Quantitative Modeling of Sr, Ca, Rb, and K in the Bjerkrem-Sogndal Layered Lopolith (S.W. Norway)

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Abstract. A systematic approach with graphic techniques is used to establish a quantitative model of fractional crystallization process in igneous layered complexes. Modeling of the evolution of $Sr-Ca$ in plagioclase and $K-Rb$ in plagioclase and whole rock coming from the Bjerkrem-Sogndal layered lopolith (Rogaland-S.W. Norway) is taken as an example. The relationships in logarithmic coordinates can be decomposed in a succession of segments. This permits identifying the Rayleigh law as controlling the process. A step by step solution is used to determine the parameters of the model which lead to the adjustment of the calculated evolution to the observed trend. Evidence in favour of an open system crystallization of the cumulate rocks permits determining the equilibrium partition coefficients between the various minerals and the liquid. The mean cumulate corresponding to a phase of crystallization of the intrusion is determined by averaging the mineral composition of the rocks belonging to that phase. The concentrations of the major elements Ca and K are used instead of activities. The adopted plagioclase-magma partition coefficients are close to those measured in anorthositic rocks for the same range of plagioclase composition between megacrysts and liquid. For an anorthite content of respectively 50, 43 and 31, D_{Sr}^{plag} is equal to 2.0, 2.3 and 3.9, D_{K}^{plag} varies between 0.40 and 0.25, D_{Rb}^{plag} is either constant (ca. 0.10) or increases from 0.12 to 0.25, D_{Ca}^{plag} is supposed to remain at an approximately constant value of 1.48. The fraction of residual liquid in the intrusion is 0.47 at the end of the anorthositic-leuconoritic phase, and 0.21 at the end of the cpx-noritic phase.

Introduction

The study of igneous layered intrusions and of cumulate rocks has greatly contributed to the development of modern petrology. We believe that these rocks can also considerably enrich trace element geochemistry. Integration of data obtained in the field, at the microscope and in the chemical laboratory indeed permits to establish physical chemical models which quantitatively describe the magmatic differentiation phenomenon.

Relatively few studies, such as that of Paster et al. (1974) on the Skaergaard, have been devoted to igneous layered complex. This paper is an attempt to calculate a model of the crystallization of the Bjerkrem-Sogndal massif (S.W. Norway). It will be shown that the phenomenon can be identified with a fractional crystallization process. A differentiation index based on a physical chemical law will be calculated by means of an estimation of the different partition coefficients, on condition that several hypotheses are accepted, namely on the formation of cumulate rocks.

The Bjerkrem-Sogndal Massif

The Bjerkrem-Sogndal massif belongs to the Rogaland anorthositic province (S.W. Norway). It is a layered lopolith (Michot, 1960, 1965) which displays a complete series of rocks from anorthosite to quartz mangerite (hypersthene mesoperthite-granite).

The lower part of the complex is composed of anorthosites, leuconorites and norites. The rocks are characterized by typical igneous cumulate textures (igneous lamination, banding, small-scale rhythmic units). Five macroscale rhythmic units (I to V) of several hundred meters in thickness have been defined by Michot (1960, 1965). Each rhythm starts with anorthosite or leuconorite at the base and grades upwards into more mafic terms.

Jotunitic (hypersthene monzodioritic) cumulates lie above the fifth rhythm and constitute a rapid transition towards the upper part of the massif which comprises mangerites (hypersthene mesoperthite-monzonite) and, on top, quartz mangerites of massive (unlayered) character.

Variations in the contents of the mafic minerals are schematically represented on Figure 1. The colorimetric index increases rhythmically and cumulates at the beginning of the jotunitic transition. The nature and the chemical composition of the cumulate minerals (Michot, 1965; Duchesne, 1972a and b) also show a rhythmic evolution. Clinopyroxene and apatite join the association plagioclase-orthopyroxene-oxide minerals in the upper part of rhythm III, disappear at the base of rhythm IV and appear again in the intermediate part of rhythm IV to remain permanent in rhythm V and upwards. A similar behaviour – a "false entrance" in the intermediate part of rhythm III – also characterized the magnetite (Duchesne, 1972a). As for the major and trace element variations, the trends observed on the scale of a rhythm develop in
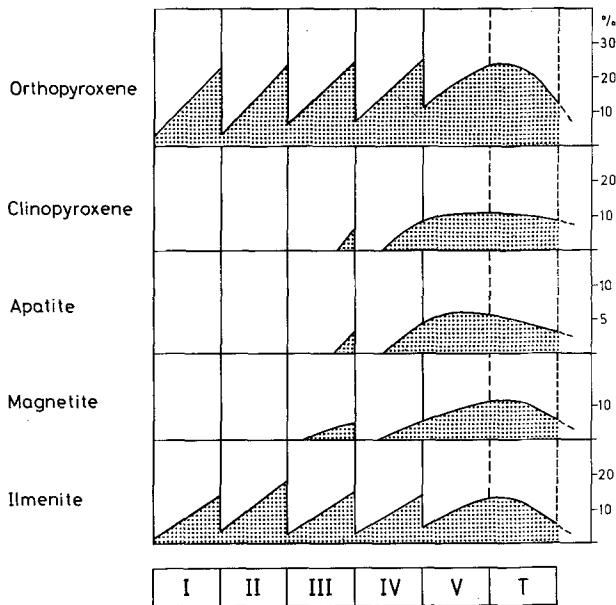
Fig. 1. Schematic evolution of the contents in mafic minerals plotted against the structural height in the five rhythmic units (I to V) and in the transitional jotunites (T). The thickness of the different units is represented by segments of equal length [after Michot (1965) and Duchesne (1972a) with slight modifications]. Olivine (Fe-rich) can occur in the transitional jotunites (10 to 20 % in content). It is absent from the rhythms except for a brief occurrence near the base of rhythm IV (not represented on the figure)

Fig. 2. Sr versus Ca in plagioclases from the Bjerkrem-Sogndal massif (logarithmic coordinates). The trend is schematically circumscribed by a dashed line. The following symbols are related to the rhythmic structure: o Rhythms I and II; ⊕ Rhythm III (base and intermediate parts); ⊖ Upper part of rhythm III; ⊙ Lower part of rhythm IV; ⊖ Rhythm IV (intermediate and upper parts); ⊕ Rhythm V; ● Transitional jotunites. Points 0, 1 and 2 (full star) refer to the calculated plagioclases of model I at stages 0, 1 and 2 (Table 2)

directions identical with those defined on the scale of the massif, but the evolution is of a lesser amplitude.

This body of data has led Duchesne (1972a) to conclude that the changes in the chemical composition of the magma at the base of the rhythms were recurrences of the basic character superimposed on a continuous fractional crystallization process. It has been suggested that repeated influxes of fresh magma have flowed into the magmatic chamber and mixed with progressively lower temperature residual magma. The can explain that the amplitude of the recurrence, depending upon the proportions of the fresh magma added to the residual magma, can vary from rhythm to rhythm: e.g. it is large at the base of rhythm IV, but of little amplitude at the base of rhythm V.

The mechanism controlling the recurrences is however not the chief concern here. Whatever it might be, the important point is the possibility to leave aside the recurrent character of the differentiation and to reconstitute the sequence of rocks which were generated by the sole fractional crystallization process. The sequence starts with anorthosites, leuconorites and norites — reconstituting what we call here the anorthositic-leuconoritic phase — in which plagioclase (unzoned, An_{50} to An_{43}) is associated with orthopyroxene and oxide minerals (ilmenite first alone, then accompanied by magnetite). It passes upwards to norites — belonging to the here called clinopyroxene (cpx)-noritic phase — in which clinopyroxene and apatite appear as additional cumulate minerals and plagioclase becomes more albitic (An_{45} to An_{40}); higher in the norites, potassium feldspar appears first as antiperthite, then progressively as a cumulate mineral and the rocks grade into jotunitic cumulates, transitional to the rocks of the upper part.

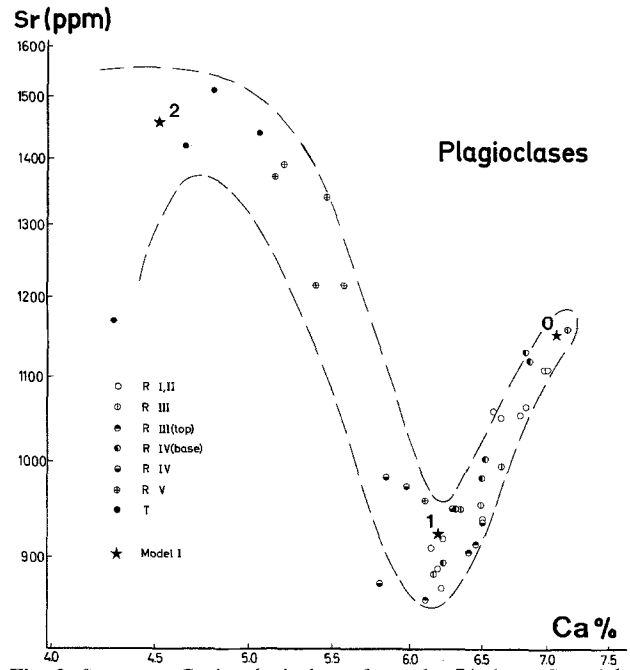The composition of the initial magma is not known. No chilled

margin has ever been recognized. A jotunitic composition has however been considered as likely by Michot (1965). Moreover, isotopic data (Demaiffe et al., 1977) indicate that the intrusion did not remain a closed system during the consolidation of its upper part; contamination by material coming from the envelope has taken place. These two features of the Bjerkrem-Sogndal massif makes questionable the reconstitution of the major element evolution by classical methods and emphasizes the usefulness of the informations which can be drawn from model involving trace elements.

Sr — Ca and K — Rb Evolution

The Sr and Ca contents in plagioclases separated from the rocks are reported in the Appendix and plotted on Figure 2.

The Sr and Ca contents show a zig zag pattern in which three trends can be recognized. The first one, starting from the highest Ca content, characterizes the plagioclases of the anorthositic-leuconoritic phase and shows a positive slope. The second one with a negative slope corresponds to the cpx-noritic phase. It culminates and starts to decrease again in jotunitic rocks.

A first qualitative interpretation of this behaviour was given by Duchesne (1971). It was based on a close correlation between the trend on which a plagioclase plots and the nature of the minerals with
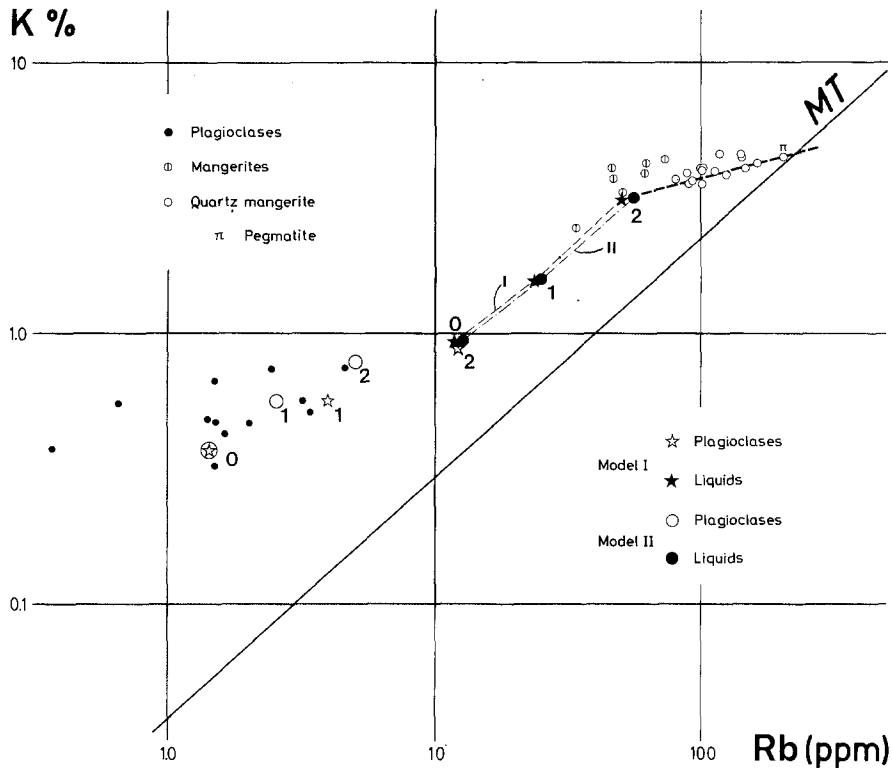
Fig. 3. K versus Rb in plagioclases and total rocks from the Bjerkrem-Sogndal massif. ●: plagioclases; ⊕ mangerites and mangeromonzonites; ○: quartz mangerites [*II*: pegmatitic quartz mangerite (64120)]. *MT*: main trend of Shaw (1968) describing the K — Rb fractionation in average magmatic rocks. Model I: plagioclase (☆) and liquids (★) at stages 0, 1 and 2 (see Table 3). Model II: plagioclase (○) and liquids (●) at stages 0, 1 and 2 (see Table 3)

which it is associated in the rock. If one admits that the first two trends can somewhat overlap near the change in direction, it can be seen that the plagioclases associated with orthopyroxene and oxide minerals plot on the first trend, those with clinopyroxene and apatite on the second one, and those with cumulate K-feldspar on the third one. In the first trend the evolution is mainly controlled by plagioclase which is the only mineral to subtract Sr and Ca from the magma. The two elements have plagioclase-magma partition coefficients higher than 1 and thus both elements decrease in successive residual liquids. The appearance of clinopyroxene and apatite changes the evolution because both minerals subtract relatively more Ca and less Sr than the other minerals. And again as soon as K-feldspar plays a role a situation similar to the first one is restored because the mineral subtracts much more Sr than Ca.

It is one of the purposes of the present paper to give a more quantitative bearing to this interpretation, which in fact will prove to be only partially correct. The calculations will show that the increase of the Sr partition coefficient in plagioclase and of the proportion of mafic minerals also contributed to the change in direction of the trend.

Before continuing it is necessary to refute an interpretation which could appear plausible. The plagioclases of rhythm V all plot on the second trend and contrast with those of rhythms I to III. One

could thus think that rhythm V is different from the other ones by the process of differentiation and/or by the nature of the influx which has taken place at its base. This interpretation is however not consistent with the data of rhythm IV whose plagioclases plot on the first trend in the lower part of the rhythm, and plot distinctly on the second trend in the upper part (cpx and apatite-bearing norite!). Moreover the field relations, the petrological data and the chemical evolution of the minerals in rhythm V compared to the previous ones give no evidence that rhythm V results from a specifically different mechanism. The recurrence at the base of V is simply not of sufficient amplitude to bring back an association without clinopyroxene and apatite (Fig. 1).

Some plagioclases and whole rocks (mangerites and quartz mangerites) were also analysed for Rb and K. The results are reported in the Appendix and plotted in Figure 3.

Cumulate plagioclases do not plot on a well defined trend but form a rather loose group, some of them reaching extremely low Rb values and high K/Rb ratios (up to 10,000). On the other hand, the rocks and specially the quartz mangerites form a linear trend passing by the most acidic rock, a pegmatitic variety of quartz mangerite. The slope is smaller than that of the main trend of Shaw (1968) which means a more rapid decrease of the K/Rb ratio.

Identification of the Differentiation Process

Modeling of the $Sr-Ca$ and $K-Rb$ behaviours first implies that the law governing the evolution can be identified, and then that the various parameters of the equation of the law can be determined in order to carry out the best fit between the results of the calculations and the observations.

Though sophisticated methods have been suggested to solve this kind of problem (e.g. Allègre et al., 1977; Minster et al., 1977), we shall simply use here graphic techniques taken from Treuil and Joron (1975) in which the best fit is obtained by visual estimation after several trials and errors.

It must be recalled that, basically, if a trend relating the evolution of the concentrations c_i^{liq} and c_{i+1}^{liq} of two elements i and i+1 in a liquid is a straight line in a bilogarithmic diagram, the Rayleigh logarithmic law—i.e. the law of fractional crystallization in a closed system—can be identified as governing the process. It can indeed be easily show that, if the $\bar{D}_i$ and $\bar{D}_{i+1}$ are the bulk partition coefficients of the elements i and i+1 between the solid phases and the liquid, the slope of the straight line is $\bar{D}_i - 1/\bar{D}_{i+1} - 1$, provided $\bar{D}_i$ and $\bar{D}_{i+1}$ do not vary. Similarly the evolution of the concentrations c_i^j and c_{i+1}^j in one of the solids j in equilibrium with the liquid is also a straight line by definition of the solid-liquid partition coefficient $D_i^j = c_i^j/c_i^{liq}$. As $\bar{D}_i = \sum_j \mu_j D_i^j$ and μ_j is the weight fraction of mineral j in the fractionating assemblage of minerals, the constancy of $\bar{D}_i$ implies that of D_i^j and μ_j. This is only possible in a small interval of temperature and mineral compositions (Neumann et al., 1954) and also, as already pointed out by Albarède (1976), for a slight curvature of the cotectic lines relating the crystallizing minerals along the liquid line of descent. If the D_i^j are constant, a change of slope in the trend is thus only due to appearance or disappearance of a mineral in the assemblage. It is however unrealistic to assume the constancy of the D_i^j during crystallization, chiefly for Sr whose D_{Sr}^{plag} is known to vary considerably with the anorthite content of the plagioclase since the work of Philpotts and Schnetzler (1970). One can however assume that the variation of D_i^j can be approximated by a succession of discrete jumps between which the value remains constant. In such a case a change in slope of a trend can be due to a variation of the mineralogy and/or of the D_i^j.

In the Bjerkrem-Sogndal massif the $K-Rb$ trend in the acidic rocks can be assimilated to a straight line (Fig. 3) and the $Sr-Ca$ trends in plagioclases (Fig. 2) to a succession of straight lines; the Rayleigh law can thus be identified as controlling the differen-

tiation. Calculation of the parameters will show that it is also necessary to let the D_i^j vary stepwise.

Discontinuous Variation of the D_i^j

The determination of the various parameters in the case of a stepwise variation of the D_i^j requires to write in a more complex form the equation relating the individual samples of a series to the degree of crystallization.

Let us consider that the evolution can be decomposed in several stages ρ ($\rho = 0, ..., \lambda$), each stage being characterized by parameter f_ρ, the fraction of liquid left ($0 < f < 1$).

The partition coefficient of the ith element between the jth solid (j = 1, ..., k) and the liquid is, at stage ρ, $D_{i,\rho}^j = c_{i,\rho}^j/c_{i,\rho}^{liq}$ with $c_{i,\rho}^j$ and $c_{i,\rho}^{liq}$ the concentrations (strictly speaking activities) of the ith element in the jth mineral and in the liquid respectively.

The bulk partition coefficient is

$$\bar{D}_{i,\rho} = \sum_{j=1}^{k} D_{i,\rho}^j \mu_{j,\rho} \quad \text{with} \quad \sum_{j=1}^{k} \mu_{j,\rho} = 1.$$

During the ρth stage, the Rayleigh equation is

$$c_{i,\rho}^{liq} = c_{i,\rho-1}^{liq} f_\rho^{\bar{D}_{i,\rho-1}-1}. \tag{1}$$

The fraction of liquid F left at the end of stage ρ is $F_\rho = f_\rho f_{\rho-1} \cdots f_1$.

By definition of the partition coefficient, Equation (1) gives

$$\frac{c_{i,\rho}^j}{D_{i,\rho}^j} = \frac{c_{i,\rho-1}^j}{D_{i,\rho-1}^j} f_\rho^{\bar{D}_{i,\rho-1}-1}. \tag{2}$$

The bulk partition coefficient $\bar{D}_{i,\rho-1}$ can also be written, if $D^{j/1} = c_i^j/c_i^1$,

$$\bar{D}_{i,\rho-1} = D_{i,\rho-1}^1 \left(\mu_{1,\rho-1} + \sum_{j=2}^{k} D_{i,\rho-1}^{j/1} \mu_{j,\rho-1} \right). \tag{3}$$

In these equations, $c_{i,\rho}^j$ and $c_{i,\rho-1}^j$ are "data" (the concentrations measured); the μ_j and $D_{i,\rho-1}^{j/1}$ can be calculated provided some conditions on the formation of the cumulate rocks are realized; $D_{i,\rho-1}^1$ is obtained by resolution of the previous step. We have two unknowns left, $D_{i,\rho}^j$ and f_ρ, and the problem is undetermined. We need additional constraints to solve it. A solution can be found if the function linking the partition coefficient and the degree of crystallization is known as demonstrated by Greenland (1970). This is not the case here and we must stay at a more general level.

Cumulate Minerals and Adcumulus Growth

We consider that the composition of the mineralogical assemblage which controls the evolution in a phase of crystallization of the intrusion is equal to the average mineralogical composition of the rocks belonging to that phase. This means admitting that our sampling is representative and that the influence of crystal sorting by gravity is cancelled by the sampling. Moreover we assume that crystallization of a possible interstitial liquid trapped by crystal accumulation has not modified the mineral proportions and element concentrations.

There are good reasons for accepting our sampling method: the averages[1] obtained (see below) are relatively little dispersed and give a geologically realistic estimation; we have avoided banded rocks in which crystal sorting has played a major role. As for the intercumulus liquid, we have good evidence in favour of an open system crystallization — adcumulus growth of Wager and Brown (1968). The distribution coefficients of the rare earth elements (REE) between plagioclase and apatite remain constant in the course of crystallization of the cpx-noritic phase (Roelandts and Duchesne, 1977). Such an observation cannot be reconciled with a closed system crystallization. When fractionating, the intercumulus liquid becomes strongly enriched in normative apatite and REE. The solubility of REE in apatite being very high compared to that in plagioclase and other minerals, the major part of the REE is precipitated by the apatite formed by intercumulic crystallization. It follows a more important increase of REE in apatite than in plagioclase and a variation of the apatite-plagioclase distribution coefficient with the amount of intercumulus liquid. Similarly the regularity in the evolution of Sr and Ca in the plagioclase of the anorthositic-leuconoritic phase is also incompatible with crystallization of variable amounts of entrapped liquid. We are thus allowed to assume that the minerals remained in equilibrium with each other and the liquid during the consolidation of the cumulate. Consequently the $D_{i,\rho}^{j/1}$ measured between the minerals are interpreted as equilibrium partition coefficients.

Modeling the Sr − Ca Evolution

Since Equation (2) is underdetermined, its solution requires applications of the following additional constraints.

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[1] Normative compositions are used here instead of modal compositions. The coarseness of the rocks makes the point counting technique difficult. There is no significant difference between the mode and the norm because hydrous minerals are absent

Table 1. Sr and Ca contents in the cumulate minerals of a Bjerkrem-Sogndal norite

Mineral (j)	Sr (ppm)	Ca (%)	$D_{Sr}^{j/plag}$	$D_{Ca}^{j/plag}$
Plagioclase (plag)	950	6.28		
Clinopyroxene (cpx)	32	15.5	0.03	2.47
Orthopyroxene (opx)	<10	0.76	negl.	0.12
Apatite (ap)	390	39.4	0.41	6.29
Oxides (oxid)	negl.	negl.	negl.	negl.

Specimen no. 6444
From Duchesne, 1971

D_{Sr}^{plag} will be accepted to increase with the decrease of the anorthite content of the plagioclase in agreement with Philpotts and Schnetzler (1970), Jensen (1973), Vernières et al. (1977), etc. A value of 2.3 ±0.2 has been measured by Duchesne and Demaiffe (1978) between plagioclase megacrysts ($An_{47\pm2}$) and the parental liquid of the Hidra massif. Here we have adopted values close to this one for plagioclases of similar composition and a value of about 4 for plagioclase of An_{30}. The rate of increase seems reasonable in view of the value usually adopted for acidic plagioclases [e.g. Vitrac and Allègre (1975): $D_{Sr}^{plag} = 7$; see also the compilation of Jensen (1973)].

As for Ca, the value of 1.48 ±0.04 found by Duchesne and Demaiffe (1978) is taken for D_{Ca}^{plag}. This constitutes a very rough approximation because it is calculated as the ratio of concentrations instead of activities. This value is kept constant or nearly constant (variation from 1.50 to 1.46) during the evolution from An_{50} to An_{30}. The hypothesis is plausible: there exists a well known positive correlation in porphyritic volcanic rocks between the An content of plagioclase phenocrysts and the basicity of the mesostasis; moreover, in the range of An values considered here, trajectories of the Ca evolution in the plagioclase and in the liquid can be considered approximately parallel in Bowen's diagrams Albite-Anorthite and Albite-Diopside-Anorthite (Morey, 1964).

The $D_{Sr}^{j/plag}$ and $D_{Ca}^{j/plag}$ are calculated using the compositions of the different minerals in a typical noritic cumulate (Duchesne, 1971) (Table 1). As in the case of D_{Ca}^{plag}, we assimilate concentrations and activities.

Calculations

A. Anorthositic-Leuconoritic Phase

Average mineralogical composition (8 samples): (0.74 ±0.11) plag + (0.14 ±0.07) opx + (0.12 ±0.04) oxid.

Estimated mean cumulate: 0.74 plag + 0.16 opx + 0.10 oxid.

Table 2. Models for the evolution of Sr and Ca in plagioclase

	Model I			Model II			Model III			Model IV		
	0	1	2	0	1	2	0	1	2	0	2	3
c_{Sr}^{plag}	1150	920	1460	1150	920	1465	1164	945	1465	1157	910	1465
c_{Ca}^{plag}	7.06	6.20	4.51	7.06	6.18	4.79	7.00	6.43	4.79	7.03	6.43	4.79
c_{Sr}^{liq}	575	400	374	575	460	419	613	450	396	609	444	380
c_{Ca}^{liq}	4.71	4.25	3.09	4.71	4.23	3.28	4.80	4.40	3.28	4.82	4.40	3.28
D_{Sr}^{plag}	2.0	2.3	3.9	2.0	2.0	3.5	1.9	2.1	3.7	1.9	2.05	3.85
D_{Ca}^{plag}	1.50	1.46	1.46	1.50	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46
F	1.00	0.47	0.21	1.00	0.47	0.24	1.00	0.47	0.19	1.00	0.46	0.25
No. of segments in the noritic phase	1			3			4			4		

Ca content in % and Sr content in ppm

Substituting in Equation (3),

$$\bar{D}_{Sr,0} = D_{Sr,0}^{plag}(0.74 + 0.16 \times 0.01 + \cdots) = 0.74\, D_{Sr,0}^{plag},$$

$$\bar{D}_{Ca,0} = D_{Ca,0}^{plag}(0.74 + 0.16 \times 0.12 + \cdots) = 0.76\, D_{Ca,0}^{plag}.$$

Let us choose $D_{Sr,0}^{plag} = 2.0$, $D_{Ca,0}^{plag} = 1.50$ and $D_{Ca,1}^{plag} = 1.46$. Let us fix point 0 (Fig. 2 and Table 2) as representing the first plagioclase in equilibrium with the initial liquid and point 1 the plagioclase which characterizes the beginning of the cpx-noritic phase (that is the first rocks with clinopyroxene and apatite). Substituting the values in Equation (2) written for Sr and Ca respectively, it follows that $D_{Sr,1}^{plag} = 2.3$ and $F_1 = f_1 = 0.47$.

B. Cpx-Noritic Phase

Average mineralogical composition (8 samples): (0.50 ± 0.05) plag $+ (0.21 \pm 0.03)$ opx $+ (0.08 \pm 0.01)$ cpx $+ (0.06 \pm 0.02)$ ap $+ (0.15 \pm 0.02)$ oxid.

Estimated mean cumulate: 0.45 plag + 0.22 opx + 0.09 cpx + 0.04 ap + 0.20 oxid.

$$\bar{D}_{Sr,1} = D_{Sr,1}^{plag}(0.45 + 0.22 \times 0.01 + 0.09 \times 0.034 + 0.04 \times 0.41 + \cdots)$$
$$= 0.47\, D_{Sr,1}^{plag},$$

$$\bar{D}_{Ca,1} = D_{Ca}^{plag}(0.45 + 0.22 \times 0.12 + 0.09 \times 2.47 + 0.04 \times 6.29 + \cdots)$$
$$= 0.95\, D_{Ca,1}^{plag}.$$

Let us choose point 2 (Fig. 2 and Table 2) as representing the plagioclase at the end of the noritic phase (beginning of crystallization of potassium feldspar). Since $D_{Sr,1}^{plag} = 2.3$ and $D_{Ca,1}^{plag} = 1.46$, it follows that $D_{Sr,2}^{plag} = 3.9$, $f_2 = 0.45$ and $F_2 = 0.21$.

Discussion

Points 0, 1 and 2 can vary somewhat within the trend. Those calculated here have been obtained after several trials and errors in which the D_i^{plag} values

were allowed to vary slightly around the values initially chosen. The fitting of the calculated trajectory to the measured points can be considered satisfactory in the anorthositic-leuconoritic phase.

In order to improve the fitting into the cpx-noritic phase and to reduce the amplitude of the jump of D_{Sr}^{plag} from 2.3 to 3.9, we have investigated the effect of dividing the trajectory into 3 or 4 segments. If the positions of points 0 and 1 are not modified, the method gives lower values for $c_{Sr,3}^{liq}$ and still higher values for $D_{Sr,3}^{plag}$. On the other hand, if a slight modification of the position of points 0, 1 and 2 and of the partition coefficients is admitted, a series of possible models can be reconstructed. They are plotted on Figure 4 and detailed in Table 2. The F values at the end of the cpx-noritic phase are not very different from the values initially calculated.

Modeling the K—Rb Evolution

The linear trend (Fig. 3) which characterizes the rocks of the upper part of the massif is considered to represent the evolution of magmatic liquids on the basis of the following arguments.

Contrary to the rocks of the lower part of the massif, these rocks do not show any cumulate structure at micro or mesoscales. The major element compositions plot on liquid lines of descent in classical variation diagrams (Duchesne, unpublished data). The evolution of the liquids can grossly be explained by the crystallization of a cumulate of mangeritic composition and some mangerites show a REE pattern with a positive Eu anomaly typical of feldspathic cumulates (Demaiffe et al., 1977).
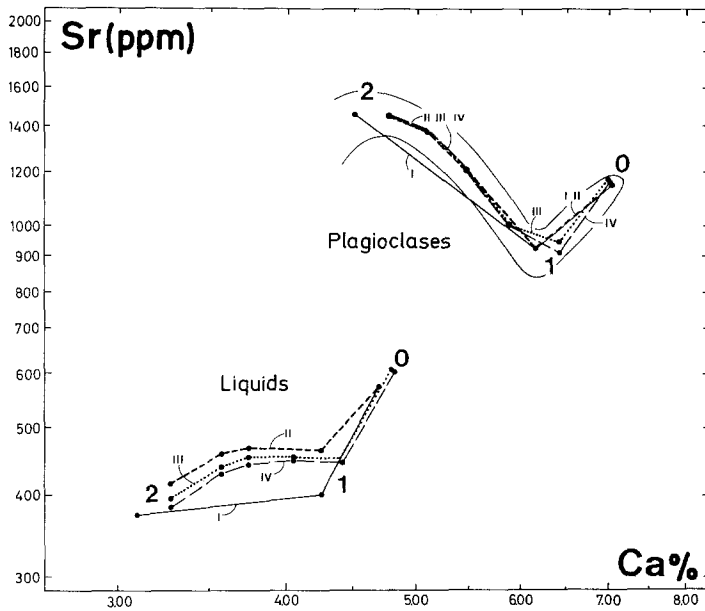
Fig. 4. Various calculated models (I to IV) describing the evolution of plagioclase-liquid pairs in a Sr versus Ca diagram (see Table 2). 0 beginning of the anorthositic-leuconoritic phase; 1 and 2 beginning and end of the cpx-noritic phase

Table 3. Models for the evolution of K and Rb in plagioclase

	Model I			Model II		
	0	1	2	0	1	2
c_K^{plag} (%)	0.38	0.57	0.91	0.38	0.58	0.8
c_K^{liq} (%)	0.95	1.60	3.2	0.95	1.60	3.20
c_{Rb}^{plag} (ppm)	1.4	3.9	12	1.4	2.5	5
c_{Rb}^{liq} (ppm)	12	23	50	13	25	55
D_K^{plag}	0.40	0.35	0.29	0.40	0.36	0.25
D_{Rb}^{plag}	0.12	0.17	0.25	0.11	0.10	0.09
F	1.00	0.47	0.21	1.00	0.47	0.21

We can thus admit that the starting point of the trend (the basic end of the liquid line of descent) gives the composition of the liquid after the formation of the cumulates of the first two phases, that is $c_{K,2}^{lig}$ and $c_{Rb,2}^{liq}$. These values are introduced in equation (2) written for K and Rb as well as f_1 and f_2 given by the Sr—Ca model. The bulk partition coefficient becomes $\bar{D}_{i,\rho} = \mu_{plag,\rho} D_{i,\rho}^{plag}$ since plagioclase is the only mineral which significantly concentrates K and Rb.

The choice of the points representing the plagioclase at stages 0, 1 and 2 is somewhat arbitrary. The K — Rb values do not plot on a well defined trajectory as is the case for the Sr—Ca relation. A perturbing process, likely of deuteric origin, seems to superimpose its effects on those of fractional crystallization.

Two plausible models have been calculated (Table 3 and Fig. 3). We have chosen to let D_K^{plag} values decrease in both cases with the basicity of the plagioclase following Dupuy and Allègre (1972). As

for D_{Rb}^{plag}, an increase has been assumed in model I — following Dupuy and Allègre (1972) and Vitrac and Allègre (1975) — and a decrease in model II.

Both models are satisfactory and the partition coefficients adopted for the most basic plagioclase are consistent with those determined on megacryst-liquid system, i.e. $D_K^{plag} = 0.47 \pm 0.05$ and $D_{Rb}^{plag} = 0.15 \pm 0.06$ (Duchesne and Demaiffe, 1978).

A model for the linear trend of the acidic rocks can be calculated using the $D_{K,2}^{plag}$ and $D_{Rb,2}^{plag}$ obtained in the two previous models and assuming values of the partition coefficients of K and Rb in the potassium feldspar of 2.54 and 0.80 respectively, close to those of Dupuy and Allègre (1972). The cumulate which fractionates is a mangerite with a (normative) association of 28 % potassium feldspar +45 % acidic plagioclase and 27 % other minerals (K and Rb free). In this model the pegmatitic quartz mangerite (Π on Fig. 3), which is the most acidic rock of the trend, corresponds to 3 % of liquid left.

Duchesne and Demaiffe (1978) have however shown that granitic gneisses belonging to the metamorphic envelope of the anorthositic complex also plot within the same trend or on its extension towards higher K and Rb contents. It has moreover been demonstrated (Demaiffe et al., 1977) that mangerites and quartz mangerites of Bjerkrem-Sogndal show a $^{87}Sr/^{86}Sr$ initial ratio of 0.7105, higher than those of the anorthosite-norite rocks (0.705–0.706). Contamination by supracrustal material of granitic composition and high strontium isotopic initial ratio can explain this feature. Such mixing process would be represented on the K — Rb diagram by a very flat curve, not significantly different from a straight line.

It thus appears that the K − Rb trend can be interpreted by two different mechanisms between which we cannot discriminate. REE data moreover indicate that they all might have acted together (Demaiffe et al., 1977).

Conclusions

The available data lead to an internally consistent model. The various working hypotheses implied by modeling can thus be accepted, namely, the hypothesis on the formation of the anorthosite-norite series by fractional crystallization, that on the solidification of cumulate rocks by adcumulus growth, and that on the use of concentrations of major elements instead of activities. To which extent these last two hypotheses can be generalized to other layered intrusions or to different types of magma remains however to be proved. In any cases, a Bjerkrem-Sogndal model can be useful to ascertain the partition coefficients of other trace elements in the other minerals, as for instance the REE in apatite (Roelandts and Duchesne, 1977).

The model also shows that the plagioclase-magma partition coefficients determined by Duchesne and Demaiffe (1978) for Sr, Rb and K are acceptable. It is consistent with the hypothesis put forward by these authors on an equilibrium between the plagioclase megacrysts found in various anorthositic bodies and a liquid similar to the jotunitic parental magma of the Hidra massif (Duchesne et al., 1974). The K_2O content (ca. 1.2 %) obtained for the Bjerkrem-Sogndal original magma also favours a jotunitic composition, in agreement with the suggestion of Michot (1965).

From a more general point of view the systematic approach using graphic techniques appears promising in order to define in layered intrusions a differentiation index based on physical chamical laws, not on empirical rules.

Appendix: List of Samples and Chemical Data

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
Plagioclases						
Rhythm I						
6671	Anorthosite, Klungland-Helleland road	6.97	1110	0.46	1.5	3070
6672	Leuconorite, id.	6.98	1110	0.42		
6405	id.	6.78	1055	0.43		
64155	Anorthosite, N. Bjerkrem	6.14	910	0.60		
64101	Leuconorite, E. Saglandsvatn	6.22	920	0.52		
66189	Anorthosite, N.W. Bjerkrem	6.50	940	0.51	3.4	1500
66190	id.	6.18	890	0.56	3.2	1750
64103a	Leuconorite, E. Saglandsvatn	6.21	870	0.56		
Rhythm II						
64160	Anorthosite, Ravnefjell	6.58	1060	0.46		
6408	Leuconorite, Klungland-Helleland road	6.82	1065	0.39		
6409	id.	6.63	1055	0.40		
Rhythm III						
66187	Anorthosite, Mögedalsvatn	7.14	1160	0.39	0.36	10800
6434	Leuconorite, Klungland-Helleland road	6.63	995	0.40		
6436	id. (Stromstad)	6.48	955	0.40		
6437A	id.	6.33	950	0.46		
6461 bis	Norite, N. Stromstad	6.15	885	0.42		
6674	*Cpx-leuconorite*, Klungland-Helleland road	6.40	905	0.55	0.64	8600
6675	*Cpx-norite*, id.	6.46	915	0.56		
6427	*id.*	6.10	860	0.58		
Rhythm IV						
6448	Anorthosite, E. Teksevatn	6.82	1130	0.45		
64107	Anorthosite, Bentjafjell	6.50	985	0.50		

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
6423	Olivine bearing leuconorite, Svalestad	6.86	1120	0.465	2.0	2330
6420b	Leuconorite, Helleland	6.22	895	0.45		
6422	id.	6.30	950	0.45		
6476	Norite, Bakka	6.51	1005	0.23		
6444	*Cpx-noritic layer*, S. Teksevatn	6.28	950	0.32	1.5	2130
6442	*Cpx-leuconorite*, id.	5.80	875	0.71		
6443	*Cpx-norite*, id.	6.50	935	0.34		
6695	*Cpx-leuconoritic layer*, Ollestad	5.97	975	0.48	1.4	3430
PM 5063	*Cpx-norite*, id. (coll. P. Michot)	5.84	985	0.55		

Rhythm V

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
66107	*Cpx-leuconorite*, Refsland station	6.09	970	0.53		
66102	*Antiperthitic cpx-norite*, Helleland-Hestad road	5.56	1215	0.66	1.5	4400
6460	id.	5.45	1340	0.74		
66103	id.	5.38	1215	0.81		
64133	*id.*, Hestad-Helleren road	5.14	1370			
6458	*Antiperthitic olivine-bearing cpx-norite*, Helleland-Hestad road	5.19	1390	0.80	2.4	3330

Jotunitic transition

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
64131	*Antiperthitic cpx-jotunite*, Hestad-Helleren road	5.05	1440	0.79		
64132	*Olivine-bearing cpx-jotunite*, Hestad-Helleren road	4.29	1170	1.66		
6457	*id.*, Helleland-Hestad road	4.64	1420	0.95		
64136	*id.*, Hestad-Helleland road	4.80	1510	0.95		
64134	*Hypersthene monzonite*, Hestad			0.45	4.4	1020

Whole Rocks

Mangerites

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
66209	Olivine-bearing mangeromonzonite, Slåten			3.83	47	820
72104	id.			4.15	46	900
66253	id., Helleland-Hestad road			2.53	33	770
66137	Olivine-bearing mangerite, Herveland			3.98	60	660
6470	id., E. Herveland			3.37	51	660
Pa 66P	id., Herveland (coll. P. Pasteels)			4.31	62	700
T II	id.			4.47	72	620

Quartz mangerites

		Ca (%)	Sr (ppm)	K (%)	Rb (ppm)	K/Rb
7377-6	Quartz mangerite, Langhus-Dyping road			3.76	78	480
7379-8	id.			3.98	87	460
7380-9	Hypersthene quartz monzonite, id.			3.64	88	410
7313-12	Quartz mangerite, Langhus-Sandsmork road			4.07	99	410
7312-12b	Hypersthene quartz monzonite, id.			4.09	111	370
7373-13	Quartz mangerite, id.			3.61	100	360
7373-14	id.			4.65	138	340
64110	id., Steine road			3.91	124	320
6687	Olivine-bearing quartz mangerite, Steine			4.60	138	330
66261	id., W. Orsland			4.65	113	410
Pa 66N	id., Orrestad (coll. P. Pasteels)			4.12	100	410
7315	Quartz mangerite, E. Dypingsvatn			4.17	144	290
64118	id.			4.34	159	270
64120	Pegmatitic quartz mangerite, id.			4.60	198	230

Sr, Ca and K in plagioclase were analyzed by XRF with precisions (s) of 1.4%, 1.3% and 2.5% respectively (Duchesne, 1968). Matrix influence on the Sr determination was corrected by using a method described in Duchesne (1965). Rb in plagioclase was analyzed by NAA with a precision of $s = 2\%$ (Roelandts, person. comm.). Rb in whole rock was determined by XRF using the intensity of the background to monitor the matrix effect. The accuracy of the Sr and Rb determination was checked by isotope dilution (Demaiffe, person. comm.). K in whole rock was determined by a combination of XRF and flame photometry. The minerals were separated with dense liquids and a Frantz magnetic separator (Duchesne, 1966)

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