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Technical note

Scale effects in physical piano key weirs models

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ABSTRACT

With inertia and gravity representing the dominant forces for most open channel flow applications (e.g. weir flow), Froude similitude is commonly used for scaling hydraulic performance data from the model to prototype structures. With weir flow, as the upstream head decreases, however, the relevance of surface tension and viscosity forces can increase to the point when the model and prototype similitude is not fully achieved through Froude scaling. Such discrepancies are referred as size-scale effects, and among other things, can result in variations in the head–discharge relationship, nappe trajectory, and air entrainment. Published criteria for avoiding significant size-scale effects for free flow over linear weirs have suggested that minimal heads of ~ 0.02 to 0.07 m be respected, independently of the model size. In this study, the size-scale effect, minimum upstream head, and Weber number limits are investigated for four piano key weirs with geometric model scales of 1:1, 1:7, 1:15, and 1:25.

Keywords: Physical modelling; piano key weir; scale effects; surface tension; viscous effects; Weber number

1 Introduction

Hydraulic performance data derived from laboratory-scale physical models have historically been the foundation for most prototype weir and spillway designs. Nowadays despite the progress in numerical modelling, hydraulic models remain one of the principal engineering tools to design and optimize complex hydraulic structures. To attain full model-prototype similitude, the geometric, kinematic and dynamic similitudes must be achieved. Simply reproducing the hydraulic structure and corresponding flow domain boundary geometries at different size scales achieves geometric similitude. Kinematic similitude means that the fluid flow patterns are similar between the model and prototype that can be achieved when all relevant forces have the same prototype-to-model force ratios (dynamic similitude). Though gravity and inertia represent the

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dominant forces associated with free-surface flow similitude (i.e. Froude similitude), other potentially relevant fluid dynamic forces include viscous forces (Reynolds similitude) and surface tension (Weber similitude). It is not possible, however, to maintain dynamic similitude with respect to gravity, viscous, and surface tension forces when the same fluid (e.g. water) is used for both the model and prototype applications (Ettema, Arndt, Roberts, & Wahl, 2000; Kobus, 1980; Novak, Guinot, Jeffrey, & Reeve, 2010). Consequently, model-scale hydraulic performance data intended for prototype application (Froude scaling) should therefore be limited to flow conditions when viscous and surface tension effects are truly negligible. The challenge, however, lies in knowing what those limits are.

Relative to weir flow, the relevance of surface tension and viscous forces increases as the flow depths (and corresponding velocities) decrease. Pfister, Battisacco, De Cesare, and Schleiss (2013) and Matthew (1991) reported that the presence of viscous and surface tension effects at the model scale (low upstream head conditions) tends to overestimate Froude-scaled prototype heads at the same scale-equivalent discharge (Q). Bretschneider in Kobus (1980), Ettema et al. (2000), Novak et al. (2010), Heller (2011), and Pfister et al. (2013) presented limiting upstream head criteria for avoiding significant scale effects for head–discharge relationships or downstream jet trajectory corresponding to a variety of weir crest geometries. Bretschneider in Kobus (1980) states that a discharge coefficient determined in a physical scale model can be applied directly to the corresponding prototype if the overflow head is greater than 0.02 m. He mentions also that overflow heads greater than 0.06 m are needed to reproduce the jet trajectory over sharp crested weirs. Based on the Bureau of Reclamation's experience, Ettema et al. (2000) suggest a minimum head of 0.075 m to study spillway's design operating range. Citing previous research, Novak et al. (2010) quote a head at least equal to 0.04 to 0.06 m to reproduce the shape of the nappe over a sharp-edge notch. Based on a study of cylindrical weirs, Pfister et al. (2013) found that the limiting head is 0.03 m for a crest radius between 0.005 m and 0.3 m and the limiting criterion should be more severe for crest radius smaller than 0.005 m.

Regarding piano key weirs (PKW), a recent evolution of traditional labyrinth weirs (Lempérière & Ouamane, 2003; Machiels, Erpicum, Dewals, Archambeau, & Pirotton, 2011), some researchers that used scale models to analyse the discharge capacity also mention scale effects. For instance, Machiels et al. (2011), Leite Ribeiro, Bieri et al. (2012) and Machiels, Pirotton, Archambeau, Dewals, and Erpicum (2014) consider in their studies only data for which the Weber number is higher than 50 or "sufficiently large". Pfister, Erpicum, Machiels, Schleiss, and Pirotton (2012) and Leite Ribeiro, Pfister, Schleiss, and Boilat (2012) do not consider data for which the upstream head is lower than 0.03 or 0.05 m, respectively.

In an effort to better understand the effects of surface tension and viscous effects on PKW Froude-scale modelling and the corresponding minimum values of the upstream head H above which these effects can be considered negligible, the discharge characteristics of four geometrically similar PKWs were evaluated (one prototype structure and three laboratory-scale models). The total upstream head H is defined as the upstream water depth h, measured relative to the weir crest, plus the velocity head ($V^2/(2g)$) at the measurement location. In this study, H was used to characterize the upstream head. It is worth noting, however, that the velocity head or kinetic energy term was typically quite small relative to h. Therefore, in most cases h could be substituted for H without affecting the result.

2 Experimental method

Three scale models (1:7, 1:15 and 1:25) corresponding to a specific prototype PKW geometry (Escouloubre Dam, France, Fig. 1) were fabricated using PVC and evaluated under similar hydraulic conditions based on Froude similitude in the Engineering Hydraulics Laboratory at the University of Liège. Each PKW model was installed in separate, geometrically similar, rectangular horizontal flumes that were 8.4 m wide with a 25 m long upstream approach section (prototype dimensions). The Escouloubre type-A PKW includes one inlet key, two outlet keys, and linear weir sections connecting the PKW to the adjacent chute walls. The PKW sidewalls have a trapezoidal crest

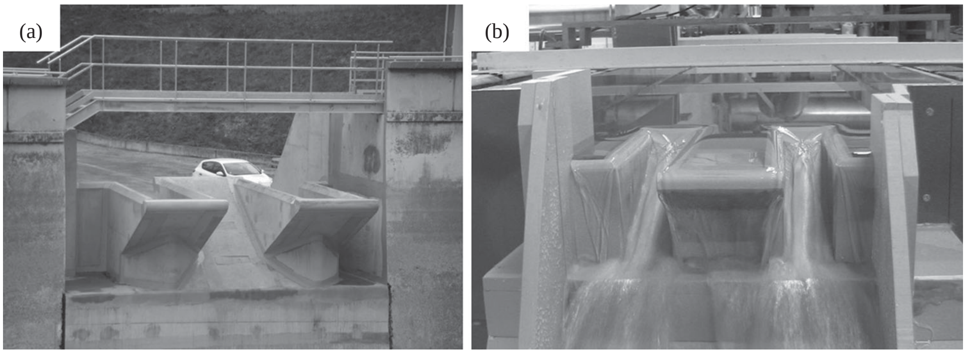

Figure 1 (a) Upstream view of the prototype PKW (Escouloubre Dam, France – courtesy of EDF-CIH); and (b) downstream view of the corresponding 1:15 laboratory scale model

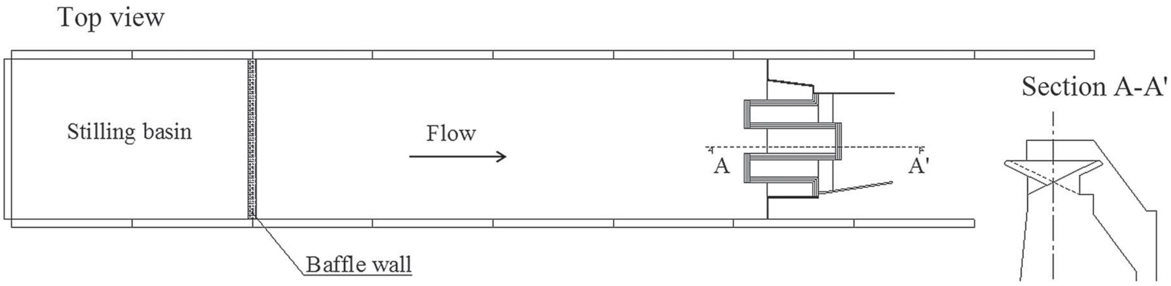

Figure 2 Experimental facility

profile; the inlet and outlet key apex crest shape is quarter round (Fig. 1). The prototype structure has a design discharge of 10 $\text{m}^3\,\text{s}^{-1}$ at an upstream head of 0.5 m. The PKW height is 1.77 m, its width is 5.11 m and the developed crest length is 21.91 m.

The approach flow conditions were analogous to contracted weir flow. The abutment wall of the dam created horizontally contracting flow and the elevated weir apron produced vertically contracting flow. Consequently, the approach flow was not explicitly ponded or channelized, but rather something in between. The experimental set-up is shown in Figs 1 and 2. Flow baffles were installed between the water source and the test flume to insure approach flow uniformity. Flow depth data, relative to the crest elevation, were collected using an ultrasound probe (accuracy better than 1 mm), 10 m (prototype dimension) upstream of the PKW at the flume centre. Discharges were measured using electromagnetic flow meters (accuracy better than 0.5%) installed in the inlet piping. The flow meter diameters were 150 mm for model discharges higher than 5 $\text{l}\,\text{s}^{-1}$ and 50 mm for the smaller discharges.

Prototype head–discharge data were also collected in the field for four different discharge conditions (i.e. 1.2, 2.8, 5.0, and 10.0 $\text{m}^3\,\text{s}^{-1}$). The water level was measured using a staff gauge installed on the right bank; the discharge rates were estimated using hydroelectric turbine output upstream. The accuracy of the prototype data is unknown and, in this study, their application is limited to visual, qualitative comparisons only.

3 Experimental results

A number of hydraulic performance variations were observed between the different scale models, including: prototype head associated with flow initiation, head–discharge similitude, and nappe aeration/trajectory behaviours. As would be expected, surface tension effects were more prominent with the smallest PKW model (1:25), relative to the larger scale models at comparative prototype-scaled heads. If H is not sufficiently large to overcome the surface tension forces at the crest–water–air interface, then a positive H condition with no discharge will exist. As H increases, local surface tension forces are eventually overcome and weir flow is initiated, but still limited. Local surface tension forces can vary with local weir crest surface roughness, resulting in only part of the weir crest length initially passing discharge at very low upstream heads. When H increases sufficiently to overcome the surface tension everywhere, the entire crest will be engaged in conveying discharge. Figure 3 shows the 1:25 scale model at three different very low head discharges. Figure 3a and 3b, 3c and 3d, and 3e and 3f correspond to common discharges, respectively. In Fig. 3a and 3b, surface tension forces are sufficient to prevent discharge from passing over sections of the sidewall weir and the downstream apexes. In Fig. 3c and 3d, the sidewall weirs are fully engaged and the downstream apexes are only partially engaged. In Fig. 3e and 3f, H is sufficiently large to overcome surface tension forces at the crest and the entire weir crest is engaged. As can be seen in Fig. 3f, however, the effects of surface tension are still present, causing the nappe flow to clinging to the underside of the downstream apex overhang, as well as separate from the chute wall on the downstream of the left PKW apex (right side of image). Because surface tension effects increase with decreasing H and decreasing radii of flow curvature, the surface tension scale effects were most significant with the 1:25 scale model, as expected.

For each scale model, 18 head–discharge data points were collected for prototype-scale discharges ranging from 0.7 to 15.5 $\text{m}^3\,\text{s}^{-1}$. Using Froude similitude scaling relationships:

$$H_2 = H_1 \mathsf{S} \tag{1}$$

$$Q_2 = Q_1 \mathsf{S}^{5/2} \tag{2}$$

the model head–discharge data were scaled to the prototype size for comparison (Fig. 4). In Eqs (1) and (2), S is the length scale factor, and Q is flow discharge. Note that only head–discharge data corresponding to flow conditions were the full crest length was engaged were included in the analysis.

Assuming an uncertainty of 1 mm for h at each model scale, the relative uncertainty of the h for the 1:25, 1:15 and 1:7 head data scaled to the prototype scale are 25, 15 and 7 mm, respectively. Given the low approach velocities to the PKW, kinetic term contribution to the head is small and the uncertainty in h is the uncertainty on H. Comparing the prototype scale specific head–discharge data curves in Fig. 4, the values of H at common Q values were considered to be equivalent if they were within error bands equal to ± the scaled H-measurement uncertainty values. The vertical dashed lines in Fig. 4, which correspond

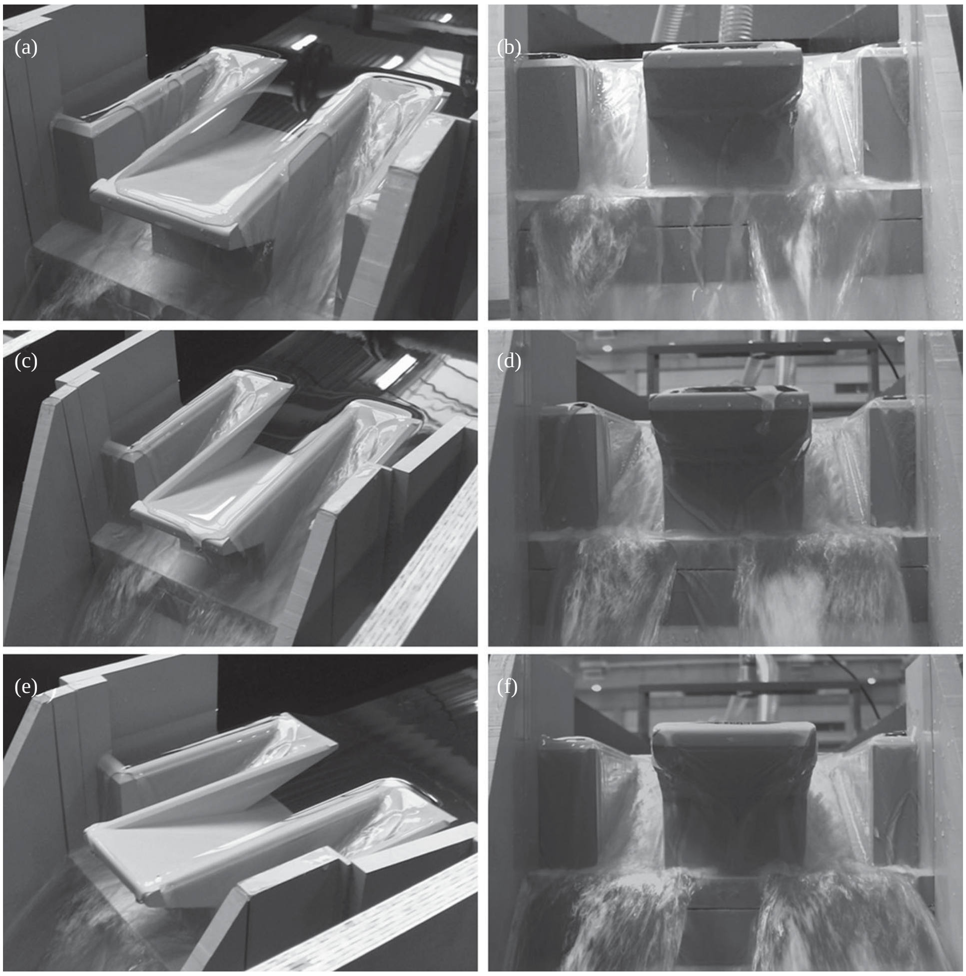

Figure 3 Examples of surface tension effects on small-head PKW flow at the 1:25 scale. (a) and (b), (c) and (d), and (e) and (f) photo pairs are at common discharges, respectively, with the discharge increasing slightly through the photo sequence

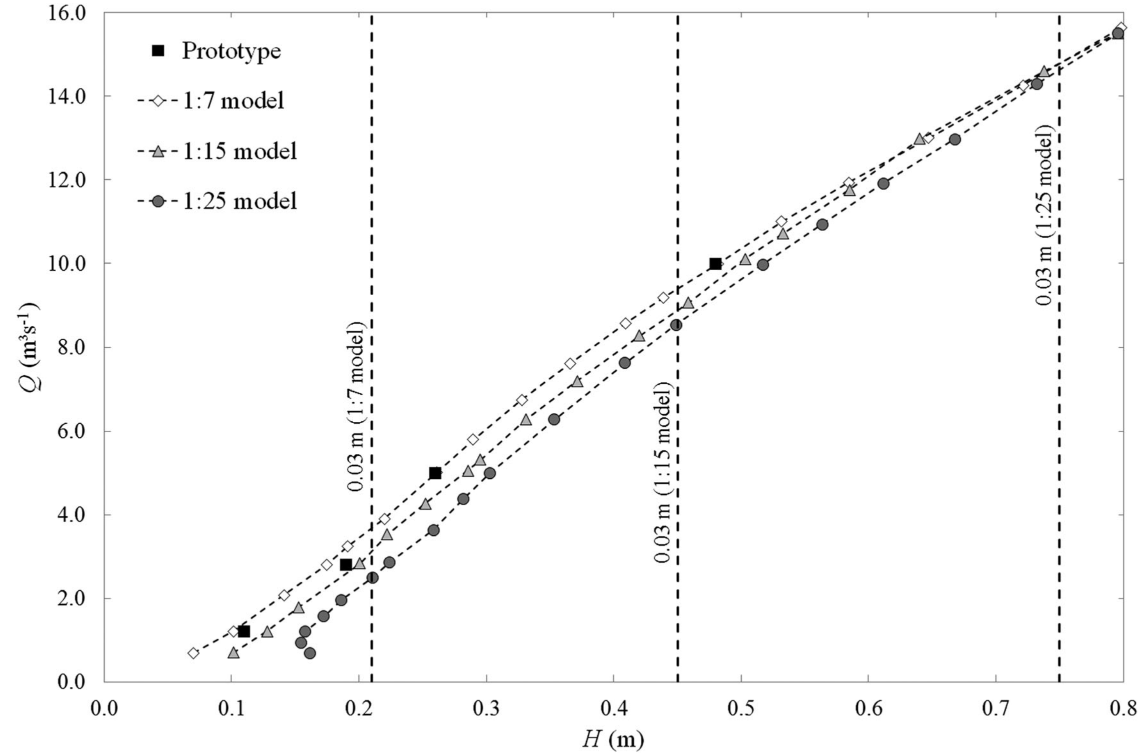

Figure 4 Models rating curves scaled at prototype scale and prototype data. The limiting criteria of 0.03 m model upstream head are shown for the three models with dashed lines

to 0.03 m at the respective model scales, indicate the minimum upstream model head values at which the 1:15 and 1:25 model data matched the 1:7 model head–discharge data (i.e. no viscous nor surface size scale effects), which is consistent with the lower minimum head limit reported by Pfister et al. (2013) for cylindrical weirs. Though there is no significant difference in relative size between the 1:25 and 1:15 scale PKW weirs, it is interesting to note that the minimum head requirement to avoid viscous and surface tension effects is constant (independent of model size scale for the model sizes tested). When scale effects are present, the scaled physical model discharge capacity underestimates the prototype or larger scale model behaviour. This confirms the margin of safety provided by scale models for the evaluation of discharge coefficient, as mentioned by Bretschneider in Kobus (1980).

Surface tension and viscous effects can also influence nappe behaviour with changing model scales. In free surface flow, surface tension forces work to maintain the air–water interface. As flow turbulence increases, the air–water interface (water surface) becomes less smooth (more irregular), which leads to air entrainment and aerated nappe flow. The ratio of surface tension-to-viscous forces increases with decreasing model size scale, resulting in a more stable air–water interface and less nappe flow aeration. This effect can be seen in Fig. 5. For all three prototype discharges (10.0, 5.0, and 1.2 $m^3\ s^{-1}$), the nappe is visibly aerated. At 10.0 and 5.0 $m^3\ s^{-1}$, the amount of air entrainment appears to decrease with decreasing model size; the 1:25 scale model showing very limited nappe flow aeration. For the 1.2 $m^3\ s^{-1}$ case, all three model scales show limited nappe air entrainment in relation to the prototype. The minimum head value of 0.06 m given by Bretschneider in Kobus (1980) and Novak et al. (2010) in order to maintain similitude for nappe trajectories is consistent with these observations (Fig. 5).

For non-vented weir flows, the volume of the air cavity that develops between the nappe and the downstream weir wall is affected, in part, by the amount of air entrained in the nappe. As the nappe aeration levels decrease, less air is provided to the air cavity and the pressure in the air cavity decreases. This is

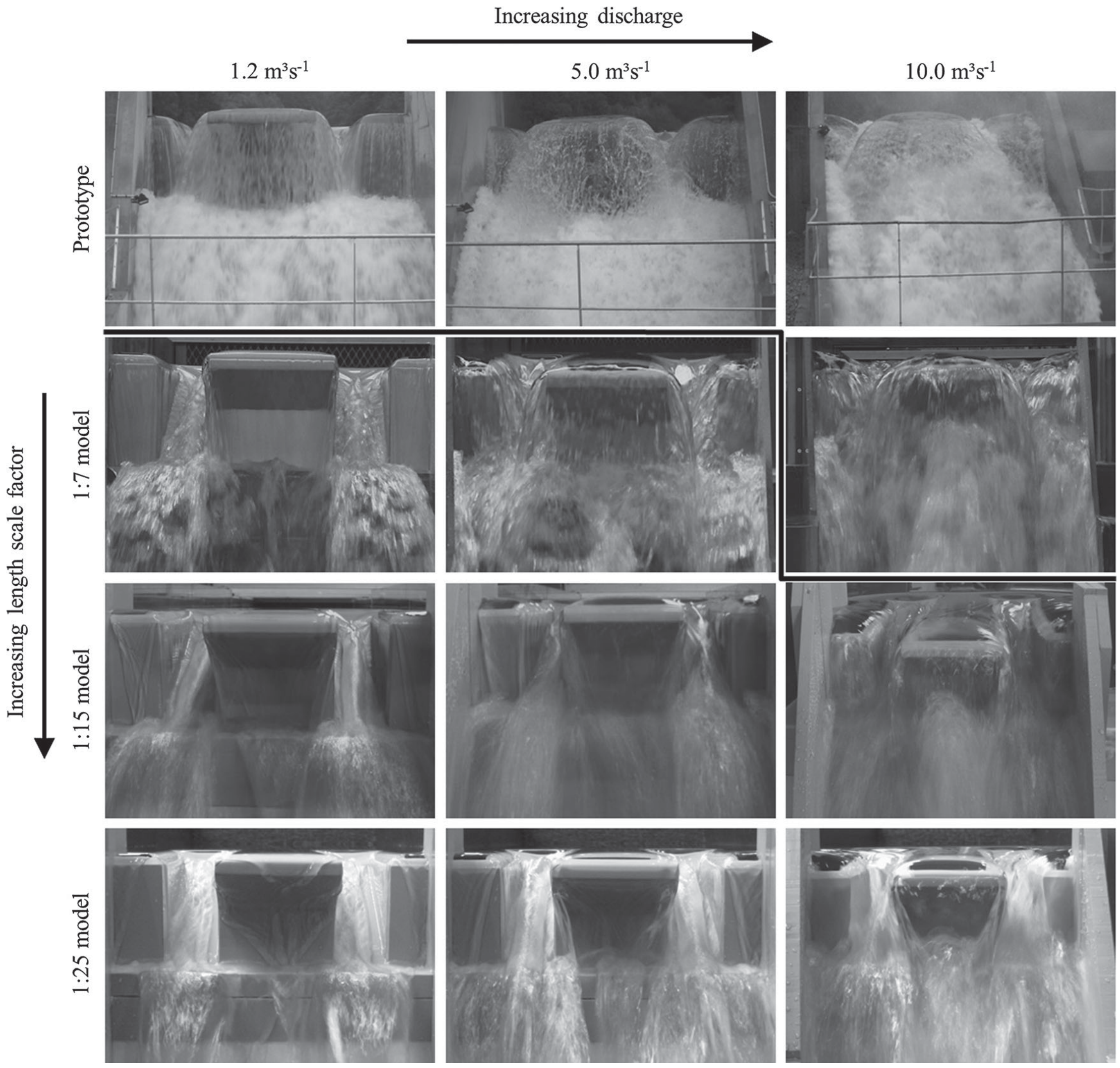

Figure 5 Photographic overview of nappe trajectory and aeration depending on model scale (rows: prototype, 1:7, 1:15, 1:25) and prototype equivalent discharge (columns: 1.2, 5.0, 10.0 $m^3\ s^{-1}$). Black line = limit between model heads higher (top right) and lower (bottom left) than 0.06 m

evident in the 10.0 m^3 s^{-1} case. As the nappe aeration decreases with decreasing model size, the reduced air void volume and increasingly negative pressure causes the nappe trajectory to reduce. For the 1:25 model, negative pressures in the air cavity cause the nappe trajectory to contract to a width narrower than the weir apex width. In some cases, the smaller models produce clinging nappe flow (i.e. the nappe remains in contact with the downstream weir wall, which in the case of PKW is an overhang), while the large-scale model(s) and prototype may not.

4 Discussion

In the absence of explicit methods for quantifying the effects of surface tension on free surface flows (e.g. flow over weirs), empirical methods are typically employed, such as identifying a minimum Weber number (W) value above which the surface tension effects can be considered negligible. Thus, the Weber number W:

$$\mathsf{W} = \frac{\rho V^2 L}{\sigma} \tag{3}$$

was used in an effort to introduce a non-dimensional limiting criterion in this study. In Eq. (3), ρ is fluid density, σ is surface tension, V is the characteristic velocity, and L is the characteristic length. Using critical depth $h_c = 2H/3$ at the weir crest as the characteristic length and the critical velocity $V_c = \sqrt{2gH/3}$ as the characteristic velocity, the number W becomes:

$$\mathsf{W} = \frac{4\rho g H^2}{9\sigma} \tag{4}$$

If the water temperature remains constant, then W becomes solely a function of H. The resulting minimum Weber number analysis, as it relates to weir flow surface tension effects with W defined by Eq. (4), becomes an alternative method for restating what has already been observed with respect to the minimum $H = 0.03$ m required to avoid surface tension affects. The minimum W value, above which surface tension effects are considered negligible for the water temperature and PKW geometry evaluated in this study and corresponding to $H = 0.03$ m, is 54.

5 Conclusion

This study evaluated head–discharge relationships and nappe flow characteristics of three geometrically similar Froude scale models (1:7, 1:15 and 1:25) and a prototype PKW. For the scale model sizes evaluated, it was determined that a minimum upstream total head of 0.03 m was required to avoid head–discharge size scale effects related to surface tension and viscous effects. The corresponding minimum Weber number was 54. The minimum H of 0.06 m was required to maintain a geometrically similar nappe trajectory profile for the weirs evaluated in this study. Future studies should consider alternative PKW design configurations as well as a broader range of laboratory-scale, geometrically similar models.

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Notation

g = gravity acceleration (m s^{-2})
h = upstream flow depth measured relative to the weir crest (m)
h_c = critical flow depth (m)
H = total upstream head, $H = (h + V^2/2g)$ (m)
Q = volumetric discharge (m^{3}s^{-1})
L = characteristic length of the flow (m)
V = flow velocity (m s^{-1})
V_c = flow velocity at critical depth (m s^{-1})
W = Weber number (–)
S = length scale factor (–)
ρ = fluid density (kg m^{-3})
σ = surface tension (N m^{-1})

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