen as embedded in a larger symmetry, such as in grand unified theories, one may expect that at some high energy scale the three couplings, $\alpha_{1,2,3} = \kappa_{1,2,3}\,\alpha_{y,w,s}$, unify for adequate normalization factors $\kappa_{1,2,3}$. In the case we consider the embedding in a string framework it is even possible to unify the gravitational coupling together with the gauge couplings at a scale around 10^{17} GeV.

Using the renormalization group equations at one-loop level to evolve the SM gauge couplings and considering the $\mathsf{SU}(5)$ normalization, $\kappa_1 = 5/3$, $\kappa_2 = 1 = \kappa_3$, one finds $\alpha_1 = \alpha_2$ around 10^{13} GeV and $\alpha_2 = \alpha_3$ around 10^{17} GeV, which encourages us to search for gauge unification within this interval. To do so, we can play with the normalization factors (see e.g. Ref. [1]) or include extra particles. However, the consideration of extra fermions beyond the SM content leads us to the issue of anomaly cancellation.

An anomaly appears when the symmetry of the classical Lagrangian is broken by quantum effects. If we want to have a renormalized theory and guarantees its gauge invariance one needs

to get rid of the so-called triangular anomalies [2, 3, 4]. They are determined from the one-loop triangle diagram considering the chiral fermion running in the loop and the gauge bosons in the external lines. Only chiral massless fermions contribute to anomalies and the left- and right-handed fermions contribute with opposite sign. Additionally, only fermions in complex representations [5] are problematic; the ones in real and pseudo-real representations have no anomaly. For more details on anomalies see e.g. Ref. [6].

Besides the triangle anomaly, a chiral theory has a global $SU(2)$ gauge anomaly [7], called also Witten's anomaly, and a mixed gauge-gravitational anomaly [8, 9]. The absence of the mixed gauge-gravitational anomaly guarantees the general covariance of the theory while the absence of the Witten's anomaly is required to enable the computation of the fermion integral in an invariant way. According to the Witten's anomaly, a consistent theory is the one where the sum, over all representations, of the trace of the squared $SU(2)$ generators need to be an integer number and hence any theory with an $SU(2)$ gauge group needs to have an even number of Weyl doublets in order to be mathematically consistent.

The SM is free of anomalies: the cubic $SU(3)$ anomaly vanishes because the group is non-chiral however the cubic $SU(2)$ anomaly vanishes because it has only real representations; the cancellation of remaining triangle anomalies is achieved among the generations [4, 10, 5]. Only when the three anomaly conditions (triangular, gravitational and Witten's anomaly) are satisfied do we have an unique determination of the massless fermion representations and their hypercharge relations [11]. In fact, the fifteen Weyl states of the SM do not constitute the simplest anomaly-free set of fermions; the set $(3,1)_Q \oplus (\overline{3},1)_{-Q} \oplus (1,2)_q \oplus (1,2)_{-q}$ is simpler however it is a vector-like set.

In the literature, one can find several works addressing the study of anomalies in the case where extra chiral fermions were added to the SM e.g. Refs. [12, 13, 14, 15, 16, 17]. Ref. [18] was studied the minimal vector- like fermion content necessary, beyond the SM, to unify the gauge couplings and the gravitational coupling in the context of weakly coupled heterotic string theory. Here, in this work, as done in Ref. [19], we search for the minimal sets of anomaly-free chiral fermions, beyond the SM, and study whether or not the unification of the gauge couplings holds. The study was extended to string scale in Ref. [20].

This paper is organized as follows: in section 2 we search for the minimal sets of chiral fermions with arbitrary quantum numbers, beyond the SM particle content, that are free of anomalies, we study then whether these solutions lead to unification of the gauge couplings. In section 3, we extend the study of the previous section to chiral fermion sets that belong to $SU(5)$ multiplets. We draw the conclusions in section 4.

2. Minimal anomaly-free chiral fermion sets beyond SM
In this section we study which is the minimal chiral fermion content that can be added to the SM and is anomaly-free. For such propose, and in order to preserve the parity symmetry, we require that our particle content is vector-like from the $U(1)_{em}$ and $SU(3)$ point of view.

Once the SM is by itself anomaly-free, we need just to consider the contributions coming from the new chiral fermions that we are interested in. Hence the contributions of the new

chiral fermions must verify the following anomaly conditions

$$[\mathsf{SU}(3)\text{-}\mathsf{SU}(3)\text{-}\mathsf{SU}(3)]: \qquad \sum_R A_3(R)\,d_2(R) \;=\; 0\,, \tag{1a}$$

$$[\mathsf{SU}(3)\text{-}\mathsf{SU}(3)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R\,t_3(R)\,d_2(R) \;=\; 0\,, \tag{1b}$$

$$[\mathsf{SU}(2)\text{-}\mathsf{SU}(2)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R\,t_2(R)\,d_3(R) \;=\; 0\,, \tag{1c}$$

$$[\mathsf{U}(1)\text{-}\mathsf{U}(1)\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R^3\,d_2(R)\,d_3(R) \;=\; 0\,, \tag{1d}$$

$$[\text{gravity}\text{-}\text{gravity}\text{-}\mathsf{U}(1)]: \qquad \sum_R y_R\,d_2(R)\,d_3(R) \;=\; 0\,, \tag{1e}$$

written with respect to the standard model gauge group; $d_i(R)$, y_R, $A_i(R)$ and $t_i(R)$ are, respectively, the dimension, the hypercharge, the cubic anomaly index and the Dynkin index of the representation R with respect to the subgroup G_i. Without loss of generality, we consider that all new chiral fields added are left-handed, from now on, we represent the chiral multiplets as $(d_3(R), d_2(R))_{y_R}$.

The cubic anomaly index, $A_i(R)$, for a representation R of a group G is given by [21],

$$\mathrm{tr}\left(\left\{T_R^a, T_R^b\right\}T_R^c\right) \;=\; A(R)\,\mathrm{tr}\left(\left\{T^a, T^b\right\}T^c\right), \tag{2}$$

where T^a are the generators of the fundamental representation of G and T_R^a are the generators of the representation R of G. The cubic anomaly takes integer values and verifies the relation $A(\overline{R}) = -A(R)$ ($\overline{R}$ is the conjugate of R) which means that real representations, like all $\mathsf{SU}(2)$ representations, have vanishing cubic anomaly index.. The Dynkin index, t_R, of a representation R, can be determined through the relation

$$t_m(R)\delta^{ab} = \mathrm{tr}\left(T_R^a\,T_R^b\right), \tag{3}$$

where $t_m(\overline{R}) = t_m(R)$ holds. For the fundamental representation of $\mathsf{SU}(N)$ groups the Dynkin index is normalized to $1/2$ and for $\mathsf{U}(1)_Y$ is given by $t_1(R) = y_R^2$. We list in Table 1 the cubic anomaly index for $\mathsf{SU}(3)$ and the Dynkin index for $\mathsf{SU}(2)$ and $\mathsf{SU}(3)$ (for more details on Dynkin and cubic anomaly indices computation see e.g. [20, 22]).

The sum of the $\mathsf{SU}(2)$ Dynkin indices over all the representations present in a theory is constrained by the Witten's anomaly to be an integer number. For a question of simplicity, we will consider $\mathsf{SU}(2)$ representations up to dimension 5 which implies that the number of chiral $\mathsf{SU}(2)$ doublets is even (see Table 1).

Looking at the conditions given in Eqs. (1), one sees that they are invariant under an overall rescaling of the hypercharges, y_R, independently of the number of multiplets considered. This hypercharge overall normalization is important when we impose the condition of having vector-like particles after electroweak symmetry breaking and must then be determined in a proper way.

In order to know how many chiral multiplets do we need and how they look like under the SM gauge group, to be an anomaly-free set, let us look over Eqs (1). In the case we consider one multiplet, the only possible solution is to have a chiral fermion in the adjoint representation of $\mathsf{SU}(3)$ (the octect) with hypercharge zero. In the case we consider two chiral fermions, R_1 and R_2, we end up with the relation $y_{R_1}^2 = y_{R_2}^2$ among the hypercharges of the two fermions, resulting in two sub-cases: $y_{R_1} = y_{R_2}$ and $y_{R_1} = -y_{R_2}$. For $y_{R_1} = y_{R_2}$ the only possible

Table 1: The cubic anomaly index for $\mathsf{SU}(3)$, A_3, and the Dynkin index for $\mathsf{SU}(2)$ and $\mathsf{SU}(3)$, t_2, t_3, respectively (smallest irreducible representations considered).

$\mathsf{SU}(2)$-irrep	**2**	**3**	**4**	**5**	**6**	**7**
t_2	$\frac{1}{2}$	2	5	10	$\frac{23}{2}$	28
$\mathsf{SU}(3)$-irrep	**3**	**6**	**8**	**10**	**15**	**15'**
t_3	$\frac{1}{2}$	$\frac{5}{2}$	3	$\frac{15}{2}$	10	$\frac{35}{2}$
A_3	1	7	0	27	14	77

solution arises for vanishing hypercharge; to satisfy the condition in Eq. (1a) either the cubic anomaly of each multiplet must be zero (adjoint representation) or the cancellation occurs between multiplets, as for example in the set $(\mathbf{6}, \mathbf{1})_0 \oplus (\overline{\mathbf{3}}, \mathbf{7})_0$. Working out the equations for the sub-case $y_{R_1} = -y_{R_2}$, we end up with multiplets of equal $\mathsf{SU}(2)$ dimension; requiring to have vector-like particles under $\mathsf{SU}(3)$ after electroweak symmetry breaking leads to vector-like pairs of the form $(\mathbf{d}, \mathbf{d'})_y \oplus (\overline{\mathbf{d}}, \mathbf{d'})_{-y}$.

In the case of three chiral multiplets the anomaly-free conditions are given by,

$$a_3(R_1)\, d_2(R_1) \,+\, a_3(R_2)\, d_2(R_2) \,+\, a_3(R_3)\, d_2(R_3) \,=\, 0\,,$$

$$z\, y_{R_1} t_3(R_1)\, d_2(R_1) \,+\, z\, y_{R_2} t_3(R_2)\, d_2(R_2) \,+\, z\, y_{R_3} t_3(R_3)\, d_2(R_3) \,=\, 0\,,$$

$$z\, y_{R_1} t_2(R_1)\, d_3(R_1) \,+\, z\, y_{R_2} t_2(R_2)\, d_3(R_2) \,+\, z\, y_{R_3} t_2(R_3)\, d_3(R_3) \,=\, 0\,,$$

$$z^3\, y_{R_1}^3\, d_2(R_1)\, d_3(R_1) \,+\, z^3\, y_{R_2}^3\, d_2(R_2)\, d_3(R_2) \,+\, z^3\, y_{R_3}^3\, d_2(R_3)\, d_3(R_3) \,=\, 0\,,$$

$$z\, y_{R_1} d_2(R_1)\, d_3(R_1) \,+\, z\, y_{R_2} d_2(R_2)\, d_3(R_2) \,+\, z\, y_{R_3} d_2(R_3)\, d_3(R_3) \,=\, 0\,.$$

In order to see whether it is possible to have anomaly-free sets with three chiral multiplets, we restrict our search to representations with $\mathsf{SU}(3)$ dimensions $\mathbf{d} \equiv d_3(R) \leq 10$ and of $\mathsf{SU}(2)$ dimensions $d_2(R) \leq 5$, and rational hypercharges. For this restrictions we obtain four different sets of solutions, those presented in Table 2. Looking at the results, we note that the $\mathsf{SU}(3)$ dimensions, $\mathbf{d}$ are equal within each set and can take the values 1, 3, 6, 8 and 10. The exceptions are the sets P1 and P4 where the $\mathsf{SU}(3)$ dimension is restricted, by the Witten's anomaly [7], to be an even number. The fact that, the $\mathsf{SU}(3)$ dimensions are equal within each set is a consequence of the choice, $d_3(R) \leq 10$; if we relax this constraint we find solutions with completely different $\mathsf{SU}(3)$ dimensions, such as $(\mathbf{15}, \mathbf{1})_{z/6} \oplus (\overline{\mathbf{6}}, \mathbf{2})_{-z/3} \oplus (\mathbf{1}, \mathbf{3})_{z/2}$, but in that case we have much higher dimensions. Let us remember that z is an overall rescaling on the hypercharge and its value is of major importance in determining which of the solutions lead to vector-like particles (with respect to the electric charge) at low energy. In order to fix the value of z, we need to verify, for each set, the condition for the cancellation of the electric charges,

$$\sum_{p=1}^{3} \sum_{j_p} \left[j_p + y_p(z) \right]^m = 0\,, \tag{4}$$

where m is any odd positive integer number and $j_p = -s_p, -s_p + 1, \ldots, s_p - 1, s_p$ with $s_p = (d_2(R_p) - 1)/2$. For $m = 1$ or 3, Eq. (4) is automatically satisfied; the value of z is then determined for $m = 5$ which leads to $|z| = 0, 1$, or 3, independently of the set. We found that, for $\mathbf{d} = 1$ or 8 all values of z are possible however when $\mathbf{d} = 3, 6$ or 10 only $|z| = 1$ is viable.

Table 2: Minimal solutions for anomaly-free chiral fermion sets with $\mathsf{SU}(3)$ dimension $\mathbf{d} \leq \mathbf{10}$ and $\mathsf{SU}(2)$ dimension $d_2(R) \leq 5$. The $\mathsf{SU}(3)$ dimension for sets P1 and P4 is forced to be, by the Witten's anomaly, an even number.

Set	Particle content		
P1	$(\mathbf{d}, \mathbf{1})_{5z/6}$ $\oplus$	$(\mathbf{d}, \mathbf{2})_{-2z/3}$ $\oplus$	$(\overline{\mathbf{d}}, \mathbf{3})_{z/6}$
P2	$(\mathbf{d}, \mathbf{1})_{7z/6}$ $\oplus$	$(\mathbf{d}, \mathbf{3})_{-5z/6}$ $\oplus$	$(\overline{\mathbf{d}}, \mathbf{4})_{z/3}$
P3	$(\mathbf{d}, \mathbf{1})_{3z/2}$ $\oplus$	$(\mathbf{d}, \mathbf{4})_{-z}$ $\oplus$	$(\overline{\mathbf{d}}, \mathbf{5})_{z/2}$
P4	$(\mathbf{d}, \mathbf{2})_{4z/3}$ $\oplus$	$(\mathbf{d}, \mathbf{3})_{-7z/6}$ $\oplus$	$(\overline{\mathbf{d}}, \mathbf{5})_{z/6}$

2.1. Gauge coupling unification

We discuss now whether it is possible to achieve unification of the gauge couplings for some of the anomaly-free sets found above. For such propose, we will work in the framework of the SM extended by the chiral fermion sets in Table 2. In our approach we consider only the Higgs and gauge content of the SM; if some additional Higgs or gauge fields are present we take them it do not affect the running.

The evolution of gauge couplings $\alpha_i \, (i = 1, 2, 3)$ is governed by the renormalization group equations that have an exact solution at one-loop level, given by

$$\alpha_U^{-1} = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi} B_i \ln\left(\frac{\Lambda}{M_Z}\right). \tag{5}$$

At the unification scale one must have,

$$\alpha_U \equiv \kappa_1 \, \alpha_y(\Lambda) = \kappa_2 \, \alpha_w(\Lambda) = \kappa_3 \, \alpha_s(\Lambda), \tag{6}$$

with $\alpha_U \lesssim 1$ to ensure that we are in the perturbative regime [23].

In the presence of N chiral fermions with intermediate mass scales $M_I, I = 1, \ldots, N$, the parameter B_i in Eq. (6) is given by,

$$B_i = \frac{1}{\kappa_i}\left(b_i^{\mathrm{SM}} + \sum_{I=1}^{N} b_i^I r_I\right), \tag{7}$$

varying between M_Z and the unification scale Λ. The parameter r_I is called "running weight" and is defined as,

$$r_I = \frac{\ln\left(\Lambda/M_I\right)}{\ln\left(\Lambda/M_Z\right)}, \tag{8}$$

taking values in the interval $0 \leq r_I \leq 1$. The one-loop beta coefficients b_i^{SM} account for the SM contribution while b_i^I account for the contributions of the intermediate particles above the threshold M_I.

The one-loop beta coefficient, b_i^R, for one chiral fermion in a representation R can be computed through the expression:

$$b_i^R = \frac{2}{3}\, t_i(R) \prod_{j \neq i} d_j(R). \tag{9}$$

In the case we have non-chiral particles, the factor 2/3 must be substituted (1/6 for real scalars, 1/3 for complex scalars, $-11/3$ for gauge bosons and 4/3 for vector-like fermions). For the SM particle content we have $b_1^{\mathrm{SM}} = 41/6$, $b_2^{\mathrm{SM}} = -19/6$ and $b_3^{\mathrm{SM}} = -7$.

To determine whether the sets in Table 2 lead to unification of the gauge couplings, we solve Eqs. (6) using the equations above for $N = 3$ chiral multiplets with the corresponding beta coefficient. Hence one has to solve two equations to determine four unknowns: the unification scale and the intermediate mass scales of the three extra particles, expressed in terms of the running weights r_1, r_2 and r_3. We adopt to vary, without lost of generality, the running weights of the second, r_2, and third, r_3, chiral-fermion in Table 2 inside the allowed range and then determine the scale of the remaining particle through the expression,

$$r_1 = \frac{B\,B'_{12} - B'_{23}}{\Delta^1_{23} - B\Delta^1_{12}}, \tag{10}$$

where

$$B'_{ij} = B'_i - B'_j, \qquad B'_i = \frac{1}{\kappa_i}\left(b_i^{\mathrm{SM}} + \sum_{I=2}^{N} b_i^I r_I\right), \tag{11}$$

and

$$\Delta^I_{ij} = \frac{b_i^I}{\kappa_i} - \frac{b_j^I}{\kappa_j}\,. \tag{12}$$

and the unification scale via the relation

$$\ln\left(\frac{\Lambda}{M_Z}\right) = \frac{\widetilde{B}}{B_1 - B_2}\,, \tag{13}$$

where the constants B and $\widetilde{B}$, completely defined in terms of parameters at M_Z scale, are given by [24, 25]

$$B \equiv \frac{\sin^2\theta_w - \dfrac{\kappa_2\,\alpha}{\kappa_3\,\alpha_s}}{\dfrac{\kappa_2}{\kappa_1} - \left(1 + \dfrac{\kappa_2}{\kappa_1}\right)\sin^2\theta_w}, \qquad \widetilde{B} \equiv \frac{2\pi}{\alpha}\left[\frac{1}{\kappa_1} - \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)\sin^2\theta_w\right]. \tag{14}$$

Using the experimental data at $M_Z = 91.1876$ GeV [26] scale

$$\begin{aligned}
\alpha^{-1} &= 127.944 \pm 0.014\,,\\
\alpha_s &= 0.1185 \pm 0.0006\,,\\
\sin^2\theta_w &= 0.23126 \pm 0.00005\,,
\end{aligned} \tag{15}$$

and choosing $\kappa_1 = \kappa_2 = \kappa_3 = 1$ we get

$$B = 0.308 \pm 0.001, \quad \widetilde{B} = 431.4 \pm 0.1\,. \tag{16}$$

The obtained results are shown in plots of Figures 1-4; there we present the allowed ranges for the intermediate mass scales (ordered as they appear in Table 2) and the unification scale (the black bar in the plots) as functions of the SU(3) dimension. In our search we accepted solutions for which the unification scale is higher than $\Lambda \geq 5 \times 10^{15}$ GeV in order to be consistent with proton decay bounds.

For all sets of solutions present in Table 2 there is no gauge coupling unification when $z = 0$. For the set P1, there is gauge coupling unification only for the hypercharge normalization $z = 3$ and $\mathbf{d} = 8$ (Figure 1), in this case all intermediate scales take values above 10^{13} GeV.

For set P2 and $z = 1$ (left panel of Figure 3) we have unification for $\mathbf{d} = 3$, 6, or 8. In this case, the mass of the SU(2) singlet (blue bar) can take values around the TeV scale or even lower,

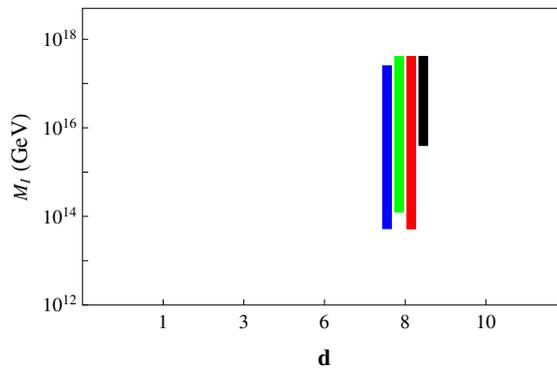Figure 1: The allowed range for the intermediate scales M_I and the unification scale Λ as functions of the $\mathsf{SU}(3)$ dimension $\mathbf{d}$ for the solution P1 of Table 2 with $z = 3$. The colored bars correspond to the energy scales of $(\mathbf{d}, \mathbf{1})_{\frac{5}{2}}$ (blue), $(\mathbf{d}, \mathbf{2})_{-2}$ (green), $(\overline{\mathbf{d}}, \mathbf{3})_{\frac{1}{2}}$ (red) and Λ (black).
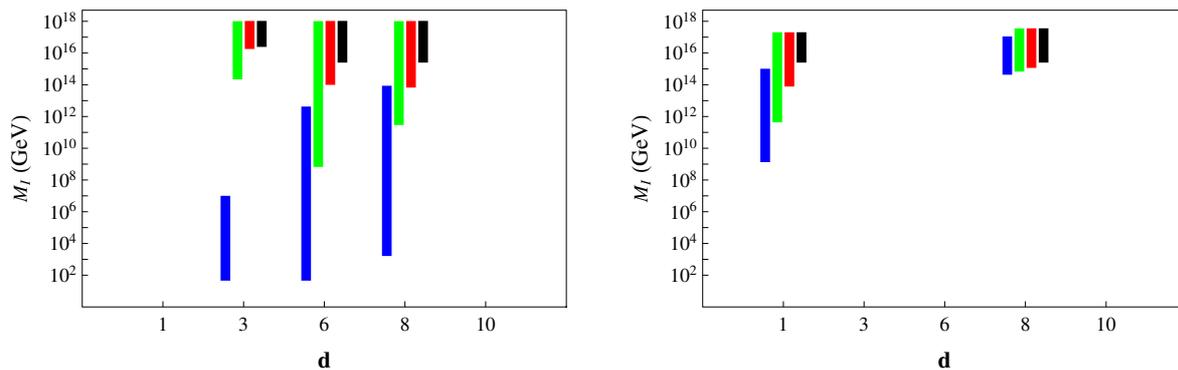

Figure 2: The allowed range for the intermediate scales M_I and the unification scale Λ as functions of the $\mathsf{SU}(3)$ dimension $\mathbf{d}$ for the solution P2 of Table 2 with $z = 1$ (left panel) and $z = 3$ (right panel). For each value of $\mathbf{d}$, the colored bars correspond to the energy scales of $(\mathbf{d}, \mathbf{1})_{7z/6}$ (blue), $(\mathbf{d}, \mathbf{3})_{-5z/6}$ (green), $(\overline{\mathbf{d}}, \mathbf{4})_{z/3}$ (red) and Λ (black).

the remaining two masses $((\mathbf{d}, \mathbf{4})_{-1}$ in green and $(\overline{\mathbf{d}}, \mathbf{5})_{1/2}$ in red) are above 10^9 GeV. For $z = 3$ (right panel of Figure 3) we found solutions only for $\mathbf{d} = 1$ and 8; with this normalization, the unification is achieved for quite high intermediate scales, in particular, for $\mathbf{d} = 8$ the intermediate scales are above 10^{14} GeV. The pattern of solutions found for P3 is very similar to the one found for P2, with the exception that the set P3 has an additional solution for $\mathbf{d} = 10$ and $z = 1$. For set P4 one has unification for $z = 1$ (left panel of Figure 4) with any even $\mathsf{SU}(3)$ dimension while for the normalization $z = 3$ (right panel of Figure 4) we found unification only for $\mathbf{d} = 8$. For $z = 1$ the intermediate scales have values above 10^{10} GeV while for $z = 3$ take values higher than 10^{15} GeV.

It is interesting to note that, we achieve unification of the gauge couplings despite the additional chiral fermions have unusual quantum numbers under the SM gauge group, in comparison with those present in the embedding of the SM in a $\mathsf{SU}(5)$ or $\mathsf{SO}(10)$ groups.

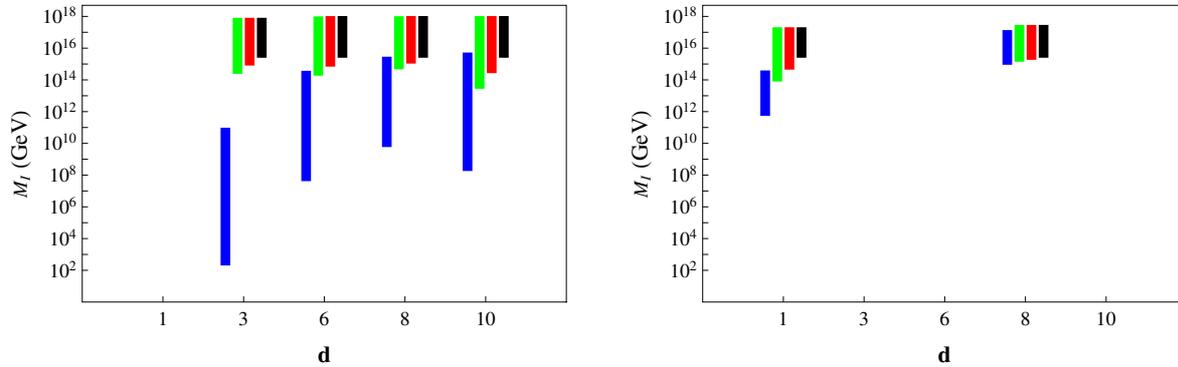One can find the study of the unification of the gravitational and gauge couplings at string scale in Ref. [20].

Figure 3: The allowed range for the intermediate scales M_I and the unification scale Λ as functions of the SU(3) dimension **d** for the solution P3 of Table 2 with $z = 1$ (left panel) and $z = 3$ (right panel). For each value of **d**, the colored bars correspond to the energy scales of $(\mathbf{d}, \mathbf{1})_{3z/2}$ (blue), $(\mathbf{d}, \mathbf{4})_{-z}$ (green), $(\overline{\mathbf{d}}, \mathbf{5})_{z/2}$ (red) and Λ (black).
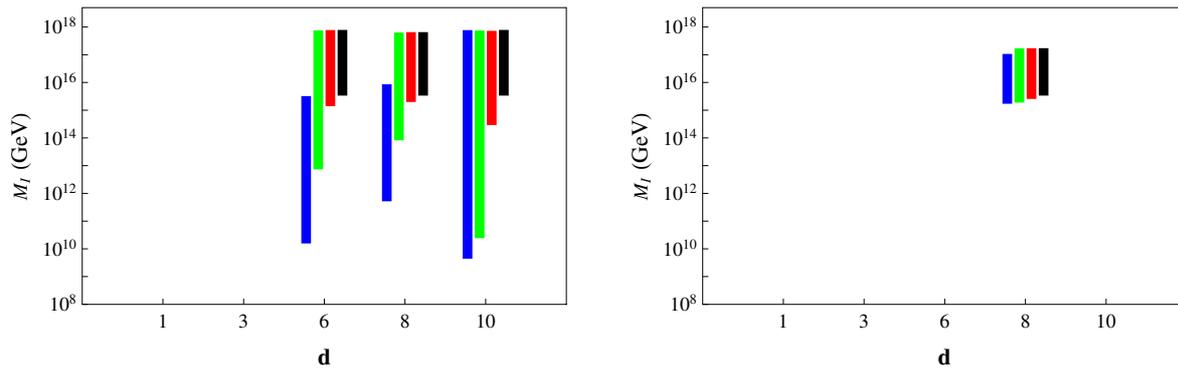

Figure 4: The allowed range for the intermediate scales M_I and the unification scale Λ as functions of the SU(3) dimension **d** for the solution P4 of Table 2 with $z = 1$ (left panel) and $z = 3$ (right panel). For each value of **d**, the colored bars correspond to the energy scales of $(\mathbf{d}, \mathbf{2})_{4z/3}$ (blue), $(\mathbf{d}, \mathbf{3})_{-7z/6}$ (green), $(\overline{\mathbf{d}}, \mathbf{5})_{z/6}$ (red) and Λ (black).

3. Anomaly-free chiral fermion sets: SU(5)-inspired

Following the results of previous section together with the idea of grand unified theories, we can ask now which is the minimal anomaly-free chiral set of fermions besides the SM if we consider SU(5) as a gauge group. In our search, we consider SU(5) multiplets with dimensions less than 70 and we require as before that we got vector-like particles with respect to the color and electric charge after the symmetry breaking.

Considering the SU(5) representations with dimensions less than 70 (see Ref. [27]) we end up with 21 different multiplets, the ones listed in Table 3.

Writing the anomaly constraints of Eqs. (1) for the 21 different multiplets in Table 3 together

Table 3: Particle multiplets within $\mathsf{SU}(5)$ representations with dimension less than 70.

Label	Multiplet	$\mathsf{SU}(5)$-rep	Label	Multiplet	$\mathsf{SU}(5)$-rep	Label	Multiplet	$\mathsf{SU}(5)$-rep
1	$(\mathbf{1},\mathbf{2})_{1/2}$	5, 45	8	$(\mathbf{1},\mathbf{4})_{-3/2}$	35	15	$(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	45
2	$(\mathbf{3},\mathbf{1})_{-1/3}$	5, 45, 50	9	$(\overline{\mathbf{3}},\mathbf{3})_{-2/3}$	35, 40	16	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	45, 50
3	$(\mathbf{1},\mathbf{1})_{1}$	10	10	$(\overline{\mathbf{6}},\mathbf{2})_{1/6}$	35, 40	17	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	45
4	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	10, 40	11	$(\overline{\mathbf{10}},\mathbf{1})_{1}$	35	18	$(\mathbf{8},\mathbf{2})_{1/2}$	45, 50
5	$(\mathbf{3},\mathbf{2})_{1/6}$	10, 15, 40	12	$(\mathbf{1},\mathbf{2})_{-3/2}$	40	19	$(\mathbf{1},\mathbf{1})_{-2}$	50
6	$(\mathbf{1},\mathbf{3})_{1}$	15	13	$(\mathbf{8},\mathbf{1})_{1}$	40	20	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	50
7	$(\mathbf{6},\mathbf{1})_{-2/3}$	15	14	$(\mathbf{3},\mathbf{3})_{-1/3}$	45	21	$(\mathbf{6},\mathbf{1})_{4/3}$	50

with the requiring that our set is vector-like with respect to $\mathsf{SU}(3)$ and $\mathsf{U}(1)_{\mathrm{em}}$:

$$
\begin{aligned}
\mathbf{1}_1 &: \quad n_1 + n_3 + n_6 - n_8 - n_{12} = 0\,, \\
\mathbf{1}_2 &: \quad n_6 - n_8 - n_{12} - n_{19} = 0\,, \\
\mathbf{1}_{-3} &: \quad n_8 = 0\,, \\
\mathbf{3}_{-1/3} &: \quad n_2 + n_5 - n_9 + n_{14} = 0\,, \\
\overline{\mathbf{3}}_{-2/3} &: \quad n_4 - n_5 + n_9 - n_{14} + n_{16} = 0\,, \\
\overline{\mathbf{3}}_{-5/3} &: \quad n_9 + n_{16} = 0\,, \\
\mathbf{3}_{-4/3} &: \quad n_{14} - n_{15} = 0\,, \\
\mathbf{6}_{-2/3} &: \quad n_7 - n_{10} - n_{20} = 0\,, \\
\overline{\mathbf{6}}_{-1/3} &: \quad n_{10} + n_{17} + n_{20} = 0\,, \\
\overline{\mathbf{6}}_{-4/3} &: \quad n_{20} - n_{21} = 0\,, \\
\mathbf{8}_1 &: \quad n_{13} + n_{18} = 0\,, \\
\overline{\mathbf{10}}_1 &: \quad n_{11} = 0\,,
\end{aligned}
\tag{17}
$$

we end up with the following two equations,

$$
\begin{aligned}
2n_1 + 9n_2 + 3n_3 + 17n_4 - 9n_5 - 5n_6 + 16n_7 - 18n_{10} + 8n_{13} &= 0\,, \\
54n_1 + 243n_2 + 81n_3 + 459n_4 - 243n_5 - 135n_6 + 432n_7 - 486n_{10} + 216n_{13} &= 0\,,
\end{aligned}
\tag{18}
$$

where the numbers in bold in Eq. (17) refer to $\mathsf{SU}(3)$ dimension and the numbers in subscript to the electric charges; n_I is the number of specie of the particle I ($I = 1, ..., 21$).

We list the solutions in Table 4; the ones with less than eleven multiplets and less than seven different species per set. The first set, S1, has four different species and have a total of seven multiplets; the sets from S2 to S8 are composed by five different species and the remaining sets have six different species. The sets S2 and S7 correspond, respectively, to one and two additional SM generations. Note that, for the cases where n_i is found to be negative we wrote the adequate conjugate multiplet, in comparison to what is written in Table 3.

Let us now study the unification of the gauge couplings. To do so, we need to have a well defined model i.e., we need to have the adequate scalar content to perform the right symmetry breaking. Since, the anomaly study done in this section consider multiplets of $\mathsf{SU}(5)$, we will consider the minimal scalar content need to break the $\mathsf{SU}(5)$ group. The breaking of the $\mathsf{SU}(5)$

Table 4: Anomaly-free chiral sets obtained from $\mathsf{SU}(5)$ representations with dimension less than 70. In the last column it is indicated whether the sets lead ($\checkmark$) to gauge coupling unification or not ($-$). For the sets marked with $*$ the unification scale is very constrained.

Set	Particle content										GUT
S1	$3\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{2}$				$-$
S2	$(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{3},\mathbf{1})_{2/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$		$-$
S3	$(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{1},\mathbf{3})_{1}$	$\oplus$	$(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{-2}$		$-$
S4	$2\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{2/3}$	$\oplus$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$		$-$
S5	$(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{1}$	$\oplus$	$(\mathbf{1},\mathbf{3})_{1}$	$\oplus$	$3\,(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{2}$		$-$
S6	$2\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$3\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{1},\mathbf{3})_{-1}$	$\oplus$	$2\,(\mathbf{1},\mathbf{2})_{3/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{-2}$		$-$
S7	$2\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\mathbf{3},\mathbf{1})_{-1/3}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$2\,(\mathbf{3},\mathbf{1})_{2/3}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$		$\checkmark$
S8	$2\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$2\,(\mathbf{1},\mathbf{3})_{1}$	$\oplus$	$2\,(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$2\,(\mathbf{8},\mathbf{2})_{-1/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{-2}$		$\checkmark$
S9	$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\oplus$	$(\mathbf{3},\mathbf{3})_{2/3}$	$\oplus$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$*$
S10	$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$\oplus$	$(\mathbf{8},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	$\oplus$	$(\mathbf{8},\mathbf{2})_{1/2}$	$\checkmark$
S11	$(\mathbf{3},\mathbf{1})_{2/3}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\oplus$	$(\mathbf{3},\mathbf{3})_{2/3}$	$\oplus$	$(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$\oplus$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$\checkmark$
S12	$(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{2})_{1/6}$	$\oplus$	$(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$(\mathbf{6},\mathbf{1})_{1/3}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{2}$	$-$
S13	$(\overline{\mathbf{6}},\mathbf{1})_{2/3}$	$\oplus$	$2\,(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	$\oplus$	$2\,(\mathbf{8},\mathbf{2})_{-1/2}$	$\oplus$	$(\mathbf{6},\mathbf{3})_{1/3}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{-4/3}$	$\checkmark$
S14	$2\,(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$\oplus$	$2\,(\mathbf{3},\mathbf{3})_{2/3}$	$\oplus$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	$\oplus$	$(\mathbf{6},\mathbf{1})_{4/3}$	$\checkmark$
S15	$2\,(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$\oplus$	$2\,(\mathbf{3},\mathbf{2})_{1/6}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{2/3}$	$\oplus$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	$\checkmark$
S16	$2\,(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{2})_{1/6}$	$\oplus$	$2\,(\mathbf{3},\mathbf{3})_{-1/3}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	$\oplus$	$(\mathbf{6},\mathbf{3})_{1/3}$	$\oplus$	$(\overline{\mathbf{6}},\mathbf{1})_{-4/3}$	$\checkmark$
S17	$(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$\oplus$	$2\,(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$\oplus$	$2\,(\mathbf{3},\mathbf{2})_{1/6}$	$\oplus$	$(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{2}$	$\checkmark$
S18	$(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{3})_{1}$	$\oplus$	$3\,(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$(\overline{\mathbf{8}},\mathbf{2})_{-1/2}$	$\oplus$	$(\mathbf{1},\mathbf{1})_{2}$	$-$
S19	$3\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$3\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{1},\mathbf{3})_{1}$	$\oplus$	$(\mathbf{1},\mathbf{2})_{-3/2}$	$\oplus$	$(\mathbf{8},\mathbf{1})_{1}$	$\oplus$	$(\overline{\mathbf{8}},\mathbf{2})_{-1/2}$	$*$
S20	$2\,(\mathbf{1},\mathbf{2})_{1/2}$	$\oplus$	$2\,(\mathbf{1},\mathbf{1})_{-1}$	$\oplus$	$2\,(\mathbf{1},\mathbf{3})_{-1}$	$\oplus$	$2\,(\mathbf{1},\mathbf{2})_{3/2}$	$\oplus$	$(\overline{\mathbf{8}},\mathbf{1})_{-1}$	$\oplus$	$(\mathbf{8},\mathbf{2})_{1/2}$	$*$

group down to the SM gauge group is done by the adjoint scalar representation, **24**, and then the SM is broken by the vacuum expectation value of the neutral component of the Higgs field in 5-dimensional representation. Hence, the minimal scalar content needed is $\Sigma_3 \sim (\mathbf{1},\mathbf{3})_0$, $\Sigma_8 \sim (\mathbf{8},\mathbf{1})_0$, $(X,Y)^\intercal \sim (\mathbf{3},\mathbf{2})_{-5/6}$ in the adjoint **24** representation and $H^{\mathrm{c}} \sim (\mathbf{3},\mathbf{1})_{-1/3}$ and $H \sim (\mathbf{1},\mathbf{2})_{1/2}$ of the 5-dimensional representation.

However, since the scalars X, Y and H^{c} can mediate proton decay, we will assume them around the unification scale and hence not contributing to the running. On the other hand, the Higgs doublet, H, is to be at the electroweak scale while the Σ_3 and Σ_8 mass scales are allowed to vary from M_Z to Λ.

To study whether any of the sets in Table 4 lead to gauge coupling unification at high energy scale, we proceed as in the previous section: we use Eqs. (10)-(13) and vary randomly the running weights r_2 to r_{21} within the interval $[0,1]$ and the running weights of the scalars Σ_3 and Σ_8, that for simplicity we choose to be equal. Then, using Eq. (10), we determine the running weight of the first particle, r_1, and via Eq. (13) we calculate the unification scale. Here, we have used the $\mathsf{SU}(5)$ normalization, $\kappa_1 = 5/3$, $\kappa_2 = \kappa_3 = 1$ instead of the "democratic" normalization used in the previous section. In this case the B and $\widetilde{B}$ constants defined in Eq. (14) take the values $B = 0.718 \pm 0.003$ and $\widetilde{B} = 185.0 \pm 0.2$.

We present the variation limits for the intermediate mass scales in the plots of Figures 5

and 6; in all the cases the unification scale (black bars) is required to be $\Lambda \geq 5 \times 10^{15}$ GeV in order to be compatible with proton decay limits. For each set of solutions, the multiplets of the same type i.e. with the same quantum numbers, are setted at the same scale.

We found possible to unify the gauge couplings for the sets S7, S8, S10, S11 and S13 to S17. The solutions that lead to unification were already anticipated in the last column of Table 4 with the sign ✓; the sets marked with the symbol ∗ (S9, S19, and S20) are those for which we found unification of the gauge couplings within a very narrow interval $\Lambda \lesssim 10^{16}$ GeV.

From the plots we can see that it is possible to obtain unification with some intermediate scales as low as the electroweak scale. The numerical limits for the unification scale and the intermediate mass scales are given explicitly in Table VI of Ref. [20].

Since the results, presented in the plots of Figures 5 and 6, show just the variation limits of the intermediate mass scales, we present in Figure 7 a concrete example for the running of the gauge couplings at one-loop level. We choose the set S7, which corresponds to the SM plus two complete generations of SM. The unification scale is $\Lambda = 1.3 \times 10^{16}$ GeV with $\alpha_U^{-1} \simeq 35$; the intermediate mass scales are $M_2 = M_5 = 560$ GeV, $M_1 = 6.9 \times 10^{10}$ GeV and $M_3 = M_4 = 1.0 \times 10^{16}$ GeV and it was assumed $M_{\Sigma_3} = M_{\Sigma_8} = \Lambda$, where $M_1 \sim (\mathbf{1}, \mathbf{2})_{1/2}$, $M_2 \sim (\mathbf{3}, \mathbf{1})_{-1/3}$, $M_3 \sim (\mathbf{1}, \mathbf{1})_{-1}$, $M_4 \sim (\mathbf{3}, \mathbf{1})_{2/3}$, $M_5 \sim (\overline{\mathbf{3}}, \mathbf{2})_{-1/6}$.

4. Conclusions

In this work we searched for the minimal sets of chiral fermions, beyond the SM, that are free of anomalies and lead to vector-like particles under SU(3) and U(1)$_{\text{em}}$ after symmetry breaking. For such propose we considered two different frameworks: in section 2 we searched for anomaly-free chiral fermions with arbitrary quantum numbers, under the SM gauge group, while in section 3 we searched for anomaly-free chiral fermions that belong to SU(5) representations with dimensions less than 70. Furthermore, we studied whether the addition of such chiral fermions to the SM allows for unification of gauge couplings at high scale, where we considered the running of the gauge couplings at one-loop level within a non-supersymmetric scenario. For the case of chiral fermions with arbitrary quantum numbers and restricting the SU(3) and SU(2) dimensions to be less than or equal to 10 and 5 respectively, we found four anomaly-free sets with three multiplets each; the results are presented in Table 2. Within each set, the SU(3) dimension of the multiplets is equal in all multiplets. The study of the unification revealed that only 16 solutions lead to successful gauge coupling unification with $\Lambda \geq 5 \times 10^{15}$ GeV, the results are shown in the plots of Figures 1-4.

We summarized the anomaly-free sets of chiral fermions that belong to the SU(5) representations in Table 4, restricting our search to multiplets contained in SU(5) representations with dimension less than 70 and to sets with up to ten multiplets and six different species. Among the twenty sets given in Table 4 only nine lead to unification of the gauge couplings; the limits for the intermediate mass scales as well as for the unification scale are given in the plots of Figures 5 and 6.

In Ref. [20] the study was performed of the unification of gauge and gravitational couplings at the string scale within both frameworks, SM supplemented by chiral fermion with arbitrary quantum numbers and by multiplets within SU(5) representations.

Before finishing some comments are in order. The gauge unification analysis done in this work was performed for non-supersymmetric scenarios at one-loop level, so it would be worthwhile to do a more complete analysis including higher loop corrections and considering supersymmetry, which will be done elsewhere. Furthermore, from the results presented in Figures 1-6 one sees that the gauge coupling unification requires that some of the new particles decouple from the theory at scales much larger than the electroweak scale. For the two considered setups, and once the charged scalars are very constrained by electroweak precision data, the generation of large masses for the intermediate fermions via vacuum expectation value of some extra scalar

fields seems unfeasible. In Ref. [20] we pointed out the possibility of generating the intermediate masses via a dynamical mechanism similar to the one done in Ref. [28], by extending the theory with mirror fermions, however this issue deserves an additional analysis.

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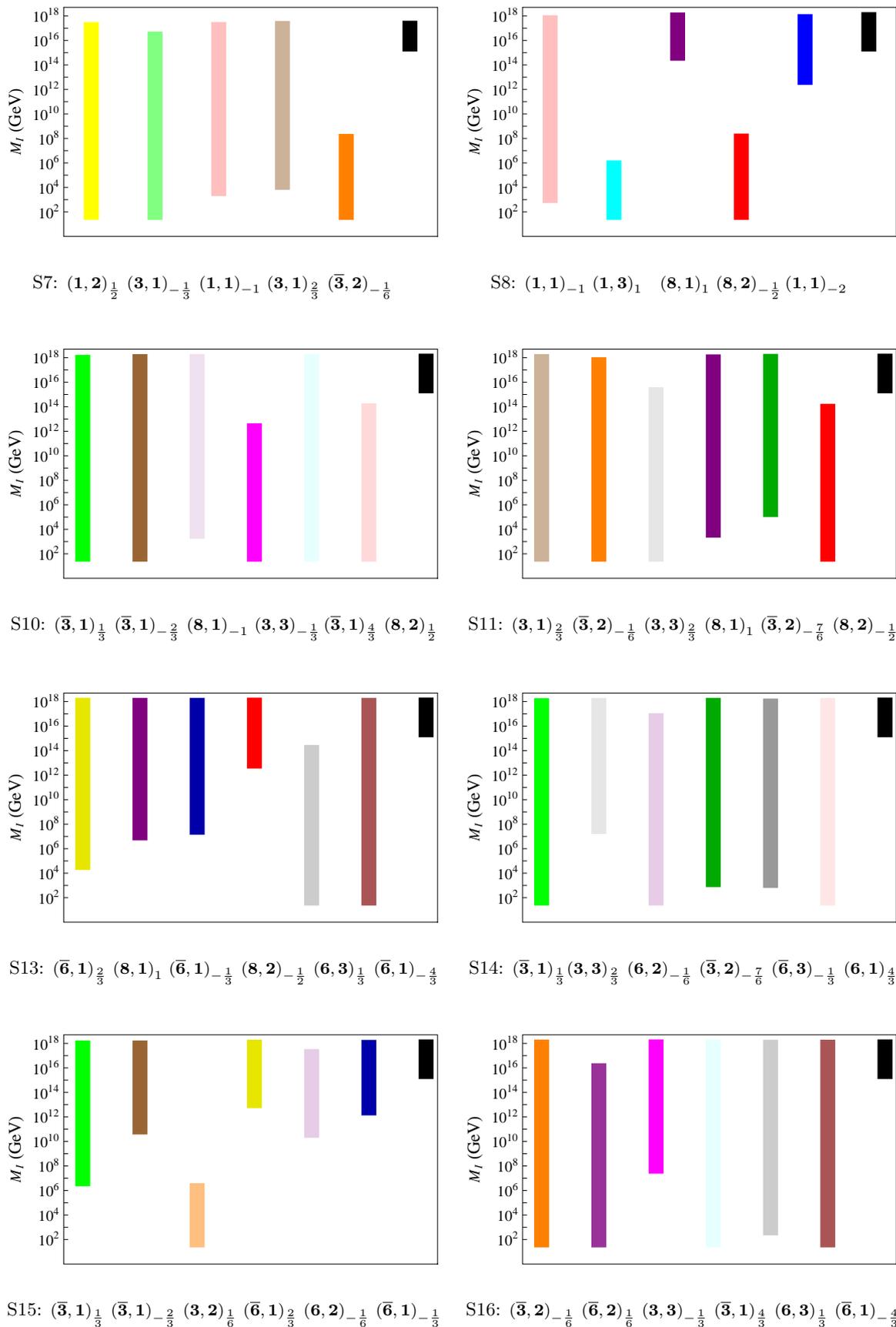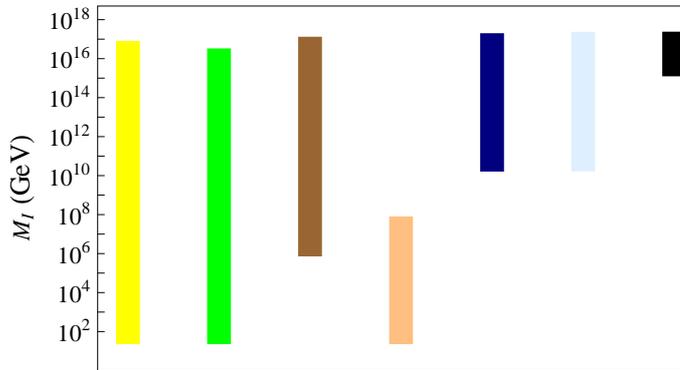
Figure 5: The allowed range for the intermediate scales, M_I, and the unification scale Λ (bars in black) for sets S7, S8, S10, S11, S13-S16, which lead to unification of the gauge couplings. For each panel, it is written below each colored bar the multiplet it refers to; the last bar in black corresponds to the unification scale.

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S17: $(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$ $(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$ $(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$ $(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$ $(\mathbf{1},\mathbf{2})_{-\frac{3}{2}}$ $(\mathbf{1},\mathbf{1})_{2}$

Figure 6: The allowed range for the intermediate scales and the unification scale Λ for the set S17. Below each colored bars is written the multiplet it refers to; the last bar in black corresponds to the unification scale.
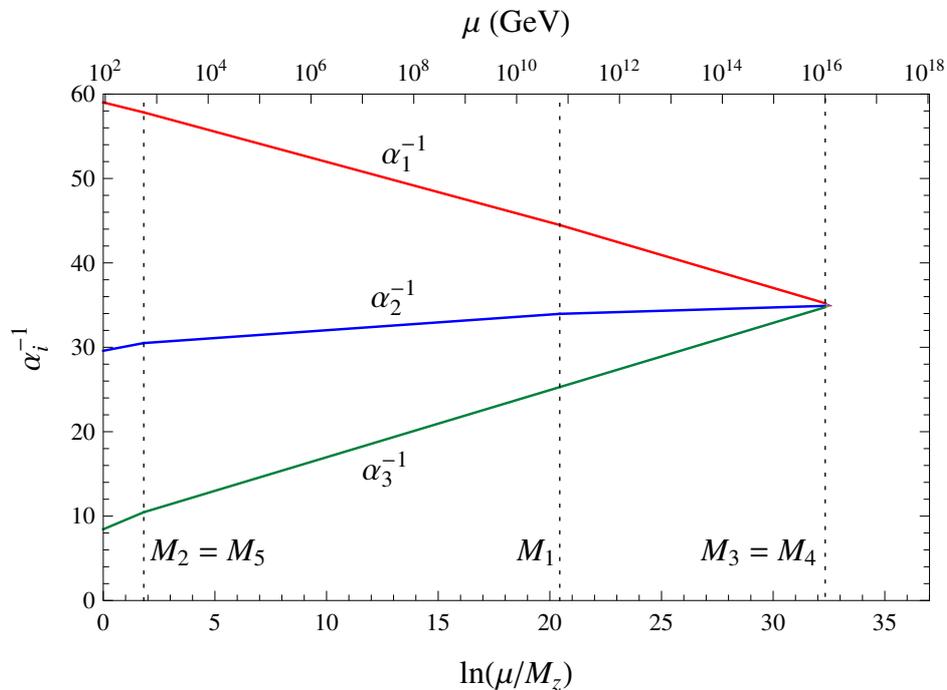

Figure 7: The gauge coupling running at one-loop level for the set S7. The unification scale is $\Lambda = 1.3 \times 10^{16}$ GeV and the intermediate mass scales are $M_2 = M_5 = 560$ GeV, $M_1 = 6.9 \times 10^{10}$ GeV and $M_3 = M_4 = 1.0 \times 10^{16}$ GeV where it was assumed $M_{\Sigma_3} = M_{\Sigma_8} = \Lambda$ ($M_1 \sim (\mathbf{1},\mathbf{2})_{1/2}$, $M_2 \sim (\mathbf{3},\mathbf{1})_{-1/3}$, $M_3 \sim (\mathbf{1},\mathbf{1})_{-1}$, $M_4 \sim (\mathbf{3},\mathbf{1})_{2/3}$, $M_5 \sim (\overline{\mathbf{3}},\mathbf{2})_{-1/6}$).