Public Education Expenditures, Growth and Income Inequality*

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Abstract

Public education is usually seen as having at least two desirable effects: fostering economic growth and reducing income inequality. This paper revisits both relations in a single model of occupational choice with an endogenous supply of teachers. First, we show that the impact of public education expenditures on economic growth depends both on the level of these expenditures and the shape of the human capital distribution. Second, our model shows that the relationship between public education spending and income inequality can be U-shaped. We provide empirical evidence for this U-shaped relationship. Finally, we calibrate our model for 8 OECD countries.

Keywords: Endogenous growth, human capital, inequality, occupational choice, public education.

JEL Classification: E24, I24, I25, O41, O47

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1 Introduction

Public education is one of the biggest items of public spending across the world. Governments in Northern America and Western Europe spent an average of 5.2% of GDP on all levels of education in 2009, while in other regions of the world, this rate ranged between 3.6% and 5.0% of GDP (UNESCO (2011)). The public sector is a major provider of education at all levels. In primary and secondary education, the enrollment rate of students in public schools in 2013 exceeded 80% in OECD countries on average (92% in the U.S.). In tertiary education, on average, 70% of students (72% in the U.S.) were enrolled in public institutions in 2013 (OECD 2015).

Robust evidence shows that investment in education, an important input of the accumulation of human capital, has positive effects on individual earnings. Following Mincer (1974), a large body of literature has empirically investigated the return to education and found a positive association between years of schooling and earnings. Empirical evidence also tend to demonstrate that education attainment is positively correlated with aggregate income growth. Public education can be used as an instrument to sustain economic growth and correct for market inefficiencies arising from human capital externalities (Lucas (1988), Azariadis and Drazen (1990) and Romer (1990)) or credit market imperfections (Galor and Zeira (1993)). In addition, public education is also perceived as reducing income inequality. By providing the same level of schooling to everyone regardless of parental income, public education could be used as a policy tool to reduce income inequality. Such a negative relationship between investment in public education and income inequality is explored theoretically by Glomm and Ravikumar (1992), Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994) and Zhang (1996). Braun (1988), Goodspeed (2000), Sylwester (2002) and Keller (2010) provide empirical evidence in favor of a negative relationship between public education expenditures and income inequality.

In contrast to the existing literature, we document a U-shaped relationship between public education spending and income inequality. This suggests that a higher level of public

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4 An exception is Glomm and Ravikumar (2003) who show that the effect of higher public education expenditures on income inequality may be positive in the short run. They nevertheless obtain a positive association in the long run.
education spending is not necessarily associated with lower income inequality (Table 1). This result is obtained by regressing income inequality on education spending as a share of GDP and its squared value in a cross-section of countries. The convex relationship between public education spending and inequality is robust to controls and holds for different measures of inequality. We can notice that all regressions in Table 1 predict that the minimum inequality is attained \textit{ceteris paribus} for a level of public education expenditures around 4\% of GDP. The 25th, 50th and 75th percentiles of public education expenditures are respectively 3.26\%, 4.35\% and 5.44\%. In Figure 1 and regressions (i) to (vi) in Table 2, we report the same U-shaped relationship for contiguous US States (all US States excluding Alaska and Hawaii and including District of Columbia).

In this context, the contribution of our paper is twofold. First, we propose an overlapping generations model of endogenous growth through human capital accumulation in which the quality of education is determined endogenously. A recent body of literature has highlighted the importance of education quality in explaining differences in income both between individuals and between countries. When the quality of education differs from one country to the other, school attainment (or years of schooling) is not an appropriate measure of relative human capital. Hanushek and Kimko (2000), Hanushek and Woessmann (2012) and Hanushek and Woessmann (2015) further show that differences in the quality of education can also explain variation in economic growth rates across countries. This suggests that factors affecting the quality of education (and not only years of schooling) should be taken into consideration when analyzing the role of education on economic growth. In this paper, we propose to model the supply side of education in a model of occupational choice focusing on public education. In particular, agents are heterogeneous in terms of their human capital and can become workers, managers or teachers. When young, agents go to school and build their human capital. Teachers’ relative wage in the

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5Our dataset includes both developing and developed countries with available data between 1991 and 2010.
6See Appendix A.1 for a description of the control variables.
7More precisely, between 3.99\% and 4.78\% of GDP.
8Data is described in Appendix A.2.
9In a recent paper, Mannelli and Seshadri (2014) show that a large share of TFP differences across countries can be explained by differences in human capital when agents can choose both the number of years of schooling but also the amount of human capital acquired per year of schooling.
10Our model shares some features with the recent growth literature with heterogeneous agents and occupational choice developed by Lucas (2009), Eeckhout and Jovanovic (2012), Alvarez, Buera, and Lucas (2013), Lucas and Moll (2014), Perla and Tonetti (2014) and Luttmer (2014). They develop growth mechanisms based on knowledge diffusion but never consider educators as a distinct occupation group.
<table>
<thead>
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<th>Dep. var.:</th>
<th>Top 10%</th>
<th>Top 20%</th>
<th>10/10 ratio</th>
<th>20/20 ratio</th>
<th>gini</th>
<th>gini SWIID</th>
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<td>(641.2)</td>
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<td>pub. educ.²</td>
<td>58.14**</td>
<td>60.49**</td>
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<td>5358.6**</td>
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<td>(6561.3)</td>
<td>(2340.7)</td>
<td>(28.81)</td>
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<td>GDP ($1000)</td>
<td>-0.00520**</td>
<td>-0.00506**</td>
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<td>-0.466**</td>
<td>-0.00509*</td>
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<td>2.740***</td>
<td>579.6**</td>
<td>204.3***</td>
<td>3.397***</td>
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<tr>
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<td>(228.1)</td>
<td>(68.97)</td>
<td>(0.854)</td>
<td>(0.763)</td>
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<tr>
<td>Constant</td>
<td>0.493***</td>
<td>0.652***</td>
<td>51.65**</td>
<td>23.68***</td>
<td>0.611***</td>
<td>0.597***</td>
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<tr>
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<td>(0.0627)</td>
<td>(0.0655)</td>
<td>(21.12)</td>
<td>(7.119)</td>
<td>(0.0812)</td>
<td>(0.0550)</td>
</tr>
</tbody>
</table>

Table 1: Country regressions.  
Notes: Robust standard errors in parentheses. *, ** and *** respectively denote significance at the 10%, 5% and 1% level.  
Variables are described in Appendix A.1 and are averaged over the period 1991-2010.
economy endogenously determines the quality of education (teachers’ human capital) and affects the growth rate of the economy.\textsuperscript{11} We derive an explicit relationship between the growth rate of the economy and teacher quality, which has potential implications for empirical research. Current empirical work on the link between public education (measured by public education expenditures as a share of GDP) and economic growth finds mixed results.\textsuperscript{12} Our model suggests that such empirical work should control for the quality of the educational sector. In particular, we show that economies with a higher level of inequality (measured by the variance of the human capital distribution) can generally attract relatively better teachers for a given level of public education spending. This implies that one should control for higher-order moments of human capital distribution when investigating the relationship between public education expenditures and the rate of economic growth.

Our second contribution relates to the role of public education in mitigating income inequality. The increase in income inequality since the late 1970s has recently attracted a lot of attention both in academic and policy circles.\textsuperscript{13} The role of several public policies in shaping the evolution of income inequality has been investigated in the literature.\textsuperscript{14} In this paper, we theoretically show that investment in public education does not necessarily decrease income inequality. An increase in the relative wage of teachers, financed by higher public education expenditures, augments teacher quality and increases the speed of human capital accumulation and growth, but can also lead to higher levels of inequality in the long run.\textsuperscript{15} Figure 2 and regressions (vii) to (xii) in Table 2 show that the same U-shaped relationship holds between inequality and education quality (as measured by teacher wage) across US states. Our model identifies several forces which can potentially lead to such a result. Introducing an endogenous supply of teachers together with the non-degenerate distribution of individual human capital and the complementarity of workers and manager skills allows for a non-monotone effect of public education spending on income inequality.

\textsuperscript{11}See Hanushek and Rivkin (2010) for a literature review of the positive impact of higher teacher’s quality on students’ achievement and earnings.


\textsuperscript{14}See for instance Piketty, Saez, and Stantcheva (2014) and Guvenen, Kuruscu, and Ozkan (2014) for a discussion of the role of income taxation and David, Manning, and Smith (2016) and Card and DiNardo (2002) for minimum wage.

\textsuperscript{15}Card and Krueger (1992) use teacher wages to measure education quality and find a positive correlation between education quality and return to education.
The effect of a change in public education expenditures affects the shape of human capital distribution (through altered education quality) as well as occupational choice. Keeping the distribution of human capital and the mass of agents in each occupation fixed, an increase in education expenditures (financed by an increase in taxation) results in a lower demand for workers which leads to a downward pressure on wages and increased inequality. On the other hand, profits are also negatively affected by an increase in tax which tends to decrease inequality. In addition, changes in relative wages lead agents to make different occupational choices. The mass of workers and managers decreases which affects relative wages. A lower supply of workers leads to higher wages while fewer managers decreases the demand for workers, and hence puts downward pressure on wages. Eventually, the reallocation of agents to teaching modifies the shape of human capital distribution. The overall effect of an increase in education expenditures on income inequality depends on which effects dominate. We find a U-shaped relationship between education expenditures and income inequality for some parameter sets. This implies that for low level of public education, better education quality benefits the poor relatively more while the reverse happens at high initial levels of public education expenditures. The policy implication is the following: if a country is on the decreasing part of the U-shaped relationship, it can increase public spending to foster growth and reduce income inequality. In this case, growth and reduction in income inequality go hand in hand. If a country is on the increasing part of the U-shaped relationship, there is a tradeoff between growth and the reduction in inequality. Calibrating our model for 8 OECD countries, we find that some countries (among which the US) could increase growth and decrease income inequality by increasing public education spending.

The remainder of the paper is organized as follows. Section 2 presents the static model of occupational choice between three occupations (worker, teacher and manager). Section 3 embeds this occupational choice in a dynamic model in which growth is driven by human capital accumulation. Section 4 calibrates the model on data from 8 OECD countries. Section 5 concludes.
\begin{table}
\centering
\begin{tabular}{lcccccccccccccccc}
\hline
 & \textit{Top 10\%} & \textit{Top 1\%} & \textit{gini} & \textit{theil} & \textit{rmeandev} & \textit{atkin05} & \textit{Top 10\%} & \textit{Top 1\%} & \textit{gini} & \textit{theil} & \textit{rmeandev} & \textit{atkin05} \\
\hline
\textit{pub. educ.} & 405.6*** & 192.1*** & 254.1*** & 1991.0*** & 492.2*** & 352.7*** & (88.76) & (59.67) & (87.36) & (393.9) & (122.3) & (71.37) \\
\textit{Teach.wage} ($1000$) & & & & & & & -0.0262*** & -0.0124* & -0.0267*** & -0.129*** & -0.0340*** & -0.0220*** & (0.00920) & (0.00699) & (0.00932) & (0.0445) & (0.0115) & (0.00775) \\
\textit{Teach.wage} ($1000$) & & & & & & & 0.000289*** & 0.000121 & 0.000278*** & 0.00143*** & 0.000382*** & 0.000250*** & (0.000101) & (0.0000769) & (0.0000103) & (0.0000484) & (0.0000126) & (0.0000840) \\
\hline
\textit{GDP} ($10^6$) & 81.05* & 75.41** & 146.2*** & 384.8* & 82.01 & 54.25 & -14.54 & -12.03* & 0.292 & -100.3** & -13.82 & -17.25** & (44.81) & (29.43) & (38.36) & (219.4) & (60.87) & (40.16) & (9.155) & (7.052) & (7.868) & (45.74) & (11.15) & (7.812) \\
\textit{growth} & 0.00654 & 0.0644 & 0.0663* & 0.0271 & -0.00720 & -0.0122 & -0.0501 & -0.0107 & -0.0238 & -0.190 & -0.0507 & -0.0365 & (0.0365) & (0.0486) & (0.0370) & (0.207) & (0.0517) & (0.0352) & (0.0748) & (0.0600) & (0.0627) & (0.0889) & (0.0641) \\
\textit{pub. exp.} & 0.203 & 0.173 & 0.272* & 0.481 & 0.206 & 0.0645 & 0.358 & 0.370* & 0.524*** & 1.242 & 0.327 & 0.160 & (0.151) & (0.135) & (0.145) & (0.698) & (0.187) & (0.127) & (0.224) & (0.209) & (0.197) & (1.056) & (0.239) & (0.170) \\
\textit{school} & -0.250*** & -0.0850 & -0.371*** & -0.611** & -0.351*** & -0.126** & -0.118 & -0.0663 & -0.280 & 0.0753 & -0.156 & 0.00408 & (0.0876) & (0.0742) & (0.136) & (0.289) & (0.125) & (0.0491) & (0.169) & (0.0900) & (0.186) & (0.682) & (0.228) & (0.121) \\
\textit{pop. growth} & 0.908* & 1.118* & 0.525 & 4.028* & 0.558 & 0.484 & 2.303*** & 1.284*** & 1.287 & 11.36*** & 2.587** & 1.903*** & (0.452) & (0.569) & (0.606) & (2.063) & (0.649) & (0.345) & (0.849) & (0.628) & (0.990) & (3.551) & (1.128) & (0.602) \\
\textit{Constant} & 0.899*** & 0.498*** & 1.316*** & 2.507*** & 1.385*** & 0.555*** & 0.870*** & 0.355* & 1.265*** & 2.360*** & 1.454*** & 0.555*** & (0.147) & (0.0869) & (0.122) & (0.674) & (0.191) & (0.124) & (0.203) & (0.204) & (0.207) & (1.073) & (0.241) & (0.179) \\
\hline
\textit{Observations} & 48 & 48 & 48 & 48 & 48 & 48 & 49 & 49 & 49 & 49 & 49 & 49 \\
\textit{R}^2 & 0.79 & 0.55 & 0.64 & 0.86 & 0.73 & 0.86 & 0.58 & 0.31 & 0.44 & 0.71 & 0.58 & 0.75 \\
\hline
\end{tabular}
\caption{Inequality regressions (US states).}
\textbf{Notes:} Robust standard errors in parentheses. *, ** and *** respectively denote significance at the 10%, 5% and 1% level. Variables are described in Appendix A.2.
\end{table}
A static model of occupational choice

For expositional purposes, we start with a static (one-period) model of occupational choice. We embed this static model in an overlapping generations structure in Section 3. Agents can choose between three different jobs: worker, teacher and manager. There is a measure one of agents in the economy and they are heterogeneous in their level of human capital (h) with cdf $F : \mathbb{R}_+ \to [0, 1]$. There is a single consumption good in the economy.

2.1 The Agent problem

The choice of occupation by agents is driven by the return to the three potential jobs which is itself a function of human capital. We assume that agents cannot perform more than one job. If an agent decides to become a worker, she inelastically supplies one unit of labor and receives a wage $w$ which is determined endogenously. A teacher receives a wage $w^T$. Teachers do not directly participate in production. Firms produce output by combining one manager with a set of workers. A firm’s production is determined by the
manager’s span of control as in Lucas (1978). The production function of a manager with a level of human capital $h$ is given by:

$$y(h) = Ahn(h)^\alpha$$ \hspace{1cm} (1)

where $A$ is the productivity parameter common to all managers, $\alpha \in (0, 1)$ and $n(h)$ is the labor demand of a firm employing a manager with human capital $h$.

The profit function for the firm as a function of the managers level of human capital $h$ is then given by:

$$\pi(h) = A(1 - \tau)hn(h)^\alpha - wn(h)$$ \hspace{1cm} (2)

where $w$ is the wage paid to workers and $\tau$ is a tax on production. This tax is used by the government to finance public education spending and in particular to pay teachers’ wages.

Profit maximization implies the following labor demand:

$$n(h) = \left[\frac{(1 - \tau)A\alpha h}{w}\right]^\frac{1}{1-\alpha}$$ \hspace{1cm} (3)
and associated profit:

$$\pi(h) = (1 - \alpha) [(1 - \tau)Ah]^{\frac{1}{1-\alpha}} \left( \frac{\alpha w}{w} \right)^{\frac{\alpha}{1-\alpha}}$$ (4)

The profit of managers is convex in the level of human capital, decreasing in the level of the tax and \( \lim_{h \to 0} \pi(h) = 0 \). We assume that the profit of the firm is entirely paid to managers as wage.\(^{16}\) In the remainder of the paper, a manager’s wage and the profit of her firm are used interchangeably.

We assume the following utility function:

$$u(c, h) = c - 1_T \gamma(h)$$ (5)

where \( c \) is consumption, \( 1_T \) takes the value one if the agent is a teacher and zero otherwise and \( \gamma(h) \) is a utility cost of working as a teacher.\(^{17}\) We assume that this cost is strictly decreasing and strictly convex in the level of human capital with \( \lim_{h \to 0} \gamma(h) = \infty \) and \( \lim_{h \to \infty} \gamma(h) = 0 \).

Agents consume their wage or profit so that the utility for an agent with human capital \( h \) associated with the three occupations is given by:

$$u^W(h) = w \text{ as a worker}$$ (6)
$$u^T(h) = w^T - \gamma(h) \text{ as a teacher}$$ (7)
$$u^M(h) = (1 - \alpha) [(1 - \tau)Ah]^{\frac{1}{1-\alpha}} \left( \frac{\alpha w}{w} \right)^{\frac{\alpha}{1-\alpha}} \text{ as a manager}$$ (8)

Agents choose the occupation which gives the highest utility given their level of human capital.

\(^{16}\)This is obtained if we assume that there is free-entry of firms competing for managers. In this case, any manager with human capital \( h \) works for a firm which offers a wage equal to the maximum profit that a manager with human capital \( h \) can generate. All firms consequently make no profit in equilibrium.

\(^{17}\)This could, for instance, reflect the formal requirement of a particular degree in order to teach in public schools.
2.2 Government budget constraint

We assume that the government has a single role in our economy. It levies a proportional tax \( \tau \) on the production of managers whose revenues are used to pay teachers’ wages \( w^T \). We assume that the government budget is balanced so that:

\[
\int_T w^T dF(h) = \int_M \tau y(h) dF(h)
\]

where \( T \) and \( M \) respectively stand for the sets of teachers and managers in the economy. Using the optimal labor demand function in Equation (3), we can write:

\[
w^T = \tau \frac{A^{\frac{1}{1-\alpha}} \left( \frac{(1-\tau)\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \int_M h \frac{1}{1-\alpha} dF(h)}{\int_T dF(h)}
\]

2.3 Equilibrium conditions

In order to derive the equilibrium conditions, we first show that the distribution of agents over occupations for given wages for workers \( w \) and teachers \( w^T \) can be summarized by two cutoffs \( h_W \) and \( h_M \) \( (h_W \leq h_M) \). Agents with human capital respectively below \( h_W \) (above \( h_M \)) choose to be workers (managers) and agents with human capital between \( h_W \) and \( h_M \) choose to be teachers. We then show that the condition \( w^T > w + \gamma(\pi^{-1}(w)) \) has to hold in any equilibrium. This implies that, in equilibrium, there is a positive mass of agents working in each occupation and that teachers receive a higher wage than workers.

**Proposition 1** Given wages \( w > 0 \) and \( w^T > 0 \) and a human capital distribution with support \( \mathbb{R}^+ \), the optimal occupational choice of agents is defined by two cutoffs \( (h_M \geq h_W) \). Agents with human capital below \( h_W \) become workers and agents with human capital between \( h_W \) and \( h_M \) become teachers. Agents with human capital above \( h_M \) work as managers.

**Proof:** The proof starts from the limit behavior of the utility function under the three different occupations. For workers, the utility function is flat at \( w > 0 \). For teachers and managers, we have:

\[
\lim_{h \to 0} u^T(h) = -\infty \text{ and } \lim_{h \to \infty} u^T(h) = w^T
\]

\[
\lim_{h \to 0} u^M(h) = 0 \text{ and } \lim_{h \to \infty} u^M(h) = \infty
\]
This implies:
\[
\lim_{h \to 0} u^W(h) > \lim_{h \to 0} u^T(h) > \lim_{h \to 0} u^M(h) \tag{13}
\]
\[
\lim_{h \to \infty} u^M(h) > \lim_{h \to \infty} u^T(h) > \lim_{h \to \infty} u^W(h) \tag{14}
\]

Since wages are strictly positive, teacher and manager wages are continuously and monotonically increasing in human capital, teacher wage is strictly concave and profits strictly convex, which proves the existence of two cutoffs. First, there exists a cutoff \((h_W > 0)\) up to which agents find it optimal to become workers. There also exists another cutoff \((h_M \geq h_W)\) above which agents decide to become managers. This implies that only agents in the middle of the human capital distribution (between \(h_W\) and \(h_M\)) want to become teachers. To have a strictly positive mass of agents working as teachers, two conditions are needed: first, the wage of teachers \((w_T)\) must be strictly greater than that of workers since teachers suffer a positive utility cost and, second, the utility of working as a teacher with human capital \(h_W\) (i.e. \(w\)) must be strictly greater than the profit of a manager with the same level of human capital i.e., we need the teacher utility function to intersect \(w\) before the profit function (both teacher utility and profit intersect the worker wage function only once).\(^{18}\)

\[
h^* = \{h : w = w^T - \gamma(h^*)\} < h^{**} = \{h : w = \pi(h^{**})\}
\]
\[
w^T > w + \gamma(\pi^{-1}(w)) \tag{16}
\]

If Equation (16) is satisfied, \(h^* = h_W > h_M\) and there is a positive mass of teachers in the economy. Otherwise, \(h_W = h_M\) and there are workers and managers only in the economy.

**Static Equilibrium Definition:** *Given a distribution of human capital \(F : \mathbb{R}^+ \to [0, 1]\) and a tax rate \((\tau)\), a static equilibrium is a collection of wages \((w, w^T, \pi(h))\), cutoffs \((h_W, h_M)\), demand for workers \((n(h))\), and final good production \((y(h))\) such that:

1. Given wages, firms maximize profit.
2. Given wages, agents maximize utility by following a cutoff strategy in which agents

---

\(^{18}\)Since teacher utility and profits are monotonically increasing functions of human capital, \(w > 0\), \(\lim_{h \to 0} u^T(h) = -\infty\) and \(\lim_{h \to 0} u^M(h) = 0\).
Proposition 1 shows that agents with low (high) levels of human capital choose to become workers (managers). Teachers, provided that \( w^T > w + \gamma(\pi^{-1}(w)) \), are to be found in the middle of the human capital distribution. We now prove that, in equilibrium, \( w^T > w + \gamma(\pi^{-1}(w)) \) has to hold.

**Proposition 2** Given a human capital distribution with support \( \mathbb{R}^+ \) and a tax rate \( \tau > 0 \), there is a positive mass of agents working in each occupation in equilibrium i.e. \( w^T > w + \gamma(\pi^{-1}(w)) \) and \( h_W < h_M \).

**Proof:** First, we can show that the wage of workers is positive in equilibrium. If it was not positive, the managers’ demand for labor would be infinite (see Equation (3)) and, hence, the labor market condition could not be satisfied. Having proved that \( w > 0 \) in any equilibrium, we know from Proposition 1 that a strictly positive mass of agents finds it optimal to become workers and managers. To prove that there must also be a positive mass of teachers in equilibrium, we use the government budget balance condition:

\[
\int_{h_W}^{h_M} w^T \, dF(h) = \int_{h_M}^{\infty} \tau y(h) \, dF(h) \tag{17}
\]

Since there is a positive mass of workers and managers, production is positive and hence proceeds of taxes are positive. If there is no teacher in the economy, we obtain:

\[
\lim_{h_W \to h_M} w_T = \lim_{h_W \to h_M} \frac{\int_{h_M}^{\infty} \tau y(h) \, dF(h)}{\int_{h_W}^{h_M} dF(h)} = \infty \tag{18}
\]

This would contradict the assumption that no agents find it optimal to become a teacher and, hence, there must be a positive mass of teachers in the economy in equilibrium. 

To determine the equilibrium conditions of the static model, we combine the indifference conditions and the market clearing condition. Agents with human capital \( h_W \) are...
indifferent between being a teacher and a worker and agents with human capital $h_M$ are indifferent between being a teacher and a manager. In addition, wages ($w$ and $w^T$) and human capital cutoffs have to satisfy the balanced government budget and the labor market clearing condition. The equilibrium in this static model can be summarized by the following system of four equations and four unknowns:

\begin{align*}
  w &= w^T - \gamma(h_W) \\
  w^T - \gamma(h_M) &= (1 - \alpha) \left( \frac{\alpha}{w} \right) \frac{1}{\tau} \left( (1 - \tau)Ah_M \right) \frac{1}{1-\alpha} \\
  w^T &= \tau \frac{A^{\frac{1}{1-\alpha}} \left( \frac{(1-\tau)\alpha}{w} \right) \frac{1}{\tau \alpha} \int_{h_M}^{\infty} h^{\frac{1}{1-\alpha}} dF(h)}{F(h_M) - F(h_W)} \\
  F(h_W) &= \left( \frac{A\alpha(1-\tau)}{w} \right) \frac{1}{\tau \alpha} \int_{h_M}^{\infty} h^{\frac{1}{1-\alpha}} dF(h)
\end{align*}

where Equations (19) and (20) come from agents’ indifference between occupations at the cutoffs $h_W$ and $h_M$, Equation (21) corresponds to the balanced government budget and Equation (22) is the labor market clearing condition.

Figure 3 shows an example of equilibrium occupational choice by agents. The dotted lines represent utility as a function of human capital under the three occupations. Workers get a wage which is independent of their level of human capital. Teachers receive a wage and incur a convex utility cost. The profit of managers is increasing and convex in human capital (see Equation (4)). The solid line represents the equilibrium utility of agents as a function of human capital i.e., the maximum of the utility across the three occupations. In equilibrium, agents with human capital below $h_W$ become workers, agents with human capital between $h_W$ and $h_M$ become teachers agents with human capital above $h_M$ are managers.

### 2.4 Comparative statics

In this section, we study how the equilibrium changes as we change the tax rate $\tau$. The objective of this paper is to eventually analyze the effect of public education spending on education quality, economic growth and income inequality in a dynamic version of the model presented in this section. In a static one-period model, education plays no role in
Figure 3: Occupational choice.
Notes: In this example, we assume $\gamma(x) = \frac{1}{x}$, $\alpha = \frac{1}{3}$, $\tau = 0.1$, $A = 1$ and $F$ is a log-normal distribution with mean and variance equal to one. Dotted lines represent utility under the three different occupations. The solid line represents the equilibrium utility derived by agents as a function of their level of human capital.
the economy as there is no human capital accumulation. We can nevertheless study how
the quality of education evolves as public spending increases. We measure the quality of
education as the aggregate human capital level of agents who decide to become teachers:

\[ S = \int_{h_w}^{h_M} h \, dF(h) \quad (23) \]

Figure 4 shows comparative statics for \( \tau \). Increasing \( \tau \) raises the incentive for agents to
become teachers as it increases teacher wage \textit{ceteris paribus}. On the other hand, it also
decreases the net profit of managers and their labor demand which also makes teaching
relatively more attractive to agents. As a consequence, the mass of teachers increases and
the masses of workers and managers decrease as the tax is raised. This means that some
workers and managers switch to teaching when the tax rate is increased. As teaching
attracts more agents, the quality of education \((S)\) also improves. However, it is apparent
from Figure 4 that there is a trade-off between production and quality of education.
This comes from the fact that improving the quality of education through higher public
education spending diverts agents from the productive sector of the economy. Manager
wage monotonically decreases with the tax rate. Worker wage and teacher wage, however,
are concave in the tax rate (inverted-U shape). The evolution of the worker wage depends
on the relative change in worker supply and demand following an increase in the tax rate.
Increasing the tax rate, increases the share of production going to teachers. On the other
hand, it also increases the mass of teachers in the economy and decreases total production,
which tends to decrease teacher wage. At low tax levels, the first effect dominates while
it is more than offset by the other two effects for high levels of tax rates generating the
concave relationship between tax and teacher wage in Figure 4. Even though teacher
wage is eventually decreasing in the tax rate, it always increases relatively to worker
and manager (mean) wage (Figure 4). This means that more public education spending
increases the relative wage of teachers so as to make it ever more attractive in our model.
Our model implies that relatively higher wages of teachers are associated with higher
levels of education quality.
Figure 4: Comparative statics for $\tau$.

Notes: In this Table, we use $\gamma(x) = \frac{1}{x}$, $\alpha = \frac{1}{3}$ and $A = 1$. $F$ is a log-normal distribution with mean and variance equal to one.
3 A dynamic model of occupational choice, growth and inequality

3.1 The dynamic model

In this section, we embed the static model into an overlapping generations model. We assume that there is at any time a mass one of young and a mass one of old agents in the economy. Agents live for two periods and only consume when old with preferences similar to those described in the one-period model in Section 2.1:

\begin{align*}
  u^W_t(h) &= w_t \text{ as a worker} \\
  u^T_t(h) &= w^T_t - \gamma_t(h) \text{ as a teacher} \\
  u^E_t(h) &= (1 - \alpha) [(1 - \tau)Ah]^{1-\alpha} \left( \frac{\alpha}{w_t} \right)^{\frac{\alpha}{1-\alpha}} \text{ as a manager}
\end{align*}

Each old agent is assumed to have one child so that there is a measure one of families composed of one young and one old agent. When young, agents go to school and build their level of human capital:

\begin{equation}
  h_{t+1} = a_t h_t^{\beta_1} S_t^{\beta_2}
\end{equation}

where \( h_t \) is the level of human capital of the old agent in the family at time \( t \), \( a_t \) is an idiosyncratic shock to the transmission of human capital to the child with distribution \( G_t(a) \) and \( S_t = \int_{hW_t}^{hM_t} h \, dF_t(h) \) is the quality of education measured as the human capital of teachers per student.

An agent’s human capital is thus a function of her parent’s human capital, the quality of the educational system when she is young and a random shock to her ability to absorb the knowledge from her parent and teachers. The relative importance of parents and education in the formation of human capital is captured by \( \beta_1 \) and \( \beta_2 \). We assume that \( \beta_1 + \beta_2 = 1 \) and \( \beta_1 \in (0, 1) \). An imperfect transmission of knowledge through the shock \( a \) allows for social mobility across generations.
Dynamic Equilibrium Definition: Given an initial distribution of human capital $F_0 : \mathbb{R}^+ \rightarrow [0,1]$, a distribution for the shock $a_t (G)$ and a tax rate ($\tau$), a dynamic equilibrium is a sequence of wages ($w_t, w_t^T, \pi_t(h)$), cutoffs ($h_{W,t}, h_{M,t}$), demand for workers ($n_t(h)$), education quality ($S_t$) and final good production ($y_t(h)$) so that, at every period:

1. Given wages, firms maximize profit.
2. Given wages, agents maximize utility by following a cutoff strategy in which agents with human capital in $[0, h_{W,t})$ become workers, agents with human capital in $[h_{W,t}, h_{M,t})$ are teachers and agents with human capital above $h_{M,t}$ work as managers.
3. Labor market clears: $\int_{h_{M,t}}^{\infty} n_t(h) \, dF_t(h) = \int_0^{h_{W,t}} \, dF_t(h)$
4. Government budget is balanced: $\int_{h_{W,t}}^{h_{M,t}} w_t^T \, dF_t(h) = \int_{h_{M,t}}^{\infty} \tau y_t(h) \, dF_t(h)$
5. Education quality is given by: $S_t = \int_{h_{W,t}}^{h_{M,t}} S_t \, dF_t(h)$
6. Human capital in the economy evolves as: $h_{t+1} = a_t h_t \beta_1 S_t^{\beta_2}$

In the remainder of the paper, we make the following assumptions regarding initial conditions, functional form of $\gamma_t(h)$ and the distribution of $a_t$:

$$\gamma_t(h) = \frac{\bar{h}_t^2}{\bar{h}}$$

(28)

$$\log(h_0) \sim \mathcal{N}(\mu_0, \sigma_0^2)$$

(29)

$$\log(a_t) \sim \mathcal{N}(\mu_a, \sigma_a^2)$$

(30)

where $\bar{h}_t$ is the mean human capital level of old agents at time $t$.

The choice of the functional form of $\gamma_t(h)$ enables us to obtain a strictly positive difference in wages between workers and teachers in the long run. The shock to the transmission of human capital is assumed to be the same across agents and across time. The distributional assumptions lead to the existence of a balanced growth path. Given these assumptions, we can show that the distribution of human capital at any time $t$ has a log-normal distribution. In particular,

$$\log(h_{t+1}) \sim \mathcal{N}(\mu_a + \beta_1 \mu_t + \beta_2 \log(S_t), \sigma_a^2 + \beta_1^2 \sigma_t^2)$$

(31)
In the long run, the variance of \( \log(h_t) \) converges to \( \frac{\sigma_a^2}{1-\beta_1} \). If \( \sigma_a^2 = 0 \), the distribution of human capital converges to a degenerate distribution, in which case there is no income inequality in the long run.

**Balanced Growth Path Definition:** A balanced growth path is a dynamic equilibrium in which:

1. \( h_{W,t}, h_{M,t}, S_t, Y_t = \int_{h_{M,t}}^\infty y_t(h) dF_t(h), w_t \) and \( w_T^T \) all grow at the same rate \( g \).
2. \( \sigma^2 = \frac{\sigma_a^2}{1-\beta_1} \)
3. The masses of workers, teachers and managers are constant.

In a balanced growth path, the growth rate of the economy \( (g) \) is given by:

\[
1 + g = e^{\mu_a} \left( \frac{S_t}{e^{\mu_t}} \right)^{1-\beta_1} \quad (32)
\]

\[
g \approx \mu_a + (1 - \beta_1) \left[ \log(S_t) - \mu_t \right] \quad (33)
\]

Long-term growth in the economy is increasing in the quality of education \( (S_t) \) relative to the human capital level in the economy. This has two main consequences. First, increasing the quality of public education leads to higher growth rates. Second, two countries with the same level of public education expenditures can have different rates of growth depending on the shape of the distribution of human capital in the economy. Figure 5 shows the growth rate of an economy with similar public education expenditures (as a share of GDP) but different shapes for the human capital distribution. In particular, we vary the variance of the shock \( (\sigma_a) \). \( \sigma_a \) affects the variance and the tail of the human capital distribution without directly affecting economic growth (Equations (31) and (32)). This enables us to identify the effect of the shape of the human capital distribution on the endogenously determined quality of teachers and on economic growth. Economies with a fatter right tail of the human capital distribution (higher \( \sigma_a \)) attracts higher quality teachers for a same level of public education expenditures. This, in turn, results in a higher level of growth. In other words, the growth rate of the economy is not only a function of the average level of human capital in the economy but also of higher-order effects.
moments of the human capital distribution through their effect on the quality of teachers.\textsuperscript{19} This has implications for empirical studies trying to identify the effect of public education expenditures on economic growth. In particular, one needs to control for the shape (tail) of the human capital distribution in the economy when estimating the effect of public education expenditures on economic growth. This may explain why several empirical studies fail to find a robust significant effects of public education expenditures on economic growth.

\subsection{Education spending, growth and inequality}

In this section, we focus on balanced growth paths and compare them for different values of the tax rate $\tau$. In this model, the tax rate $\tau$ can also be interpreted as public education spending as a share of GDP. We measure inequality using the $10/10$ ratio i.e. the ratio of (before-tax) incomes of the top $10\%$ of the income distribution to the bottom $10\%$ of the income distribution.

Figure 6 confirms that the growth rate of the economy is an increasing function of the \textsuperscript{19}In a different context, Perla and Tonetti (2014) find a similar positive correlation with the thickness of the tail of the productivity distribution and economic growth.
share of GDP devoted to public education. Economies with higher tax rates $\tau$ attract more human capital in the education sector which accelerates growth in the economy. Regarding inequality, our model implies that the relationship between public education spending as a share of GDP and inequality (as measured by the 10/10 ratio) may be non-monotone. In particular, this relationship may have a U shape which means that there exists a tax rate which minimizes inequality.\footnote{This also suggests that there is eventually a trade-off between economic growth and income inequality in our model and that the relationship between economic growth and inequality is non-monotone.}

The effect of a tax change on inequality (measured by the 10/10 ratio) can be decomposed into a direct effect of taxes on managers’ profit and a general equilibrium effect which is the result of the tax change on equilibrium occupational choice, labor demand and supply, and hence on wages. Given our choice of parameters, there is no teacher in either the top or bottom 10% of the income distribution (over the relevant range of taxes) and the 10/10 ratio (before tax) can be written as:

$$10/10 \text{ ratio}_t = \frac{(1 - \alpha (1 - \tau)) A^{1-\alpha} \left( \frac{\alpha (1 - \tau)}{w_t} \right)^{\frac{\alpha}{1-\alpha}} \int_{F_{t}^{-1}(0.9)}^{\infty} h^{1-\alpha} dF_t(h)}{0.1 w_t}$$  \hspace{1cm} (34)$$

$$= 10(1 - \alpha (1 - \tau)) A^{1-\alpha} \left( \frac{\alpha (1 - \tau)}{w_t} \right)^{\frac{\alpha}{1-\alpha}} \int_{F_{t}^{-1}(0.9)}^{\infty} h^{1-\alpha} dF_t(h)$$  \hspace{1cm} (35)$$

$$= 10(1 - \alpha (1 - \tau)) A^{1-\alpha} \left( \frac{\alpha (1 - \tau)}{w_t} \right)^{\frac{\alpha}{1-\alpha}} \Phi(\tau)$$  \hspace{1cm} (36)$$

where $\Phi(\tau) = \int_{F_{t}^{-1}(0.9)}^{\infty} \left( \frac{h}{w_t} \right)^{1-\alpha} dF_t(h)$ depends on $\tau$ through the effect of taxes on human capital distribution, occupational choice and wages. In a balanced growth path, $\Phi(\tau)$ converges to a constant.

In a balanced growth path, the derivative of the 10/10 ratio with respect to the tax rate is:

\footnote{The parameter values for the numerical example in Figure 6 have been chosen so that there is no teacher in either the top or bottom 10% of the distribution at any tax rate. This implies that the non-monotone relationship between public education spending and inequality is not due to a change in the occupational composition of the top or bottom of the income distribution.}
Figure 6: Comparative statics at the steady state: $\tau$.

Notes: In this Table, we use the following parameter values: $A=1$, $\alpha = 0.2$, $\beta_1 = 0.4$, $\varphi = 0.01$, $\mu_a = 3$ and $\sigma_a = 0.85$. 
\[ \frac{\partial 10/10 \text{ ratio}}{\partial \tau} = -10A^{1-\alpha} \alpha^{1-\alpha} \frac{\tau(1-\tau)^{2\alpha-1}}{1-\alpha} \Phi(\tau) + 10(1-\alpha(1-\tau))A^{1-\alpha}(\alpha(1-\tau))^{\alpha} \Phi'(\tau) \]

Direct profit effect

General Equilibrium effect

(37)

The direct profit effect looks at the change in the (before tax) profit of managers keeping the distribution and wages fixed. The general equilibrium effect takes into account the change in human capital distribution, occupational choice and wages following a change in tax rate. The direct effect on profit is always negative as an increase in the tax rate decreases managers’ profit ceteris paribus. Depending on whether the general equilibrium effect is positive or negative, the total effect of a change in education spending on inequality can be positive or negative. The sign of the general equilibrium effect depends on the effect of a tax change on labor supply (workers) and demand (managers) which, in turn, depends on the occupational choice of agents. Figure 6 (Panel E) shows that the general equilibrium effect eventually becomes positive and offsets the direct profit effect so that inequality starts increasing as the tax is raised. We can further decompose the general equilibrium effect into two parts: one related to the direct effect of a change in education spending on wages and the other related to the change in the distribution of occupations in the economy. Using Equation (22) for equilibrium wage, we can write:

\[ \Phi(\tau) = \frac{F_t(h_{W,t}) \int_{F_t^{-1}(0,9)}^{\infty} h^{1-\alpha} dF_t(h)}{(A\alpha(1-\tau))^{\frac{1}{1-\alpha}} \int_{h_{M,t}}^{\infty} h^{1-\alpha} dF_t(h)} \]  

\[ = \frac{\Psi(\tau)}{(A\alpha(1-\tau))^{\frac{1}{1-\alpha}}} \]  

(38)

(39)

where \[ \Psi(\tau) = \frac{F_t(h_{W,t}) \int_{F_t^{-1}(0,9)}^{\infty} h^{1-\alpha} dF_t(h)}{\int_{h_{M,t}}^{\infty} h^{1-\alpha} dF_t(h)}. \]

The general equilibrium effect (gee) can then be decomposed as:
\[ g(e) = 10(1 - \alpha(1 - \tau)) \cdot A^{1-\alpha}(\alpha(1 - \tau))^{1-\alpha} \left( \frac{\Psi(\tau)}{(A\alpha)^{1-\alpha}(1-\alpha)(1-\tau)^{2-\alpha}} + \frac{\Psi'(\tau)}{[A\alpha(1-\tau)]^{1-\alpha}} \right) \]

\[ = 10(1 - \alpha(1 - \tau)) \cdot \frac{\Psi(\tau)}{(1-\alpha)\alpha(1-\tau)^2} + 10(1 - \alpha(1 - \tau)) \cdot \frac{\Psi'(\tau)}{\alpha(1-\tau)} \]

\[ \text{Direct wage effect} \quad \text{Distributional effect} \]

The direct labor demand effect captures the direct effect of a change in the tax rate on inequality i.e. the effect of the decreased demand for workers by managers after a tax increase keeping constant the distribution, the mass of managers and the supply of workers. This effect always increases inequality as measured by the 10/10 ratio. The distributional effect measures the role of changing occupational choice as well as of human capital distribution (since modifying public education spending changes the quality of education, it also alters the human capital distribution).

The distributional effect can, in turn, be decomposed into two parts: one relates to the change in labor supply (mass of workers) and the other to both changes at the top of the human capital distribution and in the mass of managers.

\[ \Psi(\tau) = \frac{\int_{F_{t}^{-1}(0.9)}^{\infty} h^{1-\alpha} dF_{t}(h)}{\int_{h_{M,1}}^{\infty} h^{1-\alpha} dF_{t}(h)} \]  \[ = F_{t}(h_{W,1})\Omega(\tau) \]

\[ \text{where } \Omega(\tau) = \frac{\int_{F_{t}^{-1}(0.9)}^{\infty} h^{1-\alpha} dF_{t}(h)}{\int_{h_{M,1}}^{\infty} h^{1-\alpha} dF_{t}(h)} . \]

Panel C of Figure 6 shows that the mass of workers is decreasing in the tax rate. Everything else kept constant, a decrease in the supply of labor results in an increase in wages and hence in a decrease in income inequality. The second term, which relates to the distribution of managers, is increasing (Panel F of Figure 6). This term shows that, ceteris paribus, a change in the human capital distribution resulting from better school quality benefits the managers at the top of the distribution.

Overall, the effect of a change in public education spending as a share of GDP on inequality can be decomposed into 4 different parts:
\[
\frac{\partial 10/10 \text{ ratio}}{\partial \tau} = \begin{cases} 
Direct \text{ profit effect} + Direct \text{ labor demand effect} & \leq 0 \\
+ \begin{cases} 
Labor \text{ supply effect} + Manager \text{ distribution effect} & \geq 0 
\end{cases} & \geq 0
\end{cases}
\] (43)

The sign of the derivative in Equation (43) depends on which effects dominate.

The effect of increasing education spending on inequality is ambiguous and depends on the parameter values. Section 4 calibrates our model to data from 8 OECD countries in order to determine whether some countries are in the decreasing part of the inequality-public education relationship. In such a case, a country would have the opportunity to increase its growth rate without increasing inequality.

Figure 7 shows the relationship between education spending and alternative measures of inequality. In particular, we report the ratio of the income of the top and bottom 20% (20/20 ratio), the Gini coefficient, the share of income held by the richest 20% and 10%. In all cases, the relation between the tax rate and inequality is U-shaped. This shows that the non-monotone relationship between education spending and inequality reported in Figure 6 holds for other measures of inequality as well.

4 Quantitative exercise: 8 OECD countries

In this section, we provide numerical examples of the relationship between public education spending, economic growth and inequality for a sample of 8 OECD countries Australia, Canada, France, Germany, Italy, Japan, United Kingdom and the US. For each country, we set \( A = 1 \) and jointly calibrate the other parameters of the model \((\alpha, \beta_1, \psi, \mu_a \text{ and } \sigma_a)\) to match 5 moments in the data: inequality, real GDP growth rate, the fraction of teachers in the economy, the relative wage of teachers to other workers and intergenerational income mobility. We obtain data on the share of workers in the education and other sectors as well as corresponding wages from the International Labor Organization. Data on inequality and public education spending as a share of GDP comes from the World Bank. For all these variables, we use average values over the period 1991-2010.\(^{21}\) Real GDP growth over the period 1991-2010 is obtained from the Penn World

\(^{21}\)The period is chosen for two reasons: first, 20 years approximately correspond to one period in our model and, second, data outside this time window is very scarce.
Figure 7: Alternative measures of inequality.

Notes: In this Table, we use the following parameter values: $A=1$, $\alpha = 0.2$, $\beta_1 = 0.4$, $\varphi = 0.01$, $\mu_a = 3$ and $\sigma_a = 0.85$. 
Tables. Corak (2006, 2013) provide a summary of estimates of intergenerational income elasticity from the empirical literature.

The parameter values used in this section are reported in Table 3. Table 4 compares the targeted moments in the model and in the data. Using these calibrated parameter values, we perform the following experiment: we vary the share of GDP spent on public education and compute the corresponding change in inequality and growth predicted by our model. The results of this experiment are reported in Figure 8.

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<td>1.030</td>
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</table>

Table 3: Parameter values

Each panel of Figure 8 represents one country. Inequality is measured on the left axis (10/10 ratio) and annual growth rate on the right axis. The vertical line corresponds to actual data over the period 1991-2010 in terms of public education spending (as a share of GDP), inequality and real GDP growth. What our results indicate is that there is some heterogeneity across countries. Growth in our model is generated by human capital accumulation through public education. Higher levels of public education spending tend to increase the quality of public education and hence lead to faster growth in the long run. The effect of increased public education quality on inequality is ambiguous and depends on the dominating effects as described in Equation (43). As exemplified in Section 3.2, the relationship between public education spending (or quality) and inequality is potentially U-shaped. As a consequence, countries in the decreasing part of this U-shaped relationship can potentially raise their growth rate and decrease inequalities by increasing their public education expenditures. On the other hand, countries in the increasing part of the U face a trade-off between growth and inequality. According to our calibration, Italy, the UK and the US are in the decreasing part of the U-shaped relationship between public education spending and inequality while Australia, Canada, Germany, France and Japan are in the increasing part.
Figure 8: Calibration results: 10/10 ratio.

Notes: The dashed line represents inequality (left axis) and the solid line growth (right axis) for different levels of public education spending. The vertical line indicates actual data on public education spending, economic growth and inequality.
Table 4: Targeted moments: model vs data

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.57</td>
<td>0.57</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Ratio 10/10</td>
<td>11.1</td>
<td>11.1</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Share of teachers</td>
<td>0.07</td>
<td>0.08</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>Teacher wage relative to other workers</td>
<td>1.14</td>
<td>1.14</td>
<td>1.08</td>
<td>1.09</td>
</tr>
<tr>
<td>Income elasticity</td>
<td>0.26</td>
<td>0.26</td>
<td>0.19</td>
<td>0.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>Japan</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.13</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Ratio 10/10</td>
<td>12.9</td>
<td>12.9</td>
<td>9.2</td>
<td>9.2</td>
</tr>
<tr>
<td>Share of teachers</td>
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<td>0.09</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Teacher wage relative to other workers</td>
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<td>1.02</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>Income elasticity</td>
<td>0.50</td>
<td>0.50</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 5 reports the effect of an increase by one percentage point of the share of GDP spent on public education on growth and inequality predicted by our model. Our model prediction for the change in annual growth rate following a one-percentage-point increase in public education spending ranges from 0.1% for the US to 0.49% for Japan. On the other hand, the prediction regarding inequality goes from a decrease by −1.10% of the 10/10 ratio for the US to an increase by 0.92% for Japan. The last column reports the semi-elasticity between inequality and growth, i.e., the percentage change in inequality associated with a one-percentage-point increase in growth. This measures the trade-off between growth and inequality that different countries face in our model. Higher positive values correspond to a stronger trade-off, i.e., a higher cost in terms of increased inequality for a given increase in growth. Negative values imply that more growth would be associated with lower inequality. Our calculations indicate that Germany faces the strongest trade-off and the US the highest “free-lunch” in terms of growth and inequality.

Overall, our quantitative examples suggest that there may be heterogeneity across countries in terms of the trade-off (or the absence thereof) between growth and inequality through public education spending. Some countries may be able to raise their economic growth rate while decreasing inequalities at the same time. In Appendix A.3, we report similar results using different measures of inequality. We find similar heterogeneity across countries and systematically find the US and the UK facing no trade-off between growth.
and inequality (except when inequality is measured by the Gini coefficient).

## 5 Conclusion

Public education is often seen as promoting economic growth through human capital accumulation and as decreasing inequality by providing the same level of education to all. This paper revisits the role of public education expenditures on these two important aspects: growth and inequality. First, we provide a model of occupational choice with an endogenous supply of teachers. We derive an explicit relationship between relative teacher quality and the rate of economic growth. We further show that the same level of public education expenditures does not have the same impact on growth in two economies differing in their distribution of human capital. In particular, countries with a fatter right tail of the distribution of human capital can attract teachers of relatively better quality for the same level of public education expenditures. This suggests that empirical studies aiming at identifying the role of public education (expenditures) on economic growth should control for higher orders of the human capital distribution (beyond the mean level of human capital).

Second, we empirically show that the relationship between public education expenditures and income inequality is not monotone (U-shaped) in a cross-section of countries and across US states. We obtain the same result in our model of occupational choice when substitution between different types of workers is not perfect. In particular, managers and workers are assumed to be imperfect substitutes in production. Managers’ human capital
determines their span of control and their demand for workers. Increased public education expenditures directly affects income at the top of the distribution (through increased taxation). In addition, better educational quality affects the shape of the human capital distribution and occupational choice. This, in turn, modifies the supply and demand for labor and hence income inequality. Depending on which effect dominates, raising public education expenditures can have a negative or positive effect on income inequality generating an overall U-shaped relationship between public education and income inequality. In particular, we show that increasing public education spending eventually benefits agents at the top of the human capital distribution relatively more, even though it raises the production efficiency of all agents. From that perspective, our model has different policy implications depending on whether the economy belongs to the decreasing or increasing part of the U-shaped relationship between public education and inequality. Countries in the decreasing part could reduce inequality and increase growth through increased public education spending. Countries on the increasing part face a trade-off between growth and inequality. Increasing public education expenditures would lead to higher growth and higher income inequality. The optimal level of public education depends in this case on the preference of the country in terms of growth and inequality.

Finally, we calibrate our model to 8 OECD countries. Our results suggest that some countries (among which the US and the UK) may be in the decreasing part of the public education-inequality relationship. This implies that they could decrease income inequality and increase growth through an increase in public education expenditures. Other countries face a trade-off between growth and inequality.
References


——— (2013): Inequality from Generation to Generation: the United States in Comparison, chap. in Robert Rycroft (editor), The Economics of Inequality, Poverty and Discrimination in the 21st Century. ABC-CLIO.


A Appendix

A.1 Data description: cross-country regressions

This appendix describes the data used in Table 1. We use 6 different measures of inequality. Five of them are obtained from the World Bank: the Gini coefficient (gini), the ratio of income at the top 10% of the income distribution to the bottom 10% (10/10 ratio), the ratio of income from the top 20% of the income distribution to the bottom 20% (20/20 ratio), the share of income going to the top 10% of the income distribution (Top 10%) and to the top 20% (Top 20%). We also use an alternative measure of the Gini coefficient from the Standardized World Income Inequality Database (gini SWIID). Data on public education spending as a share of GDP (pub. educ.) is obtained from the UNESCO database. Real GDP per capita (gdp), growth rate of real GDP (growth), public expenditures as a share of GDP (pub. exp.), openness measured by the share of imports and exports in GDP (open) and the growth rate of the population (pop. growth) come from the World Bank database. Private credit by deposit money banks and other financial institutions to GDP (priv. cred.) is used as a measure of the financial development of the country and is obtained from the IMF International Financial Statistics. We also control for the average level of education measured by the average total number of years of schooling (school) from the Barro and Lee database (Barro and Lee (2013)). We use the measure of civil liberties (civil right) computed by Freedom House. Data are averaged over the period 1991-2010.

A.2 Data description: US states regressions

We obtain US state-level income inequality data from Frank (2014) covering top 10% and top 1% income shares, Gini coefficients, Theil indexes, the relative mean deviation (rmeandev) and the Atkinson index with 5% inequality aversion parameter (atkin05) as well as the share of the population with at least a high school degree (school).\footnote{Data is available at http://www.shsu.edu/eco_mwf/inequality.html.} We also use data on state-level on public education spending as a share of GDP (pub. educ.), total state and local public expenditures (pub. exp.)\footnote{Source: usgovernmentspending.com.} and GDP and income growth data from the Bureau of Economic Analysis regional economic accounts. We collect data on average teacher wages (teach. wage) at the state level from the National Education Association.
Population growth data comes from the US Census Bureau (https://www.census.gov). Data are averaged over the period 2004 - 2010. District of Columbia is excluded from the sample when working with public expenditures.
A.3 Country analysis: alternative measures of inequality

Figure 9: Calibration results: 20/20 ratio.
Notes: The dashed line represents inequality (left axis) and the solid line growth (right axis) for different levels of public education spending. The vertical line indicates actual data on public education spending, economic growth and inequality.
Figure 10: Calibration results: Top 10% share of income.

Notes: The dashed line represents inequality (left axis) and the solid line growth (right axis) for different levels of public education spending. The vertical line indicates actual data on public education spending, economic growth and inequality.
Figure 11: Calibration results: Top 20% share of income.

Notes: The dashed line represents inequality (left axis) and the solid line growth (right axis) for different levels of public education spending. The vertical line indicates actual data on public education spending, economic growth and inequality.
Figure 12: Calibration results: Gini.

Notes: The dashed line represents inequality (left axis) and the solid line growth (right axis) for different levels of public education spending. The vertical line indicates actual data on public education spending, economic growth and inequality.