Envelope approximation using equivalent static wind loads

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Introduction

Nodal Dynamic Analysis

Envelope approximation

Conclusion

Analysis of Large Structures under Random Excitations

- Engineering structures are submitted to random excitations
- Solve the equation of motion :

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{p}(t)$$

- Responses of the structure (bending moment, stresses, etc) :

$$\mathbf{z}(t) = \mathbf{O}\mathbf{x}(t)$$

- Design of the structure using extreme values :

$$\mathbf{z}_{i,max} = \mu_{z_i} + g\sigma_{z_i}$$

Equivalent analysis

- Next to that, we can compute **equivalent static wind loads** (**eswl**), $\mathbf{p}^e$, for each extreme responses and...
- by a series of static analyses under these eswl we can recover each extreme responses :

$$\mathbf{z} = \mathbf{A}\mathbf{p}_i^e \text{ with } z_i = z_{i,max}$$

- Load-Response Correlation method
- Displacement-Response Correlation method

Load-Response Correlation (LRC) method

- **Kasperski** (1991) solves the problem for a **static analysis**
- Covariance matrix between **z** and **p** is calculated by :

$$\mathbf{C_{zp}} = \mathbf{AC_p}$$

- Following **Bayes' theorem**, the most probable wind load associated to a specific extreme response, $z_{i,max}$, is :

$$\mu_{p/z} = \frac{z\sigma_{zp}}{(\sigma_z)^2} \overset{z=g\sigma_z}{\longrightarrow} \mathbf{p}_{z_{max}} = g\rho_{zp}\sigma_p$$

$$\mathbf{p}_{z_{i,max}} = g_i \rho_{\mathbf{p}\, z_i} \sigma_{\mathbf{p}} = \mathbf{p}_i^e$$

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Extension to take into account dynamic effects

- Nodal background and modal resonant analysis
  - Chen & Kareem (2001) : Equivalent static wind loads for buffeting response of bridges
  - Holmes (2002) : Effective static load distributions in wind engineering
- Full modal analysis
  - Fu (2007) : Equivalent Static Wind Loads on Long-Span Roof Structures ;

Displacement-Response Correlation (DRC) method

- We solve the problem for a **full nodal dynamic analysis**
- Covariance matrix between **z** and **x** is calculated by :

$$\mathbf{C}_{\mathbf{zx}} = \mathbf{O}\mathbf{C}_{\mathbf{x}}$$

- Following **Bayes' theorem**, the most probable displacements associated to a ith specific extreme response, $z_{max,i}$, is :

$$\mathbf{x}_i^e = g_i \rho_{\mathbf{x}\, z_i} \sigma_{\mathbf{x}}$$

- The **eswl** is simply computed as :

$$\mathbf{p}_i^e = \mathbf{K}\mathbf{x}_i^e$$

Usefullness

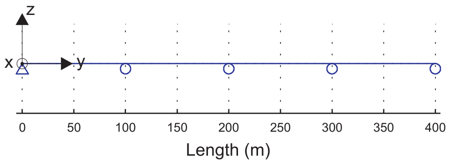
- Design of a structure is simplified with eswl $\rightarrow$ largely used in **design offices**
- Combination with other static loads is possible
- The **codes** are based on this concept

Envelope approximation using eswl

- When considering all these **static analyses** under each eswl **the envelope diagram is iteratively reconstructed...**
- Objective : use of a **minimum number of load cases**
  - which maybe can simultaneously targets several extreme responses
  - and in order to achieve an accepted underestimation level of the envelope diagram (fixed to **15%**)
- **Worked example** : Aerodynamic loading on a bridge.

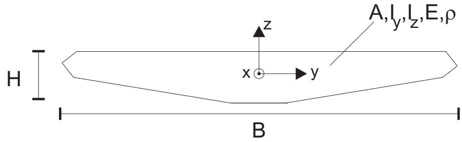
Worked example

- Four span bridge, simply supported



Finite Element model : classical beam elements ; 3DOF per node ; 12 elements by span ; 147 DOF.

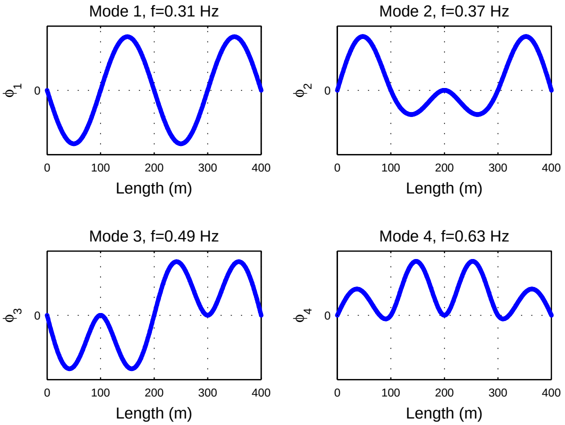
- Characteristics of the deck



$B = 30\ m$ (width) ; $H = 4\ m$; (height) ; $\Omega = 1\ m^2$ (section) ; $I_y = 10\ m^4$ (inertia) ; $E = 1e9\ N/mm^2$ (Young's modulus) ; $\rho = 2500\ kg/m^3$ (density)

Modal analysis

- Modal shapes :



- Rayleigh damping with $\xi = 1\%$ for the 1st and the 4th modes.

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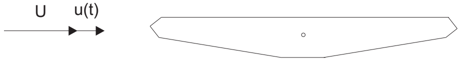
Aerodynamic loading on the deck

- **One-dimensionnal Gaussian velocity** field (quasi-steady) :



with $U = 30$ m/s (mean velocity) and $\sigma_u = 5$ m/s (standard deviation).

- **Ornstein-Uhlenbeck** for $S_u(\omega)$

- **Coherence function** : decreasing exponential

- Only the **lift aerodynamic force** is considered as :

$$f_{tot}(t) \simeq \underbrace{\frac{1}{2}\rho C_L B l U^2}_{\mu_f} + \underbrace{\rho C_L B l U u(t)}_{f(t)}$$

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Nodal Dynamic Analysis

- **Mean contribution** by a **static linear analysis** :

$$\boldsymbol{\mu}_{\mathbf{x}} = \mathbf{K}^{-1}\boldsymbol{\mu}_{\mathbf{f}}$$

where **K** is the stiffness matrix.

- **Variable contribution** by a **nodal stochastic analysis** :

$$\mathbf{S}_{\mathbf{x}}(\omega) = \mathbf{H}(\omega)\mathbf{S}_{\mathbf{f}}(\omega)\overline{\mathbf{H}}^{T}(\omega)$$

where $\mathbf{H}(\omega)$ is the nodal transfer matrix and $\mathbf{S}_{...}(\omega)$ symbolizes a PSD matrix

Design of the structure

- **Spectral moment matrix** of the response :

$$\mathbf{S_z}(\omega) = \mathbf{O}\mathbf{S_x}(\omega)\overline{\mathbf{O}}^T \overset{\int_{-\infty}^{+\infty} \ldots |\omega|^i d\omega}{\longrightarrow} m_{\mathbf{z}}^{(i)}$$

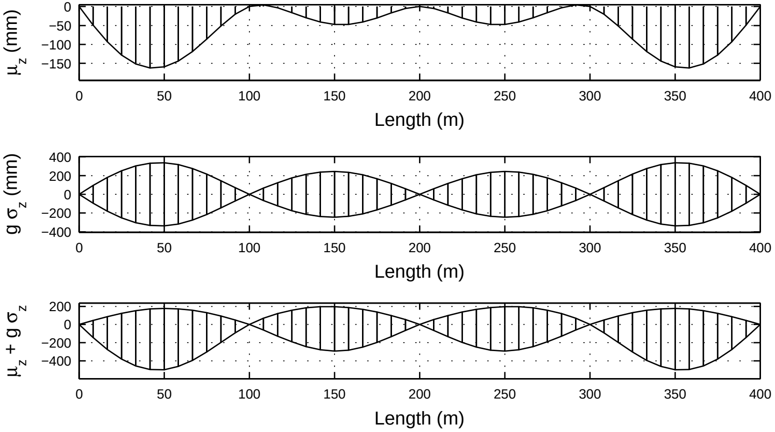
- **Mean extreme value** (using peak factor) for each response using the theory of extrema :

$$\mathbf{z}_{max} = \boldsymbol{\mu}_{\mathbf{z}} + g\boldsymbol{\sigma}_{\mathbf{z}}$$

where g and $\sigma_{\mathbf{z}}$ are obtained from different orders of the spectral moment matrix

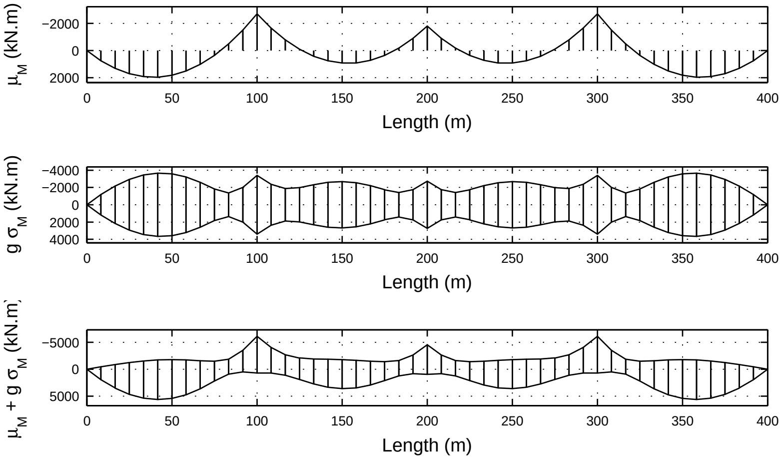
Illustration

Diagram of nodal displacements



Illustration

Diagram of bending moment



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Option 0 : Universal eswl

- Possible to compute **one eswl** to recover all of the extreme bending moments but...

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JOURNAL OF wind engineering AND industrial aerodynamics

Universal wind load distribution simultaneously reproducing largest load effects in all subject members on large-span cantilevered roof

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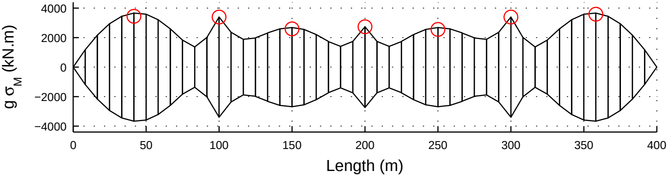
we want to keep **physical interpretations, meanings** of the static wind loads !

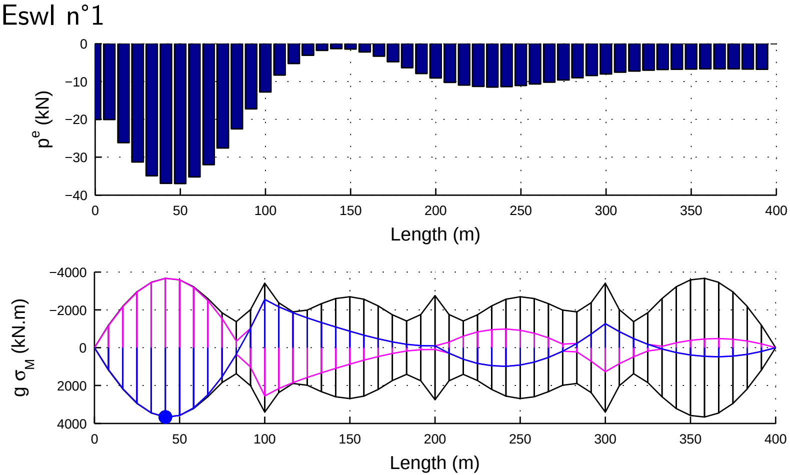
Option 1 : Engineering approach

- Selection of **critical sections** based on the engineering judgement



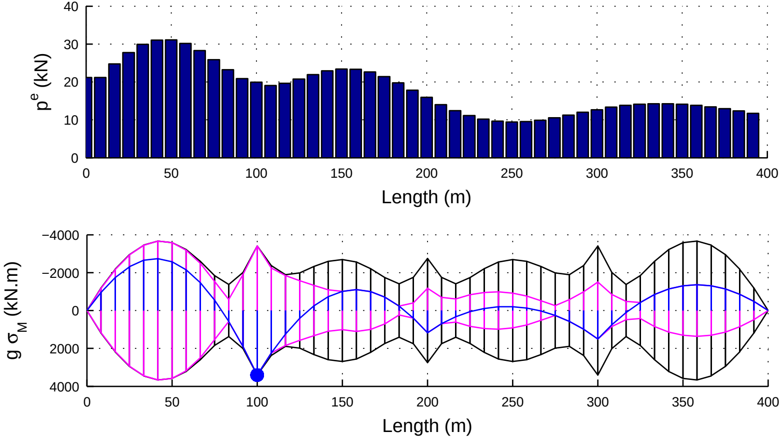
mid-spans and supports ($N^* = 7$)

Option 1 : Engineering approach



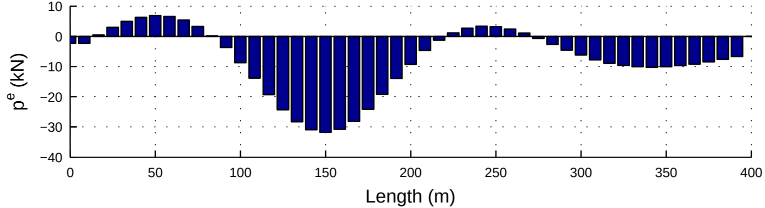
Option 1 : Engineering approach

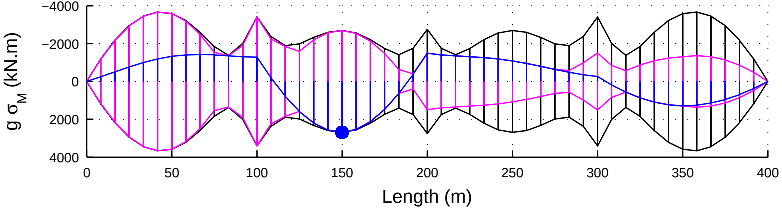
Eswl n°2



Option 1 : Engineering approach

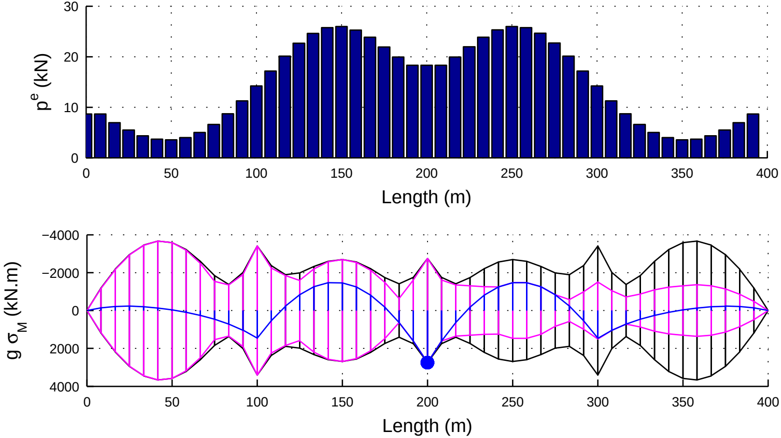
Eswl n°3





Option 1 : Engineering approach

Eswl n°4



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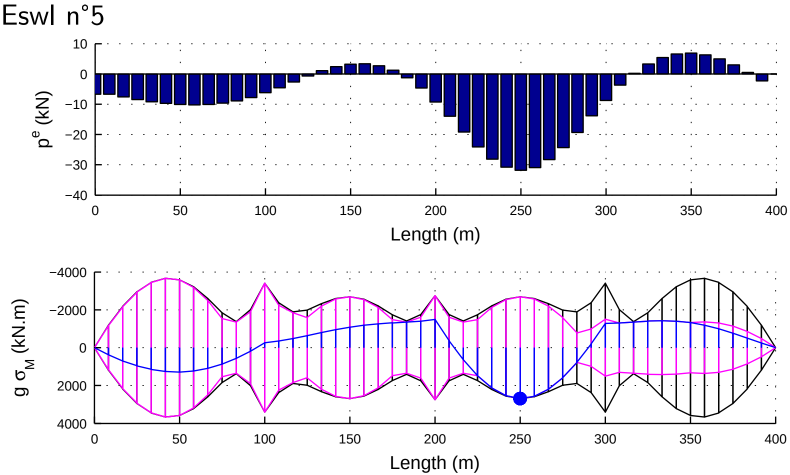

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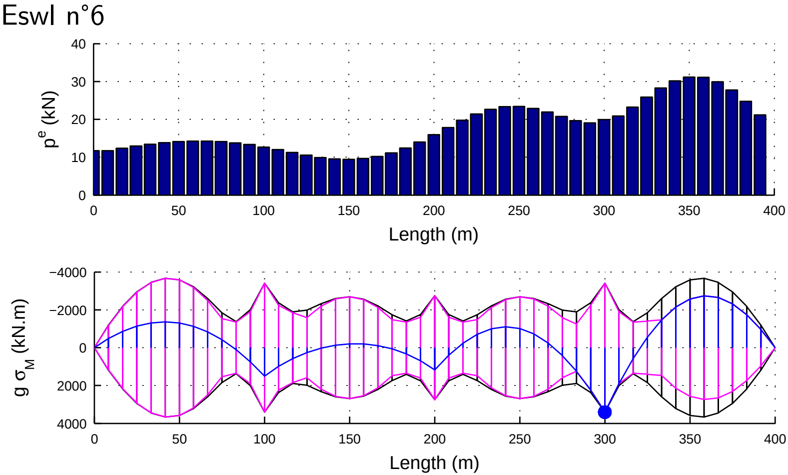

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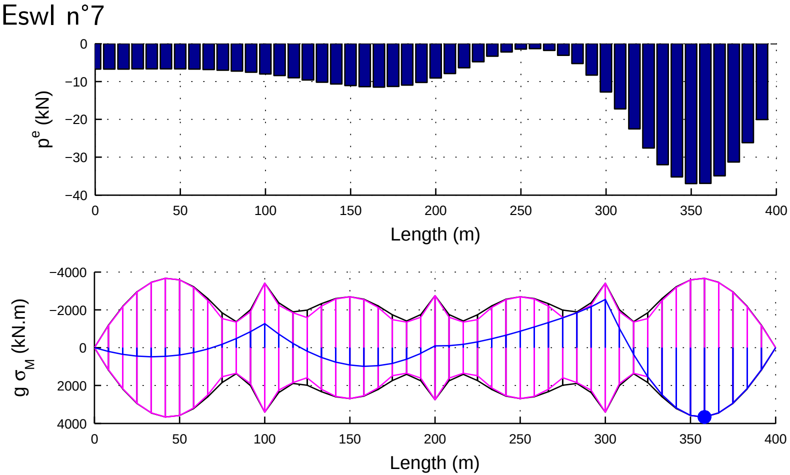
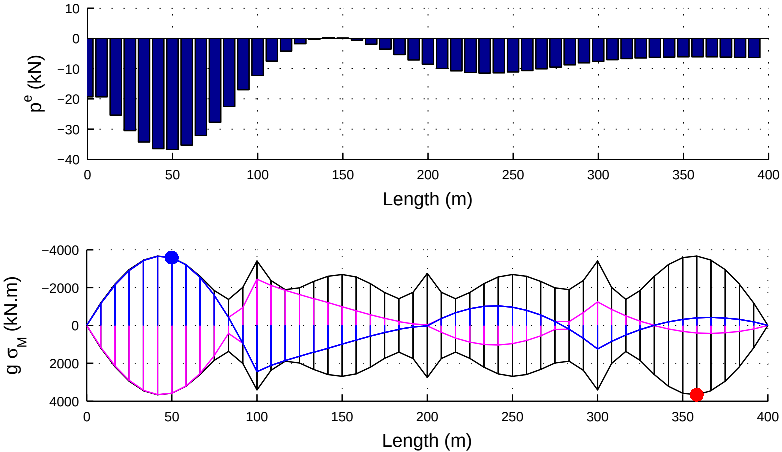


Option 2 : Iterative process

- Compute all of the eswl $\mathbf{p}_j^e$ (**N=47**)
- Selection of the eswl by an **iterative process which minimizes with the difference** to the target envelope **at each iteration.**

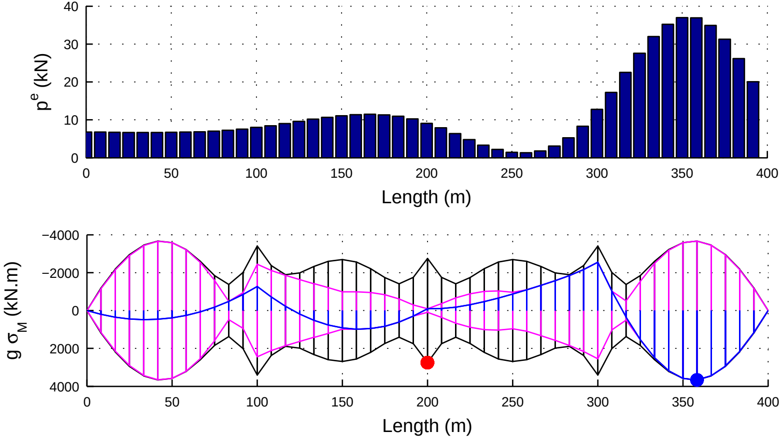
Option 2 : Iterative process

Iteration 1



Option 2 : Iterative process

Iteration 2



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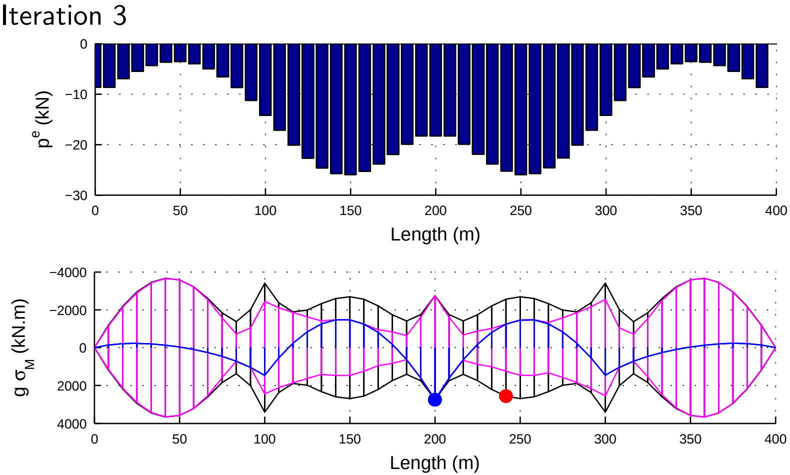
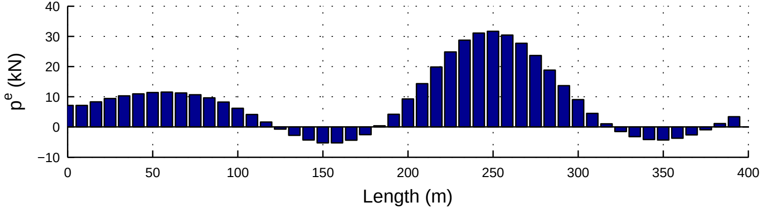
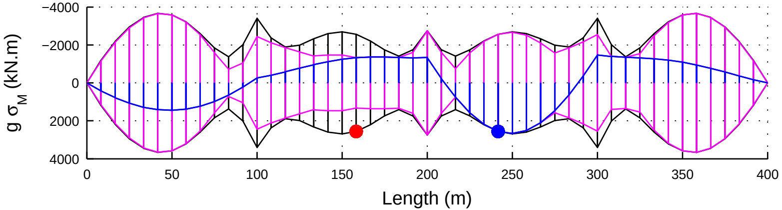
Option 2 : Iterative process



Option 2 : Iterative process

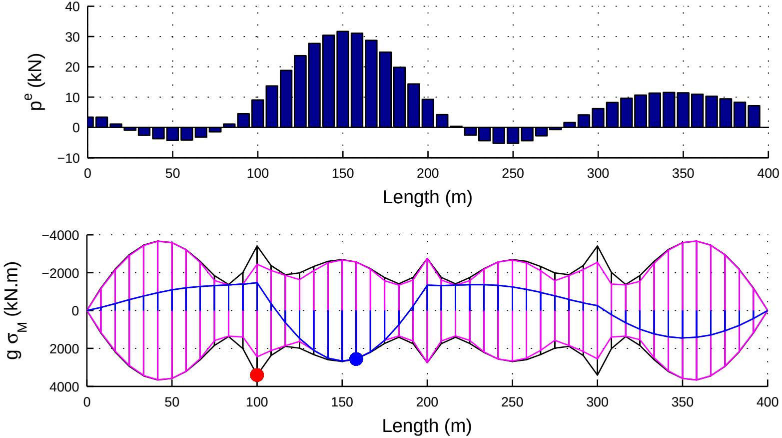
Iteration 4





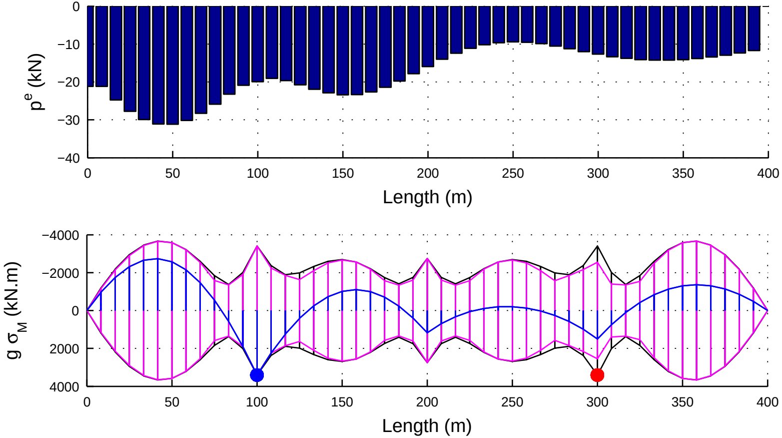
Option 2 : Iterative process

Iteration 5



Option 2 : Iterative process

Iteration 6



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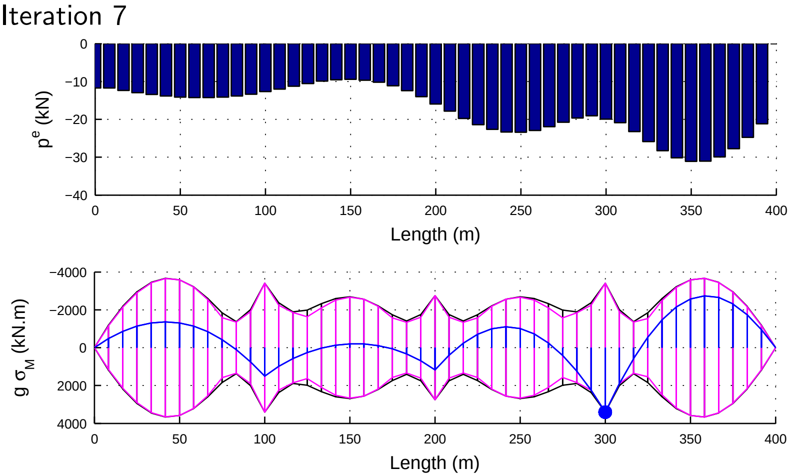
Option 2 : Iterative process



Linear combination of eswl ?

Application Research of Constrained Least-Squares Method in Computing Equivalent Static Wind Loads

Xuanyi Zhou[a], Ming Gu[b], Gang Li[c]

Abstract: This study proposes a constrained least-squares method to compute the equivalent static wind loads (ESWL) distribution for large-span roofs, which simultaneously targets several peak responses. The loading distribution is regarded as a linear combination of basic load distributions, which is based on the modified LRC method. To obtain ESWL with a reasonable magnitude range, the participation factor was limited to serve as a constraint, yielding an ESWL distribution that is a least-squares solution to a system of constrained linear algebraic equations. To verify its computational accuracy, the method is applied to a real large-span roof structure, and detailed comparisons between results from different basic load distributions were performed.

13th International Conference on Wind Engineering, July 10-15, 2011 (Amsterdam)

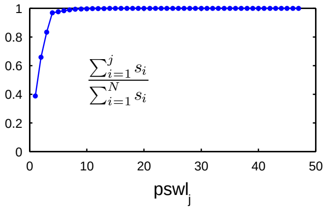
Singular Value Decomposition of the eswl matrix

- Singular value decomposition of the **eswl matrix** $\mathbf{p}^e$ ($N = 47$)

$$\underbrace{\begin{pmatrix} p^e_{12} & \cdots & p^e_{1n} \\ \vdots & \ddots & \\ p^e_{m1} & & p^e_{mn} \end{pmatrix}}_{m \times N}^{\mathbf{p}^e} = \underbrace{\begin{pmatrix} p^p_{11} & \cdots & p^p_{1r} \\ \vdots & \ddots & \\ p^p_{m1} & & p^p_{mr} \end{pmatrix}}_{m \times N^*}^{\mathbf{p}^p} \underbrace{\begin{pmatrix} s_{11} & 0 & \\ 0 & \ddots & \\ & & s_{rr} \end{pmatrix}}_{N^* \times N^*}^{\mathbf{s}} \underbrace{\begin{pmatrix} v_{11} & \cdots & v_{1n} \\ \vdots & \ddots & \\ v_{r1} & & v_{rn} \end{pmatrix}}_{N^* \times N}^{\mathbf{v}'}$$

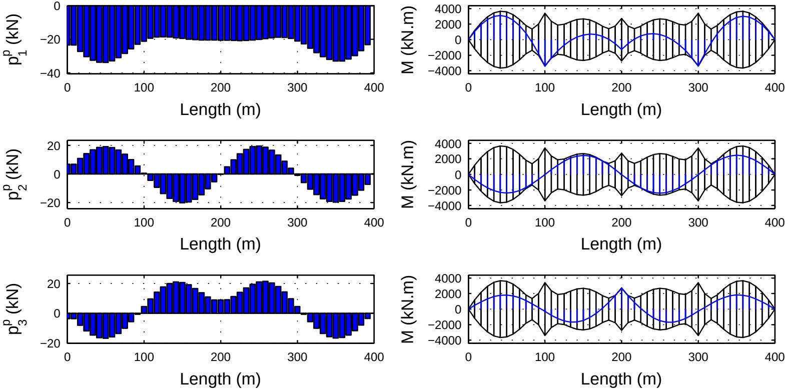
where $\mathbf{p}^p$ collects **principal static wind loads (pswl)**.

- Convergence of the decomposition :



Truncation to the **first three modes** ($N^* = 3$)

Singular Value Decomposition of the eswl matrix



- **principal swl** targets simultaneously several extreme responses
- **no need of selection** between eswl.

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How to combine ?

Application Research of Constrained Least-Squares Method in Computing Equivalent Static Wind Loads

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Option 3. Combination between pswl to obtain gswl

- **Combination** between the **principal swl** ($N^* = 3$) to obtain **global swl (gswl)** :

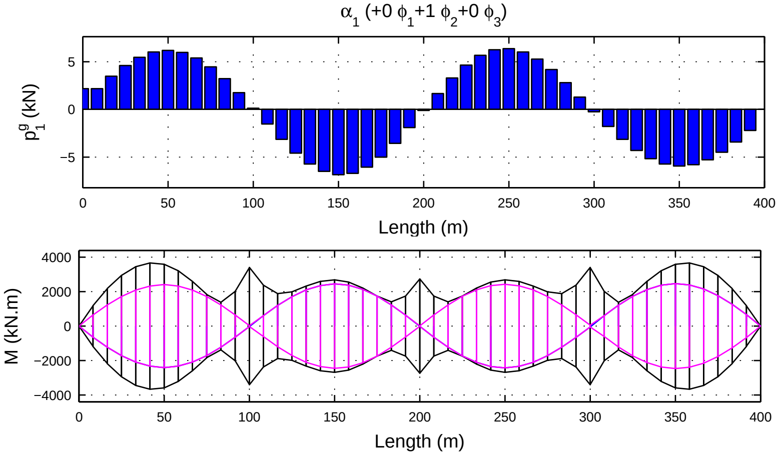
$$\mathbf{p}_i^g = \sum_{j=1}^{N^*} \mathbf{p}_{,j}^p \mathbf{q}_{j,i} \quad i = 1 : 3^{N^*} - 1$$

  where $\mathbf{q}$ collects combination $(-1; 1; 0)$ between the N^* pswl $\mathbf{p}_{,j}^p$

- The choice of the gswl to apply succesively is realized using the same iterative approach as before

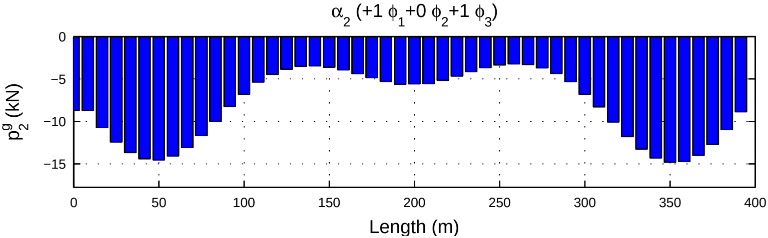
Option 3. Combination between pswl to obtain gswl

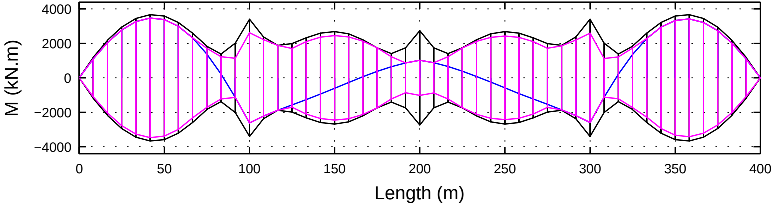
Combination 1



Option 3. Combination between pswl to obtain gswl

Combination 2





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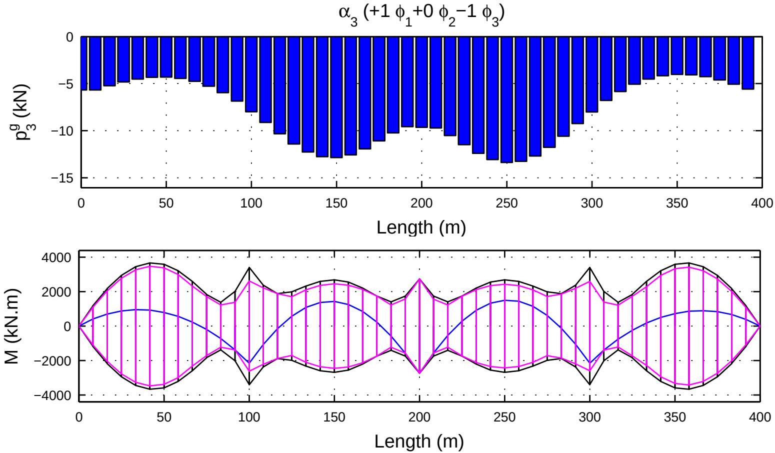
Combination 3



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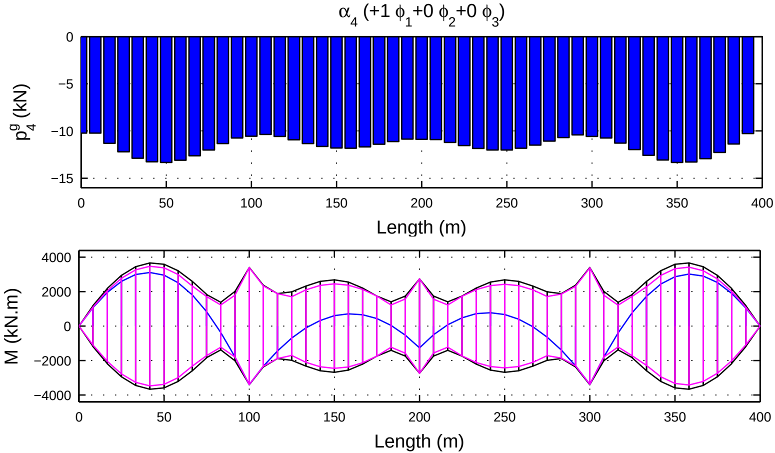
Combination 4



Introduction

Nodal Dynamic Analysis

Envelope approximation

Conclusion

Objective : use of a **minimum number of load cases** to achieve an accepted underestimation level of the envelope diagram

- Option 1 : Engineering approach. Choice of the eswl easy for a simple structure but quite complicated when considering large structures. Convergence to the target envelope is not sure.
- Option 2 : Iterative process. Works quite well but does not allow to target several extreme responses.
- Option 3 : Combination of pswl to obtain gswl
  - Computation of a **minimum number** of principal swl using SVD
  - **Combination to** obtain global swl is **straightforward**
  - These global swl targets several extreme responses and so
  - **the convergence** is increased and a minimum number of global swl is obtained

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Perspectives

- Possible **comparison** of these global swl with others computed for similar structures → **codification**
- Application to large structures

Perspectives

Thank you for your attention.

Questions ?

Read out more about me on : www.orbi.ulg.ac.be

Contact me at : N.Blaise@ulg.ac.be