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Stochastic stability of a rotational oscillator under gusty winds

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Abstract

This work studies the rotational stability of an oscillator in a random wind velocity field. Its susceptibility to autorotation is assessed by the time necessary to reach a given energy barrier departing from an initial energy level. This first passage time problem is solved by replacing the actual power spectral density of the loading by an equivalent δ-correlated noise.

1 Introduction

Although the behavior of cranes is a widely studied problem, most research works on tower cranes focus on structures in use. In opposition with previous research, Voisin performed experimental study of the susceptibility of a tower crane to autorotation when it is out-of-service, i.e. left free to rotate (Voisin, 2003).

Following his model, the crane is represented by a single degree-of-freedom model composed of a rigid jib rotating around a fixed pivot. The dimensionless and linear governing equation of this system submitted to an external force $w(t)$ and a parametric force $u(t)$ with damping coefficient ξ takes the form of a stochastic Mathieu equation:

$$\ddot{x} + \xi\dot{x} + \big(1 + u(t)\big)x = w(t). \tag{1}$$

In the following, it is assumed that the rotating oscillator is slightly damped, i.e. $\xi \ll 1$, and that the intensities of the correlated noises u and w are small. This results in a slow variation of the total energy stored in this system, which owes it the qualification of "quasi-Hamiltonian". Thereby the pendulum stability will be characterize by its energy level $H = (x^2 + \dot{x}^2)/2$ (Gitterman, 2010).

2 First Passage Time

The stability of the rotating crane is assessed by the determination of the average first-passage-time of this quasi-Hamiltonian system. This problem is solved for Gaussian white noise excitations $u(t)$ and $w(t)$ through the asymptotic expansion method developed by Moshchuk in (Moshchuk, et al., 1995a) and (Moshchuk, et al., 1995b) and applied in (Vanvinckenroye & Denoël, 2016).

In that case, it can be shown that the first passage time $U(H)$ through a chosen level of energy H_c starting from an initial energy level H is given by:

$$U(H) = \frac{4}{S_u\,(1-a)}\left[\log\frac{H_c}{H} + \frac{(1+b_c)^a - (1+b)^a}{a} - \int_b^{b_c}\frac{(1+t)^a}{t}\,dt\right] \tag{4}$$

with $a = \frac{4\,\xi}{S_u}$, $b = \frac{H\,S_u}{2\,S_w}$ and $b_c = \frac{H_c\,S_u}{2\,S_w}$.

We then consider the response of the oscillator to a correlated turbulence taking the form of an Ornstein-Uhlenbeck excitation of correlation time t_0. Inspired by the Background-Resonant decomposition of the steady response of a single-degree-of-freedom structure in a gusty wind, we approximate the first passage time under this more realistic loading as the first passage time that would be obtained under an equivalent white noise loading S_{WN} that we define as $S_{WN} = S_{OU}(\Omega^* = 2\pi/T^*)$ where Ω^* is the dimensionless natural frequency.

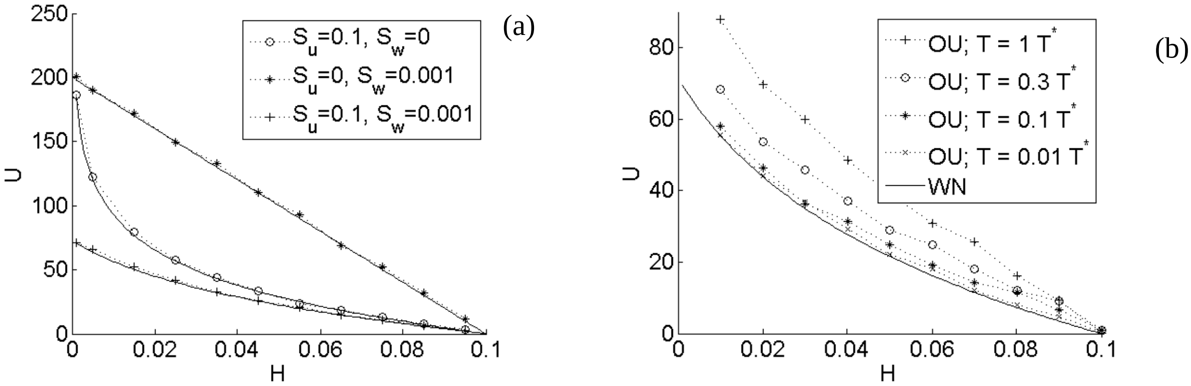

Figure 1 – First passage times for different non-correlated excitations and ξ = 0 (a) and different correlated Ornstein-Uhlenbeck (OU) excitations of intensity $S_u = 0.1$, $S_w = 0.001$ with ξ = 0 (b).

3 Results

Figure 1 (a) presents the analytical results (straight line) and simulations (dotted line) for a white noise excitation. The shape of the curve is strongly dependent on the intensity of the parametric excitation $u(t)$ compared to the intensity of the external excitation $w(t)$. As expected, the first passage time decreases with the excitation intensity. Figure 1 (b) compares the white noise excitation with the Ornstein-Uhlenbeck formulation for different correlation times. Results fit almost perfectly for a correlation time of the wind lower than 10% of the fundamental period of the oscillator, which is a typical ratio for the dynamics of tower cranes (Voisin, et al., 2004).

4 Bibliographie

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