Weak localization with interacting Bose-Einstein condensates

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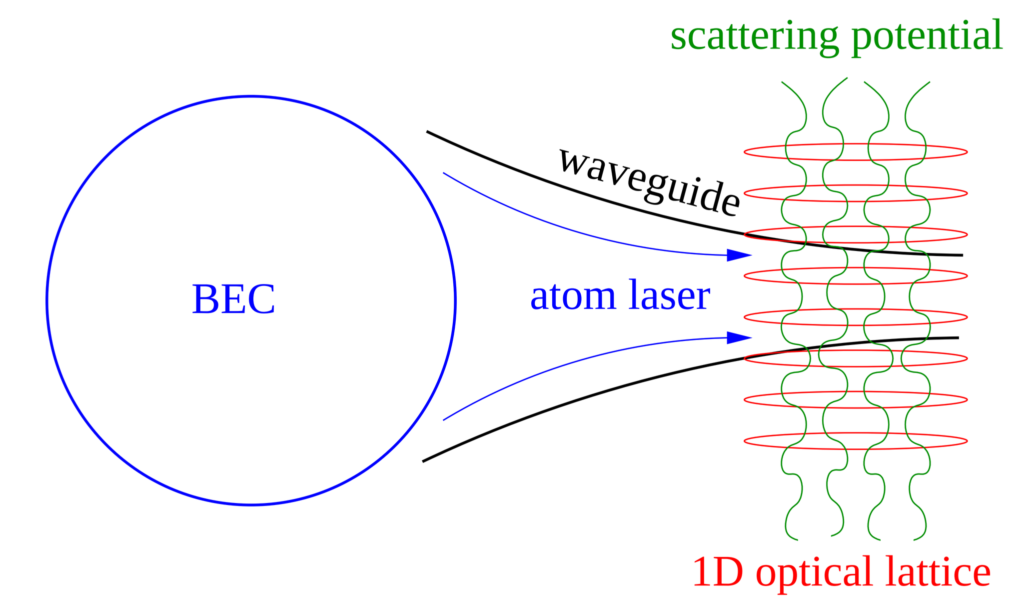
2D scattering with atom lasers



Possible experimental realization:

→ guided atom laser
W. Guerin *et al.*, PRL 97, 200402 (2006)
G. L. Gattobigio *et al.*, PRL 107, 254104 (2011)

→ 1D optical lattice perpendicular to the waveguide, in order to confine the matter-wave beam to 2 dimensions

→ 2D disorder potential (optical speckle field) or

→ 2D atom-optical billiard geometry
V. Milner *et al.*, PRL 86, 1514 (2001)
N. Friedman *et al.*, PRL 86, 1518 (2001)

→ artificial gauge field, to break time-reversal invariance
Y.-J. Lin *et al.*, PRL 102, 130401 (2009)

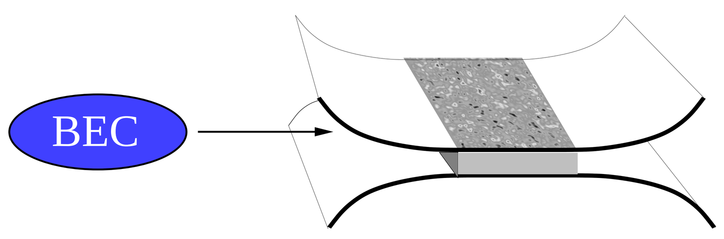
Theoretical description:

→ 2D Gross-Pitaevskii equation with a source term that models the outcoupling from a BEC reservoir ($\mathbf{r} \equiv (x, y)$)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \mathbf{A}(\mathbf{r})\right)^2 + V(\mathbf{r}) + g\frac{\hbar^2}{2m}|\Psi(\mathbf{r},t)|^2\right]\Psi(\mathbf{r},t) + S_0\chi(y)\delta(x-x_0)\exp(-i\mu t/\hbar)$$

$A(\mathbf{r}) = \frac{1}{2}B\mathbf{e}_z \times \mathbf{r}$: artificial gauge vector potential

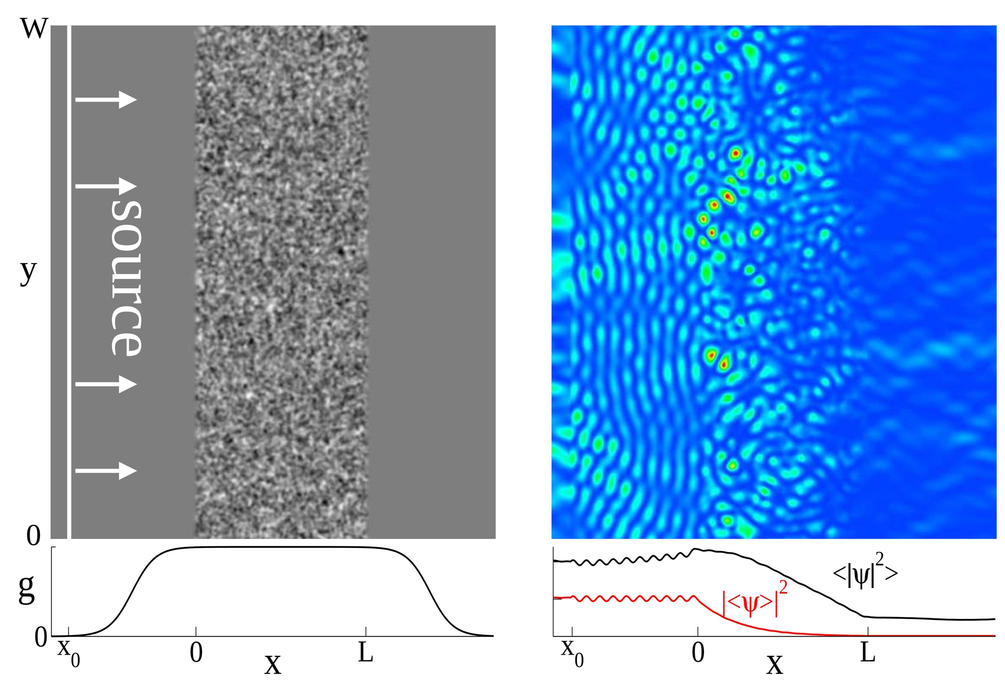
$g = 4\sqrt{2\pi}a_s/a_\perp(x) \equiv g(x)$: effective 2D interaction strength



→ numerical integration of the Gross-Pitaevskii equation in the presence of an adiabatic ramping of the source amplitude S_0

→ apply absorbing boundary conditions at the longitudinal boundaries of the spatial grid: T. Paul *et al.*, PRA 76, 063605 (2007)

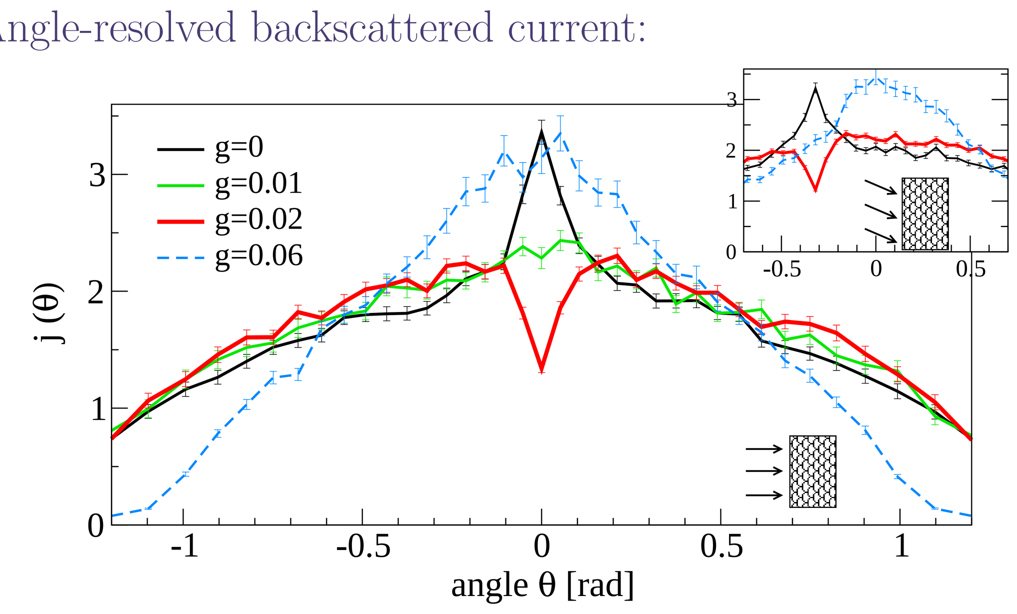
Scattering in disorder potentials



→ injection of a homogeneous plane-wave beam from the source (periodic boundary conditions along the transverse boundaries)

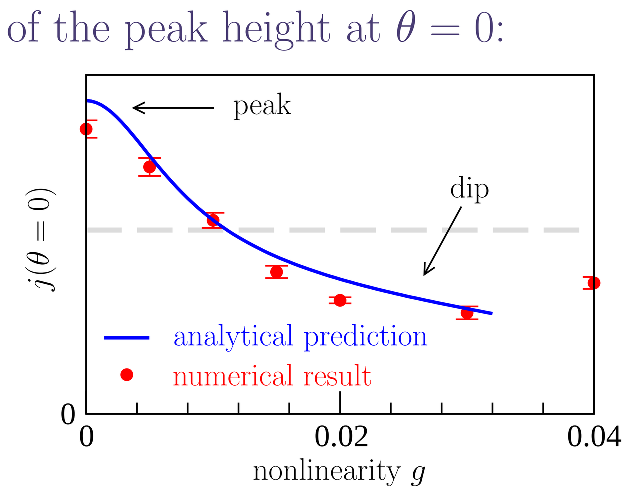
→ disorder potential with short-range (Gaussian) spatial correlation (similar results were obtained for speckle disorder)

Angle-resolved backscattered current:



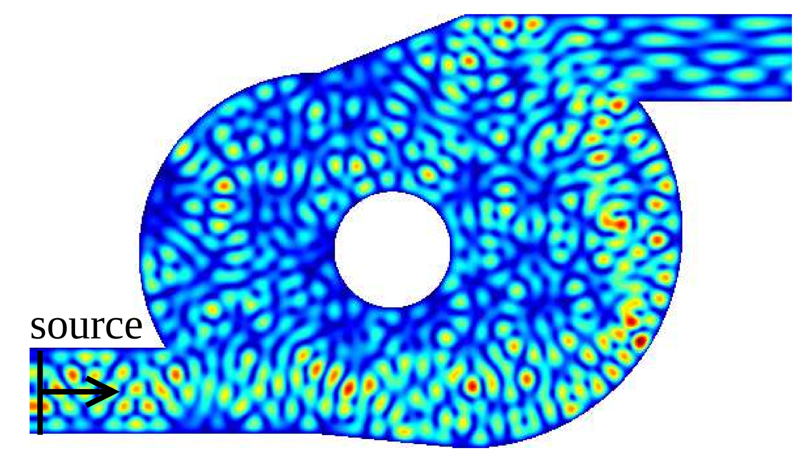
⟶ permanently time-dependent (turbulent) scattering for $g \gtrsim 0.05$

Comparison of the peak height at $\theta = 0$:



M. Hartung, T. Wellens, C. A. Müller, K. Richter, and P. Schlagheck, PRL 101, 020603 (2008)

Transport through chaotic billiards

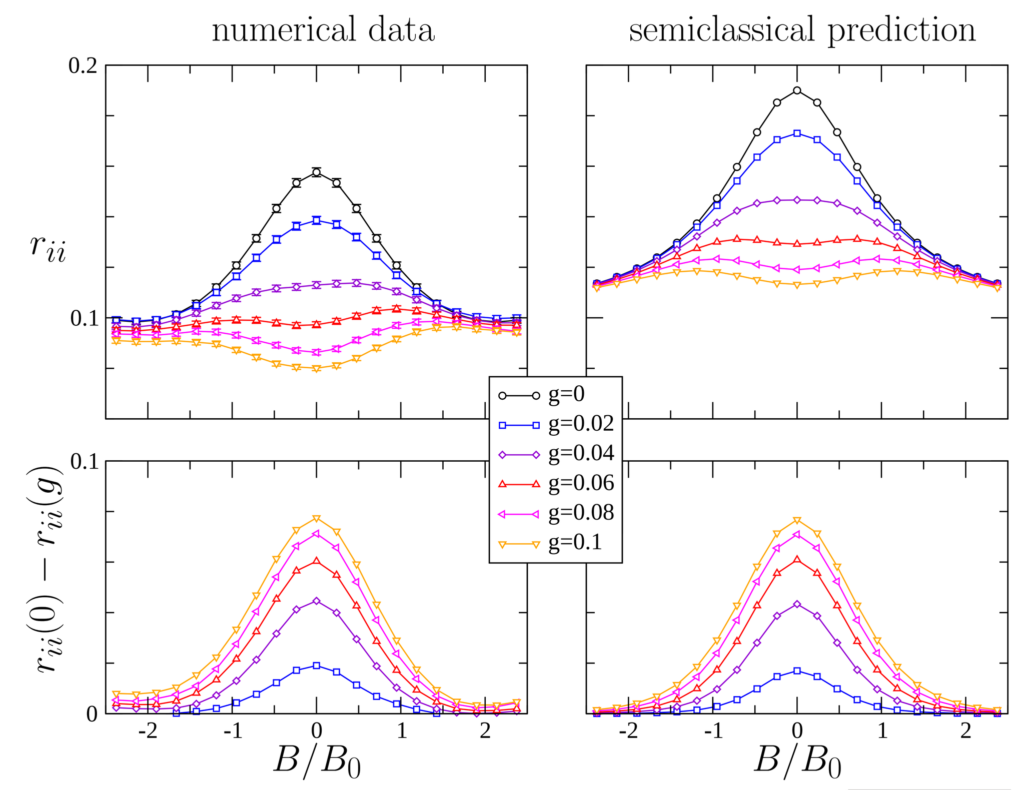


→ injection within one transverse channel of the left lead
→ homogeneous gauge field B perpendicular to the billiard
→ energy and configuration average (for several positions of the obstacle)

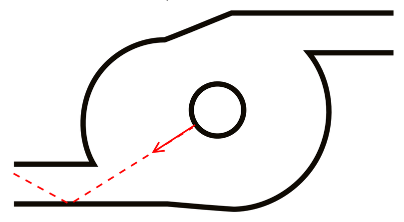
Semiclassical retro-reflection probability to incident channel:

$$r_{ii} = \frac{1}{N}\left[1 + \frac{N-1}{N}\frac{(1+B^2/B_0^2)}{(1+B^2/B_0^2)^2 + (gj^i\tau_D)^2}\right]$$

N = total number of open channels (= 10)
B_0 = characteristic gauge field scale for weak localization (obtained from directed areas accumulated along trajectories)
j^i = incident current, τ_D = mean classical survival time



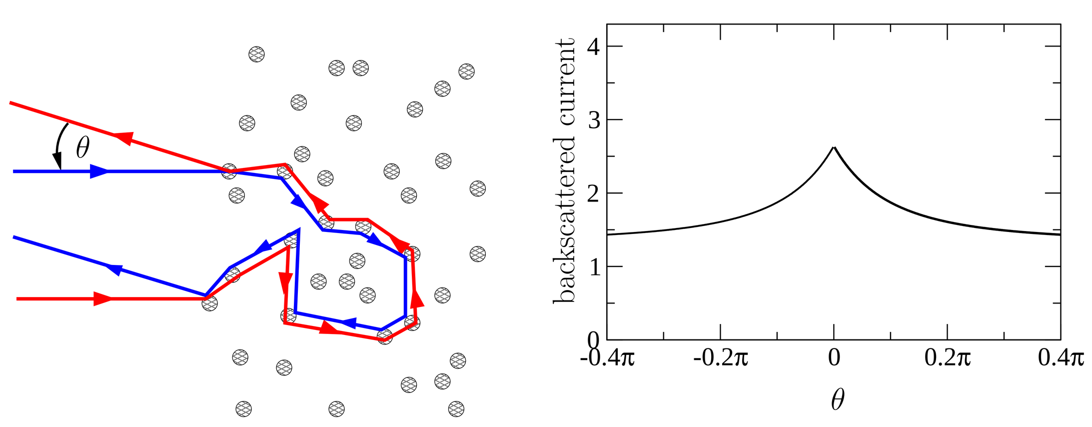
→ deviations due to non-universal short-path contributions (self-retraced trajectories)



⟶ non-universal contributions of short reflected paths give rise to signature for weak antilocalization

T. Hartmann, J. Michl, C. Petitjean, T. Wellens, J.-D. Urbina, K. Richter, and P. Schlagheck, Ann. Phys. 327, 1998 (2012)

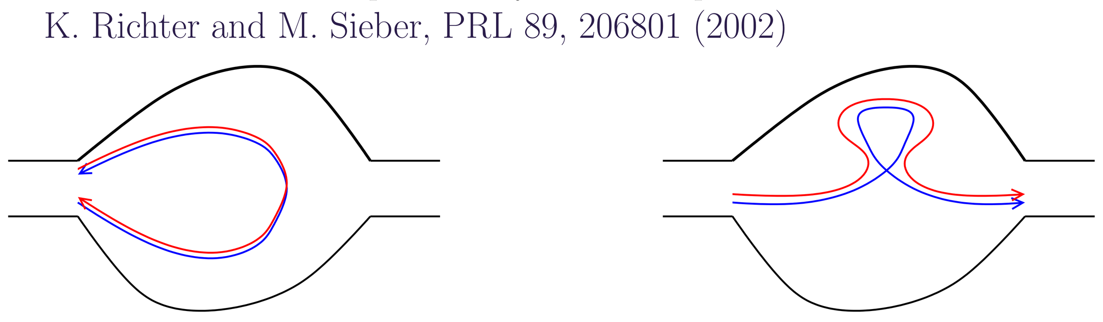
Coherent backscattering



→ constructive interference between reflected paths and their time-reversed counterparts in disordered systems or chaotic confinements

→ enhanced retro-reflection probability

→ reduced transmission probability due to loop contributions
K. Richter and M. Sieber, PRL 89, 206801 (2002)



→ dephasing in the presence of a perpendicular magnetic field

→ precursor to strong Anderson localization
D. Vollhardt and P. Wölfle, PRL 45, 842 (1980); PRB 22, 4666 (1980)

Nonlinear diagrammatic theory

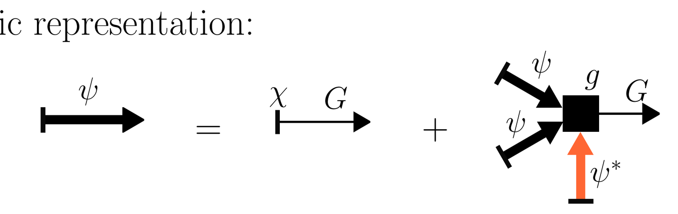
T. Wellens and B. Grémaud, PRL 100, 033902 (2008)

Basic assumption: stationary scattering state $\psi(\mathbf{r},t) = \psi(\mathbf{r})\exp(-i\mu t/\hbar)$ satisfying the nonlinear Lippmann-Schwinger equation

$$\psi(\mathbf{r}) = S_0\int G[\mathbf{r},(x_0,y'),\mu]\chi(y')dy' + \int d^2r' G(\mathbf{r},\mathbf{r}',\mu)g\frac{\hbar^2}{2m}|\psi(\mathbf{r}')|^2\psi(\mathbf{r}')$$

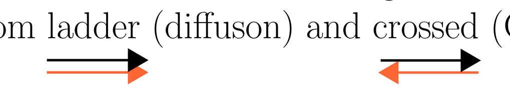
with $G(\mathbf{r},\mathbf{r}',\mu)$ = Green function of the linear scattering problem

Diagrammatic representation:

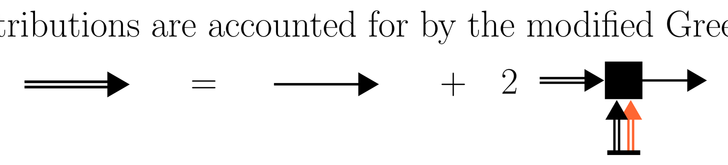


Regime of weak localization ($k \times \ell_{\rm transport} \gg 1$):

→ main contributions to average retro-reflection probability from ladder (diffuson) and crossed (Cooperon) diagrams



Ladder contributions are accounted for by the modified Green function

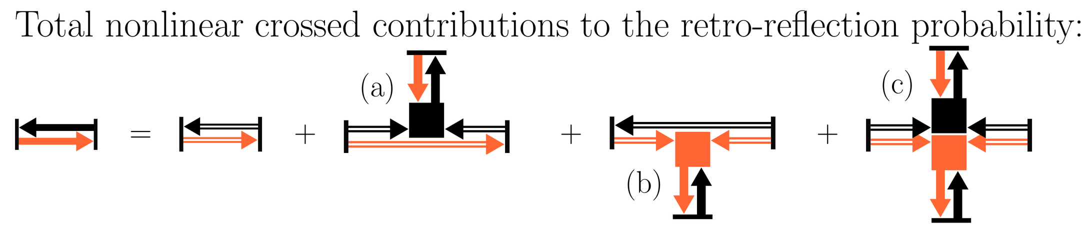


⟶ effective background potential $V_{\rm eff}(\mathbf{r}) = \langle V(\mathbf{r})\rangle + g(\mathbf{r})\frac{\hbar^2}{m}\langle|\psi(\mathbf{r})|^2\rangle$

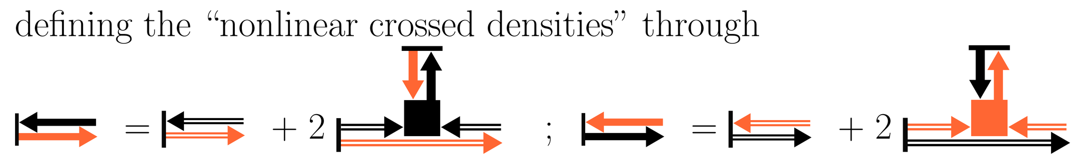
⟶ no net modification of mean densities and currents on the ladder level

Crossed contributions

Total nonlinear crossed contributions to the retro-reflection probability:

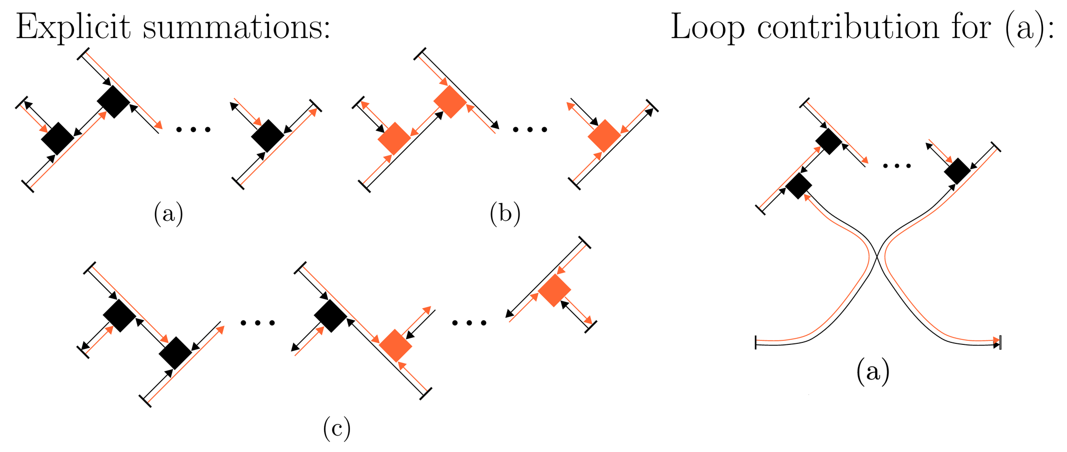


defining the "nonlinear crossed densities" through



Explicit summations:

Loop contribution for (a):



⟶ finite net modification of the retro-reflection probability: dephasing of weak localization due to the presence of interaction

⟶ current conservation is restored taking into account nonlinear chains within the loop contributions