DAMAGE DETECTION USING MODEL UPDATING AND IDENTIFICATION TECHNIQUES

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ABSTRACT

The problem for damage detection and localisation is approached employing two different techniques for fault localisation and quantification. A case study on a reinforced concrete beam with distributed damage of crack accumulation is studied. The measured frequency response functions are used as vibration data for the structure, on the basis of which the localisation procedures are performed. The considered localisation procedures are model based and use a FE model of the structure. One of the approaches suggests the use of a residual energy criterion for the fault localisation, followed by an updating procedure for an eventual quantification of the defect. The other procedure employs pattern recognition techniques for a primary localisation of the damage in a part of the structure, followed by an identification procedure for more accurate localisation and quantification. A comparison of the results and performance of both techniques is provided.

1. INTRODUCTION

The paper addresses the problem of developing non-destructive procedures for damage detection in structures, based on vibration measurement data. Early damage detection and eventual estimation of damage is an important problem, since it forms the basis of any decision for structural repair and/or part replacement. The presence of even a small damage in a structure affects its dynamic behaviour. Most of the quantitative global damage detection methods, which can be applied to complex structures, examine the changes in the vibration characteristics of a structure. There exists an extensive literature on the subject of damage detection using modal parameters as well as frequency and time domain responses (Heylen W., Lammens. S., Sas P., 1995, Mothershead J., Friswell 1993). A number of papers consider the utilisation of updating and identification procedures, based on a finite element model of the structure under consideration (Pereira, Heylen, Lammens, 1994, 1995, Heylen, Lammens, Sas, 1995).

In this paper, updating and identification procedures for damage localisation, based on the forced responses and the modal response of the structure are considered. These procedures bring the experimental response of the structure as close as possible to the model response, according to an objective function, thus changing some of the FE model parameters of the structure. These parameters supply information for the precise location and the importance of the introduced damage. First two different procedures for localisation of the damage are presented together with their respective results. One of the procedures uses pattern recognition techniques to localise the damage in a certain part of the structure. The other procedure uses a residual energy criterion that appears from the loose of equilibrium in the FE model. The localisation procedures are followed by an updating approach that can be used to evaluate the introduced damage, as well as to identify the damaged elements of the structure.

2. LOCALISATION PROCEDURES

Two procedures for damage localisation that use different techniques are explained here. Both procedures aim at localising the damage in the structure using its vibration response. One of the procedures suggests the division of the structure into a number of substructures and the determination of the damaged ones. To determine the damaged substructures the method uses a statistical pattern recognition (PR) approach. The other suggested procedure uses an equilibrium-based local error indicator.

2.1 Pattern recognition identification procedure

This procedure suggests the application of a PR method to distinguish between damaged and non-damaged substructures (Trendafilova, Heylen &Sas, 1998). The structure is divided into M substructures A_L, $L=1,2,...M$. Accordingly the following two classes are introduced:

- ♦ $\boldsymbol{C_D}$- the class of damaged substructures
- ♦ $\boldsymbol{C_N}$- the class of non-damaged substructures.

In order to distinguish between these categories feature vectors are formed using the FRF's of the intact structure $\boldsymbol{H_{ij}}$ and the FRF's of the damaged structure $\boldsymbol{H^*_{ij}}$. The following matrix, representing the relative differences of these FRF's is introduced:

$$\boldsymbol{h}_{ij} = \sqrt{\frac{(\boldsymbol{H_{ij}} - \boldsymbol{H^*_{ij}})^2}{\boldsymbol{max}(\boldsymbol{H_{ij}}, \boldsymbol{H^*_{ij}})^2}} \tag{1}$$

First for each substructure A_L only these $\boldsymbol{h}^L_{ij}$, are taken that are measured in points belonging to the substructure, where $i=1,2,..,n, j=1,2,...,m_L$, '$L$' stands for the substructure, i - for the frequencies, and j - for the DOF's, n is the number of frequencies considered and m_L is the number of DOF's corresponding to the substructure A_L. Then the first two moments of each set $\boldsymbol{h}_{ij}^L$ are used to define the feature vectors

$$\boldsymbol{f}^L : \boldsymbol{f}^L = [f_1^L, f_2^L]'$$

$$f_1^L = M(h_{ij}^L) = \frac{\sum_{i,j} h_{ij}^L}{N^L} \tag{2}$$

$$f_2^L = \sigma(h_{ij}^L) = \sqrt{\frac{\sum_{i,j}\left[h_{ij}^L - M(h_{ij}^L)\right]^2}{N^L}}$$

where $\boldsymbol{N}^L = \boldsymbol{n}.\boldsymbol{m}_L$ is the number of FRF relative differences $\boldsymbol{h}_{ij}^L$, corresponding to the substructure A_L. Thus the physical data, consistent of the FRF's for the pre-damaged state $\boldsymbol{H}_{ij}$ and the FRF's for the post-damaged state $\boldsymbol{H}_{ij}^*$, are mapped into a feature vector $\boldsymbol{f}^L$. The concept to introduce such features is that for a non-damaged structure both, the mean value and the variance of the differences between the FRF's, will be close to zero, while for a damaged structure they are not expected to be close to zero.

An algorithm, and eventually a computer code, is built that distinguishes between damaged and non-damaged structures and works on a statistical basis. It computes the probabilities $\boldsymbol{P_d}(\boldsymbol{f}^L) = \boldsymbol{Prob}(\boldsymbol{f}^L \in \boldsymbol{C_D})$ and $\boldsymbol{P_N}(\boldsymbol{f}^L) = \boldsymbol{Prob}(\boldsymbol{f}^L \in \boldsymbol{C_N})$ and on this basis categorises a substructure A_L, presented by a feature vector $\boldsymbol{f}^L$, as ***damaged*** or ***non-damaged*** . The probability $\boldsymbol{P_d}(\boldsymbol{f}^L)$, which is the probability that a substructure is damaged, can be regarded as a damage indicator $\boldsymbol{k}_L$; $\boldsymbol{0} \geq \boldsymbol{k}_L \geq 1$ for the substructure. A damage indicator $\boldsymbol{k}_L$ close to 1 means that it is very likely that A_L is damaged and accordingly a low damage indicator means, that there is a small chance that the substructure is damaged. The decision can be made automatic if it is left to the classifier, introducing a threshold value for $\boldsymbol{k}_L$. But it is better if the whole picture of damage detectors for all the substructures is taken into account and the judgement on which sub-structures are to be considered as damaged is left to the expert.

2.2 Equilibrium based damage detection

Let us assume that *modelling* errors are negligible (i.e. equations are correct, mesh is adequate) and that only model *parameter* errors (e.g. material or geometric properties) are actually present in the FE model. Under these conditions, the dynamic equilibrium equation of the experimental structure corresponding to a particular mode shape vector may be written in the frequency domain as

$$\left[K^*\right]\left\{v^*\right\} = \omega_{v^*}^2 \left[M\right]\left\{v^*\right\} \tag{3}$$

where

$\left[K^*\right],\left[M^*\right]$ are the experimental stiffness and mass matrices;

$\left\{v^*\right\},\omega_{v^*}$ is a given experimental vector and the corresponding resonance frequency.

An approximation to the expanded experimental vector $\left\{v^*\right\}$ may be found by assuming that the numerical model is close to the true structure. Using a standard formulation, an expanded vector $\{v\}$ is sought by minimising the residue of the equilibrium equation in some adequate metric:

$$([Z]\{v\})^T[\Theta]([Z]\{v\}) \tag{4}$$

Where

$[Z]=[K]-\omega_{\bar{v}}^2[M]$ is the dynamic stiffness matrix,

$\omega_{\bar{v}}$ is the identified frequency.

In order to solve problem (4), the experimental data is exploited by requiring that the expanded vectors should be similar to the reference measured shapes. For that the following constraint is added:

$$(\{v_1\}-\{\bar{v}\})^T(\{v_1\}-\{\bar{v}\})=0 \tag{5}$$

where

$\{v_1\}$corresponds to the partition of the expanded vector that has been measured,

$\{\bar{v}\}$ is the vector of measured co-ordinates.

Constraint (5) is highly restraining for the expansion process, since it implies a full confidence on the measurement process. In practice, the inclusion of noise is accepted on the measured vector $\{\bar{v}\}$ and instead of using constraint (5), a second objective to minimise is used instead

$$(\{v_1\}-\{\bar{v}\})^T[\Xi](\{v_1\}-\{\bar{v}\}) \tag{6}$$

A number of expansion techniques are based on the exact verification of equation (5) while solving problem (4). In the MECE expansion method considered here, the two problems (4) and (6) are handled simultaneously in a single cost function:

$$(\{u\}-\{v\})^T[\Theta](\{u\}-\{v\})+\alpha(\{v_1\}-\{\bar{v}\})^T[\Xi](\{v_1\}-\{\bar{v}\}) \tag{7}$$

where α is a weighting coefficient that indicates confidence in the measurements. If metric $[\Theta]=[K]^{-1}$ and $[\Xi]=[K_{red}]$ are chosen to solve problem (7), an optimisation problem may be written as follows :

Minimise

$$(\{u\}-\{v\})^T[K](\{u\}-\{v\})+\alpha(\{v_1\}-\{\bar{v}\})^T[K_{red}](\{v_1\}-\{\bar{v}\})$$

subject to

$$[K]\{u\}=\omega_{\bar{v}}^2[M]\{v\} \tag{8}$$

where $\{u\}$ is an instrument shape vector. The convenient introduction of the instrument vector $\{u\}$ in equation (8) allows the definition of an error indicator that quantifies a residual strain energy density (element-by-element, substructure-by-substructure) defined by :

$$e_s=\left.(\{u\}-\{v\})^T[K_s](\{u\}-\{v\})\middle/Vol_s\right.$$

where $[K_s]$ is the stiffness matrix of the substructure, and Vol_s is the associated volume. As can be seen, the error indicator is closely related to the proposed expansion method. The overall reliability of this indicator strongly depends on the previous expansion technique and on the energy distribution on the structure. A discussion can be in found in (Pascual, Golinval and Razeto, 1998).

3. MODEL ADJUSTEMENT METHODS

In a number of practical cases, the results from any of the above localisation procedures should be enough in the sense that the information where the damage is located and in what part of the structure it has spread could be adequate for certain purposes. If a further more precise (element-by-element) localisation and quantification of the damage is necessary, a further updating or identification procedure can be developed to determine some model parameters, that can be used to characterise the damage.

3.1 Element-by-element stiffness adjustment

One way to solve the problem for damage quantification can be the development of an identification procedure for the stiffness of the elements of the damaged substructure (Trendafilova, Heylen, Sas, 1998, Trendafilova, Heylen, 1998). As a result of the localisation procedure, it was found that a certain part of the structure (substructure) is likely to be damaged. Now an objective function to be minimised with respect to certain parameters of the elements from this part of the structure can be introduced. The element stiffness can provide information for the presence and quantity of damage in the damaged area. Accordingly the following objective function was introduced:

$$O(S_k) = \sum_{i,j} [\log(\boldsymbol{H}_{ij}(S_k) - \log(\boldsymbol{H}^*_{ij})]^3 \qquad (9)$$

where $\boldsymbol{H}_{ij}$ are the FRF's of the of the FE model, $\boldsymbol{H}^*_{ij}$ are the measured FRF's and S_k are the stiffness of the elements of the damaged part of the structure. Minimising the objective function (9) the stiffness S^*_k of these elements for the damaged model are obtained. The stiffness change (decrease) for the damaged elements can provide information about which are the damaged elements and the amount of the damage introduced in each element.

3.2. Damage model

In order to reduce the design space, a three parameter damage model is used (based on the proposed in (Peeters et al., 1998).

Let $y = |x| / (L/2)$ be the position with respect to the length of the beam. The stiffness (represented by the Young modulus) will be given by the following formula:

$$E(y, \alpha, \beta, n) = E_0 [1 - (1-\alpha) f \cos^2 t] \qquad (10)$$

where E_0 is the initial value for the undamaged beam,

$$f = \frac{\beta^k}{\beta^k + \left(y \big/ \beta \right)^k} \quad (11)$$

and

$$t = \pi \big/ 2 \left(y \big/ \beta \right)^n$$

k defines the filter (11). α , β , and n are the parameters and represent the maximum level, the length and the distribution of damage. The model assumes a symmetric distribution of damage.

3.3. Modal frequency shift based model updating

In this procedure, the considered cost function is the mean value of the relative differences between natural frequencies:

$$J = 1 \big/ m \sum_{i=1}^{m} abs(\omega_{\bar{v}_i} - \omega_{\phi_i}) \big/ \omega_{\bar{v}_i} \quad (12)$$

where m is the number of paired modes. In order to build the cost function, a mode pairing process is necessary, which is performed using the MAC technique.

4.RESULTS FROM THE TEST CASE

The procedures described above were tested using an experimental laboratory case: a reinforced concrete beam with dimensions 6m*0.2m*0.25m (see Figure 1).The beam has been damaged by applying static charges and presents cracks along the lower face. The boundaries can be considered to be a free-free condition. Excitation was applied in one of the corners to excite vertical and torsion modes. Accelerations are measured on the top and on both sides of the beam at 62 points. Ten experimental modes were identified. A detailed explanation of the set-up can be found in references (Peeters et al.,1998).

4.1 Localisation procedures

- Method 1

In order to apply the PR method described above, the beam was divided into three equal areas. The following damage indexes k_L (L=1, 2, 3) were obtained for the areas :

k_1=0.27;	k_2=0.88,	k_3=0.23

These results are shown on the FE model of the beam used on Fig.2. They indicate that the middle area of the beam is most likely to be damaged. There is a small chance between 20% and 30% for the rest two areas to be damaged, but even if they are, the damages that they contain will be far less than the damage in A_2.

- Method 2

A 3D FE model of the beam was developed using 60*2*2 shell elements. A MECE expansion was performed using a base that considers the first 20 modes of the model. Results of the localisation are shown in figure 3. The residual energy is quite spread along the length of the beam, reaching its highest values at the middle.

4.2 Updating and identification

- Element level stiffness identification

The method presented in & 3.1 is applied and it results in the element stiffness for the damaged beam. Figure 4 presents the results. The picture shows good coincidence between the experiment and the modelled damage patterns. The MAC values for the first 10 modes are calculated in order to check the performance of the obtained damaged model. The mean MAC value is 0.995 compared 0.969 for the model before identifying the stiffness. This is considered a good enough performance for the obtained model. Figure 6 shows the MAC before and after the identification.

- Updating using the damage model

The damage model (10), (11) presented in & 3.2, applied with the objective function (12) was used to estimate the introduced distributed damage. Figure 5 presents the results. The initial mean frequency shift for the 10 experimental modes reaches 23.7%. After few iterations it reaches 2.6%, which is judged low enough. The final normalised bending stiffness distribution along the beam is shown in figure 5, from which it can be said that damage is quite generalised $\alpha = .59$, $\beta = .89$, $n = 1$. The filter parameter was set to k=5. Figure 7 presents the obtained MAC values. Figure 8 presents the frequency shifts after the updating and Figure 9 pictures the change of the damage parameters with the iterations.

5. CONCLUSIONS

This paper considers a couple of possibilities to

1• detect and localise damage in a vibrating structure /&2.1 and &2.2/ and

2• quantify the localised damage and eventually localise it more precisely finding the damaged finite elements /&3.1 and &3.2/.

The procedures are demonstrated on an experimental test case. Both localisation procedures are successful to localise the present damage identifying the intermediate part of the beam as the one with the highest energy (MECE method) and as the one most likely to be damaged (the pattern recognition method). One of the updating procedures suggests a model for the damage distribution (&3.2), thus decreasing substantially the number of the parameters to be updated, while the other one identifies the stiffness of the elements of the damaged area (&3.1). Obviously the advantage of using a damage model is the small number of parameters to be handled. The element level identification procedure has a higher number of parameters to determine and aims at higher precision in the sense that the damage can be estimated and localised down to the element level. Nonetheless both procedures come up with quite similar results, which match well with the observed experimental damage pattern.

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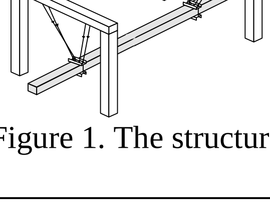


Figure 1. The structure

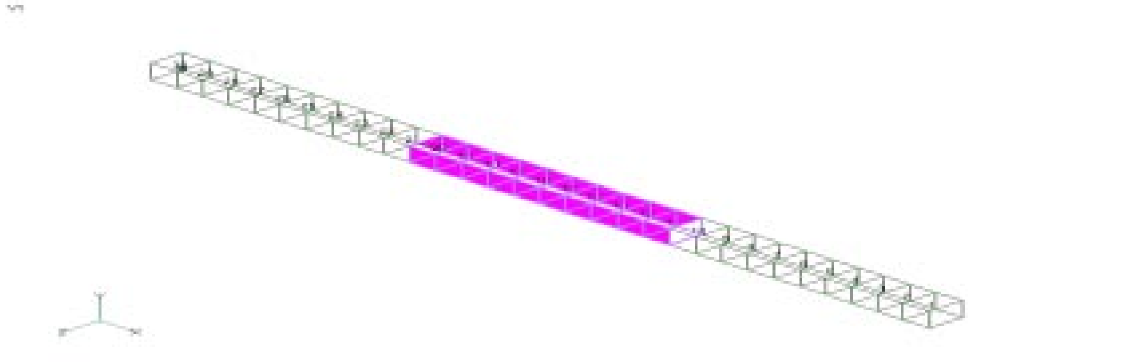


Figure 2. Damaged area (PR approach)

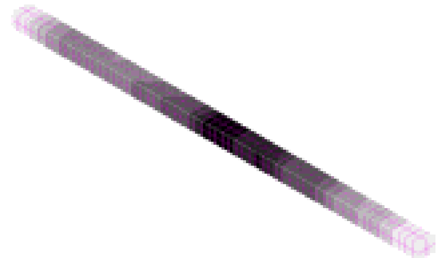


Figure 3. MECE error localization

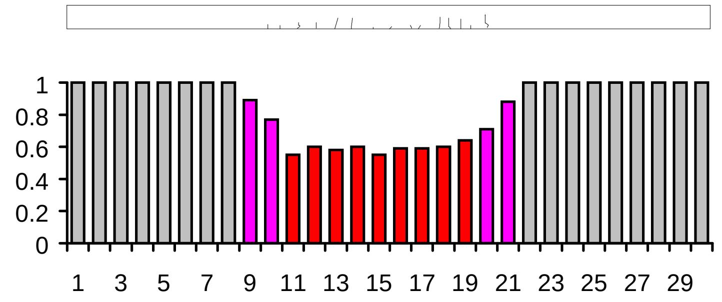


Figure 4. Updated stiffnesses (3.1 element stiffness idnetification)

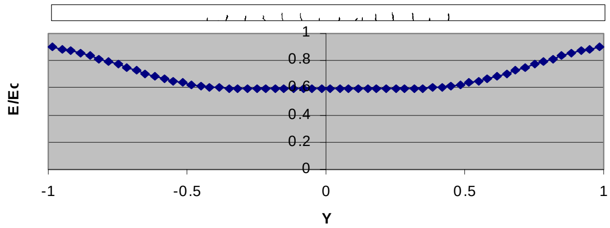
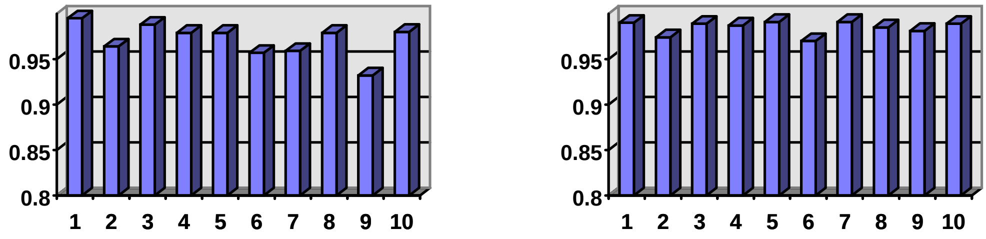


Figure 5. Updated stiffness distribution (1: initial value), according to the damage model 3.2. 3.3



a) b)

Figure 6 MAC values a) before and b) after the identification (3.1 element stiffness identification)

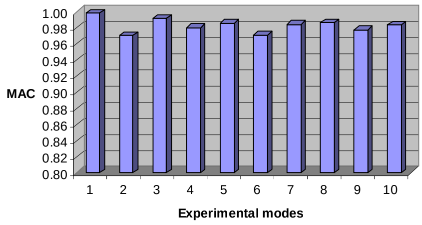


Figure 7. MAC values for the updated model v/s the damaged experimental modes according to the damage model3.2 and 3.3

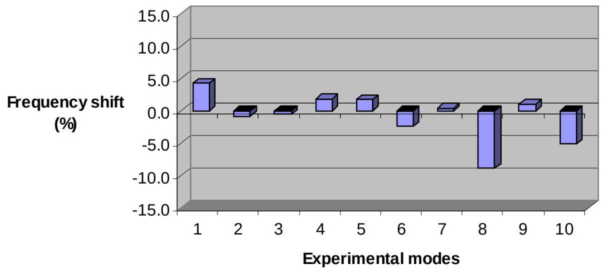


Figure 8. Frequency shifts for the updated model v/s the damaged experimental modes

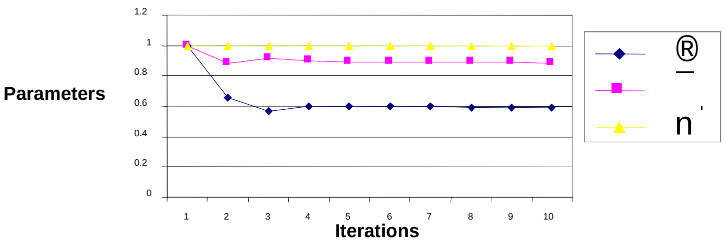


Figure 9. Evolution of the updating parameters (damage model 3.2& 3.3)