PASSIVE LINEARIZATION OF NONLINEAR SYSTEM RESONANCES

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Summary In this work we demonstrate that the addition of properly-tuned nonlinearities to a nonlinear system can increase the range over which a specific resonance responds linearly. Specifically, we seek to enforce two important properties of linear systems, namely the force-displacement proportionality and the invariance of resonance frequencies. Theoretical findings are validated through numerical simulations and experiments.

Devices used for sensing, imaging and detection are usually required to exhibit linear behavior in their dynamic range. However, nonlinearity is a frequent occurrence in physical and engineering applications. Nonlinearity may result in plethora of dynamic phenomena which can drastically limit the performance of the devices [1]. One well-established approach for enforcing linear behavior is feedback linearization, which uses feedback control to cancel the undesired nonlinearities. However, feedback linearization requires an accurate monitoring of the system's states, an actuator and an external source of energy, which complicates its practical realization.

We propose a fully passive, resonance-based approach for dealing with undesired nonlinearities in mechanical systems. Properly-tuned nonlinearities are introduced in the nonlinear system to increase the range over which a specific resonance responds linearly. Specifically, we seek to enforce two important properties of linear systems, namely the force-displacement proportionality and the invariance of resonance frequencies. Our approach relies on a principle of similarity [2] which states that the added nonlinearity should possess the same mathematical form as that of the original nonlinear system. This principle of similarity enables us to extend the linear regime over a larger range of motion amplitudes.

We consider an n-degree-of-freedom (DoF) mechanical system with concentrated nonlinearities subject to harmonic excitation:

$$\tilde{\mathbf{M}}\ddot{\mathbf{x}}+\tilde{\mathbf{C}}\dot{\mathbf{x}}+\tilde{\mathbf{K}}\mathbf{x}+\tilde{\mathbf{b}}_{nl}\left(\mathbf{x}\right)=\sqrt{\varepsilon}\tilde{\mathbf{v}}f\cos\omega t, \tag{1}$$

where ε is a small bookkeeping parameter. The vector $\tilde{\mathbf{b}}_{nl}\left(\mathbf{x}\right)$ contains both the original and additional nonlinearities, which are of polynomial nature. According to the principle of similarity [2], the additional nonlinearities should possess the same exponent as the original nonlinearity. Without loss of generality, cubic nonlinearities are considered herein.

The objective of this study is to linearize one specific resonance of system (1) through the proper design of the additional nonlinearities. To this end, the nonlinear normal mode (NNM) theory is exploited, because nonlinear resonances are known to occur in the neighborhood of NNMs [3]. First, we transform Eq. (1) into modal space through the change of variables $\mathbf{x}=\mathbf{U}\mathbf{y}$ where $\mathbf{U}$ contains the normal modes of the underlying linear system, and we define normalized modal displacements, $\mathbf{q}=\mathbf{y}/\left(\sqrt{\varepsilon}f\right)$, such that

$$\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{\Omega}\mathbf{q}+\mathbf{b}_{nl}\left(x\right)=\mathbf{v}\cos\omega t, \tag{2}$$

where $\mathbf{\Omega}=\text{diag}\left[\Omega_j^2\right]_{j=\overline{1,n}}$, $\mathbf{C}=\left[c_{ij}\right]_{i,j=\overline{1,n}}$ and $\mathbf{b}_{nl}\left(\mathbf{x}\right)=\left[\ \ldots,\quad \varepsilon f^2\sum_{h_1+\ldots+h_n=3}b_{jh_1\ldots h_n}\prod_{i=1}^n q_i^{h_i},\quad \ldots\ \right]^{\mathrm{T}}$. $\mathbf{b}_{nl}\left(\mathbf{x}\right)$ is the projection of $\tilde{\mathbf{b}}_{nl}\left(\mathbf{x}\right)$ in modal space, thus, even if $\tilde{\mathbf{b}}_{nl}\left(\mathbf{x}\right)$ is sparse, $\mathbf{b}_{nl}\left(\mathbf{x}\right)$ can be fully populated. $b_{jh_1\ldots h_n}$ are scalars, where j indicates the mode number and varies according to the rows of $\mathbf{b}_{nl}\left(\mathbf{x}\right)$, while subscripts $h_1\ldots h_n$ are in accordance with the exponents of the modal coordinates of the corresponding terms. We assume that the system features no internal resonances, i.e., natural frequencies Ω_j are incommensurate.

The NNMs are now calculated by removing damping and forcing terms in Eq. (2). Following a standard perturbation technique and limiting the solution to the fundamental harmonic, the approximate solution has the form $\mathbf{q}=\left(\mathbf{q}_0+\varepsilon\mathbf{q}_1+O\left(\varepsilon^2\right)\right)\sin\left(\left(\omega_0+\varepsilon\omega_1+O\left(\varepsilon^2\right)\right)t\right)$, where $\mathbf{q}_0=\left[\cdots,q_{j0},\cdots\right]^{\mathrm{T}}$ and $\mathbf{q}_1=\left[\cdots,q_{j1},\cdots\right]^{\mathrm{T}}$. Imposing resonance condition at order ε^0 and solving terms of order ε^1 yields for the l^{th} NNM

$$q_{j0}=0,\ q_{j1}=-\frac{3}{4}\frac{b_{j0\ldots3\ldots0}q_{l0}^3}{\Omega_j^2-\Omega_l^2}\quad\text{for } j=\overline{1,n},\ j\neq l,\quad \omega_0=\Omega_l,\ \omega_1=\frac{3}{4}\frac{b_{l0\ldots3\ldots0}q_{l0}^2}{2\Omega_l}. \tag{3}$$

q_{j0} and q_{j1} $(j\neq l)$ represent the influence of the nonresonant modes on the l^{th} mode. ω_1 is the variation of the l^{th} natural frequency with respect to the amplitude of oscillation q_{l0}. Thus, if $b_{l0\ldots3\ldots0}>0$ (<0), the resonance is of hardening (softening) type.

In order to relate the undamped, unforced NNM motions to the resonances of the damped, forced system, the energy balance criterion [4] is utilized:

$$\int_0^T\dot{\mathbf{x}}(t)^{\mathrm{T}}\tilde{\mathbf{C}}\dot{\mathbf{x}}(t)\mathrm{d}t=\int_0^T\dot{\mathbf{x}}(t)^{\mathrm{T}}\sqrt{\varepsilon}\tilde{\mathbf{v}}f\cos\omega t\,\mathrm{d}t, \tag{4}$$

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where T is the period of motion. Eq. (4) indicates that at resonance the energy dissipated by damping over a full period is equal to the input energy. Inserting the approximate solution for $\mathbf{q}$ in Eq. (4) gives

$$q_{l0} = \frac{v_l}{\Omega_l c_{ll}}, \quad q_{l1} = \frac{\Omega_l q_{l0} \left(\sum_{\substack{j=1 \\ j \neq l}}^{n} c_{lj} q_{j1} + \sum_{\substack{j=1 \\ j \neq l}}^{n} c_{jl} q_{j1} \right) - \sum_{\substack{j=1 \\ j \neq l}}^{n} q_{j1} v_j}{v_l - 2\Omega_l q_{l0} c_{ll}} + \frac{\omega_1 q_{l0}^2 c_{ll}}{v_l - 2\Omega_l q_{l0} c_{ll}} \tag{5}$$

and $x_k = \sqrt{\varepsilon} f \left(u_{kl} q_{l0} + \varepsilon \left(\sum_{j=1}^{n} u_{kj} q_{j1} \right) + O\left(\varepsilon^2\right) \right)$. Eqs. (3) and (5) completely define the l^{th} resonance of system (1) and form the basis of the design procedure developed in this paper. Based on these equations, force-displacement proportionality for coordinate x_k of the l^{th} resonance can simply be enforced through $\sum_{j=1}^{n} u_{kj} q_{j1} = 0$. Similarly, invariance of the l^{th} resonance frequency can be enforced through $\omega_1 = 0$, such that the l^{th} natural frequency $\omega_l \approx \omega_0$. Since these two conditions involve the n coefficients of the nonlinear terms $b_{j0\ldots3\ldots0}$, they can be used to design the additional nonlinearities in function of the original ones.

An experimental set-up is utilized to demonstrate the proposed idea. It comprises a cantilever beam made of steel to which a doubly-clamped beam is connected (Fig. 1a). A thin steel lamina located at the free end of the cantilever and the doubly-clamped beam itself generate two hardening nonlinearities, which can be modeled using cubic springs. A two-DoF reduced model of the system was identified experimentally. From this model, applying the aforementioned procedure, the nonlinearity of the doubly-clamped beam was designed, such that the second resonant peak satisfies force-displacement proportionality. In Fig. 1b, solid and dashed lines depict the envelopes of the normalized amplitude of the second resonant peak, respectively with and without the nonlinearity related to the doubly-clamped beam. The red circles depict the measured resonant peaks. These circles are almost aligned horizontally, which translates the fact that the displacement of the cantilever beam around the second resonance is proportional to the amplitude of the harmonic forcing. This condition is not verified if the additional nonlinearity is not included (dashed line).

Successively, in order to compensate the frequency shift of the first resonance, due to the hardening nonlinearities, a couple of permanent magnets was placed on the doubly-clamped beam and two others were placed symmetrically next to them on a fixed support. The two couples of magnets are mutually attractive in order to generate a softening nonlinearity. The amplitude of this nonlinearity was controllable by varying the distance between the two magnets. Choosing the appropriate distance, according to the analytical procedure proposed, we were able to linearize the frequency backbone of the first resonance, as illustrated in Fig. 1c (solid black lines). Dashed red lines in Fig. 1c show the frequency response of the system if the nonlinear magnetic force is neglected. The blue circles depict the resonance peaks measured experimentally.

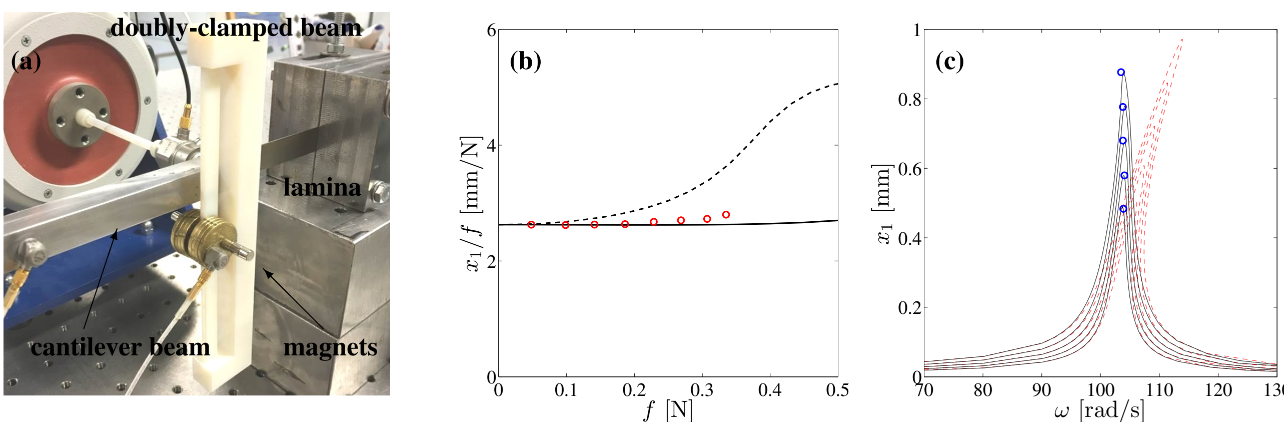

Figure 1: (a) experimental set-up. (b) first part of the experiment, envelops of resonant peak for different forcing amplitude; dashed line: nonlinearity of doubly-clamped beam neglected, solid line: nonlinearity of doubly-clamped beam included, red circles: experimental results. (c): second part of the experiment, frequency response without (dashed red lines) and with (solid black lines) magnetic nonlinear force; blue circles: experimental results.

References

[1] Postma H.C., Kozinsky I., Husain A., Roukes M.: Dynamic range of nanotube-and nanowire-based electromechanical systems. Appl Phy Lett 86:223105, 2005.

[2] Habib G., Kerschen G.: A principle of similarity for nonlinear vibration absorbers. In Proc. ASME 2015 IDETC, 2015.

[3] Vakakis A.F., Manevitch L.I., Mikhlin Y.V., Pilipchuk V.N., Zevin A.A.: Normal modes and localization in nonlinear systems, Springer, 1996.

[4] Hill T., Cammarano A., Neild S., Wagg D.: An analytical method for the optimisation of weakly nonlinear systems. Proc. EURODYN, 1981-1988, 2014.